Research Article

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Novel graph for an appropriate cross section and length for cantilever RC beams

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Abstract: Whether the design is done manually or by software, the designer will have difficulty choosing the economic and strength cross section. The designer, in this case, either relies on their experience or resorts to the method of trial and error. Especially for Cantilever beams with a long span as a result of risk deflections, it is exposed. The current theoretical study was performed on rectangular concrete cross sections of different dimensions and subjected to uniformly distributed loads. Based on a previous study, the sections are reinforced with a specific reinforcement ratio. Through an algorithm, Python 3.4 software, and an output file, the permissible deflections for each cross section were calculated according to the ACI 318M-19. Finally, the authors could draw a graph to choose the appropriate cross section for each required beam length in less time and effort.

Keywords: beam design, beam stiffness, sustained deflection, immediate deflection, beam deflection, cantilever beams

Notation

b	width of beam sections (mm)
d	adequate depth of beam sections (mm)
h	total depth of beam sections (mm)
ρ	reinforcement ratio in tension equal to (p) at output file.
ρ'	reinforcement ratio in compression.
A_s	steel area in tension (mm^2).
A_s'	steel area in compression (mm^2).
ℓ	span length of the beam and equal to (L) at output file (mm)
f_c'	ultimate compressive strength of concrete (MPa)
E_s	modulus of elasticity for steel (MPa)
E_c	modulus of elasticity for concrete (MPa)
n	modular ratio of elasticity.
I_g	gross moment of inertia for beam sections (mm^4)
I_e	effective moment of inertia (mm^4)
I_{cr}	cracking moment of inertia (mm^4)
M_n	nominal bending moment (kN m)
M_{cr}	cracking moment (kN m)
M_a	maximum bending moment (kN m)
IDL	initial dead loads without self-weight (kN)
W_{LL}	unfactored live loads (kN)
W_{DL}	unfactored dead loads (kN)
Δ_{DL}	deflection due to dead load (mm)
$(\Delta_i)_{LL}$	immediate deflection due to live load and equal to (delta_i_LL) at output file (mm)
$\Delta_{(DL+LL)}$	deflection due to live plus dead loads (mm)
$\Delta_{(cr+sh)}$	deflection due to creep and shrinkage of concrete (mm)
Δ_T	sustained deflection and equal to (delta_T) at output file (mm)
γ	factor equal to $\frac{M_n}{bd^2}$, and equal to (y) at the output file (MPa)

1 Introduction

Rectangular cross sections for concrete beams are commonly used to design reinforced concrete structures. While the design is done manually or by software, selecting the economical and strength cross sections for those beams is necessary. With the large-span cantilever beams, the selection is not easy and depends on the designer's experience with trial and error. The risk associated with the cross-sectional design becomes more severe for cantilever beams usually subjected to significant deflections. In Iraq, the construction of cantilever beams with rectangular sections appears in many hotels and commercial buildings. However, the authors did not find articles that studied the problem in detail.

What is specified at the ACI 318M-19 item 24.2 with SI unit [1] was performed to calculate the deflection. Nilson *et al.* [2] presented a graph shown in Figure 1, which is

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essential to preparing the current study. From the group of curves, the common curve 60/4 was chosen, i.e., $f_c' = 4\ \text{ksi} = 28\ \text{MPa}$ and $f_y = 60\ \text{ksi} = 414\ \text{MPa}$. Nelson *et al.* [2–7] authored textbooks that contain chapters on deflection and how to calculate it. Metwally [8] confirmed that the support location affects the immediate deflection values. Chaphalkar *et al.* [9] emphasized that modeling can be made to analyze the deflection of cantilever beams by finite element package. Marovic *et al.* [10] concluded several models for calculating the deflection of the cantilever beams with end-concentrated loads with circular and hollow cross sections. The types of deflections are most important in checking the dimensions of the selected cross sections [11–23].

The current study aims to create a relationship between rectangular cross sections and the span lengths of reinforced concrete cantilever beams. Finally, the authors define the form of this relationship through a graph that facilitates the selection of strength and economic cross sections.

2 Design procedure

2.1 Assumptions

- In the tension zone, all sections are reinforced with a variable reinforcement ratio, while the compression zone is reinforced with a minimum reinforcement content of ($\rho' = 0.002$). For the calculation of I_{cr}, the effect of compression steel has been neglected due to its proximity to the neutral axis and small quantity.
- The initial dead and live unfactored loads applied to the cantilever beams are calculated as follows:

Due to the thick slab: 0.15 m = 0.15(25) = 3.75 kN/m^2,

Due to the thick sand: 0.05 m = 0.05(17) = 0.85 kN/m^2,

Due to the thick flooring layers: 0.025 m = (20 + 24) (0.025) = 1.1 kN/m^2.

Thus, initial dead loads (IDLs) = 5.70 kN/m^2, while live load (W_{LL}) considered equal to 3 kN/m^2. Usually, the lengths of adjacent slabs range from 4 to 5 m. Using an average of 4.5 m yields:

IDL = 5.7 (4.5) = 25.65 kN/m, W_{LL} = 3 (4.5) = 13.5 kN/m.

By observing the constructed buildings in Iraq, these load values represent the worst case of loading excluding the concentrated loads.

- The approximate load values (if any) do not significantly affect the accuracy of the deflection results. The deflection of any structure depends mainly on the span length; as a result, the length is raised to the fourth power, while the load is to the first power. For example, the deflection of the cantilever beam due to the dead load is expressed as follows:

$$\Delta_{Dl} = \frac{W_{Dl}\ell^4}{8E_cI_{e1}}. \quad (1)$$

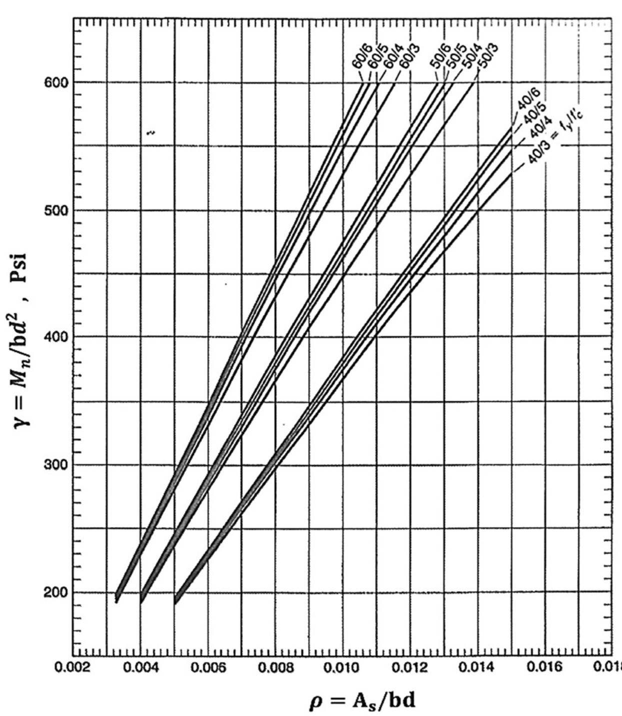

Figure 1: Strength of rectangular sections.

Table 1: Dimensions of the proposed cross sections

γ (MPa)	ρ	h (mm) 250%	γ (MPa)	ρ	h (mm) 225%	γ (MPa)	ρ	h (mm) 200%	γ (MPa)	ρ	h (mm) 175%	γ (MPa)	ρ	h (mm) 150%	b (mm)
1.38 (200 psi)	0.0034	500	2.76 (400 psi)	0.0072	450	4.14 (600 psi)	0.0110	400	4.83 (700 psi)	0.0130	350	5.52 (800 psi)	0.0150	300	200
		625			563			500			438			375	250
		750			675			600			525			450	300
		875			788			700			613			525	350
		——			——			800			700			600	400

- The Flange effect is not considered because it is in a tension zone.
- The current study does not include deep beams with more than 900 mm depth because their reinforcement is distributed according to ACI item 9.9.
- The reinforcement can be placed in one or two layers in the tension zone. The adequate depth is taken as $d = h - 60$.

2.2 Calculations

Table 1 presents typical values for cross sections bh with h taken as a percentage of b. The difference of ρ values provided to the cross sections depends on the fact that ρ is inversely proportional to the section area bh. From Figure 1, the values of $(\frac{M_n}{bd^2} = \gamma)$ will be obtained by dropping each ρ on the curve (60/4). So, the span length is expressed as follows:

$$\gamma = \frac{M_n}{bd^2} = \frac{W\ell^2/2}{bd^2}, \quad \ell = \sqrt{\frac{2\gamma bd^2}{W}}. \tag{2}$$

According to an algorithm shown in Figure 2, the authors attempted to calculate the deflection of selected cross sections. Initial calculations show that the deflection of reinforced sections with $\rho = 0.015,\ 0.013,\ 0.011,\ 0.0072$ do not match the permissible sustained deflection, as mentioned at ACI (Table 24.2.2). So, the attempt was repeated but with $\rho = 0.0034,\ 0.0040,\ 0.0050,\ 0.0060$ and $\gamma = 1.38, 1.65, 1.96, 2.38$, respectively. Appendix A presents the input file using Python 3.4 software according to the input file data shown in Table 2.

2.3 Analysis results

- ACI – Table 24.2.2 provides two permissible deflections that must be checked: immediate deflections equal to (ℓ/360) and sustained deflections equal to (ℓ/240). The calculated deflections listed in Appendix B have been checked with the permissible ones to know the pass lengths with their cross sections, as shown in Table 3. Finally, the cross-sectional selection against the required length was facilitated by the graphic relationship shown in Figure 3.
- Mainly, increasing the depth of the beam means increasing its rigidity and thus increasing the permissible length of the beam so that it does not exceed the specificity of deep beams.
- The best span length is obtained for beams with a width of 350 mm, after which increasing the width becomes

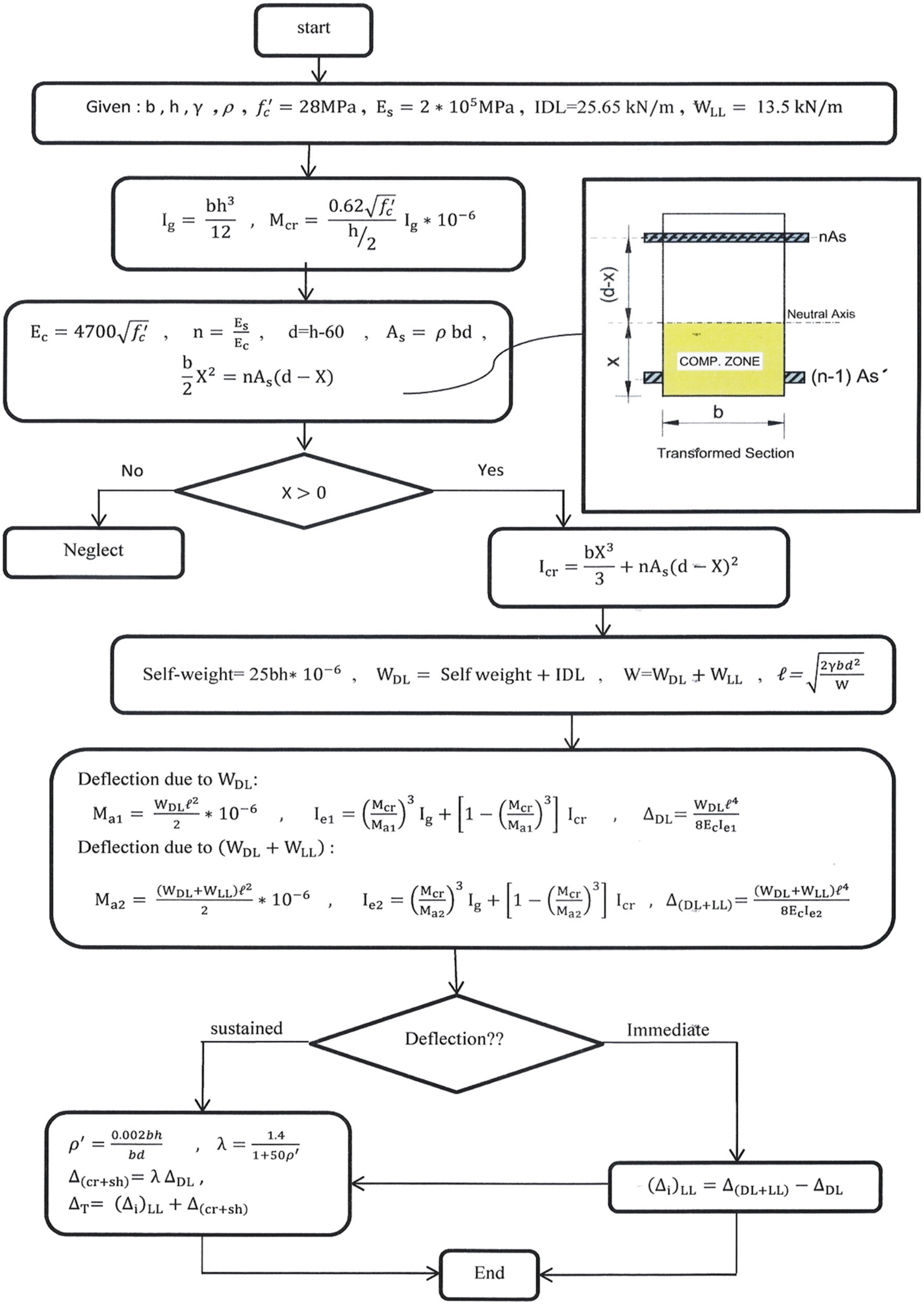

Figure 2: Flowchart to calculate beam deflections.

Table 2: Input file data

γ (MPa)				ρ				*h* (mm)		
2.38 (345 psi)	1.96 (284 psi)	1.65 (239 psi)	1.38 (200 psi)	0.0060	0.0050	0.0040	0.0034	375	250	*b* (mm)
								500		
								563		
								625		
								450	300	
								525		
								600		
								675		
								750		
								525	350	
								613		
								700		
								788		
								875		
								600	400	
								700		
								800		

useless. An increase in a width greater than 350 mms means an increase in the weight of the beam at the expense of its rigidity.

- The sustained deflection is considered the most dangerous type, a discrepancy to what was believed in more detail in refs. [8,14,15,17,19]. All the empirical results in Appendix B agreed with the ACI conditions of immediate deflection, while sustained deflection is considered a criterion for accepting pass results. For more explanation, an example can be taken from Appendix B, as shown in Table 4.

- Referring to the output file, it is noted that the allowable deflections for all sections were obtained from a trial (ρ = 0.005) against γ = 1.96 MPa. An increase in the reinforcement ratio of more than 0.005 gives an increase in length that does not meet ACI requirements. All deflections were calculated from unfactored loads based on the ACI conditions. However, the loads must be factored in when designing, and the reinforcement ratio will increase from 0.005, as shown in Appendix C. Increasing the reinforcement ratio provided that adhering to the length adopted in the current study means forming safer

Table 3: Check the results

b (mm)	*h* (mm)	ℓ (mm)	Pass results (mm)		ACI – provisions (mm)	
			$(\Delta_i)_{LL}$	Δ_T	$(\Delta_i)_{LL}$	Δ_T
250	375	1,530	2.25	4.704	4.25	6.37
	438	1,828	2.63	5.920	5.07	7.61
	500	2,118	2.96	7.074	5.88	8.82
	563	2,410	3.28	8.212	6.69	10.0
	625	2,695	3.57	9.304	7.48	11.2
300	450	2,050	3.15	7.289	5.69	8.54
	525	2,429	3.59	8.913	6.74	10.1
	600	2,802	4.00	10.47	7.78	11.6
	675	3,171	4.38	11.99	8.80	13.2
	750	3,536	4.74	13.45	9.82	14.7
350	525	2,604	4.06	10.26	7.23	10.8
	613	3,070	4.58	12.24	8.52	12.8
	700	3,523	5.05	14.31	9.78	14.6
	788	3,973	5.49	16.23	11.0	16.5
	875	4,412	5.88	18.06	12.2	18.4
400	600	2,919	4.46	10.07	8.10	12.1
	700	3,422	5.04	12.13	9.50	14.2
	800	3,915	5.55	14.10	10.8	16.3

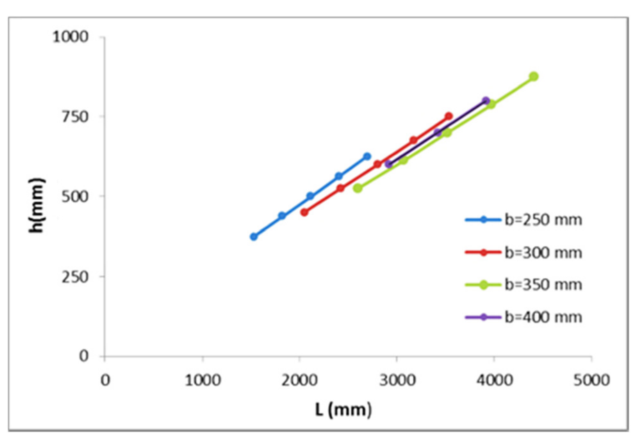

Figure 3: Optimum dimensions of R/C cantilever beams.

Table 4: Illustrative example

b (mm)	*h* (mm)	ρ	γ (MPa)	ℓ (mm)	Empirical results (mm)		ACI – provisions (mm)	
					$(\Delta_i)_{LL}$	Δ_T	$(\Delta_i)_{LL}(\ell/360)$	$\Delta_T(\ell/240)$
250	375	0.0034	1.38	1,284	1.20	1.88	O.K	O.K
250	375	0.0040	1.65	1,404	1.82	3.22	O.K	O.K
250	375	0.0050	1.96	1,530	2.25	4.70	O.K	O.K
250	375	0.0060	2.38	1,686	2.72	6.82	O.K	N.O.K

Table 5: Empirical deflections due to overload

Load increment %	Empirical results (mm)		ACI – provisions (mm)		Remark
	$(\Delta_i)_{LL}$	Δ_T	$(\Delta_i)_{LL}$	Δ_T	
5.00	4.19	11.20	7.78	11.6	O.K
7.50	4.23	11.46			O.K
10.0	4.30	11.78			N.O.K

cantilever beams to resist loads and deflections even if the values of ((f_c', f_y)) as a parameter are changed.
- Concerning the loads as a parameter, it was considered the worst distributed load identified locally. Table 5 whose calculations were made on a beam model (300 × 600 × 2,802) mm.

The distributed loads should not be increased more than 7.5% in the future. This has been tested on all pass results in Table 3 and proven correct.
- All the published articles and textbooks did not conclude Figure 2 as a simplified roadmap in calculating the various deflections exactly, instead of adopting an approximate method such as finite elements as stated in ref. [9], especially for commonly used geometric sections with (ρ = 0.005) against γ = 1.96 MPa. Also, Figure 2 shows very attractive, especially for postgraduate and undergraduate students.

3 Conclusion

- The study focused on concluding the optimum dimensions for the reinforced concrete cantilever beams with a rectangular cross section subjected to the uniformly distributed loads commonly used in building construction, excluding the concentrated loads. Due to the significant deflections, it is not easy to select cross sections of the cantilever beams, especially with a large span. To solve this problem, the authors plot a simplified graph to provide a cross section for the required beam length in less time and effort. The chart does not include deep beams with depths greater than 900 mm, with conditions specified in the ACI code. Also, the current study revealed an important economic aspect. The allowable span lengths are greater than the expected and locally common. Thus, a solution to an old problem has been developed that was not discussed in the published literature.
- The authors strongly recommend using the results in various buildings, provided that no significant concentrated loads are applied along the beams and future uniform distributed loads do not exceed 7.5% used in the current study.
- Since the increase in (ρ) increases the length of the beam, and the designer is restricted to the length adopted by this study, then any increase in (ρ) will be safer even if the values of (f_c', f_y) are changed.
- Sustained deflections are the most dangerous types of deflections.
- The authors created a simplified algorithm for calculating deflections that are not found in any published article or textbook.
- Using a beam of more than 350 mm in width is not economical.

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Conflict of Interest: The authors state no conflict of interest.

Data availability statement: Most datasets generated and analyzed in this study are in this submitted manuscript. The other datasets are available on reasonable request from the corresponding author with the attached information.

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Appendix

Appendix A. Input file

```
[(bh = [[(250,375),(250,438),(250.500).(250,563),(250.625
    [(300,750),(300,675),(300,600),(300,525),(300,450)1
    [(350,875),(350,788),(350,700),(350.613),(350,525)1
    [[(400,800),(400,700),(400,600)1
    [(py = [(0.0034,1.38),(0.004,1.65),(0.005. 1%),(0.006,2.38
    import math
    fc = 28; Es = 200,000; IDL = 25.65;WLL = I 3.5
    for bhl in bh ("####################################### print ("#######################################")
print
    for b,h in bhl
    for p,y in py
    lg = (b*pow(h ,3))/12
    Mcr = ((0.62*math.sqrt(fc))/(h/2))*1g * 0.00000 I
    (Ec = 4,700 *math.sqrt(fc n = Es/Ec
    d = h-60 As = p*b*d
    XI = ((-n*As)+math.sqrt((n*As)**2-4*(b/2)*(-n*As*d)))lb
    X2 = ((-n*As)-math .sqrt((n*As)**2-4*(b/2)*(-n*As*d)))/b (X = max(Xl,X2
    if X>O Icr = (b*pow(X.3)/3)+n*As*(d-X)**2 self_wcight = 25*b*h*O .OOOOO I WDL = self_weight+IDL W = WDLHVLL
    (L = math.sqrt(2*y*b*(d**2)/W Ma = (WDL *(L**2)/2)*0.00000 I
    Ie = pow((Mcr/Ma),3)*lg+( I-pow((Mcr/Ma),3))*lcr (delte_DL = WDL*pow(L,4)/(8*Ec*lc Ma2 = ((WDL+WLL)*(L**2)/2)
*0.00000I
    Ie2 = pow((Mcr/Ma2) ,3)*Jg+( I -pow((Mcr/Ma2),3))*1cr (delte_DL_LL = (WDL+WLL)*pow(L,4)/(8*Ec*le2 deflection#
    delta i LL = delte DL LL-delte DL (pl = 0.002*b*h/(b*d –
    (lcmbda = 1.4/(I+50*pl delta_cr_sh = lembda*dcltc_DL delta T = dclta i LL+dclta er sh
    print(b.h ,ps,L,dclta_C.LL,dclta_T.sep = "\t")
```

Appendix B. Output file

b (mm)	*h* (mm)	*p*	*y*	*L* (mm)	delta_i_LL (mm)	delta_T (mm)
250	375	0.0034	1.38	1,284.528535	1.204772808	1.889216315
250	375	0.0040	1.65	1,404.579046	1.827000915	3.224234404
250	375	0.0050	1.96	1,530.848626	2.255624256	4.704488116
250	375	0.0060	2.38	1,686.913025	2.721597045	6.825838556
250	438	0.0034	1.38	1,534.172256	1.557049476	2.561536276
250	438	0.0040	1.65	1,677.554173	2.228283574	4.199480124
250	438	0.0050	1.96	1,828.363814	2.629206231	5.920507015
250	438	0.0060	2.38	2,014.758794	3.113463796	8.383454045
250	500	0.0034	1.38	1,777.605616	1.877419335	3.219220743
250	500	0.0040	1.65	1,943.738526	2.581116647	5.132813167
250	500	0.0050	1.96	2,118.477746	2.963310207	7.074662906
250	500	0.0060	2.38	2,334.448777	3.481715347	9.864884763
250	563	0.0034	1.38	2,022.728391	2.179406299	3.878081765
250	563	0.0040	1.65	2,211.770184	2.908877273	6.056306431

250	563	0.0050	1.96	2,410.605054	3.280388006	8.21283743
250	563	0.0060	2.38	2,656.357393	3.842227813	11.32941511
250	625	0.0034	1.38	2,261.803606	2.456848024	4.515493355
250	625	0.0040	1.65	2,473.189084	3.208436785	6.943120486
250	625	0.0050	1.96	2,695.525127	3.57580832	9.304360088
250	625	0.0060	2.38	2,970.324022	4.18477208	12.73693728
300	450	0.0034	1.38	1,720.907845	1.899491023	3.189672859
300	450	0.0040	1.65	1,881.741848	2.689818385	5.19741351
300	450	0.0050	1.96	2,050.907658	3.149179882	7.289701599
300	450	0.0060	2.38	2,259.990167	3.719227507	10.28437955
300	525	0.0034	1.38	2,038.414374	2.333452058	4.122412669
300	525	0.0040	1.65	2,228.92216	3.161096716	6.514015756
300	525	0.0050	1.96	2,429.298967	3.595194997	8.913498439
300	525	0.0060	2.38	2,676.957082	4.215627041	12.36761068
300	600	0.0034	1.38	2,351.88891	2.727181776	5.034615173
300	600	0.0040	1.65	2,571.693653	3.583057672	7.785721304
300	600	0.0050	1.96	2,802.885112	4.00502623	10.4772138
300	600	0.0060	2.38	3,088.628962	4.686845969	14.37947906
300	675	0.0034	1.38	2,661.446545	3.0876788	5.92449322
300	675	0.0040	1.65	2,910.182177	3.968212988	9.017677454
300	675	0.0050	1.96	3,171.803253	4.387258686	11.99057637
300	675	0.0060	2.38	3,495.156955	5.134825763	16.33102449
300	750	0.0034	1.38	2,967.197551	3.420295323	6.792258796
300	750	0.0040	1.65	3,244.50831	4.323955165	10.21396181
300	750	0.0050	1.96	3,536.184811	4.746234304	13.45978324
300	750	0.0060	2.38	3,896.685874	5.560589567	18.22890816
350	525	0.0034	1.38	2,185.161352	2.647921466	4.745996234
350	525	0.0040	1.65	2,389.383936	3.580501887	7.499118209
350	525	0.0050	1.96	2,604.18602	4.066801826	10.25975533
350	525	0.0060	2.38	2,869.673227	4.766943236	14.23240888
350	613	0.0034	1.38	2,576.123049	3.153073087	5.963814076
350	613	0.0040	1.65	2,816.884449	4.116546697	9.19342417
350	613	0.0050	1.96	3,070.118197	4.585419233	12.33972662
350	613	0.0060	2.38	3,383.10548	5.364770151	16.90585678
350	700	0.0034	1.38	2,956.237404	3.599176564	7.128713119
350	700	0.0040	1.65	3,232.523841	4.588461835	10.80164112
350	700	0.0050	1.96	3,523.122956	5.053681076	14.31195702
350	700	0.0060	2.38	3,882.292411	5.916382436	19.44735853
350	788	0.0034	1.38	3,334.484507	4.006319426	8.26857538
350	788	0.0040	1.65	3,646.121468	5.019921686	12.36836927
350	788	0.0050	1.96	3,973.902398	5.489907668	16.23294524
350	788	0.0060	2.38	4,379.027164	6.436849677	21.92741333
350	875	0.0034	1.38	3,702.493016	4.372149225	9.359433503
350	875	0.0040	1.65	4,048.523615	5.408836445	13.86360523
350	875	0.0050	1.96	4,412.479904	5.888618307	18.06639311
350	875	0.0060	2.38	4,862.316036	6.916459073	24.29757779
400	600	0.0034	1.38	2,670.234601	3.413739737	6.516244229
400	600	0.0040	1.65	2,919.791555	4.467862884	10.07529493
400	600	0.0050	1.96	3,182.2765	4.98213654	13.55243852
400	600	0.0060	2.38	3,506.69791	5.828305797	18.59110858
400	700	0.0034	1.38	3,130.24736	3.966187244	8.010752646

400	700	0.0040	1.65	3,422.796561	5.046467123	12.13636773
400	700	0.0050	1.96	3,730.500912	5.552493174	16.0761737
400	700	0.0060	2.38	4,110.811789	6.500408425	21.83899311
400	800	0.0034	1.38	3,580.761603	4.450992282	9.443540174
400	800	0.0040	1.65	3,915.415329	5.555043299	14.10193199
400	800	0.0050	1.96	4,267.40538	6.065308054	18.48226539
400	800	0.0060	2.38	4,702.451697	7.114153209	24.94214265

Appendix C. Example

Take any passing result from Table 3. Let it be(300 × 600 × 2802 mm as well as an algorithm parameter; $f_c' = 28$ MPa, $f_y = 414$ MPa, IDL = 25.65 kN/m, and W_{LL} = 13.5 kN/m.Self-weight = 25(300) (600) ×

$$10^{-6}$$

= 4.5 kN/m

$$W_{DL}$$

= 4.5 + 25.65 = 30.15 kN/m,

Factored ultimate loads are; W_u = 1.2(30.15) + 1.6(13.5) = 57.78 kN/m

$$M_u = \frac{W_u \ell^2}{2} = \frac{(57.78)(2802)^2}{2} \times 10^{-6} = 227 \text{ kN m}$$

$$\rho = \frac{f_c'}{1.18 f_y}\left[1 - \sqrt{1 - \frac{2.36 M_u}{0.9 b d^2 f_c'}}\right] = \frac{28}{1.18(414)}\left[1 - \sqrt{1 - \frac{2.36(227)*10^6}{0.9(300)(600-60)^2(28)}}\right]. \tag{A1}$$

$\rho = 0.0074 \quad > 0.005$

Provided reinforcement ratio remains greater than 0.005 when any value of $\left(\frac{f_y}{f_c'}\right)$ in Figure 1 is substituted into equation (A1).