Research Article

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Diverse wave propagation in shallow water waves with the Kadomtsev–Petviashvili–Benjamin–Bona–Mahony and Benney–Luke integrable models

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Abstract: The shallow water wave model is one of the completely integrable models illustrating many physical problems. In this article, we investigate new exact wave structures to Kadomtsev–Petviashvili–Benjamin–Bona–Mahony and the Benney–Luke equations which explain the behavior of waves in shallow water. The exact structures are expressed in the shapes of hyperbolic, singular periodic, rational as well as solitary, singular, shock, shock-singular solutions. An efficient computational strategy namely modified direct algebraic method is employed to construct the different shapes of wave structures. Moreover, by fixing parameters, the graphical representations of some solutions are plotted in terms of three-dimensional, two-dimensional and contour plots, which explain the physical movement of the attained results. The accomplished results show that the applied computational technique is valid, proficient, concise and can be applied in more complicated phenomena.

Keywords: exact wave structures, KP–BBM equation, Benney–Luke equation, integrability, direct algebraic technique

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1 Introduction

Recently, it is a more fascinating zone for researchers and scholars to accomplish exact solutions of nonlinear partial differential equations (NLPDEs) with the assistance of computational bundles that are simply tedious and monotonous mathematical calculations. NLPDEs have gained a remarkable attention in the realm of nonlinear sciences due to their wide range of usage and applications. NLPDEs perform a great role to describe the physical mechanisms of natural phenomena and dynamical processes in ocean engineering, physics, geochemistry, fluid mechanics, geophysics, plasma physics, optical fibers and many other scientific areas. Nonlinear phenomenon is one of the greatest impressive fields for the researchers in this modern era of science. A variety of well-organized schemes are utilized to explain the complicated physical models [1–15].

The completely integrable models is very important for observing and meaning dynamical behavior of the physical systems. In the eighteenth century, the water wave model is first derived by Euler. In applied mathematics and physics by using many models, such as KdV equation, Boussinesq equations, Kadomtsev–Petviashvili (KP) equation, Davey–Stewartson equation, Benjamin–Bona–Mahony (BBM) equation is derived by Euler equations. The Benney–Luke equation is also a water wave approximation to the Euler equation. The KP equation is the two-dimensional (2D) general type of KdV equation. In a weak dispersion medium, it describes long wavelength water waves. The BBM equation is also known as regularized long-wave equation, which is an improved type of KdV equation. It is derived to model long wave propagation in nonlinear dispersion. The KP equation and the BBM equation are utilized to symbolize acoustic waves in fluids and long-wavelength surface waves in liquids, and symmetry method is a very effective method to deal with differential equations [16–23].

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However, in this article, we aim to discuss the integrable models, namely, KP–BBM and the Benney–Luke equations, in order to discuss the dynamical behavior of water waves. Different types of solutions are secured by using direct algebraic method.

The (2 + 1)-dimensional KP–BBM equation is read as ref. [24]:

$$(q_t + q_x - a(q^2)_x - bq_{xxt})_x + kq_{yy} = 0, \tag{1}$$

where a, b, k are non-zero constants. In the literature, KP–BBM equation has been examined with the assistance of different techniques [25–29].

The Benney–Luke equation is given by [30,31].

$$q_{tt} - q_{xx} + \alpha q_{xxxx} - \beta q_{xxtt} + q_t q_{xx} + 2q_x q_{xt} = 0, \tag{2}$$

where α, β are non-zero constants. Equation (2) models an approximation of the full water wave equation and is formally suitable to describe the two-way propagation of water waves in the presence of surface tension. First, equation (2) is presented by Benney and Luke [32]. There are comprehensive studies such as numerical studies, stability analysis, traveling wave solutions, analytical solutions [33–38], Cauchy problem on this equation in the literature [30,31,32,39–41].

The arrangement of this article is as follows: In Section 2, key points of the method; exact wave solutions with physical behavior in Section 3. Results and discussion in Section 4 and the conclusion is in Section 5.

2 Modified direct algebraic method (MDAM)

This section deals key points of the MDAM [42,43] in order to extract different forms of wave structures to the complex NPLDE. Let us consider the NLPDE as

$$S(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \cdots) = 0, \tag{3}$$

where S is a polynomial of $u(x, t)$ and its partial derivatives in which higher order derivatives and nonlinear terms are involved. For finding the exact solutions of equation (3), we introduce traveling wave transformation as: $u(x, t) = u(\tau)$ and $\tau = x - \alpha t$, where α represents the wave speed. After putting this transformation into equation (3), we get nonlinear ordinary differential equation (ODE) as follows:

$$D(u, u', u'', u''', \cdots) = 0, \tag{4}$$

where $'$ denotes the derivative w.r.t τ.

To extract wave structures, consider the solutions of equation (4) in the form

$$u(\tau) = A_0 + \sum_{j=0}^{m} (A_j\varphi^j + B_j\varphi^{-j})^i, \tag{5}$$

$$\varphi' = \varpi + \varphi^2, \tag{6}$$

where ϖ is a parameter. By using the homogeneous balance principle and on equating the same powers of φ^j to zero, we get the cluster of algebraic equations. The solutions of the system of algebraic equations along with the following wave structures are general solutions.

(i) If $\varpi < 0$

$$\varphi = -\sqrt{-\varpi}\tanh(\sqrt{-\varpi}\tau), \quad \text{or} \quad \varphi = -\sqrt{-\varpi}\coth(\sqrt{-\varpi}\tau),$$

(ii) If $\varpi > 0$

$$\varphi = \sqrt{\varpi}\tan(\sqrt{\varpi}\tau), \quad \text{or} \quad \varphi = -\sqrt{\varpi}\cot(\sqrt{\varpi}\tau),$$

(iii) If $\varpi = 0$

$$\varphi = \frac{-1}{\tau}.$$

We apply the aforementioned method for solving equations (1)–(2) in the next section.

3 Extraction of solutions

In this section, we will present the exact wave structures to the KP–BBM and the Benney–Luke equations by utilizing MDAM.

3.1 (2 + 1)-dimensional KP–BBM equation

In order to proceed, we use traveling wave hypothesis $q(x, y, t) = U(\eta)$, where $\eta = x + y - vt$ and $v \neq 0$. So, equation (1) becomes

$$-2aUU'' - 2aU'^2 + bvU^{(4)} + kU'' - vU'' + U'' = 0. \tag{7}$$

On making integration equation (7) twice with respect to η and taking constants equal to zero. The ODE transforms as

$$U(k - v + 1) - aU^2 + bvU'' = 0. \tag{8}$$

On utilizing the homogeneous balance rule on above equation, we obtain $m = 2$, which implies the solution of equation (8) as follows:

$$U = a_0 + a_1Z + a_2Z^2 + b_1Z^{-1} + b_2Z^{-2}, \tag{9}$$

where a_0, a_1, a_2, b_0, b_1 and b_2 are parameters. In order to find parameters, we solve equation (8) with the assistance of equation (9), and we get cluster of equations on equating same power coefficients of Z. Furthermore, by using *Mathematica*, system is solved. So, we get a set of solutions as follows:

Set 1:

$$a_0 = \frac{3(1-\nu+k)}{2a},\ a_1 = 0,\ a_2 = \frac{6b\nu}{a},$$
$$b_1 = 0,\ b_2 = 0,\ \varpi = \frac{1-\nu+k}{4b\nu}.$$

Set 2:

$$a_0 = \frac{2b(1+k)\varpi}{a(1-4b\varpi)},\ a_1 = 0,\ a_2 = 0,$$
$$b_1 = 0,\ b_2 = \frac{6b\varpi^2(k+1)}{a(1-4b\varpi)},\ \nu = \frac{k+1}{1-4b\varpi}.$$

Set 3:

$$a_0 = \frac{12b\nu\varpi}{a},\ a_1 = 0,\ a_2 = \frac{6b\nu}{a},$$
$$b_1 = 0,\ b_2 = \frac{6b\nu\varpi^2}{a},\ k = 16b\varpi\nu + \nu - 1.$$

For Set 1, we have the following solutions as:

Case 1:

When $\varpi < 0$, we get the following solutions:

Shock wave structure

$$q_1 = \left(\frac{3(k-\nu+1)}{2a}\right)\left[1 - \tanh^2\left(\frac{1}{2}\eta\sqrt{-\frac{k-\nu+1}{b\nu}}\right)\right]. \quad (10)$$

Singular wave structure

$$q_2 = \left(\frac{3(k-\nu+1)}{2a}\right)\left[1 - \coth^2\left(\frac{1}{2}\eta\sqrt{-\frac{k-\nu+1}{b\nu}}\right)\right]. \quad (11)$$

The aforementioned solutions are valid for

$$(k-\nu+1)b\nu < 0.$$

It is noted that the aforementioned results converge to particular solutions for some constant values of coefficients of hyperbolic functions. For instance, if $\sqrt{*} \to 2$, then $q_1 \to \operatorname{sech}^2(.)$ which is solitary wave type structure and also $q_2 \to \operatorname{csch}^2(.)$ which is singular wave type II structure.

Case 2:

When $\varpi > 0$, the following periodic solutions of different forms are obtained

$$q_3 = \left(\frac{3(k-\nu+1)}{2a}\right)\left[1 + \tan^2\left(\frac{1}{2}\eta\sqrt{\frac{k-\nu+1}{b\nu}}\right)\right], \quad (12)$$

and

$$q_4 = \left(\frac{3(k-\nu+1)}{2a}\right)\left[1 + \cot^2\left(\frac{1}{2}\eta\sqrt{\frac{k-\nu+1}{b\nu}}\right)\right]. \quad (13)$$

The existence criterion for the aforementioned solutions is

$$(k-\nu+1)b\nu > 0.$$

Case 3:

When $\varpi = 0$, the rational solution is expressed as

$$q_5 = \frac{3\left(\nu\left(\frac{4b}{\eta^2} - 1\right) + k + 1\right)}{2a}, \quad (14)$$

where $\eta = x - \nu t$.

For Set 2, we discuss the following cases:

Case 1: When $\varpi < 0$, we get the singular and shock wave structures, respectively,

$$q_6 = \frac{2b(1+k)\varpi}{a(4b\varpi - 1)}[3\coth^2(\sqrt{-\varpi}\,\eta) - 1], \quad (15)$$

and

$$q_7 = \frac{2b(1+k)\varpi}{a(4b\varpi - 1)}[3\tanh^2(\sqrt{-\varpi}\,\eta) - 1], \quad (16)$$

Case 2:

When $\varpi > 0$, the periodic solutions are

$$q_8 = \frac{2b(1+k)\varpi}{a(1-4b\varpi)}[3\cot^2(\sqrt{\varpi}\,\eta) + 1], \quad (17)$$

and

$$q_9 = \frac{2b(1+k)\varpi}{a(1-4b\varpi)}[3\tan^2(\sqrt{\varpi}\,\eta) + 1], \quad (18)$$

where, $\eta = \left(x - \left(\frac{k+1}{1-4b\varpi}\right)t\right)$.

For Set 3, we obtain the wave structures for different values of ϖ as mentioned below.

Case 1:

When $\varpi < 0$, we get the following mixed hyperbolic solution:

$$q_{10} = \frac{12b\nu\varpi}{a} - \frac{6b\nu\varpi}{a}[\tanh^2(\sqrt{-\varpi}\,\eta) + \coth^2(\sqrt{-\varpi}\,\eta)]. \quad (19)$$

Case 2:

When $\varpi > 0$, the periodic solution is expressed as

$$q_{11} = \frac{12b\nu\varpi}{a} + \frac{6b\nu\varpi}{a}[\tan^2(\sqrt{\varpi}\,\eta) + \cot^2(\sqrt{\varpi}\,\eta)]. \quad (20)$$

Case 3:

For $\varpi = 0$, we get rational solution

$$q_{12} = \frac{6bv}{a\eta^2}, \tag{21}$$

where $\eta = x - vt$.

By allotting different values to parameters, the physical movements of attained solutions are shown by plotting the three-dimensional (3D), 2D and contour plots given below.

3.2 Benney–Luke equation

To solve equation (2), we consider $q(x, t) = H(\eta)$, where $\eta = kx - vt$ and $v \neq 0$, so equation (2) takes the form,

$$k^2(\alpha k^2 - \beta v^2)H^{(4)} - 3k^2 vH'H'' + (v^2 - k^2)H'' = 0. \tag{22}$$

On integrating equation (22) once with respect to η and taking integration constant equal to zero, we get

$$k^2H^{(3)}(\alpha k^2 - \beta v^2) + (v^2 - k^2)H' - \frac{1}{2}3k^2vH'^2 = 0. \tag{23}$$

After getting transform $H' = U$, we can modify as

$$2k^2U''(\alpha k^2 - \beta v^2) + 2(v^2 - k^2)U - 3k^2vU^2 = 0. \tag{24}$$

Applying the balance rule on equation (24), we have $m = 2$, which implies the solution of equation (24) as follows:

$$U = a_0 + a_1Z + a_2Z^2 + b_1Z^{-1} + b_2Z^{-2}, \tag{25}$$

where a_0, a_1, a_2, b_0, b_1 and b_2 are parameters. In order to find parameters, we solve equation (24) with the assistance of equation (25), and we get cluster of equations on equating the same power coefficients of Z. Furthermore, by using *Mathematica*, system is solved. So, we get the following set of solutions.

Set 1:

$$a_0 = \frac{k^2 - v^2}{3vk^2},\ a_1 = 0,\ a_2 = -\frac{4(\beta v^2 - \alpha k^2)}{v},$$
$$b_1 = 0,\ b_2 = 0,\ \varpi = \frac{k^2 - v^2}{4\alpha k^4 - 4\beta v^2k^2}.$$

Set 2:

$$a_0 = \frac{v^2 - k^2}{vk^2},\ a_1 = 0,\ a_2 = 0,$$
$$b_1 = 0,\ b_2 = \frac{\varpi(v^2 - k^2)}{vk^2},\ \alpha = \frac{4\beta\varpi v^2k^2 + v^2 - k^2}{4\varpi k^4}.$$

Set 3:

$$a_0 = \frac{v^2 - k^2}{2vk^2},\ a_1 = 0,\ a_2 = \frac{v^2 - k^2}{4v\varpi k^2},$$
$$b_1 = 0,\ b_2 = \frac{\varpi(v^2 - k^2)}{4vk^2},\ \beta = \frac{-v^2 + 16\alpha\varpi k^4 + k^2}{16v^2\varpi k^2}.$$

For Set 1, we have the following cases as:

Case 1:

If $\varpi < 0$, then the hyperbolic function solutions are obtained as

$$q_1 = \frac{(v^2 - k^2)\left[3\tanh^2\left(\eta\sqrt{\frac{v^2 - k^2}{4\alpha k^4 - 4\beta v^2k^2}}\right) - 1\right]}{3vk^2} \tag{26}$$

and

$$q_2 = \frac{(v^2 - k^2)\left[3\coth^2\left(\eta\sqrt{\frac{v^2 - k^2}{4\alpha k^4 - 4\beta v^2k^2}}\right) - 1\right]}{3vk^2}. \tag{27}$$

The aforementioned solutions hold for

$$\frac{v^2 - k^2}{4\alpha k^4 - 4\beta v^2k^2} > 0.$$

Case 2:

If $\varpi > 0$, then the periodic solutions are

$$q_3 = \frac{(k^2 - v^2)\left[3\tan^2\left(\eta\sqrt{\frac{k^2 - v^2}{4\alpha k^4 - 4\beta v^2k^2}}\right) + 1\right]}{3vk^2}, \tag{28}$$

and

$$q_4 = \frac{(k^2 - v^2)\left[3\cot^2\left(\eta\sqrt{\frac{k^2 - v^2}{4\alpha k^4 - 4\beta v^2k^2}}\right) + 1\right]}{3vk^2}. \tag{29}$$

The validity condition for the aforementioned solutions is

$$\frac{k^2 - v^2}{4\alpha k^4 - 4\beta v^2k^2} > 0.$$

Case 3:

If $\varpi = 0$, then rational function solution can be expressed as

$$q_5 = \frac{k^2 - v^2}{3vk^2} - \frac{4(\beta v^2 - \alpha k^2)}{v\eta^2}, \tag{30}$$

where $\eta = x - vt$.

For Set 2, the following solutions are discussed for the different values of ϖ:

Case 1: If $\varpi < 0$, then the singular and shock wave solutions are obtained, respectively

$$q_6 = \frac{(v^2 - k^2)[1 - \coth^2(\sqrt{-\gamma}\eta)]}{vk^2} \tag{31}$$

and

$$q_7 = \frac{(v^2 - k^2)[1 - \tanh^2(\sqrt{-\varpi}\eta)]}{vk^2}. \tag{32}$$

Case 2:

If $\varpi > 0$, then periodic solutions are

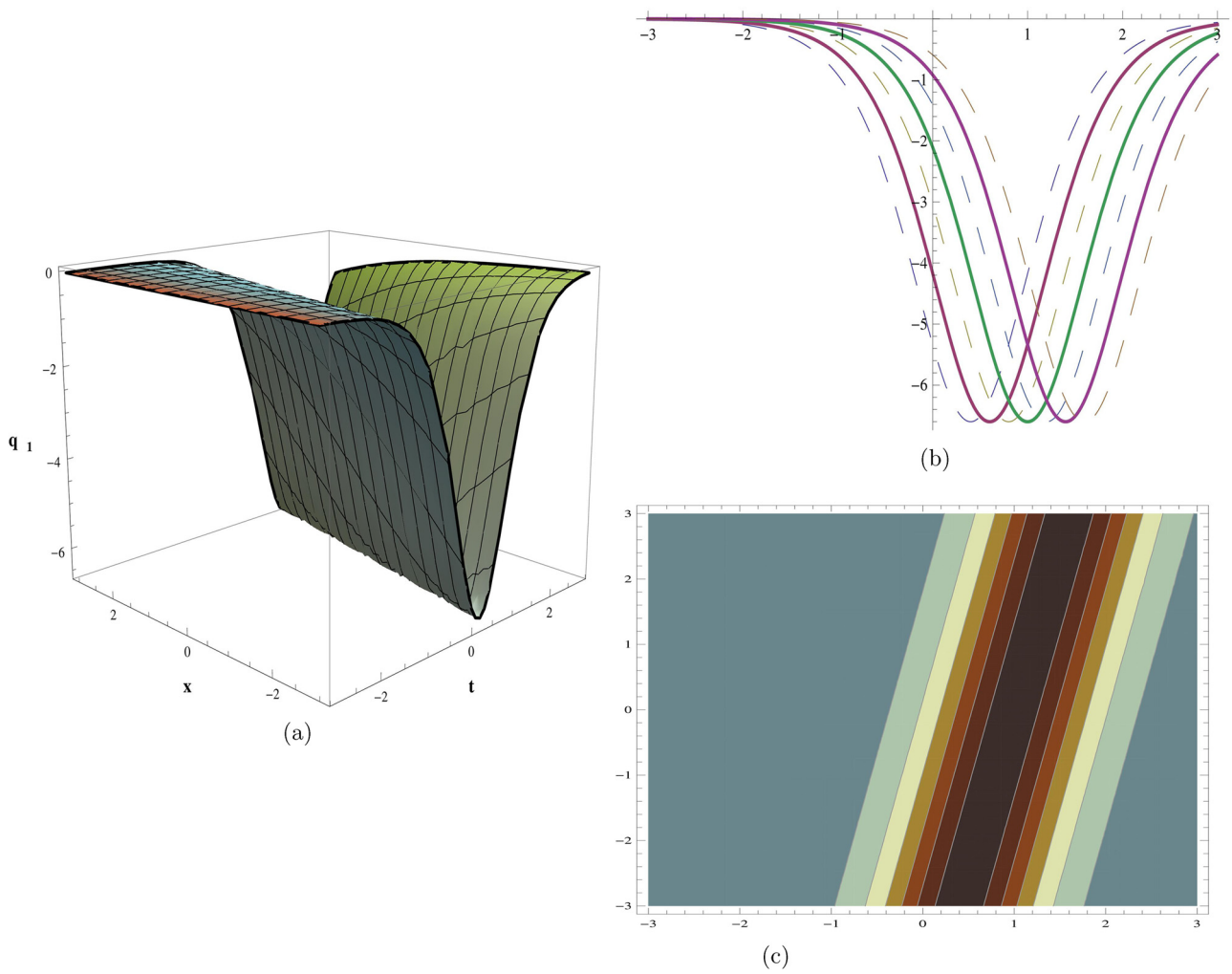

Figure 1: Plots of solution (10), under the parameters $a = 0.5$, $k = -3$, $b = 2$, $y = -1$, $v = 0.2$.

$$q_8 = \frac{(c^2 - k^2)[1 + \cot^2(\sqrt{\varpi}\,\eta)]}{ck^2} \tag{33}$$

and

$$q_9 = \frac{(c^2 - k^2)[1 + \tan^2(\sqrt{\varpi}\,\eta)]}{ck^2}. \tag{34}$$

where $\eta = x - vt$.

For Set 3, we discuss the following cases:

Case 1:

For $\varpi < 0$, shock and singular solutions in combined form are written as

$$q_{10} = \frac{v^2 - k^2}{2vk^2} - \frac{v^2 - k^2}{4vk^2}[\tanh^2(\sqrt{-\varpi}\,\eta) + \coth^2(\sqrt{-\varpi}\,\eta)]. \tag{35}$$

Case 2:

For $\varpi > 0$, we get periodic solutions

$$q_{11} = \frac{v^2 - k^2}{2vk^2} + \frac{v^2 - k^2}{4vk^2}[\tan^2(\sqrt{\varpi}\,\eta) + \cot^2(\sqrt{\varpi}\,\eta)], \tag{36}$$

where $\eta = kx - vt$.

On selecting different parameters, the graphical view in the forms of 3D, 2D and their corresponding contours for achieved solutions are sketched below.

4 Results and discussion

The obtained outcomes will be favorable to researchers to examine the most alluring utilizations of the examined equations that govern the wave propagation in shallow water. The physical movements of the solutions are demonstrated in the forms of 3D, 2D as well as contour graphs in Figures 1–9 clearly, under the selection parameters. It is

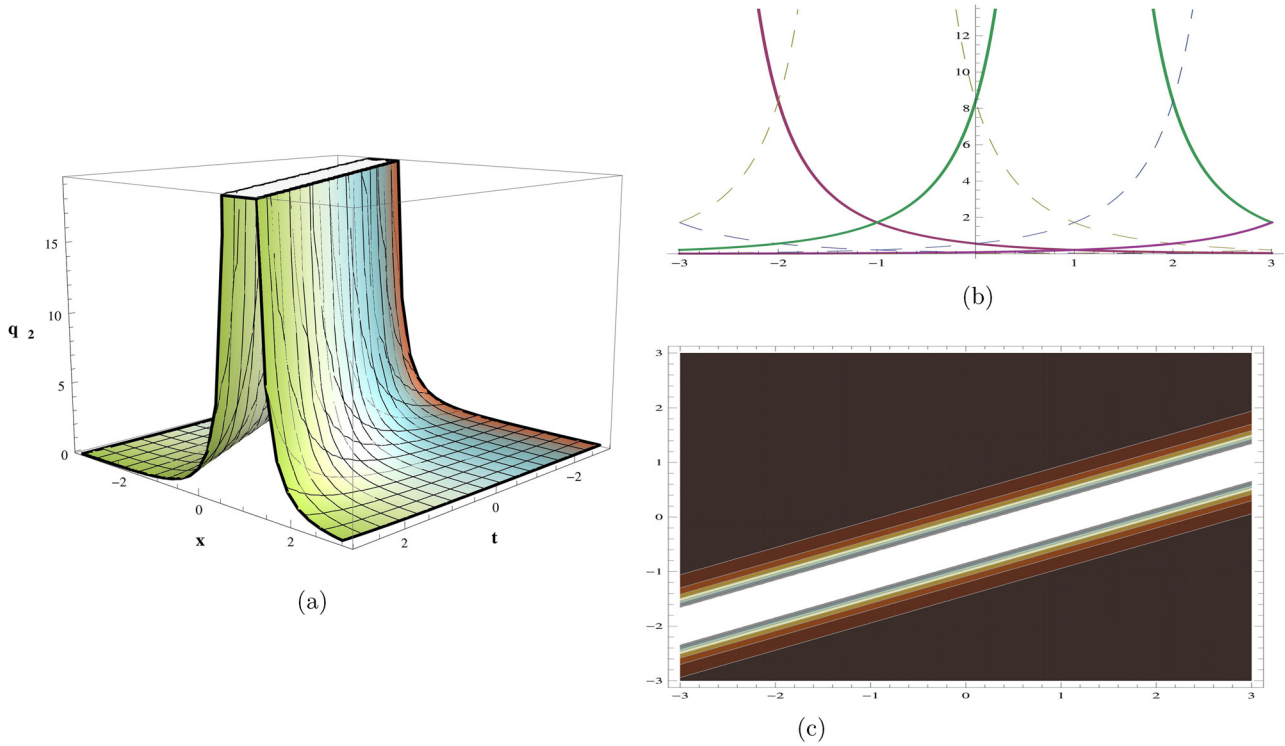

Figure 2: Plots of solution (11), under the parameters $a = 2$, $k = -1.5$, $b = 1.5$, $y = -1$, $v = 2$.

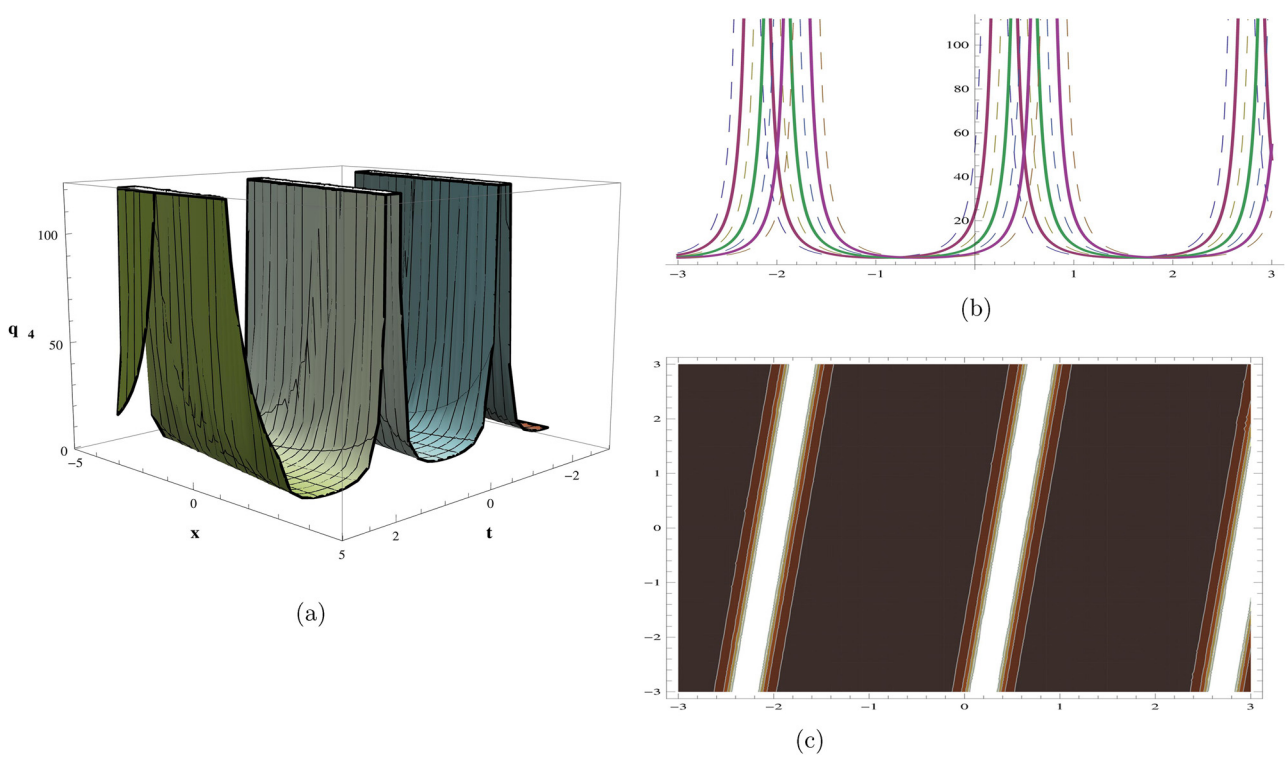

Figure 3: Plots of solution (13), under the parameters $a = 0.9$, $k = 1$, $b = 3$, $y = 2$, $v = 0.1$.

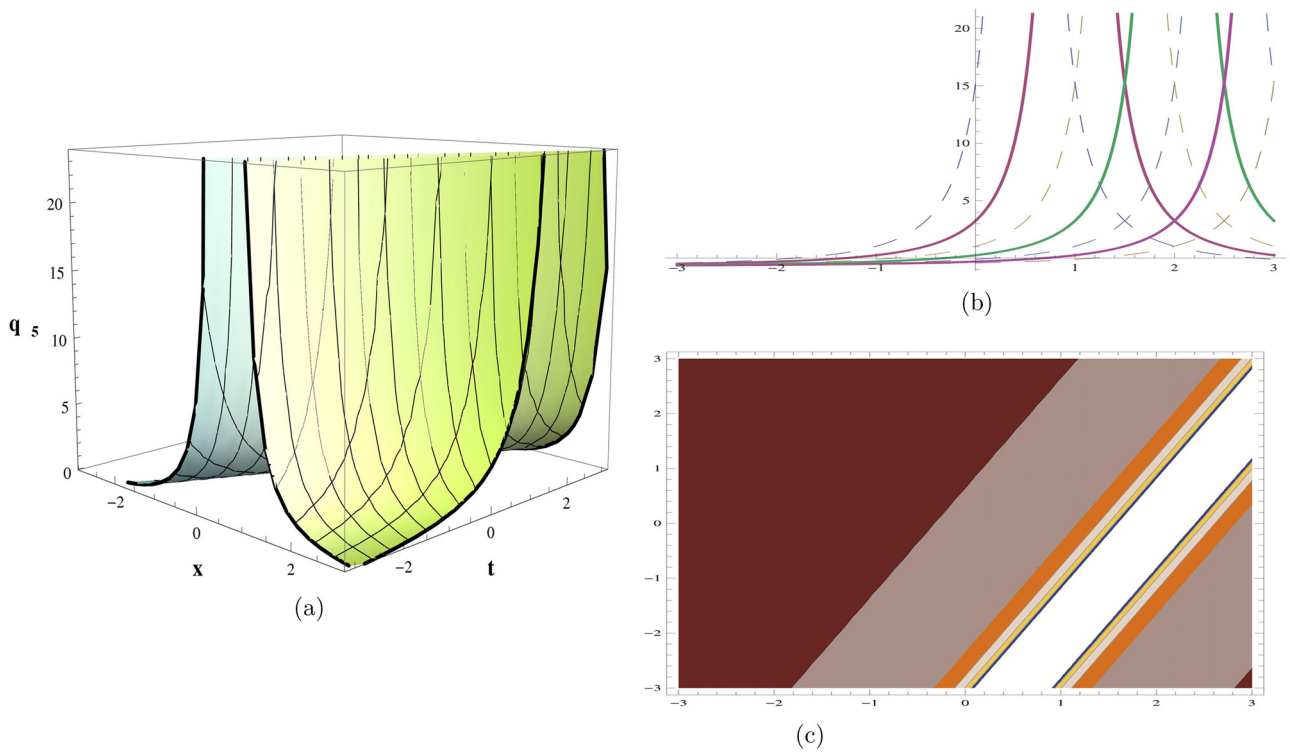

Figure 4: Plots of solution (14), under the parameters $a = 3$, $k = -2$, $b = 4$, $y = -2$, $v = 0.5$.

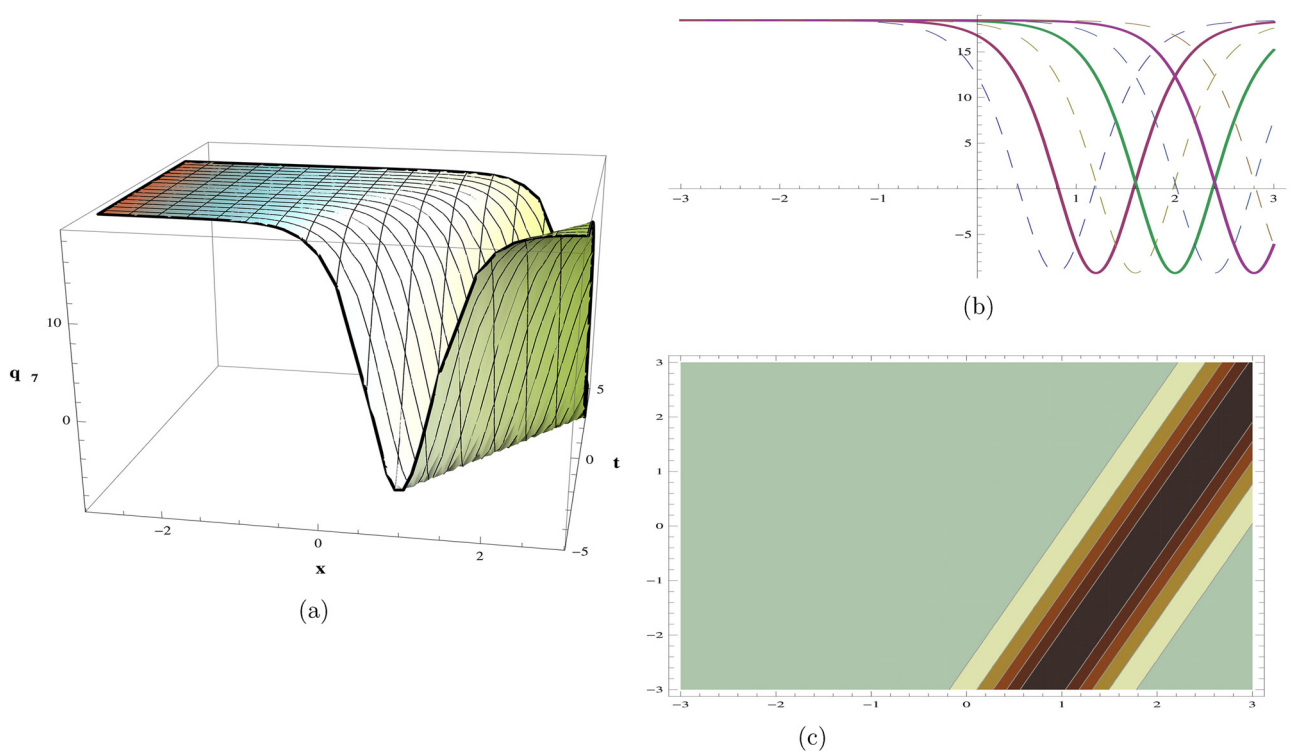

Figure 5: Plots of solution (16), under the parameters $a = 0.2$, $k = -5$, $b = 1$, $y = -2$, $\varpi = -2$, $v = 0.4$.

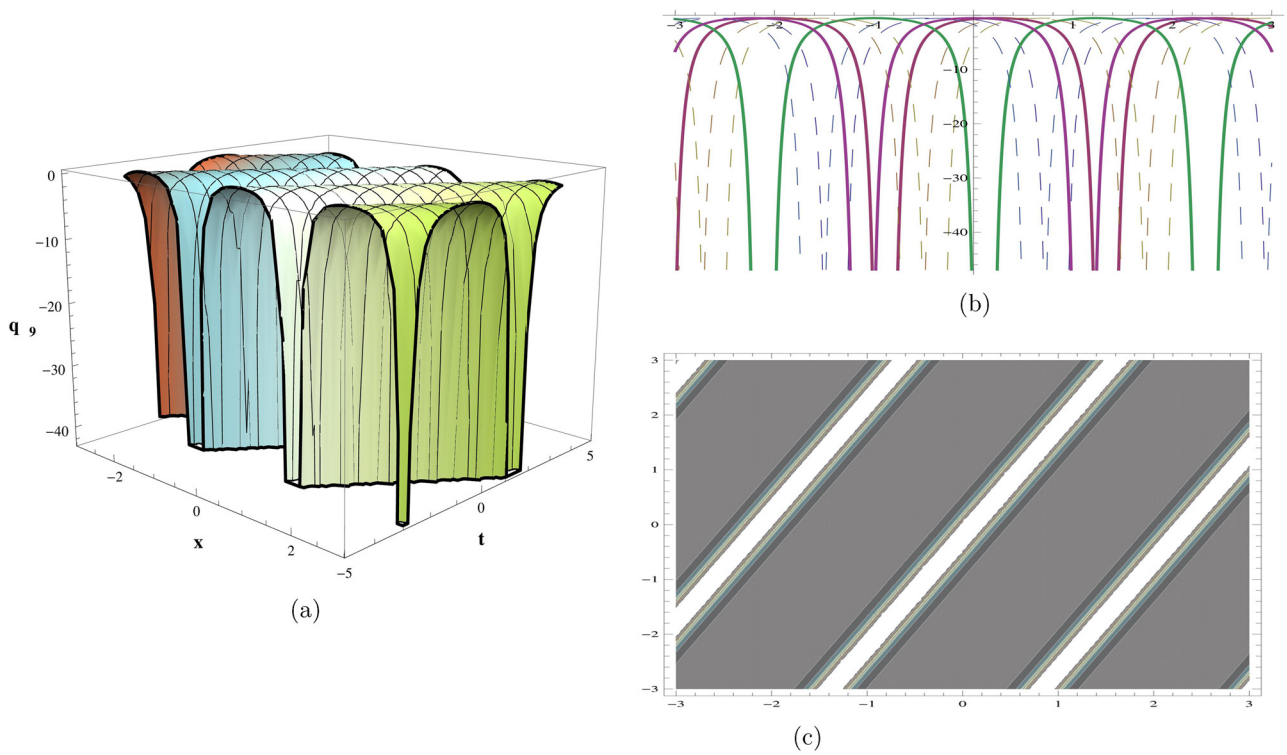

Figure 6: Plots of solution (18), under the parameters $a = 2$, $k = 1$, $b = 3$, $\varpi = 2$, $y = 1$, $v = 0.5$.

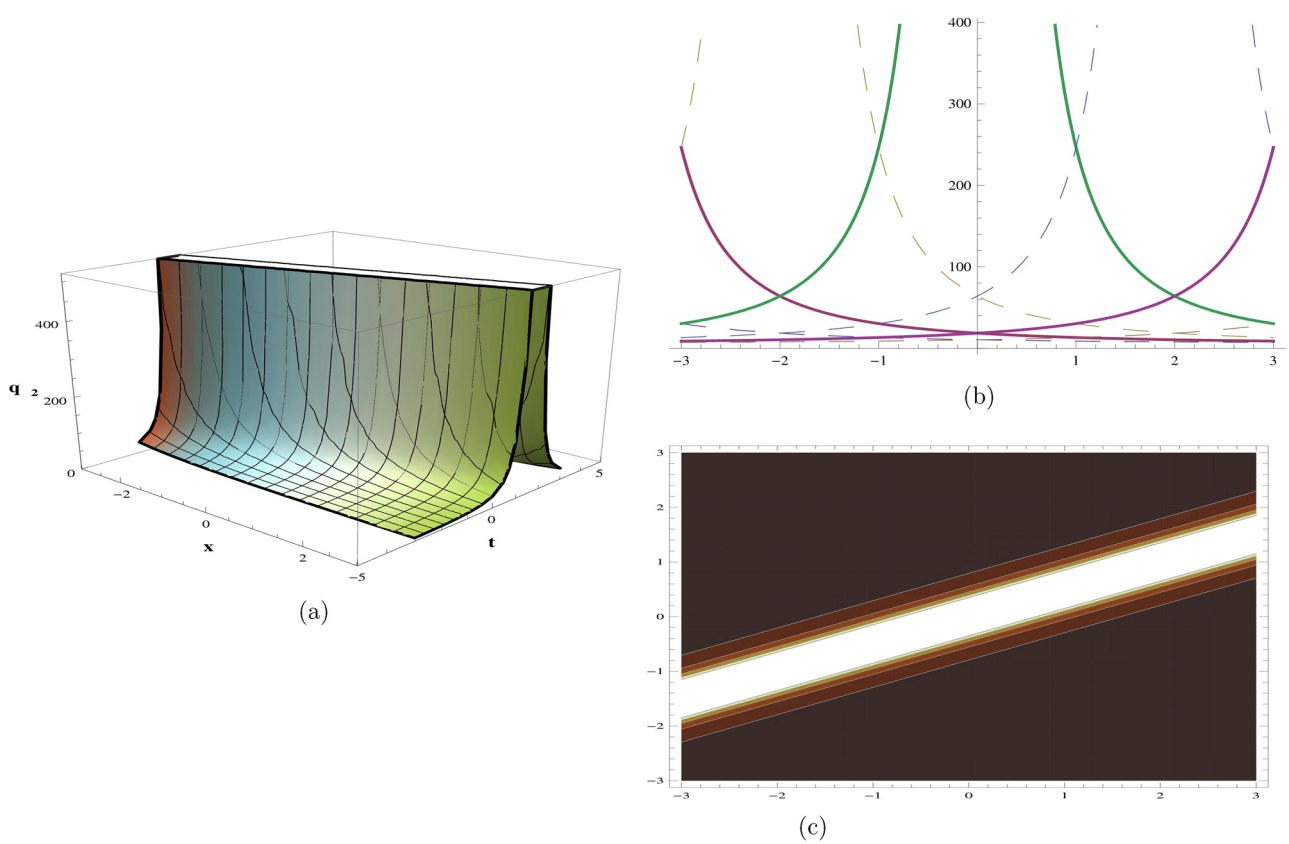

Figure 7: Plots of solution (27), on selecting the parameters $a = 0.3$, $k = 1$, $b = -2$, $\beta = -5$, $v = 2$.

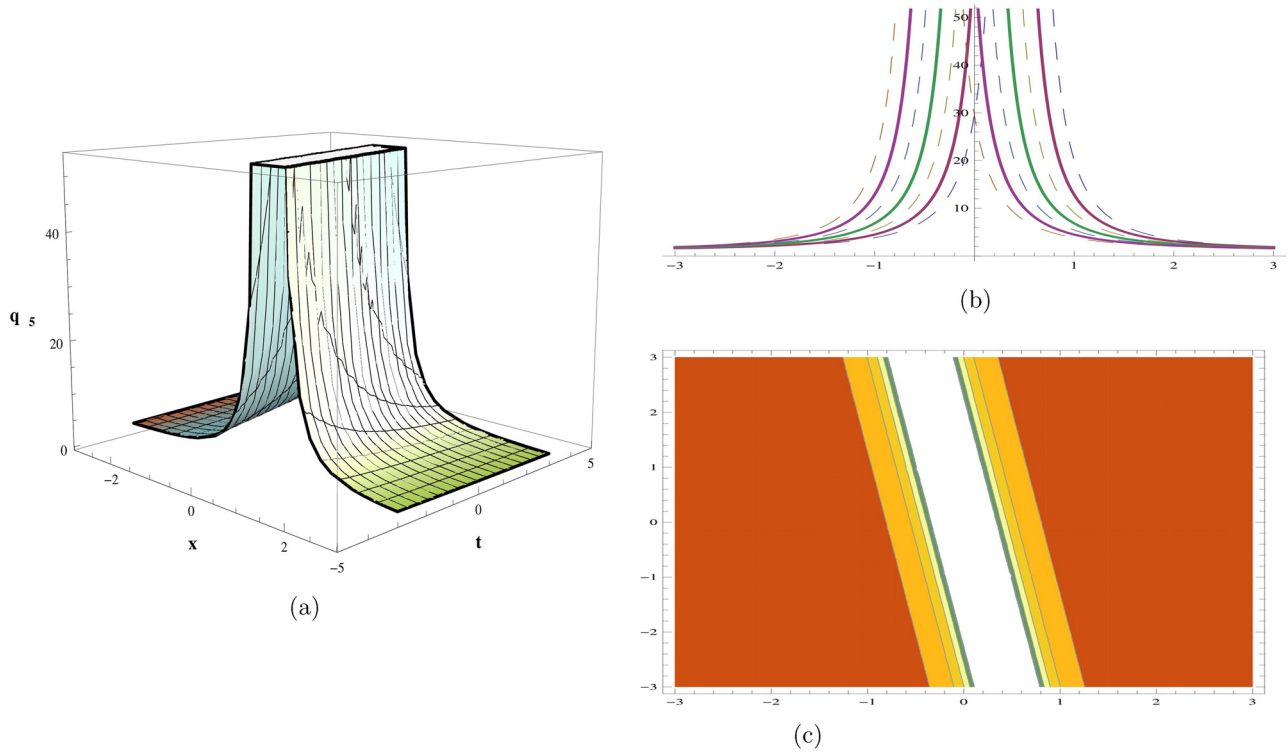

Figure 8: Plots of solution (30), on selecting the parameters $a = 0.5$, $k = -2$, $b = 2$, $\beta = 3$, $v = 3$.

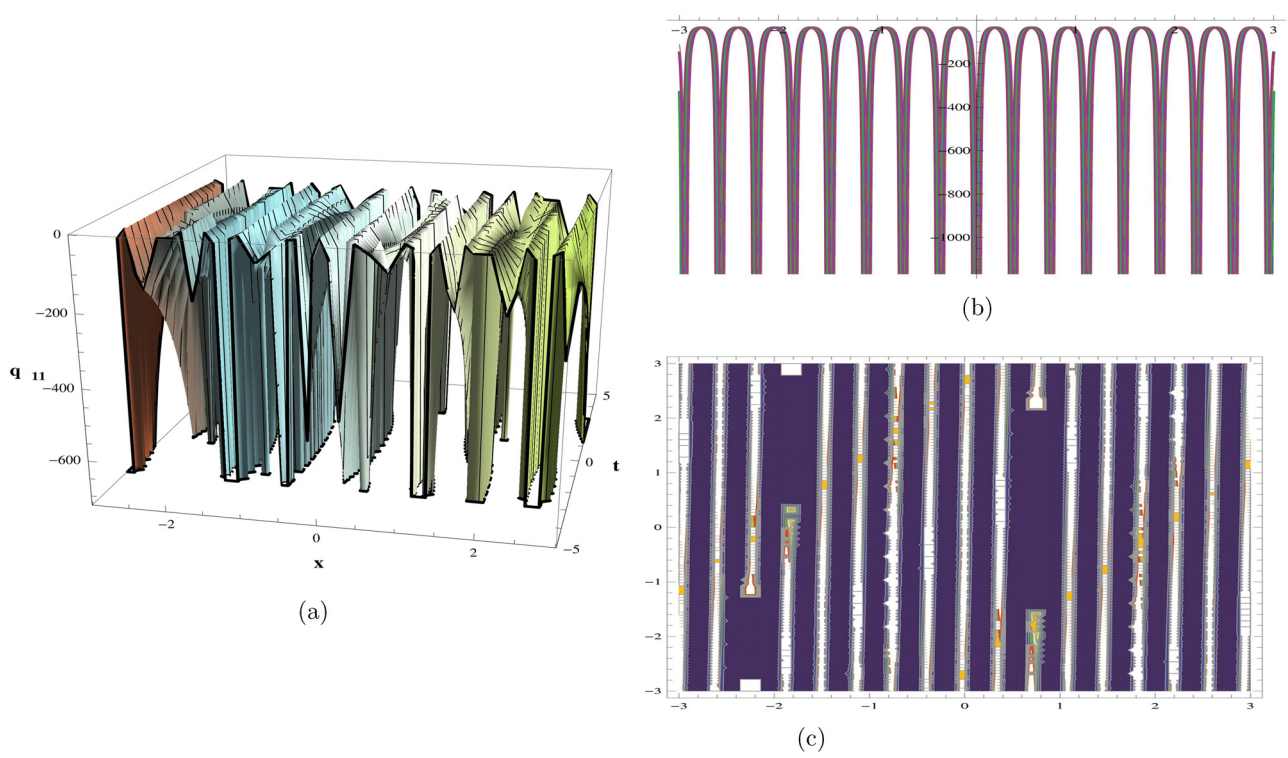

Figure 9: Plots of solution (36), on selecting the parameters $a = 1$, $k = 3$, $b = -1$, $\varpi = 2$, $v = 0.03$.

believed that the presented results in this study could be helpful in explaining the physical meaning of various nonlinear evolution equations arising in the different fields of nonlinear sciences. For example, the hyperbolic functions use in the calculation of magnetic moment, rapidity of special relativity and Langevin function for magnetic polarization. Moreover, the submitted solutions may be helpful understanding physical phenomena especially in oceanography, geophysical science.

5 Conclusion

This article secures the exact solutions in the forms of hyperbolic, periodic and rational function solutions. Different shapes of solutions like solitary, shock and singular with constraint conditions are discussed for the and the Benney–Luke equations which describe the propagation of waves in water. The methodology that is adopted to find the solutions is MDAM. Furthermore, under the selection of the parameters, the physical movement of some achieved solutions is presented by drawing 3D, 2D and relevant contour graphs.

The applied approach is a simple and efficient mathematical tool for investigating the different kinds of complicated nonlinear models in the fields of nonlinear sciences.

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