

Mathematik – Lehrbuch

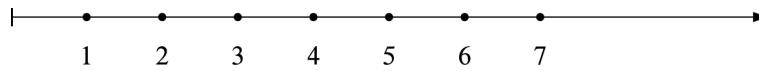
Otto Opitz / Robert Klein

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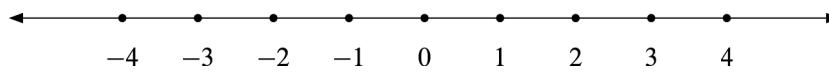
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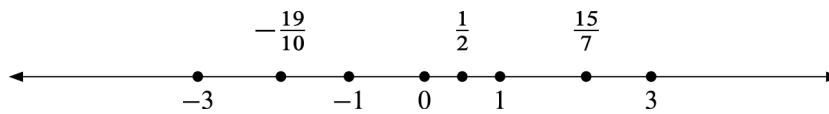
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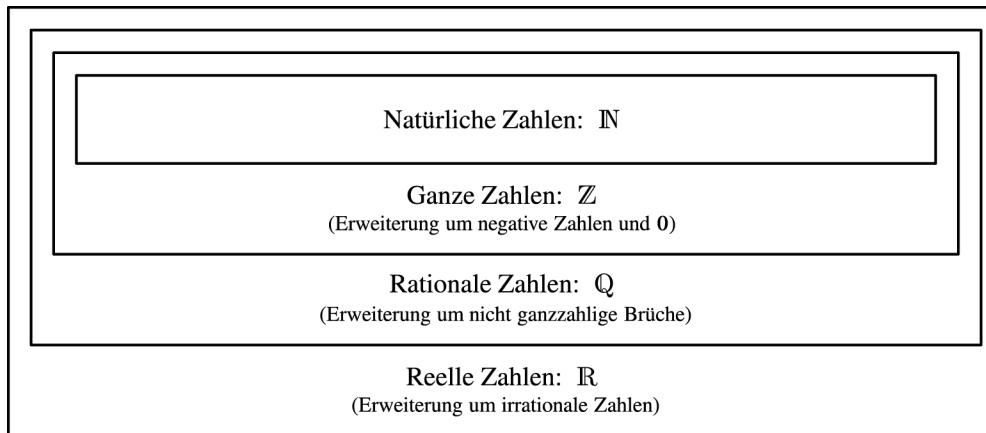
Figur 1.1: Zahlenstrahl und natürliche Zahlen



Figur 1.2: Zahlengerade und ganze Zahlen



Figur 1.3: Zahlengerade und rationale Zahlen



Figur 1.4: Hierarchischer Aufbau des Zahlensystems

Maschinenzeiten je Einheit	Produkte			
	P_1	P_2	\dots	P_n
Maschinen M_1	z_{11}	z_{12}	\dots	z_{1n}
M_2	z_{21}	z_{22}	\dots	z_{2n}
\vdots	\vdots	\vdots	\ddots	\vdots
M_m	z_{m1}	z_{m2}	\dots	z_{mn}

Figur 1.5: Doppelindizierung von Zahlen

		Binomialkoeffizienten					
$(a+b)^0$							1
$(a+b)^1$							1 1
$(a+b)^2$							1 2 1
$(a+b)^3$							1 3 3 1
$(a+b)^4$							1 4 6 4 1
$(a+b)^5$							1 5 10 10 5 1
\vdots							\vdots
$(a+b)^n$	$\binom{n}{0}$	$\binom{n}{1}$	$\binom{n}{k}$	$\binom{n}{k+1}$	$\binom{n}{n-1}$	$\binom{n}{n}$	
$(a+b)^{n+1}$	$\binom{n+1}{0}$	$\binom{n+1}{1}$	$\binom{n+1}{k+1}$		$\binom{n+1}{n}$	$\binom{n+1}{n+1}$	
\vdots	\vdots	\vdots	\vdots		\vdots	\vdots	

Figur 1.6: Zahlendreieck nach Pascal (1623–1662)

Kombinationen k -ter Ordnung aus n Objekten		
	mit Wiederholung	ohne Wiederholung ($k \leq n$)
mit Reihenfolge	n^k (Fall a)	$\frac{n!}{(n-k)!}$ (Fall b)
ohne Reihenfolge	$\binom{n+k-1}{k}$ (Fall d)	$\binom{n}{k}$ (Fall c)

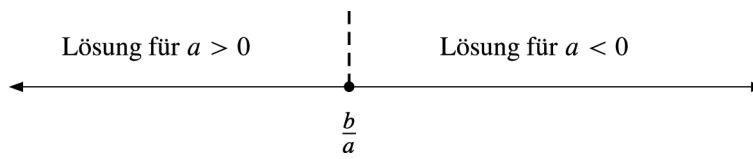
Figur 1.7: Kombinationen k -ter Ordnung aus n Objekten

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

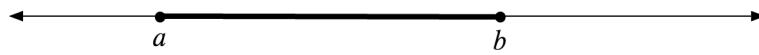
Figur 1.8: Wurfergebnisse mit zwei Würfeln

Rechenregel	Gleichung $A_1 = A_2$	Ungleichung $A_1 \leq A_2$
Vertauschen der Seiten	$A_2 = A_1$	$A_2 \geq A_1$
Addition oder Subtraktion eines Terms B	$A_1 + B = A_2 + B$ $A_1 - B = A_2 - B$	$A_1 + B \leq A_2 + B$ $A_1 - B \leq A_2 - B$
Multiplikation oder Division eines Terms $B > 0$	$A_1 \cdot B = A_2 \cdot B$ $A_1/B = A_2/B$	$A_1 \cdot B \leq A_2 \cdot B$ $A_1/B \leq A_2/B$
Multiplikation oder Division eines Terms $B < 0$	$A_1 \cdot B = A_2 \cdot B$ $A_1/B = A_2/B$	$A_1 \cdot B \geq A_2 \cdot B$ $A_1/B \geq A_2/B$

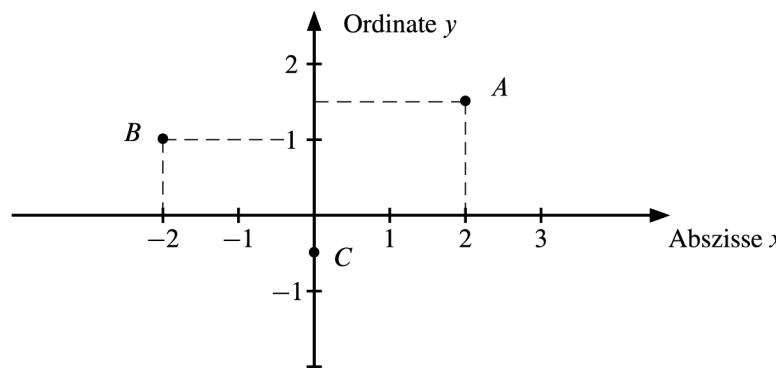
Figur 1.9: Rechenregeln zur Lösung von Gleichungen und Ungleichungen mit einer Variablen



Figur 1.10: Lösungen der Ungleichung $ax - b \leq 0$



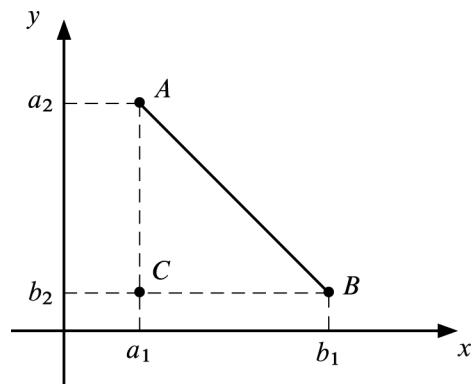
Figur 1.11: Abgeschlossenes Intervall der Zahlengeraden



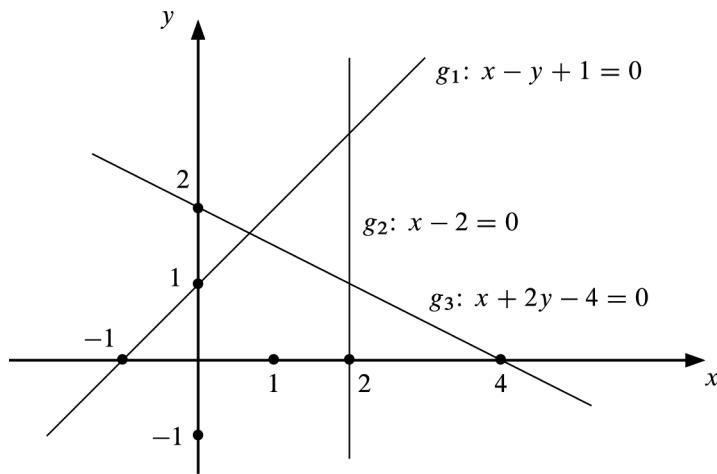
Man erhält die Koordinaten:

$$\begin{aligned}x &= 2, \quad y = 1.5 \quad \text{oder} \quad (x, y) = (2, 1.5) \quad \text{für Punkt } A \\x &= -2, \quad y = 1 \quad \text{oder} \quad (x, y) = (-2, 1) \quad \text{für Punkt } B \\x &= 0, \quad y = -0.5 \quad \text{oder} \quad (x, y) = (0, -0.5) \quad \text{für Punkt } C\end{aligned}$$

Figur 1.12: Punkte im kartesischen Koordinatensystem



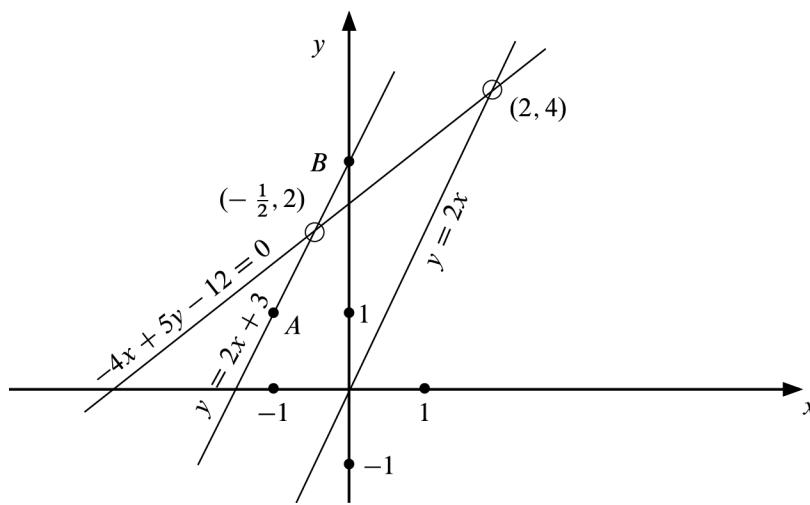
Figur 1.13: Strecke als Verbindung zweier Punkte



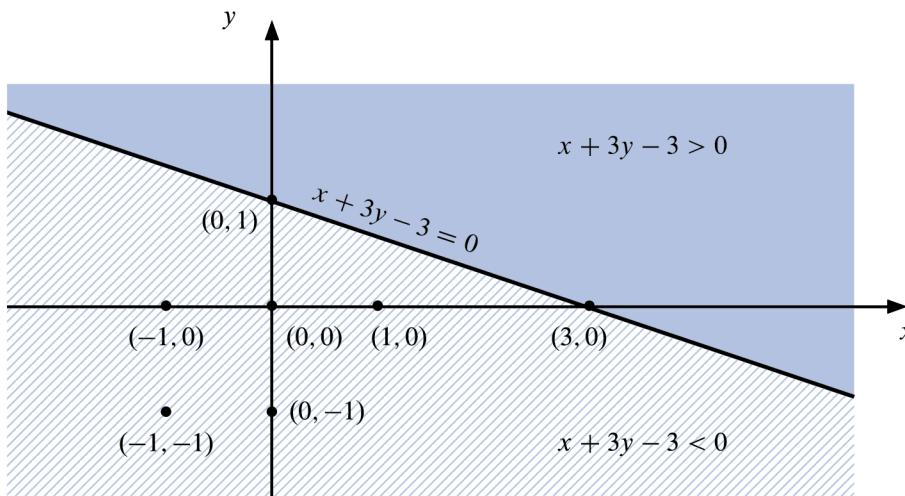
Figur 1.14: Graphische Darstellung der Gleichungen g_1 , g_2 , g_3

Konstanten	Geradengleichung	Geradenverlauf
$a = 0, b \neq 0$	$by + c = 0$	parallel zur x -Achse
$a \neq 0, b = 0$	$ax + c = 0$	parallel zur y -Achse
$c = 0$	$ax + by = 0$	durch den Nullpunkt

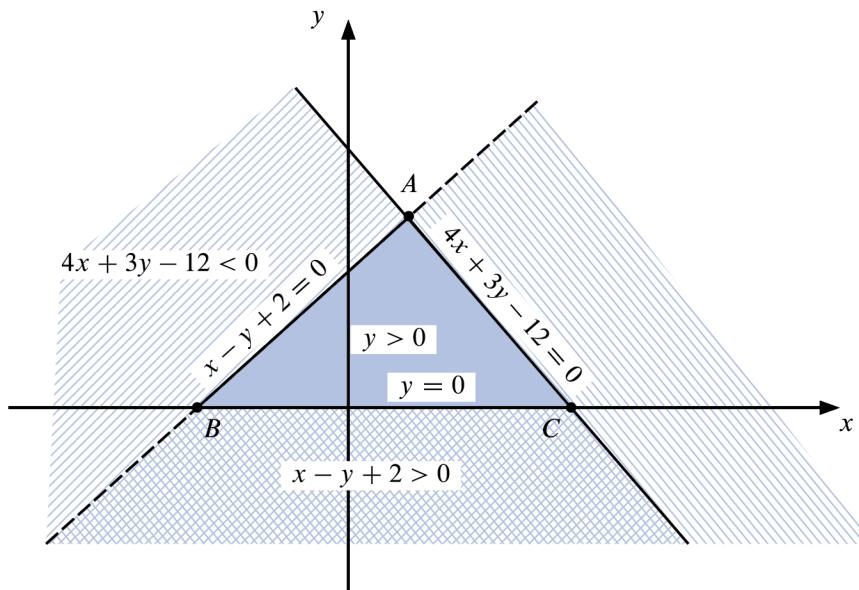
Figur 1.15: Spezielle Geraden der Ebene



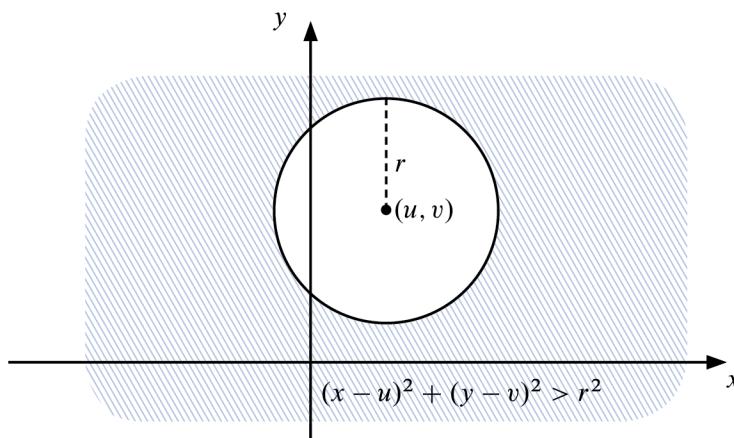
Figur 1.16: Beispiele für Geradenschnittpunkte



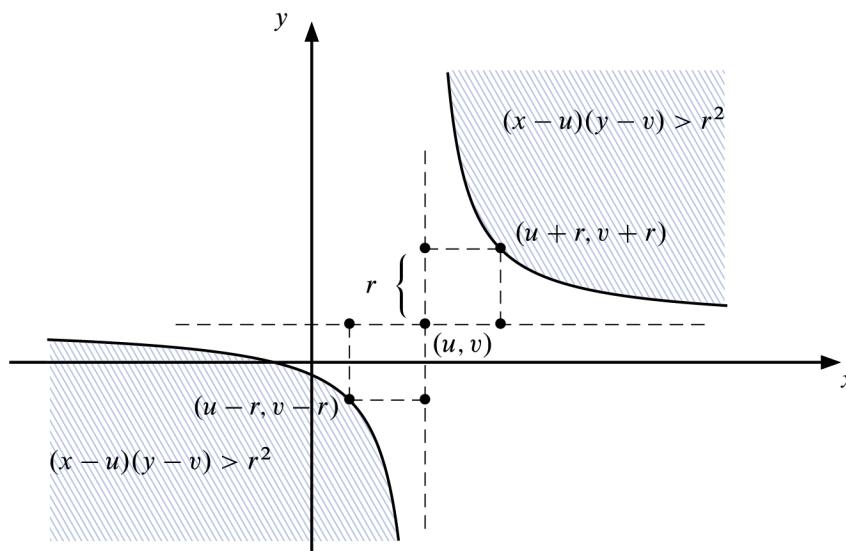
Figur 1.17: Beispiele linearer Ungleichungen und Halbebene



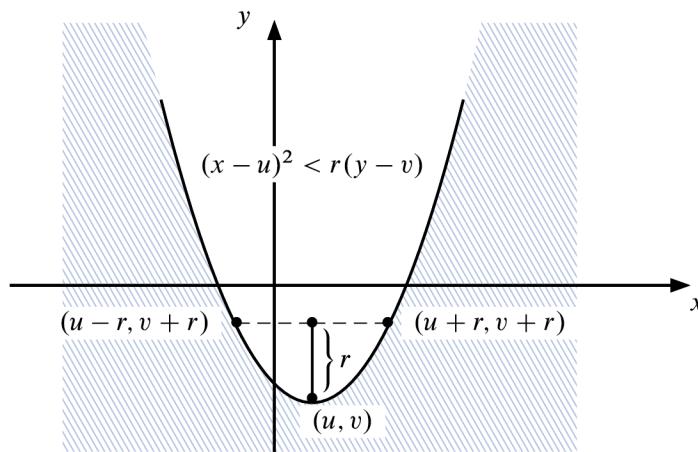
Figur 1.18: Gemeinsamer Bereich dreier Halbebenen



Figur 1.19: Kreis mit der Gleichung $(x - u)^2 + (y - v)^2 = r^2$

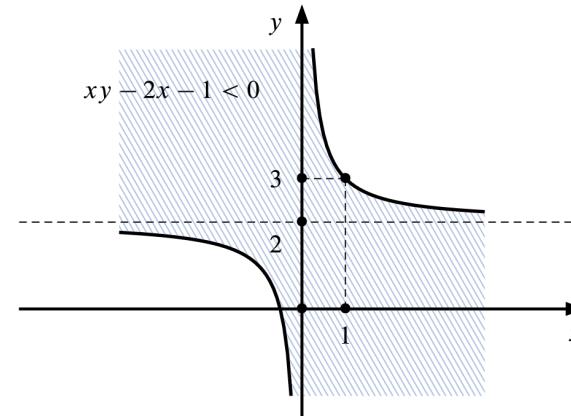
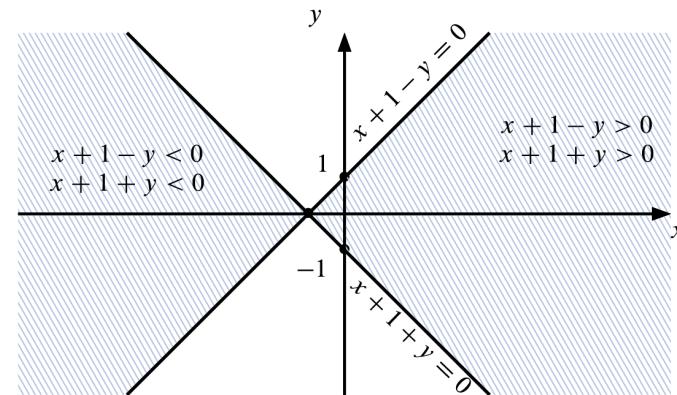
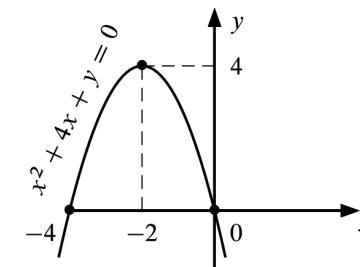


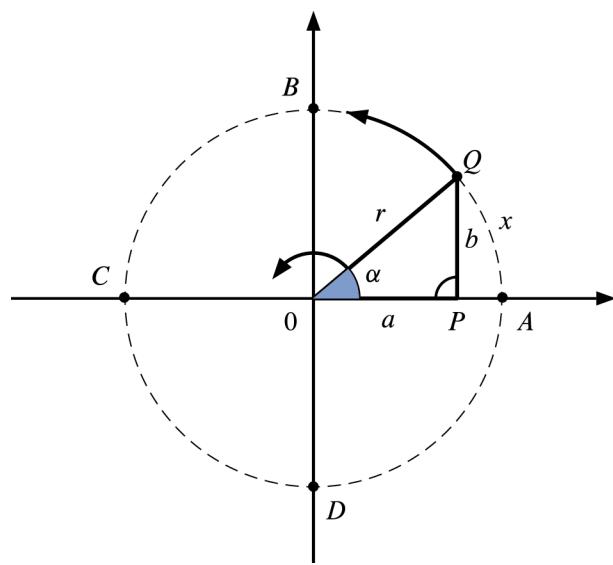
Figur 1.20: Hyperbel mit der Gleichung $(x - u)(y - v) = r^2$



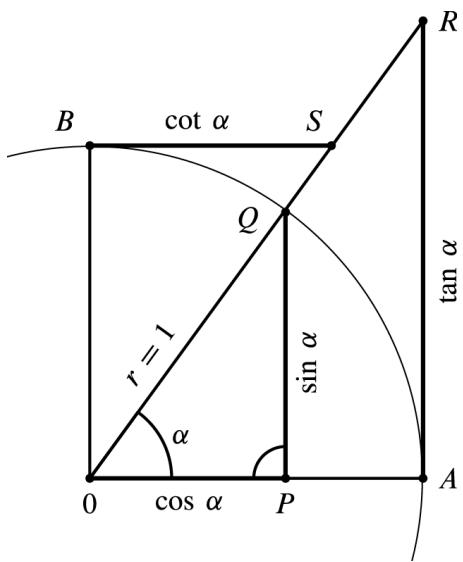
Figur 1.21: Parabel mit der Gleichung $(x - u)^2 = r(y - v)$, $r > 0$

Figur 1.22: Parabel zur Gleichung g_1 , Bereiche zu den Ungleichungen h_1, h_2





Figur 1.23: Trigonometrie am Kreis



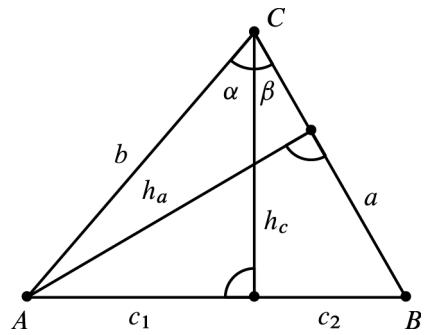
Figur 1.24: Sinus, Kosinus, Tangens und Kotangens im Einheitskreis

α	$\sin \alpha$	$\cos \alpha$	$\tan \alpha$	$\cot \alpha$
0°	0	1	0	
$\langle 0^\circ, 90^\circ \rangle$	+	+	+	+
90°	1	0		0
$\langle 90^\circ, 180^\circ \rangle$	+	-	-	-
180°	0	-1	0	
$\langle 180^\circ, 270^\circ \rangle$	-	-	+	+
270°	-1	0		0
$\langle 270^\circ, 360^\circ \rangle$	-	+	-	-
360°	0	1	0	
$\langle 360^\circ, 450^\circ \rangle$	+	+	+	+
450°	1	0		0
:	:	:	:	:

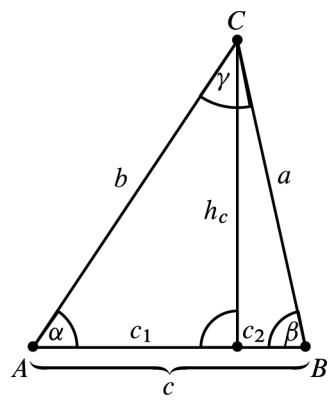
Figur 1.25: Vorzeichen und Werte von $\sin \alpha$, $\cos \alpha$, $\tan \alpha$, $\cot \alpha$

α	$\sin \alpha$	$\cos \alpha$	$\tan \alpha$	$\cot \alpha$
0°	0	1	0	
30°	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$	$\sqrt{3}$
45°	$\sqrt{2}/2$	$\sqrt{2}/2$	1	1
60°	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$\sqrt{3}/3$
90°	1	0		0

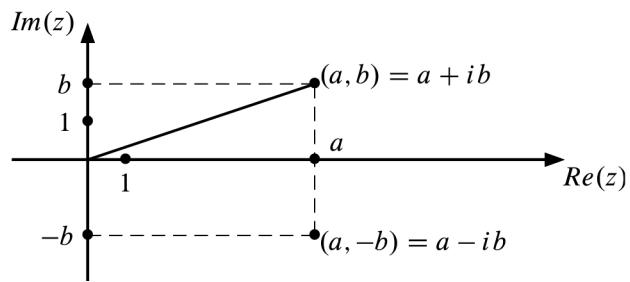
Figur 1.26: Werte für $\sin \alpha$, $\cos \alpha$, $\tan \alpha$, $\cot \alpha$ mit α aus $[0^\circ, 90^\circ]$.



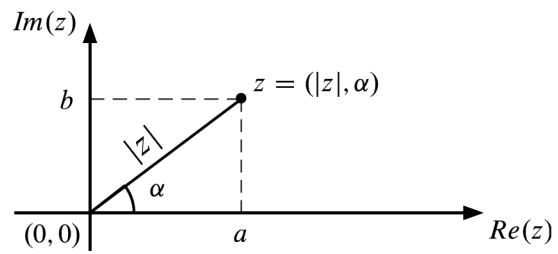
Figur 1.27: Zur Bestätigung von $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$



Figur 1.28: Zur Darstellung des Sinus- und Kosinussatzes



Figur 1.29: Geometrische Darstellung komplexer Zahlen



Figur 1.30: Polarkoordinaten für komplexe Zahlen

A	w	f
<hr/>		
\bar{A}	f	w

Figur 2.1: Wahrheitstabelle der Negation

A	w	w	f	f
B	w	f	w	f
<hr/>				
A \wedge B	w	f	f	f

Figur 2.2: Wahrheitstabelle der Konjunktion

A	w	w	f	f
B	w	f	w	f
<hr/>				
A ∨ B	w	w	w	f

Figur 2.3: Wahrheitstabelle der Disjunktion

A	w	w	f	f
B	w	f	w	f
<hr/>				
$A \Rightarrow B$	w	f	w	w

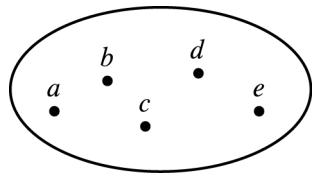
Figur 2.4: Wahrheitstabelle der Implikation

A	w	w	f	f
B	w	f	w	f
<hr/>				
A \iff B	w	f	f	w

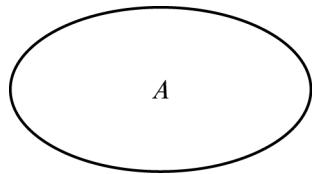
Figur 2.5: Wahrheitstabelle der Äquivalenz

Priorität	Grundrechenarten	logische Operationen
1	Potenzieren	Negation
2	Multiplizieren	Konjunktion, Disjunktion
3	Addieren	Implikation, Äquivalenz

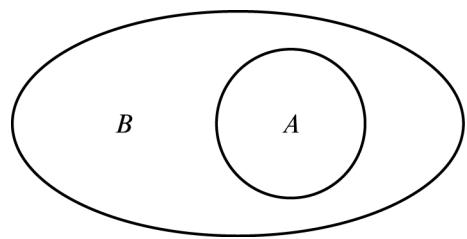
Figur 2.6: Prioritäten der logischen Operationen



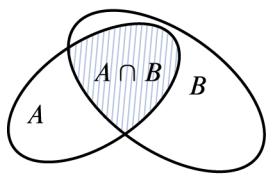
Figur 3.1: Venndiagramm der Menge $\{a, b, c, d, e\}$



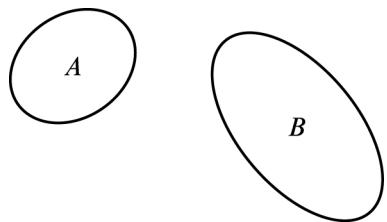
Figur 3.2: Venndiagramm der Menge A



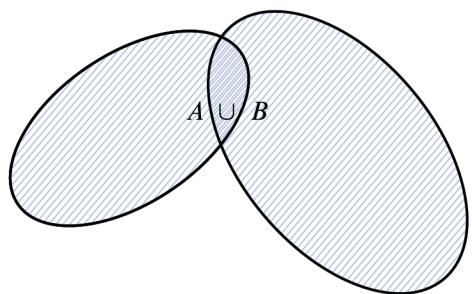
Figur 3.3: Teilmenge $A \subset B$



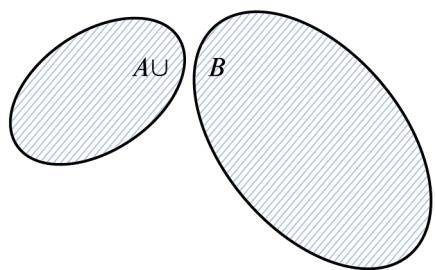
Figur 3.4: Schnittmenge $A \cap B$



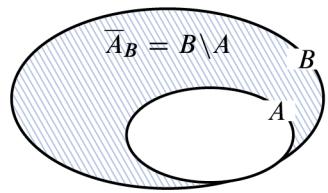
Figur 3.5: Disjunkte Mengen A, B



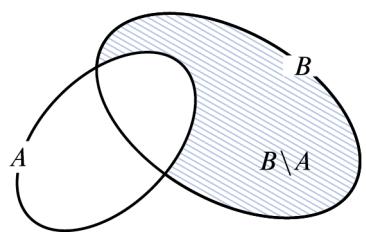
Figur 3.6: Vereinigungsmenge $A \cup B$



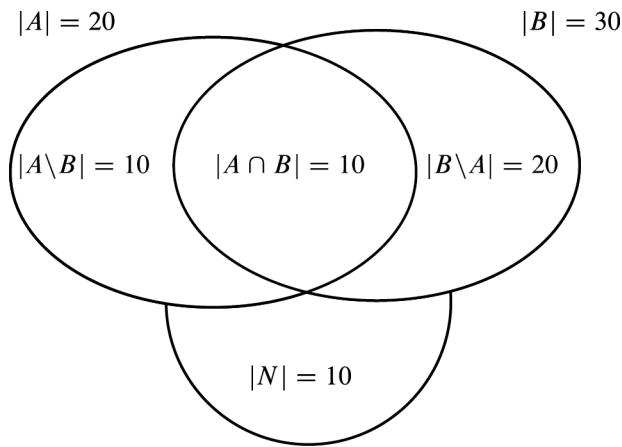
Figur 3.7: Vereinigung disjunkter Mengen A, B



Figur 3.8: Komplementärmenge von A bzgl. B



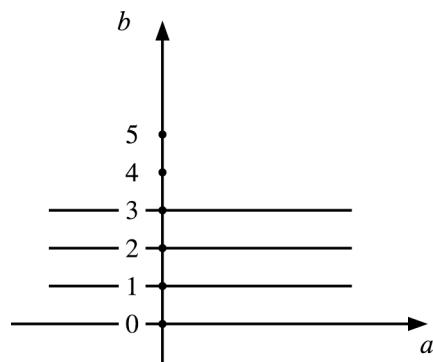
Figur 3.9: Differenzmenge von B und A



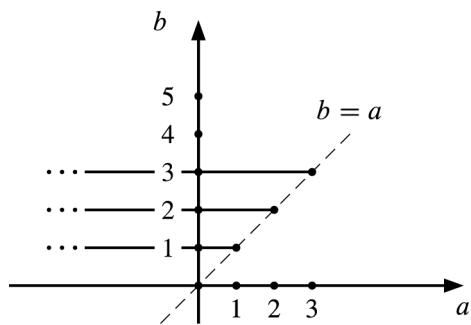
Figur 3.10: Venndiagramm zu Beispiel 3.25

Priorität	logische Operationen	Mengenoperationen
1	Negation	Komplement, Differenz
2	Konjunktion, Disjunktion	Durchschnitt, Vereinigung
3	Implikation, Äquivalenz	Teilmenge, Identität

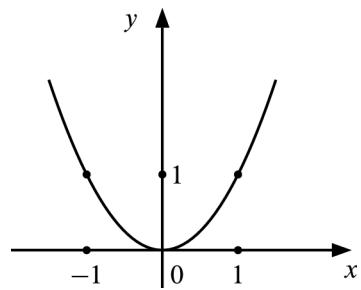
Figur 3.11: Prioritäten der Mengenoperationen



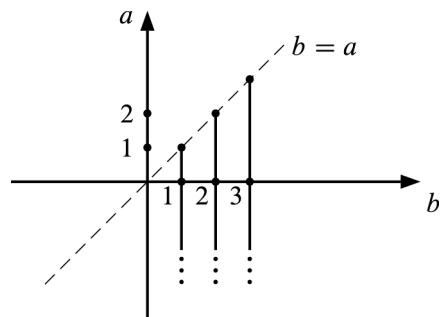
Figur 4.1: Menge $A \times B = \{(a, b) : a \in \mathbb{R}, b \in \mathbb{N}\}$



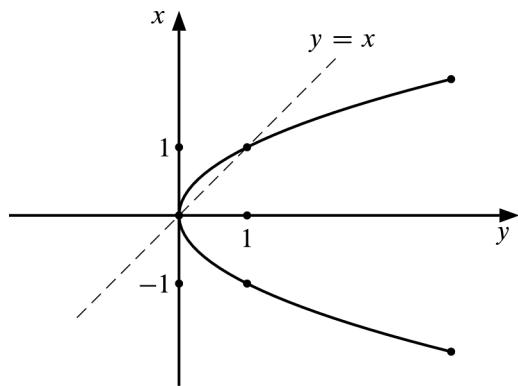
Figur 4.2: Graph der Relation $R = \{(a, b) \in \mathbb{R} \times \mathbb{N} : a \leq b\}$



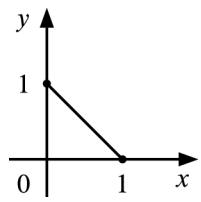
Figur 4.3: Graph der Relation $R = \{(x, y) \in \mathbb{R}^2 : y = x^2\}$



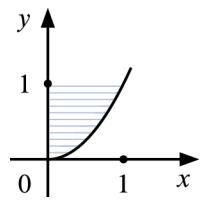
Figur 4.4: Graph der Relation $R^{-1} = \{(b, a) \in \mathbb{N} \times \mathbb{R} : a \leq b\}$



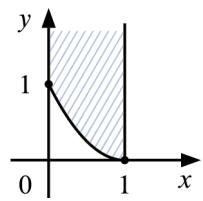
Figur 4.5: Graph der Relation $R^{-1} = \{(y, x) \in \mathbb{R}^2 : y = x^2\}$



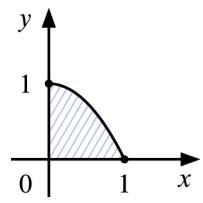
Figur 4.6: $R = \{(x, y) \in \mathbb{R}_+^2 : x + y = 1\}$



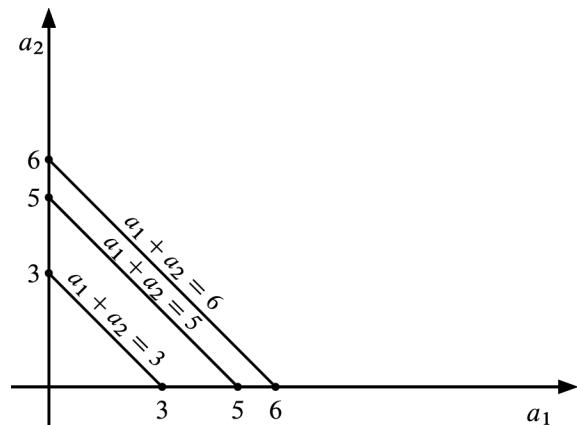
Figur 4.7: $S = \{(x, y) \in \mathbb{R}_+^2 : y \geq x^2\}$



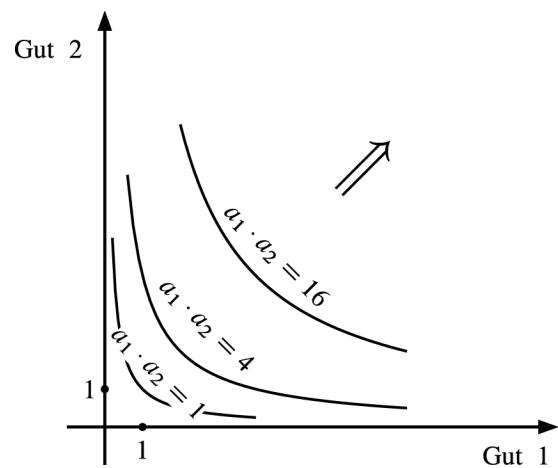
Figur 4.8: $S \circ R = \{(x, y) \in \mathbb{R}_+^2 : y \geq (1-x)^2 \wedge x \in [0, 1]\}$



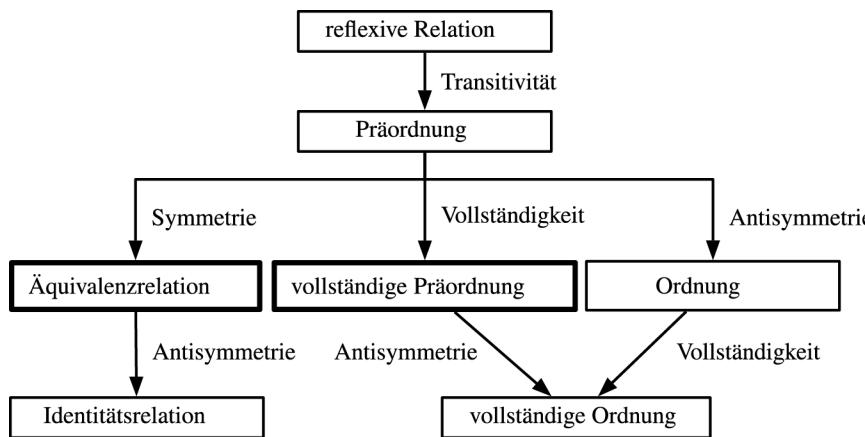
Figur 4.9: $R \circ S = \{(x, y) \in \mathbb{R}_+^2 : y \leq 1 - x^2\}$



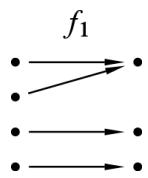
Figur 4.10: Isonutzenlinien für $a_1 + a_2 = c$



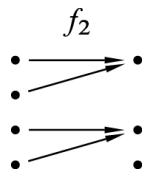
Figur 4.11: Isonutzenkurven für $a_1 a_2 = c$



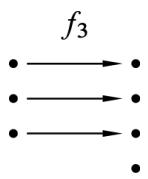
Figur 4.12: Hierarchische Anordnung von Relationen



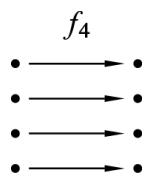
Figur 5.1: Abbildung f_1 ist surjektiv, nicht injektiv



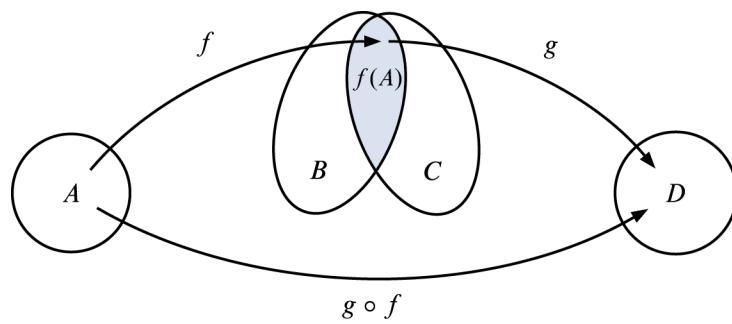
Figur 5.2: Abbildung f_2 ist weder surjektiv noch injektiv



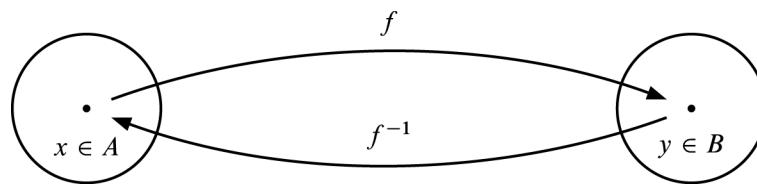
Figur 5.3: Abbildung f_3 ist injektiv, nicht surjektiv



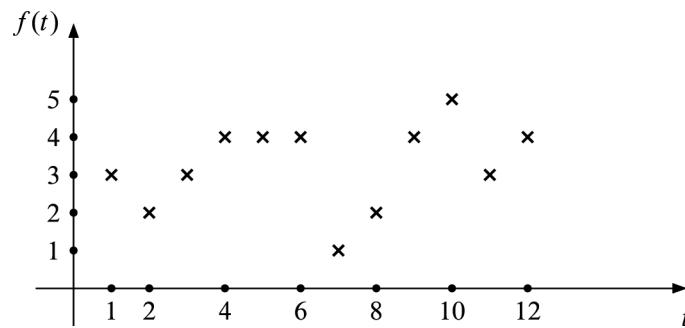
Figur 5.4: Abbildung f_4 ist bijektiv



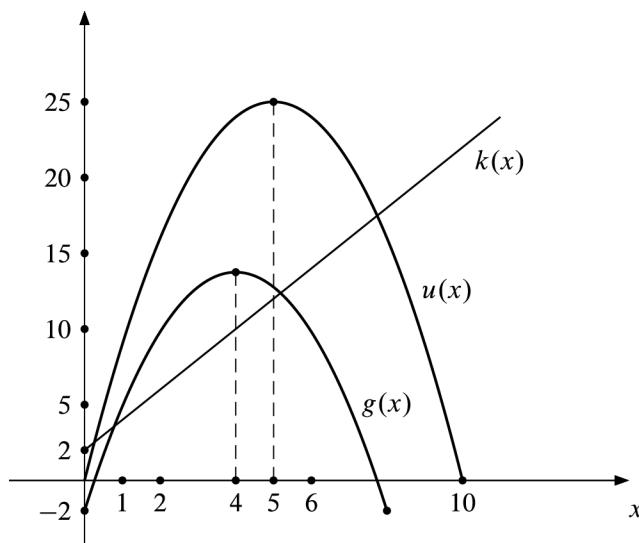
Figur 5.5: Komposition von f und g



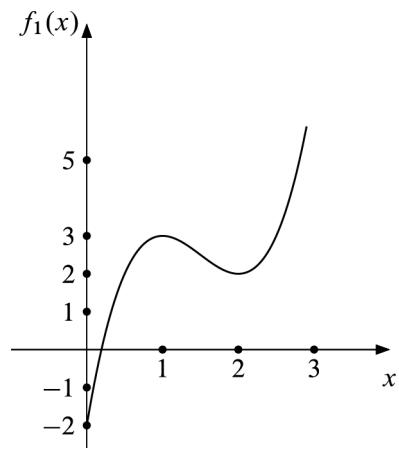
Figur 5.6: Umkehrabbildung f^{-1} von $f: A \rightarrow B$



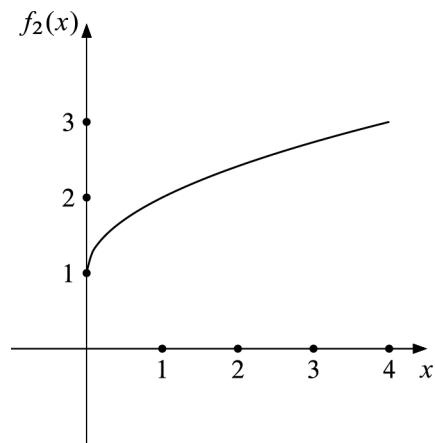
Figur 5.7: Graph der Funktion $f: \{1, \dots, 12\} \longrightarrow \{1, 2, 3, 4, 5\}$



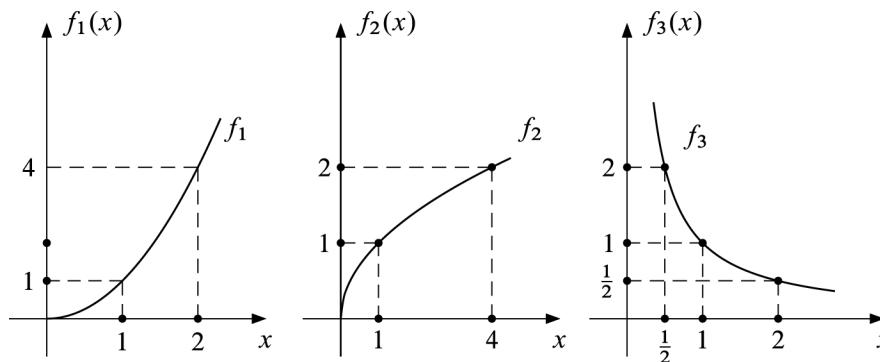
Figur 5.8: Graphen der Funktionen k , u , g mit $k(x) = 2 + 2x$, $u(x) = 10x - x^2$, $g(x) = -x^2 + 8x - 2$



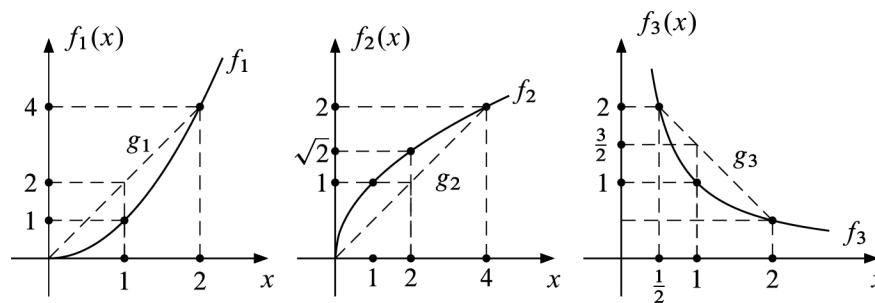
Figur 5.9: Graph der Funktion f_1



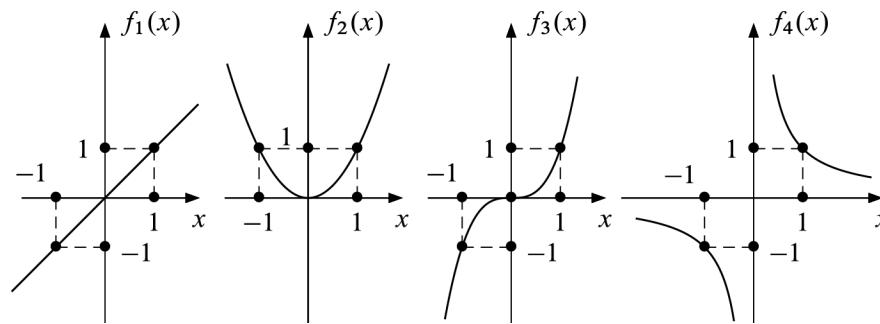
Figur 5.10: Graph der Funktion f_2



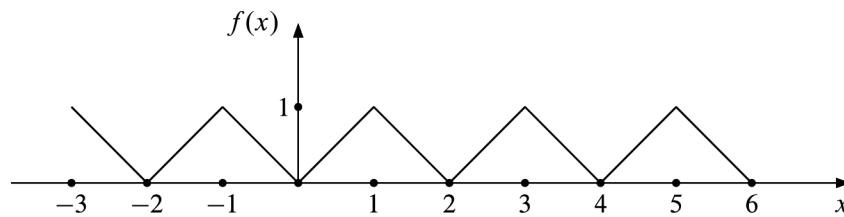
Figur 5.11: Graphen der Funktionen f_1 , f_2 , f_3 mit $f_1(x) = x^2$, $f_2(x) = \sqrt{x}$, $f_3(x) = \frac{1}{x}$



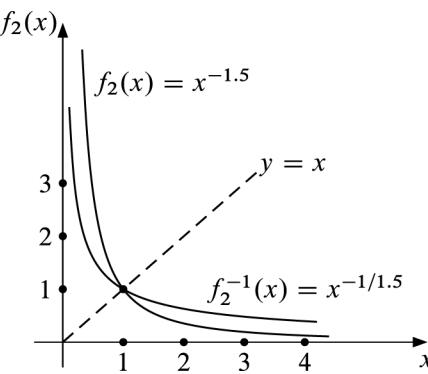
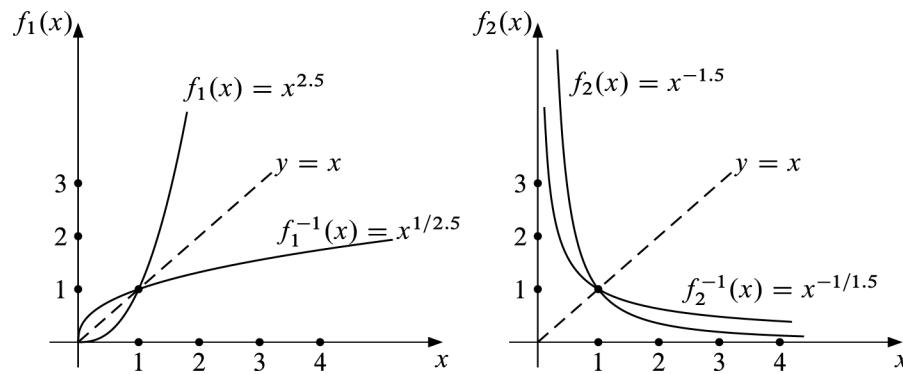
Figur 5.12: Graphen der Funktionen f_1 , f_2 , f_3 mit $f_1(x) = x^2$, $f_2(x) = \sqrt{x}$, $f_3(x) = \frac{1}{x}$



Figur 5.13: Graphen der Funktionen f_1, f_2, f_3, f_4 mit $f_1(x) = x, f_2(x) = x^2, f_3(x) = x^3, f_4(x) = \frac{1}{x}$

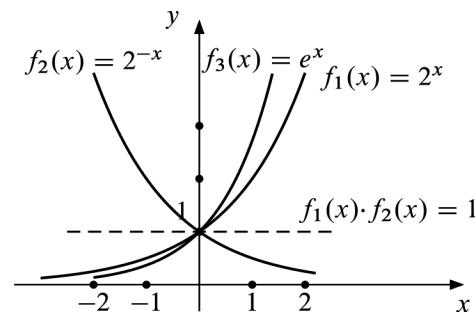


Figur 5.14: Graph einer periodischen Funktion mit $p = 2$

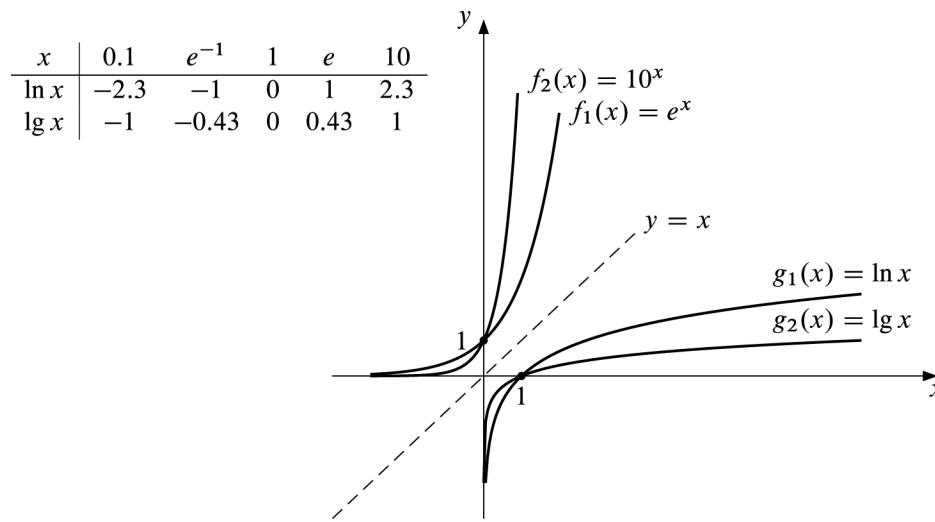


Figur 6.1: Graphen der Potenzfunktionen f_1 , f_2 , f_1^{-1} , f_2^{-1}

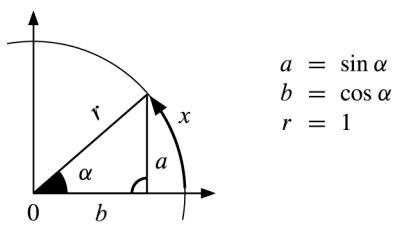
x	-2	-1	0	1	2
2^x	0.25	0.5	1	2	4
2^{-x}	4	2	1	0.5	0.25
e^x	0.14	0.37	1	2.72	7.39



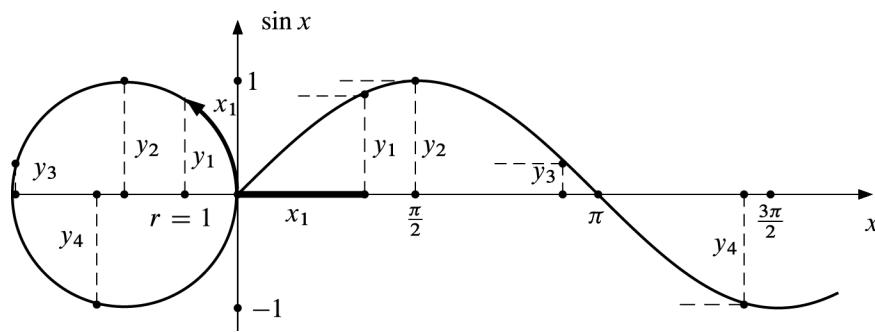
Figur 6.2: Graphen der Exponentialfunktionen f_1 , f_2 , f_3



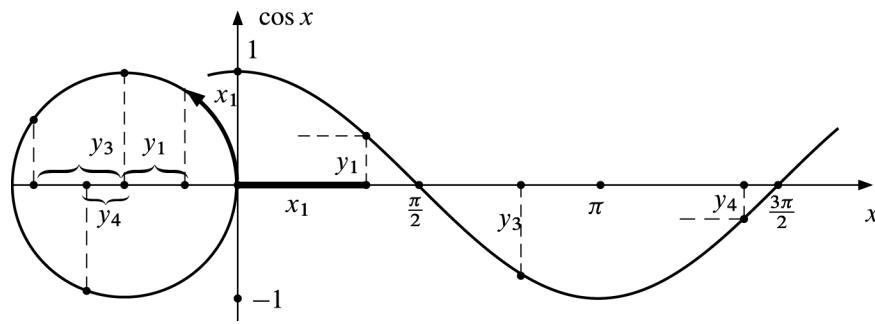
Figur 6.3: Graphen der Exponentialfunktionen f_1 , f_2 und ihrer Umkehrfunktionen g_1 , g_2



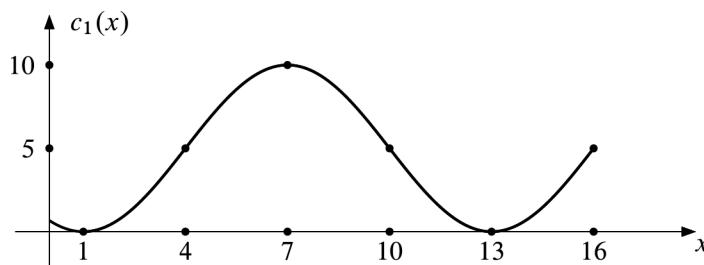
Figur 6.4: Sinus- und Kosinuswerte im Einheitskreis



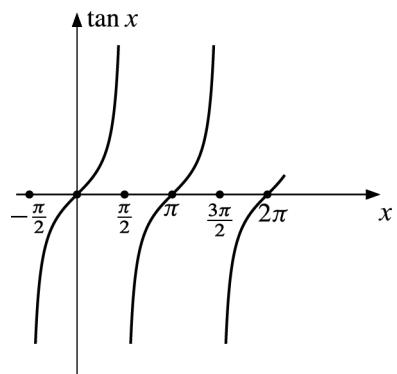
Figur 6.5: Graph der Sinusfunktion mit $y = \sin x$



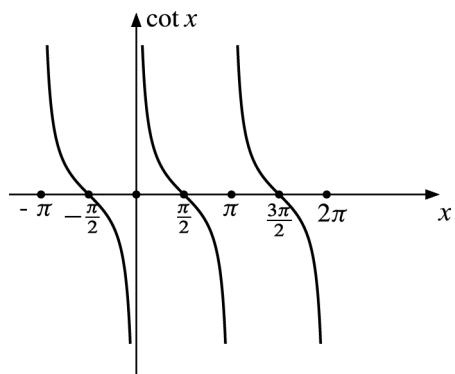
Figur 6.6: Graph der Kosinusfunktion mit $y = \cos x$



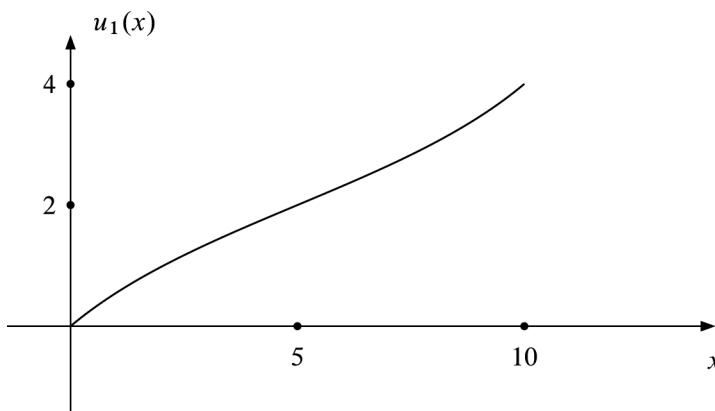
Figur 6.7: Graph der Funktion c_1



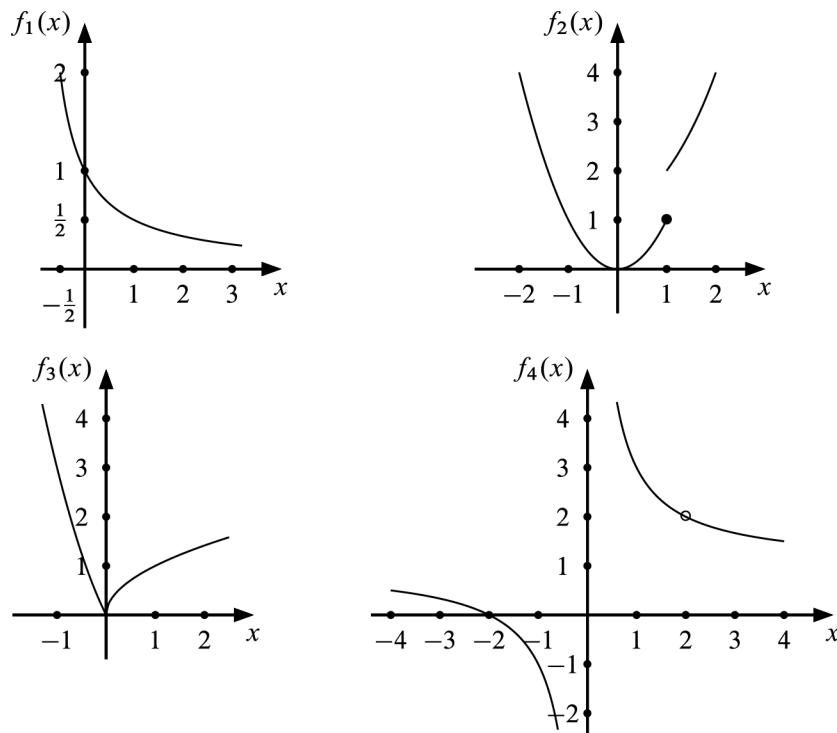
Figur 6.8: Graph der Tangensfunktion



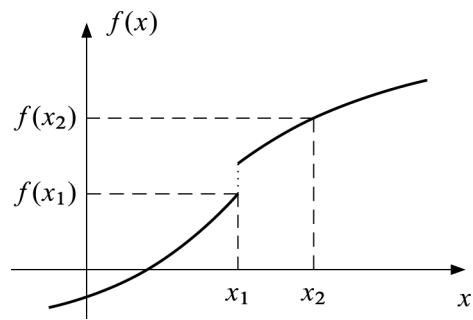
Figur 6.9: Graph der Kotangensfunktion



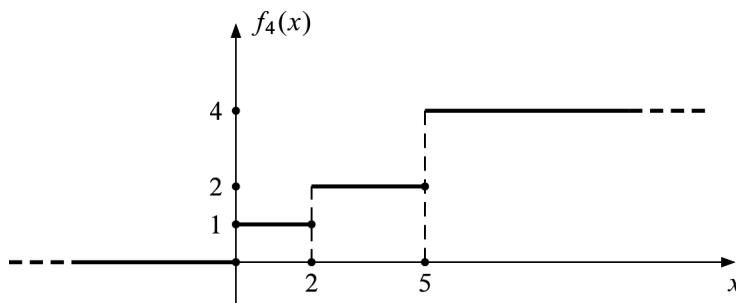
Figur 6.10: Graph der Funktion u_1



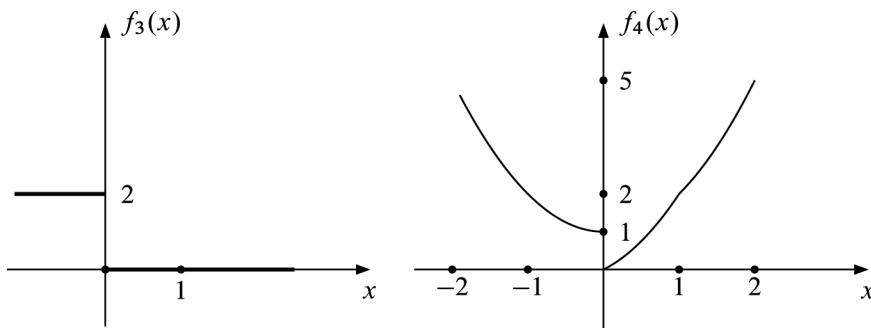
Figur 7.1: Graphen der Funktionen f_1 , f_2 , f_3 , f_4



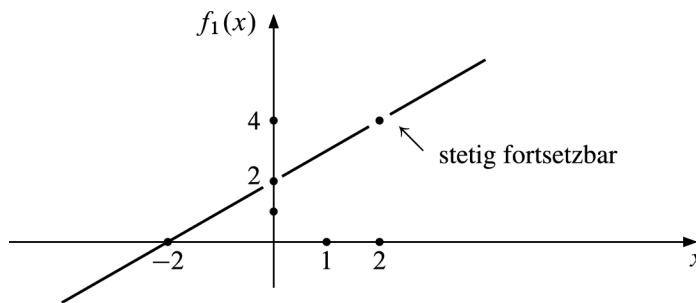
Figur 7.2: Stetigkeitsbegriff reeller Funktionen



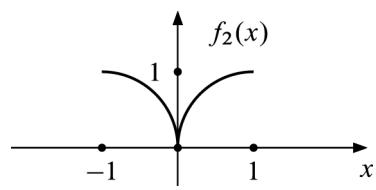
Figur 7.3: Graph der Treppenfunktion f_4



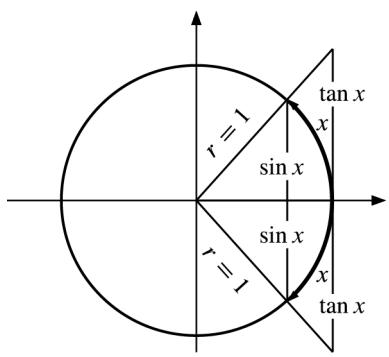
Figur 7.4: Graphen der Funktionen f_3 , f_4



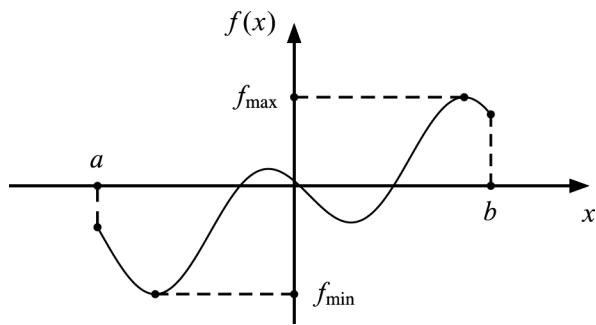
Figur 7.5: Graph der Funktion f_1



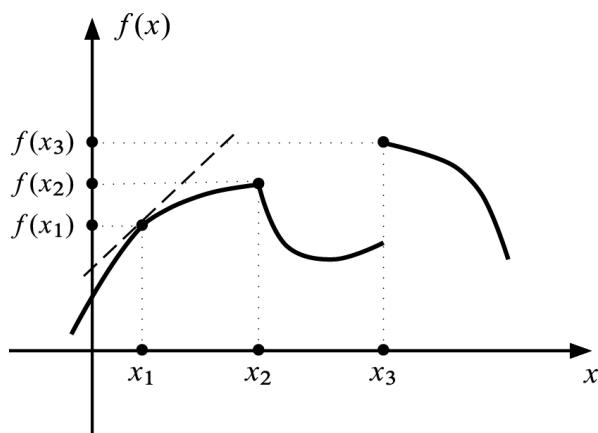
Figur 7.6: Graph der Funktion f_2



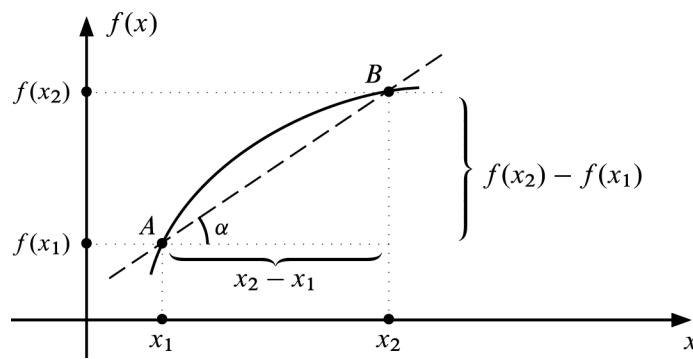
Figur 7.7: Sinus- und Tangensfunktion am Einheitskreis mit $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



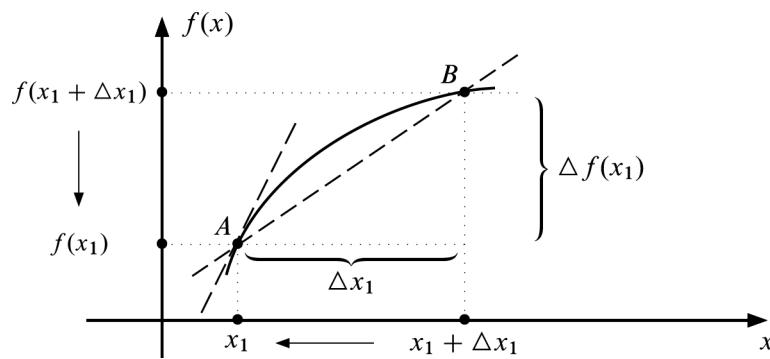
Figur 7.8: $f: [a, b] \rightarrow \mathbb{R}$ mit f stetig



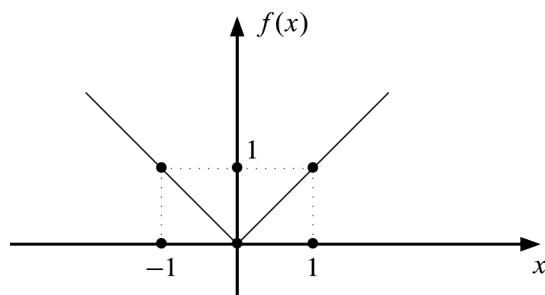
Figur 8.1: Differentiation reeller Funktionen



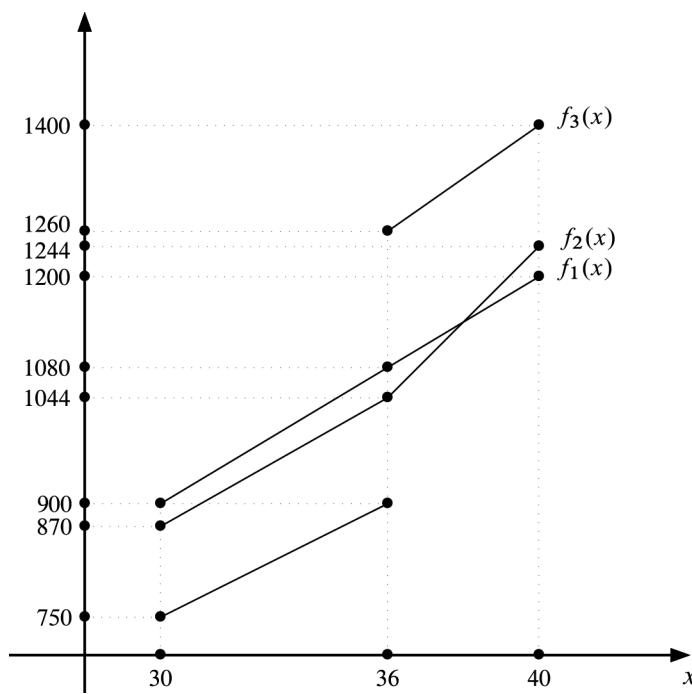
Figur 8.2: Differenzenquotient einer reellen Funktion



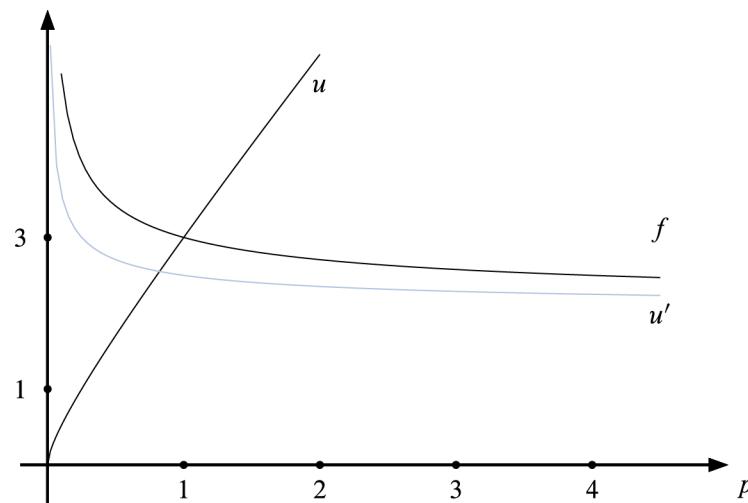
Figur 8.3: Differentialquotient einer reellen Funktion



Figur 8.4: Graph der Funktion f mit $f(x) = |x|$

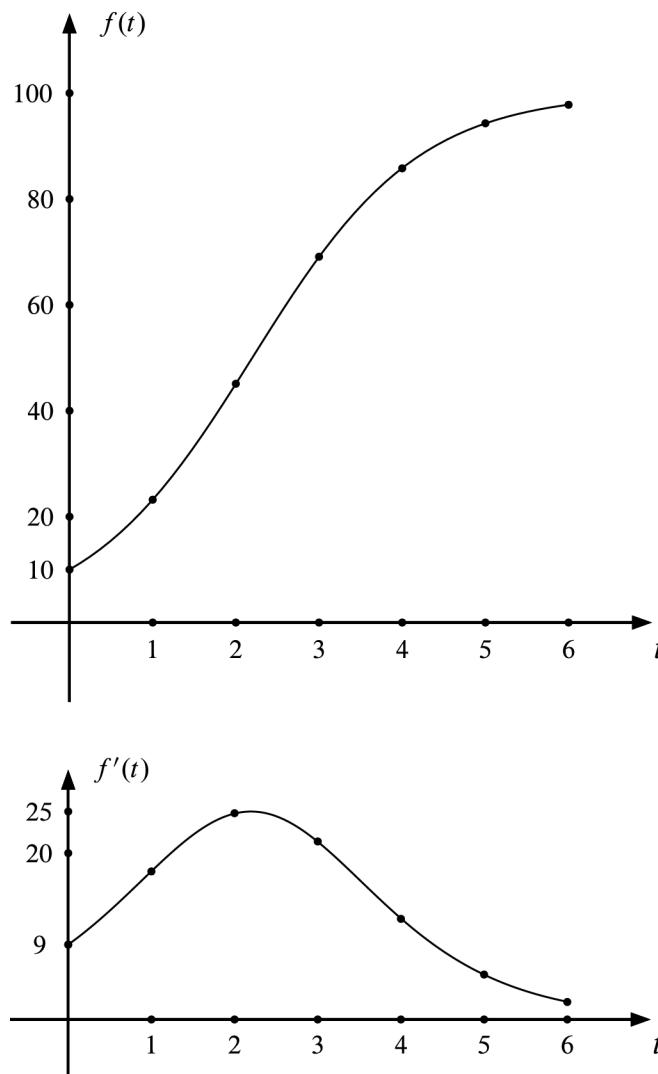


Figur 8.5: Graphen der Lohnfunktionen f_1 , f_2 , f_3

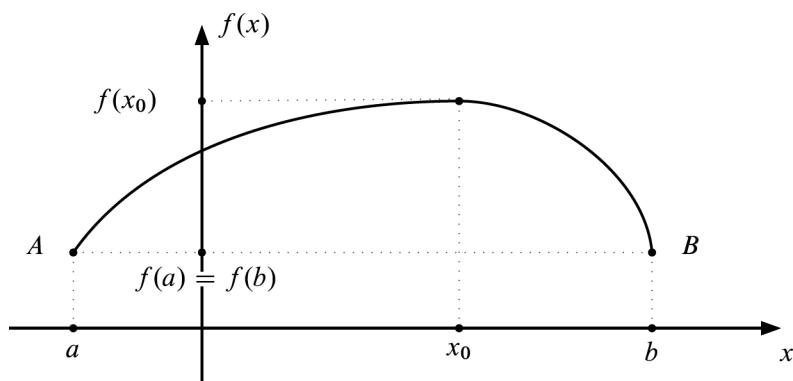


Figur 8.6: Graphen der Funktionen f , u und u' mit

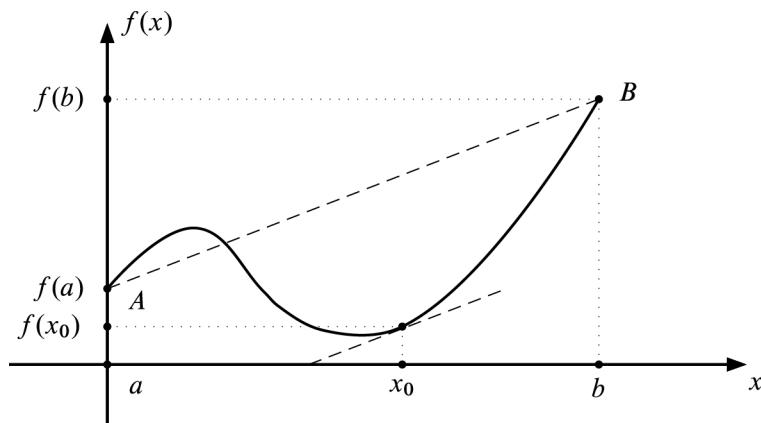
$$f(p) = 2 + \frac{1}{\sqrt{p}}, \quad u(p) = 2p + \sqrt{p}, \quad u'(p) = 2 + \frac{1}{2\sqrt{p}}$$



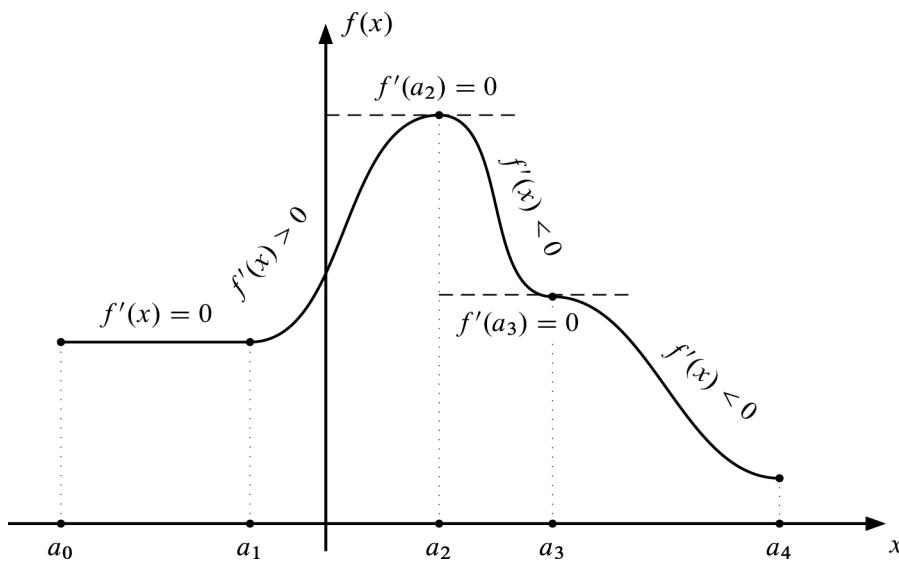
Figur 8.7: Graphen der logistischen Funktion f und ihrer Ableitung f'



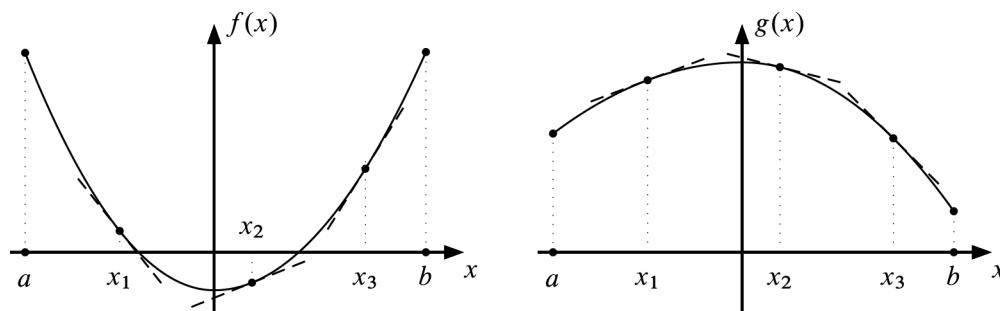
Figur 9.1: Satz von Rolle



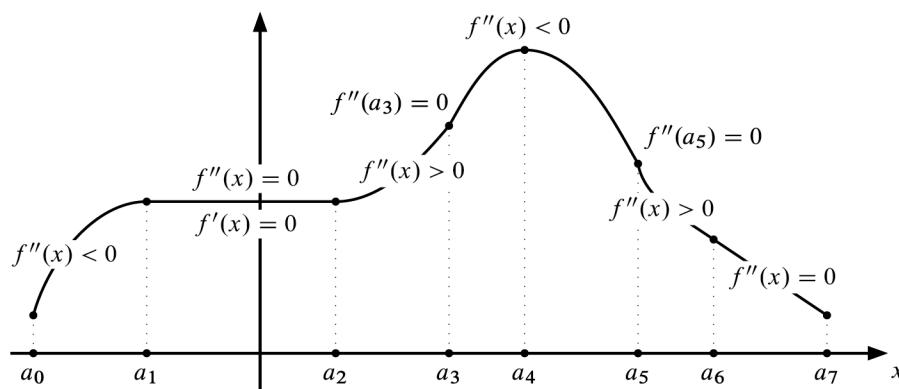
Figur 9.2: Mittelwertsatz der Differentialrechnung



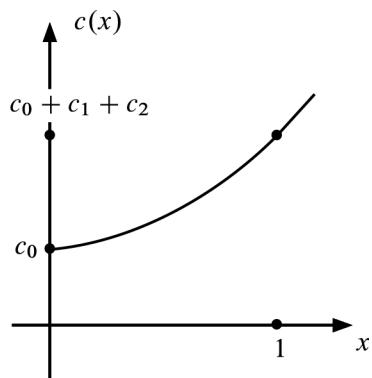
Figur 9.3: Monotonie einer differenzierbaren Funktion



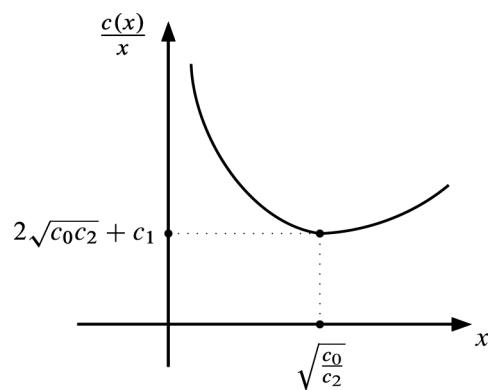
Figur 9.4: f als konvexe Funktion, g als konkave Funktion



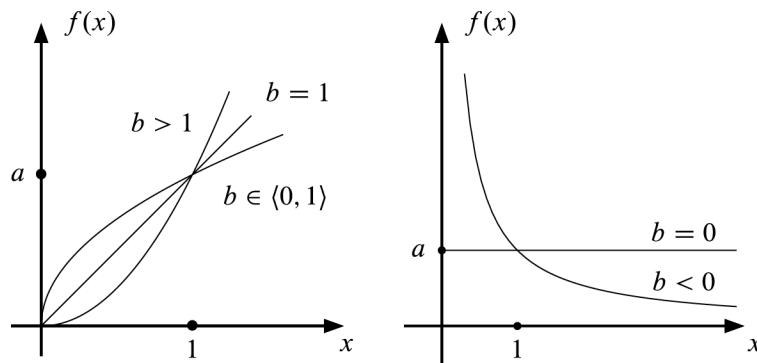
Figur 9.5: Konvexität und Konkavität einer differenzierbaren Funktion



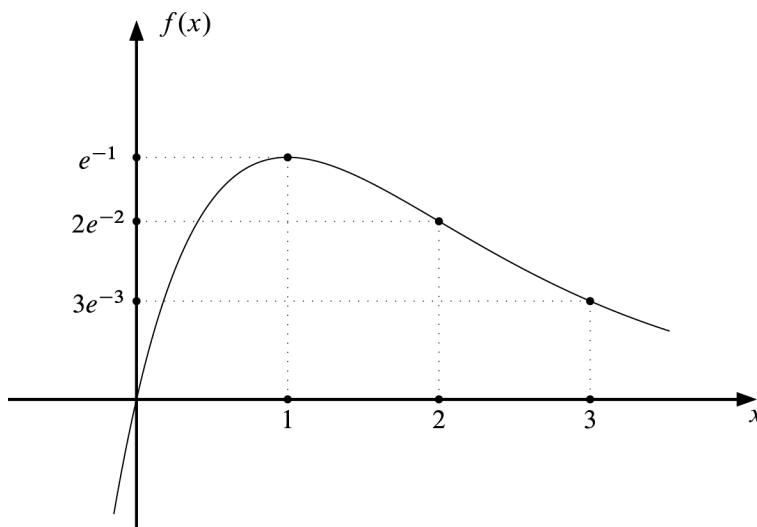
Figur 9.6: Quadratische Kostenfunktion



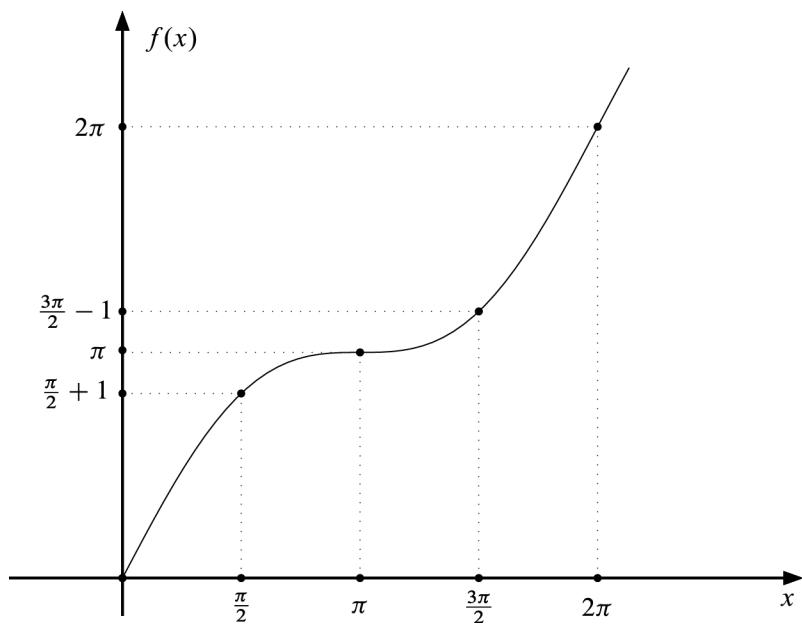
Figur 9.7: Stückkostenfunktion einer quadratischen Kostenfunktion



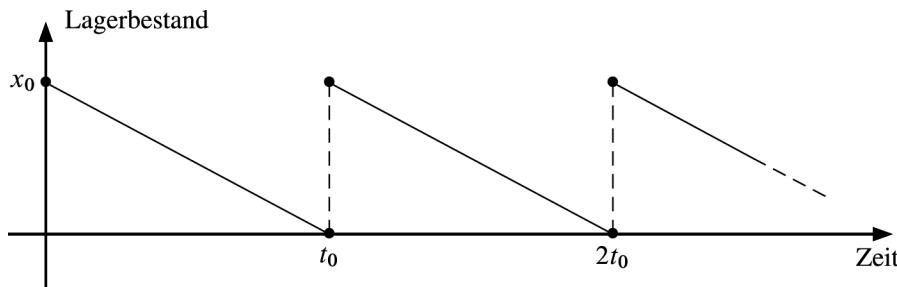
Figur 9.8: Potenzfunktion mit $f(x) = ax^b$



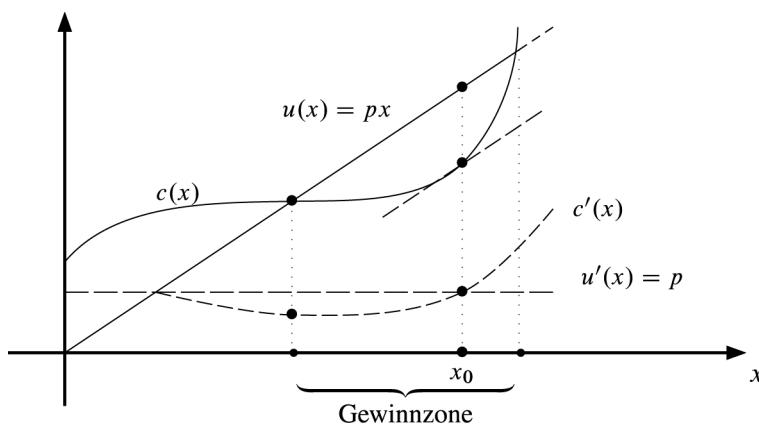
Figur 9.9: Graph der Funktion f mit $f(x) = xe^{-x}$



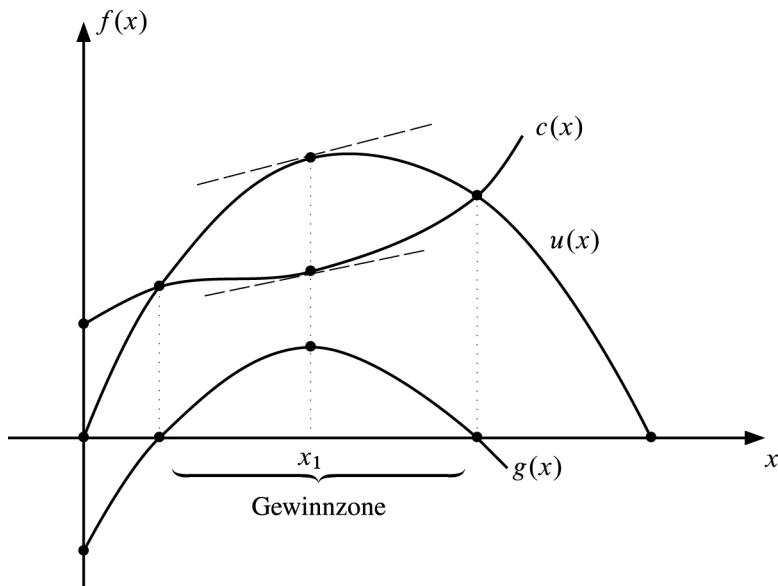
Figur 9.10: Graph der Funktion f mit $f(x) = x + \sin x$



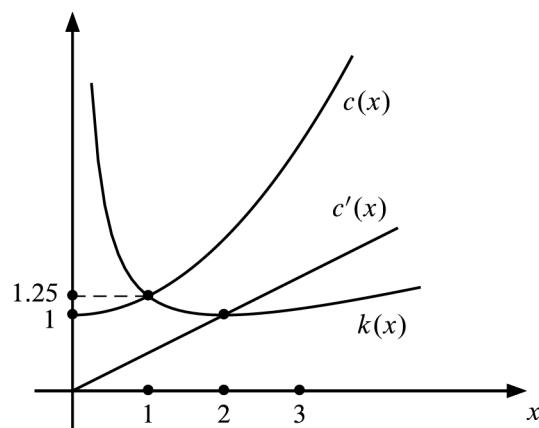
Figur 9.11: Lagerbestand bei konstanter Nachfrage pro Zeiteinheit und Bestellung in gleichen Zeitabständen $t_0, 2t_0, \dots$



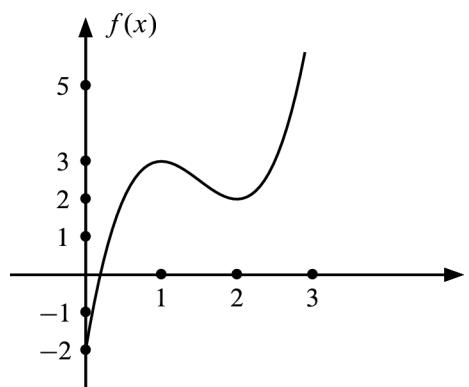
Figur 9.12: Gewinnmaximierung bei vollständiger Konkurrenz



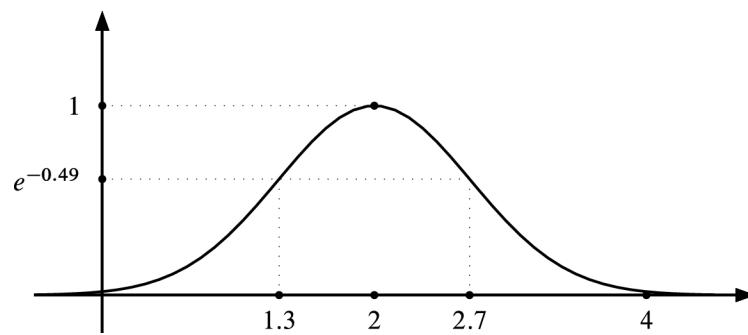
Figur 9.13: Gewinnmaximierung beim Angebotsmonopol



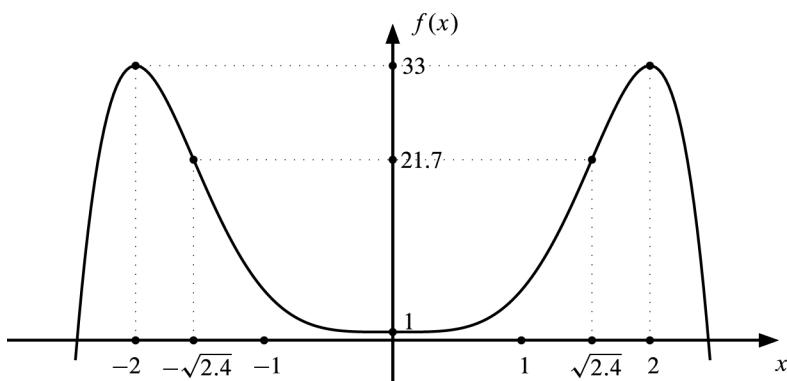
Figur 9.14: Zusammenhang von Kosten, Stückkosten und Grenzkosten



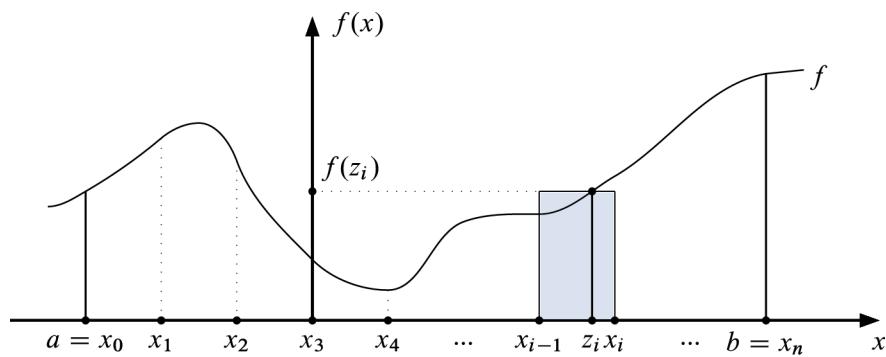
Figur 9.15: Graph der Funktion f



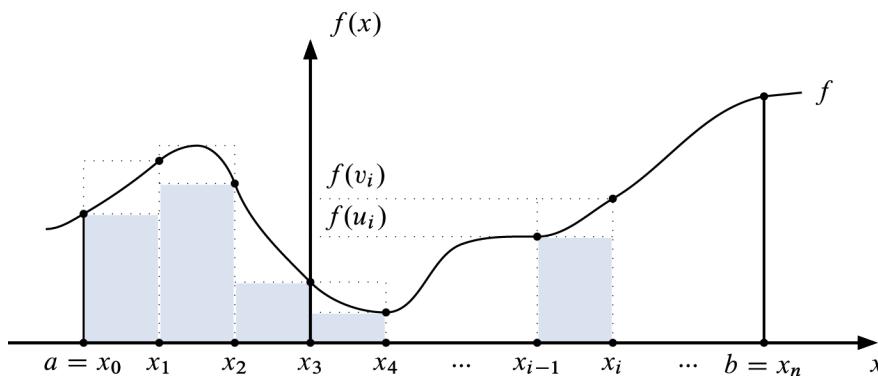
Figur 9.16: Graph der Funktion mit $f(x) = e^{-(x-2)^2}$



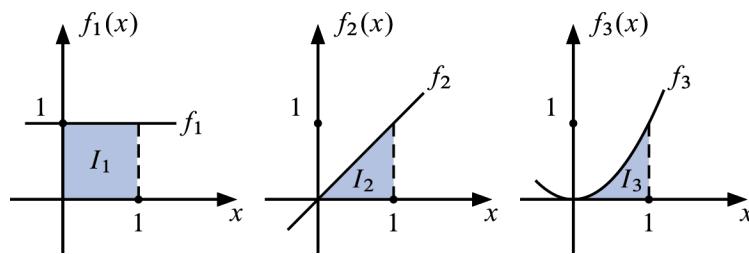
Figur 9.17: Graph der Funktion f mit $f(x) = -x^6 + 6x^4 + 1$)



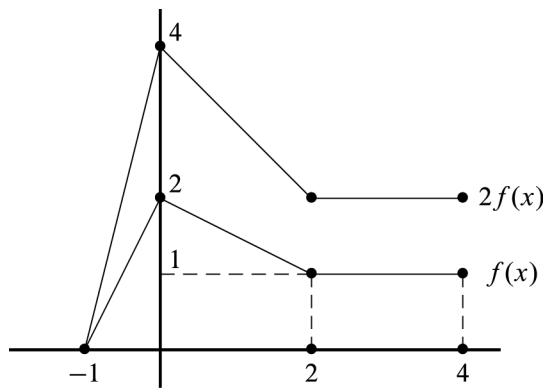
Figur 10.1: Flächeninhalt und Riemannsche Summe



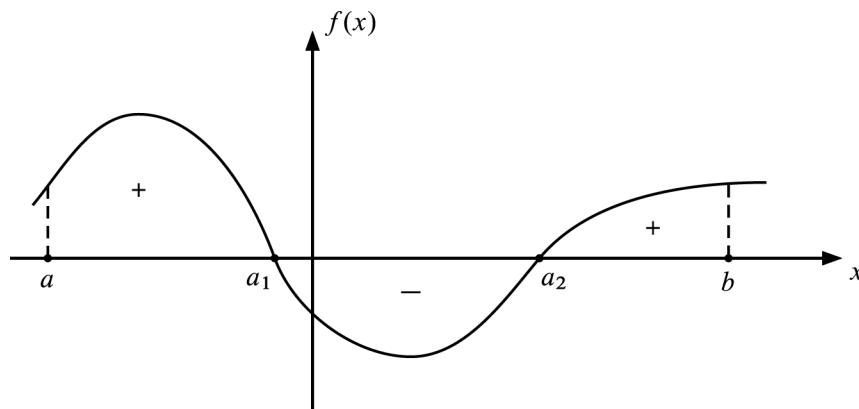
Figur 10.2: Unter- und Oberschranken des Flächeninhalts



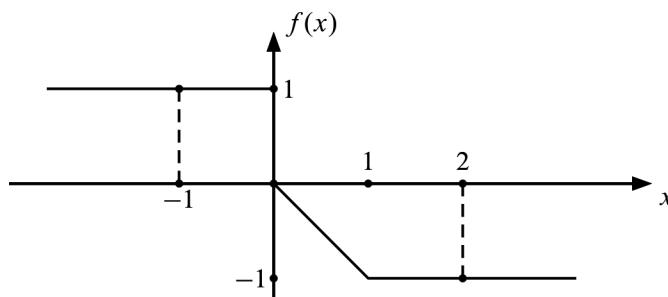
Figur 10.3: Bestimmte Integrale der Funktionen f_1, f_2, f_3 in $[0,1]$



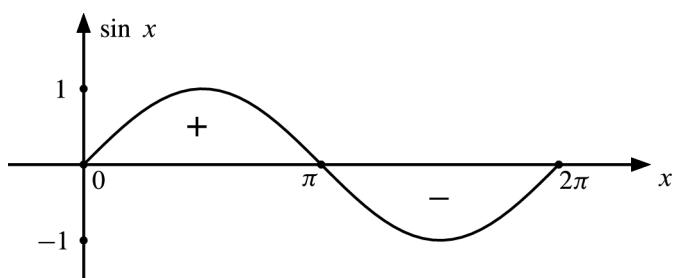
Figur 10.4: Kurvenverläufe $f(x)$, $g(x) = 2f(x)$ mit $x \in [a, b]$



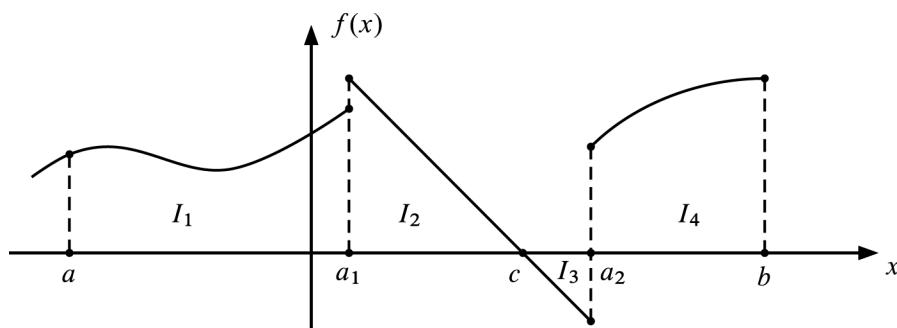
Figur 10.5: Funktion f mit $\int_{a_1}^{a_2} f(x) dx < 0$



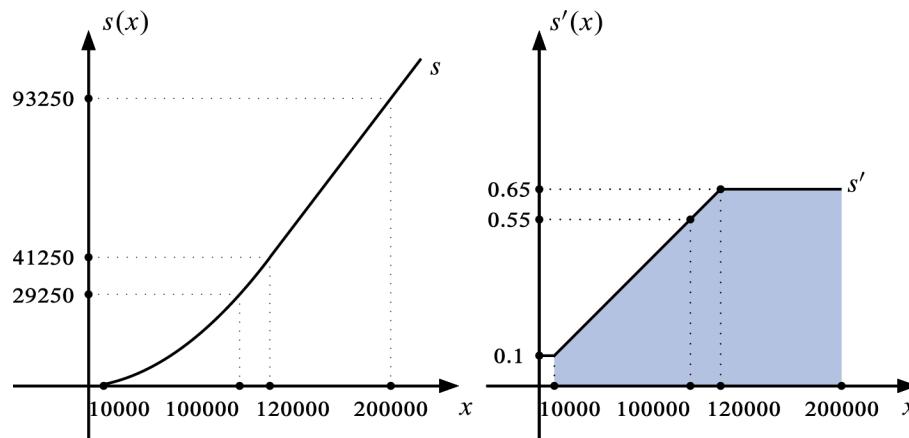
Figur 10.6: Graph der Funktion f



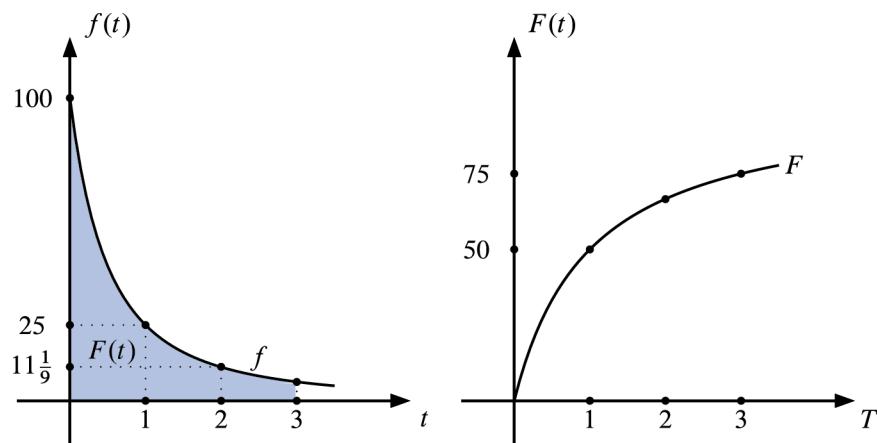
Figur 10.7: Sinusfunktion



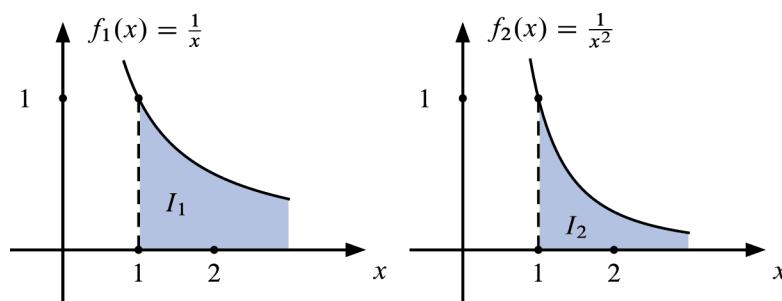
Figur 10.8: Funktion mit Sprungstellen a_1, a_2



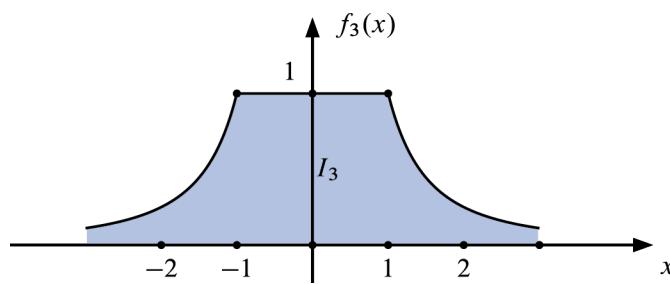
Figur 10.9: Steuersatz- und Grenzsteuersatzfunktion zu Beispiel 10.22 b



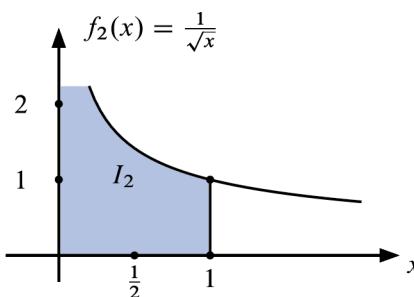
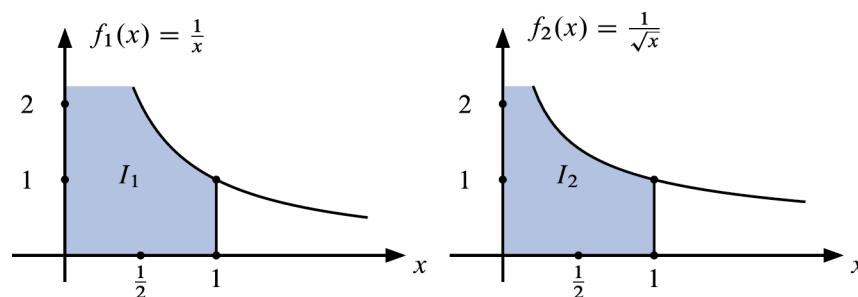
Figur 10.10: Graph von f mit $f(t) = 100(1 + t)^{-2}$ und zugehörige Stammfunktion



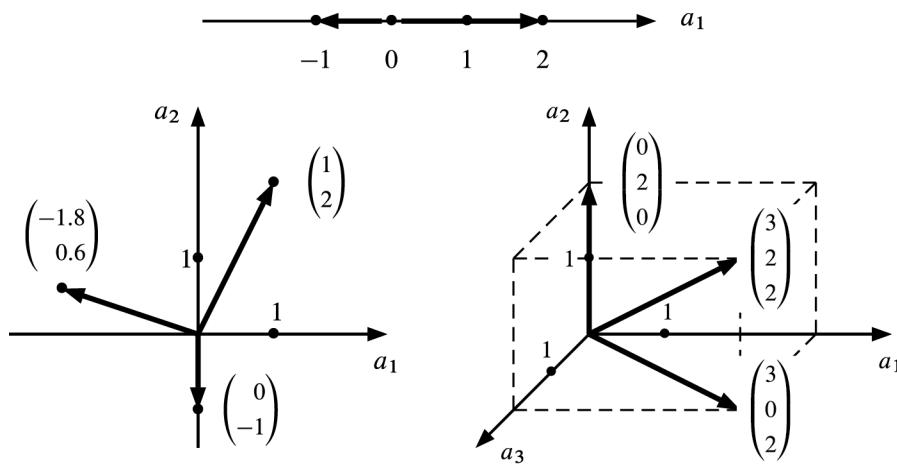
Figur 10.11: Uneigentliche Integrale im Intervall $[1, \infty]$



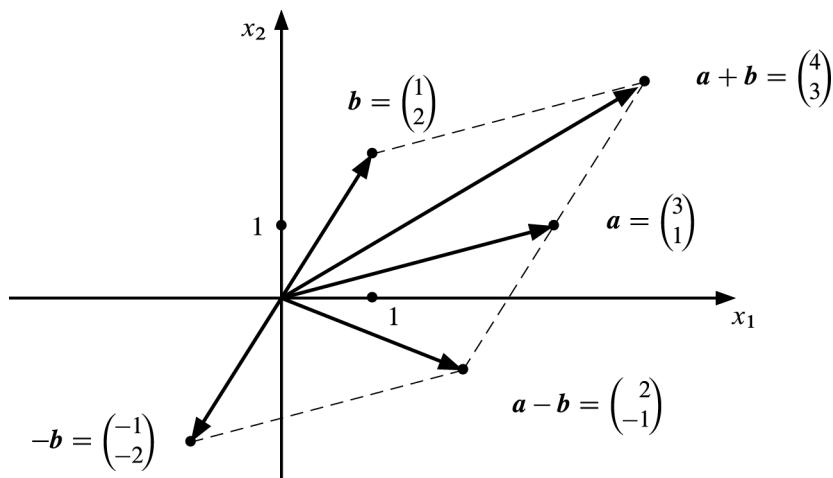
Figur 10.12: Uneigentliches Integral im Intervall $\langle -\infty, \infty \rangle$



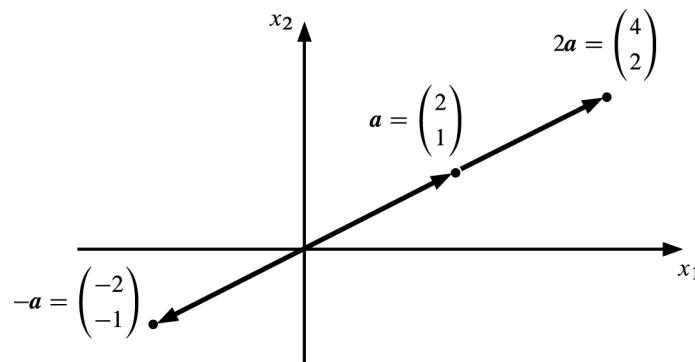
Figur 10.13: Uneigentliche Integrale im Intervall $[0, 1]$



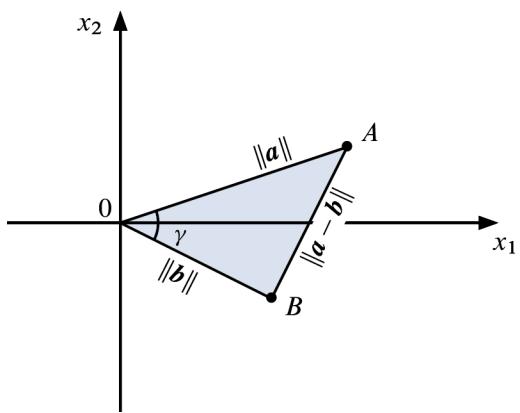
Figur 11.1: Vektoren im \mathbb{R}^1 , \mathbb{R}^2 und \mathbb{R}^3



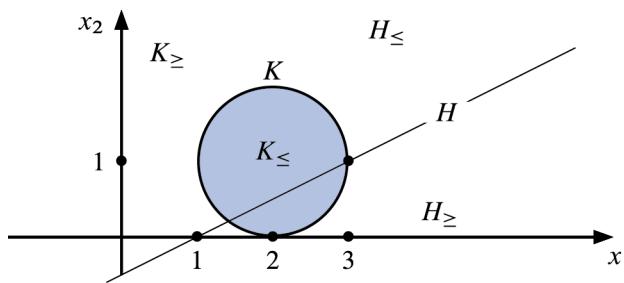
Figur 11.2: Addition und Subtraktion von Vektoren des \mathbb{R}^2



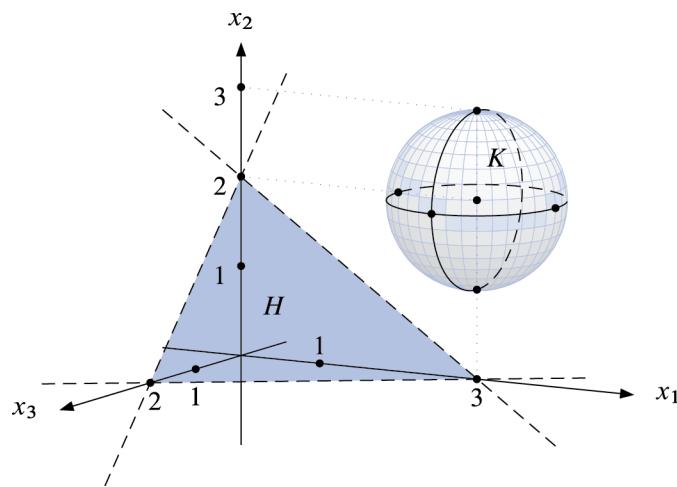
Figur 11.3: Skalare Multiplikation von Vektoren des \mathbb{R}^2



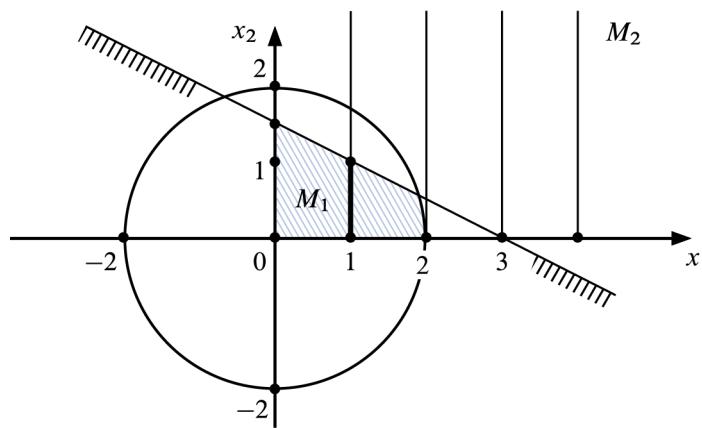
Figur 12.1: Kosinussatz im Dreieck OAB



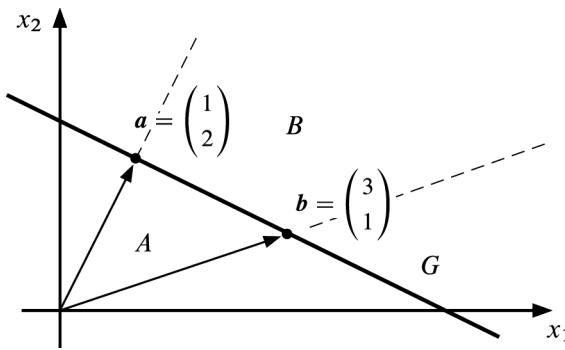
Figur 12.2: Geometrische Darstellung der Punktmenzen
 $H, H_{\leq}, H_{\geq}, K, K_{\leq}, K_{\geq}$ im \mathbb{R}^2



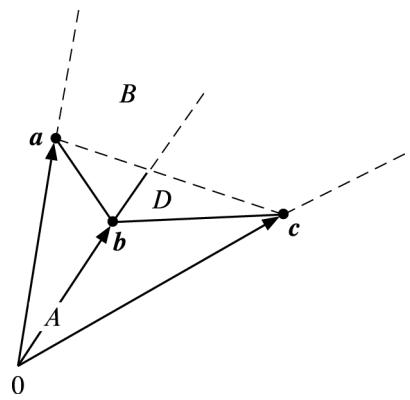
Figur 12.3: Geometrische Darstellung der Punktmengen H und K im \mathbb{R}^3



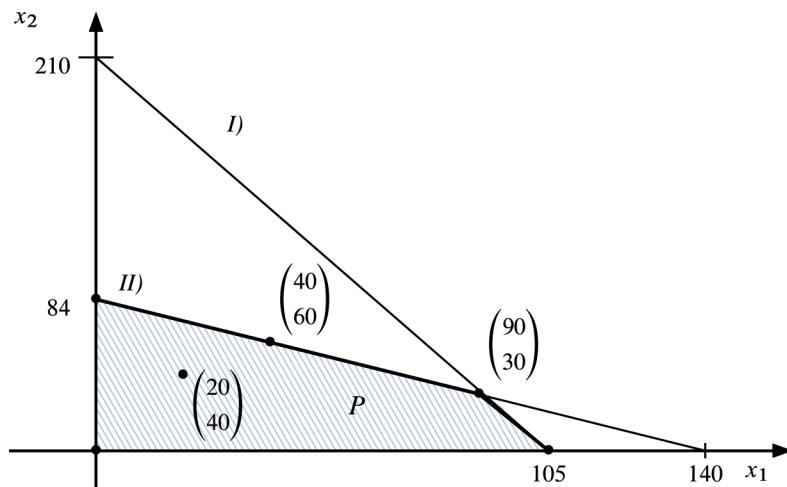
Figur 12.4: Geometrische Darstellung der Punktmengen $M_1, M_2 \subset \mathbb{R}^2$



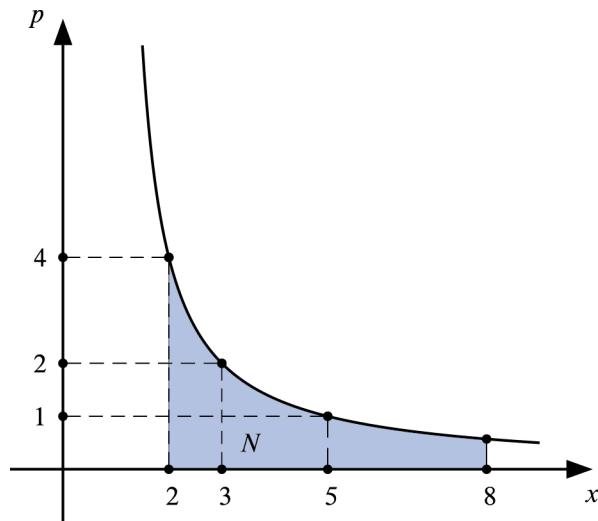
Figur 12.5: Linearkombinationen $r_1\mathbf{a} + r_2\mathbf{b}$ im \mathbb{R}^2



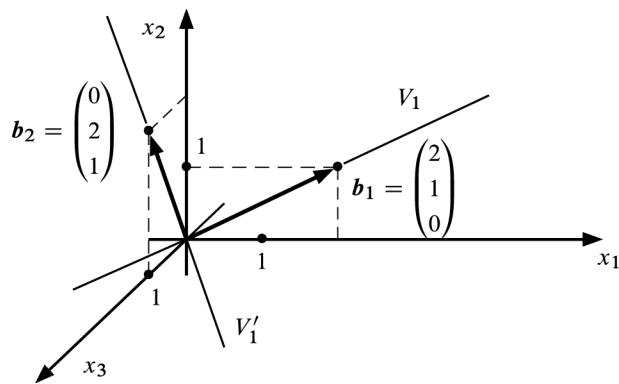
Figur 12.6: Linearkombinationen $r_1\mathbf{a} + r_2\mathbf{b} + r_3\mathbf{c}$ im \mathbb{R}^3



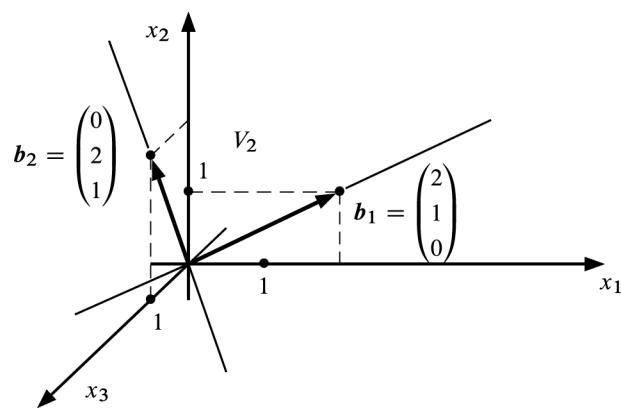
Figur 12.7: Graphische Darstellung von P



Figur 12.8: Graphische Darstellung von N



Figur 13.1: Vektorräume V_1, V_1' der Dimension 1



Figur 13.2: Vektorraum V_2 der Dimension 2

Zeile	Basis	c_k	a	Operation
(1)	b_1	r_1	p_1	
(2)	b_2	r_2	p_2	
\vdots	\vdots	\vdots	\vdots	
(k)	b_k	r_k	p_k	
\vdots	\vdots	\vdots	\vdots	
(n)	b_n	r_n	p_n	
(n+1)	b_1	0	$q_1 = p_1 - \frac{r_1 p_k}{r_k}$	(1) — $\frac{r_1}{r_k} \cdot (k)$
(n+2)	b_2	0	$q_2 = p_2 - \frac{r_2 p_k}{r_k}$	(2) — $\frac{r_2}{r_k} \cdot (k)$
\vdots	\vdots	\vdots	\vdots	\vdots
(n+k)	c_k	1	$q_k = \frac{p_k}{r_k}$	(k) · $\frac{1}{r_k}$
\vdots	\vdots	\vdots	\vdots	\vdots
(2n)	b_n	0	$q_n = p_n - \frac{r_n p_k}{r_k}$	(n) — $\frac{r_n}{r_k} \cdot (k)$

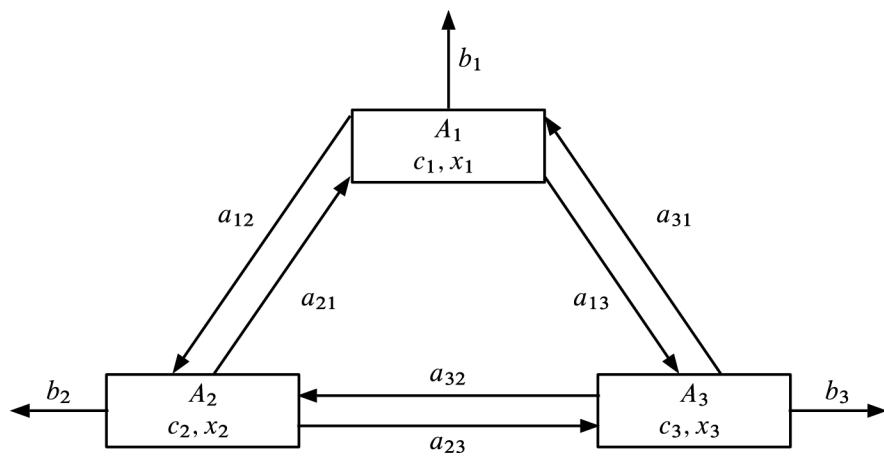
Figur 13.3: Austausch eines Basisvektors

Zeile	a^1	a^2	\dots	a^n	Operation
(1)	a_{11}	a_{12}		\dots	a_{1n}
(2)	a_{21}	a_{22}		\dots	a_{2n}
\vdots	\vdots	\vdots			\vdots
(m)	a_{m1}	a_{m2}		\dots	a_{mn}
(m+1)	1	$\frac{a_{12}}{a_{11}}$		\dots	$\frac{a_{1n}}{a_{11}}$
(m+2)	0	$a_{22} - \frac{a_{21}}{a_{11}}a_{12}$		\dots	$a_{2n} - \frac{a_{21}}{a_{11}}a_{1n}$
\vdots	\vdots	\vdots			\vdots
(2m)	0	$a_{m2} - \frac{a_{m1}}{a_{11}}a_{12}$		\dots	$a_{mn} - \frac{a_{m1}}{a_{11}}a_{1n}$

Figur 13.4: Erster Schritt des Gaußalgorithmus

Zeile	b^1	b^2	b^3	\dots	b^n	Operation
(m+1)	1	b_{12}	b_{13}	\dots	b_{1n}	
(m+2)	0	b_{22}	b_{23}	\dots	b_{2n}	
\vdots	\vdots	\vdots	\vdots		\vdots	
(2m)	0	b_{m2}	b_{m3}	\dots	b_{mn}	
(2m+1)	1	0	$b_{13} - \frac{b_{12}}{b_{22}} b_{23}$	\dots	$b_{1n} - \frac{b_{12}}{b_{22}} b_{2n}$	$(m+1) - \frac{b_{12}}{b_{22}} \cdot (m+2)$
(2m+2)	0	1	$\frac{b_{23}}{b_{22}}$	\dots	$\frac{b_{2n}}{b_{22}}$	$(m+2) / b_{22}$
(2m+3)	0	0	$b_{33} - \frac{b_{32}}{b_{22}} b_{23}$	\dots	$b_{3n} - \frac{b_{32}}{b_{22}} b_{2n}$	$(m+3) - \frac{b_{32}}{b_{22}} \cdot (m+2)$
\vdots	\vdots	\vdots	\vdots		\vdots	
(3m)	0	0	$b_{m3} - \frac{b_{m2}}{b_{22}} b_{23}$	\dots	$b_{mn} - \frac{b_{m2}}{b_{22}} b_{2n}$	$(2m) - \frac{b_{m2}}{b_{22}} \cdot (m+2)$

Figur 13.5: Zweiter Schritt des Gaußalgorithmus

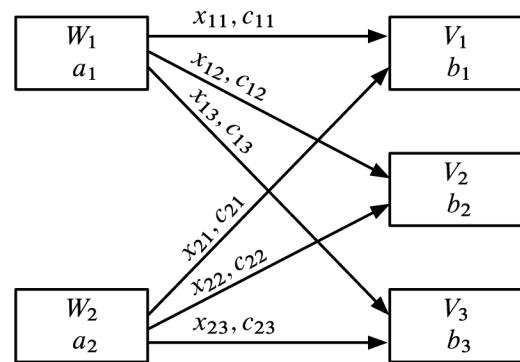


Figur 14.1: Innerbetriebliche Leistungsverflechtung der Abteilungen A_1, A_2, A_3

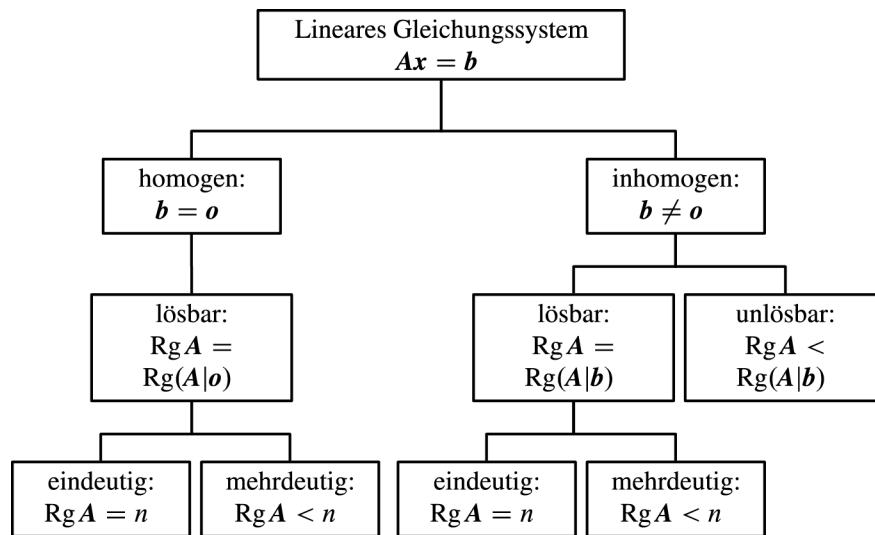
Abteilung	Sekundärkosten für abgegebene Leistungen	Primärkosten + Sekundärkosten für empfangene Leistungen
A_1	$x_1(b_1 + a_{12} + a_{13})$	$c_1 + x_2a_{21} + x_3a_{31}$
A_2	$x_2(b_2 + a_{21} + a_{23})$	$c_2 + x_1a_{12} + x_3a_{32}$
A_3	$x_3(b_3 + a_{31} + a_{32})$	$c_3 + x_1a_{13} + x_2a_{23}$

Figur 14.2: Zur Verrechnung innerbetrieblicher Leistungen

Lieferung	an			Vorrat
	V_1	V_2	V_3	
von	(x_{11}, c_{11})	(x_{12}, c_{12})	(x_{13}, c_{13})	a_1
W_1	(x_{21}, c_{21})	(x_{22}, c_{22})	(x_{23}, c_{23})	a_2
Bedarf	b_1	b_2	b_3	

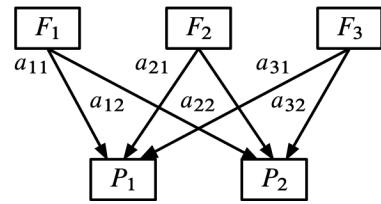


Figur 14.3: Lieferungen von den Warenlagern W_1 , W_2 an die Verkaufsstellen V_1 , V_2 , V_3

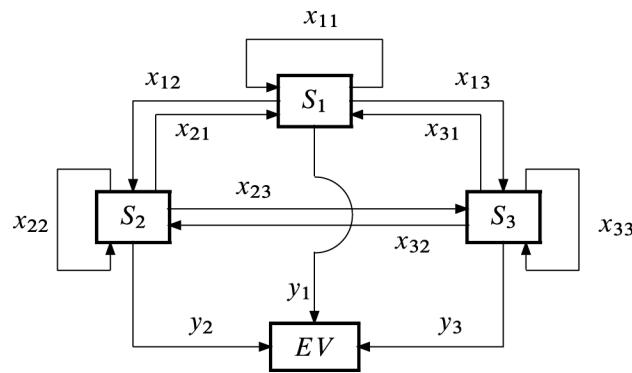


Figur 14.4: Zur Lösbarkeit linearer Gleichungssysteme

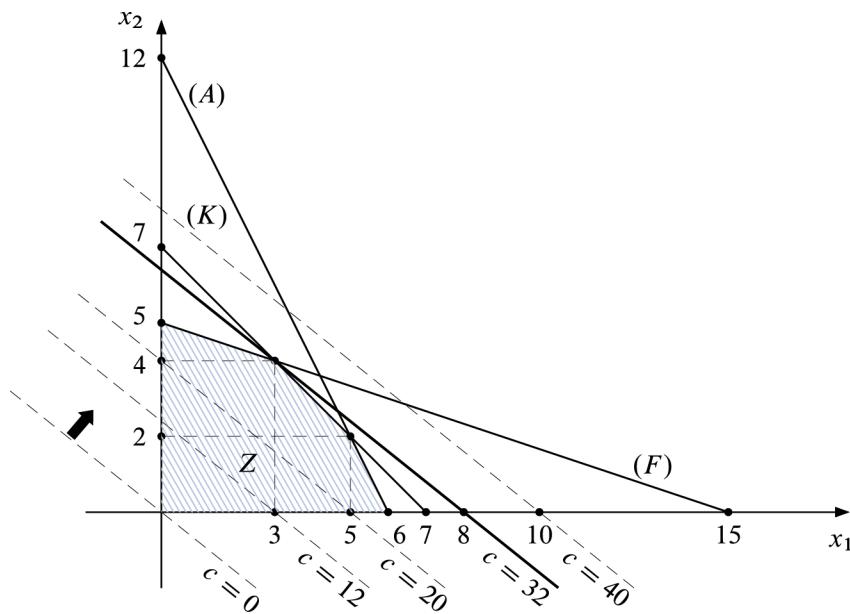
Verbrauch	für eine Einheit des Produkts	
	P_1	P_2
von Einheiten	F_1	a_{11}
der Produktions-	F_2	a_{21}
faktoren	F_3	a_{31}



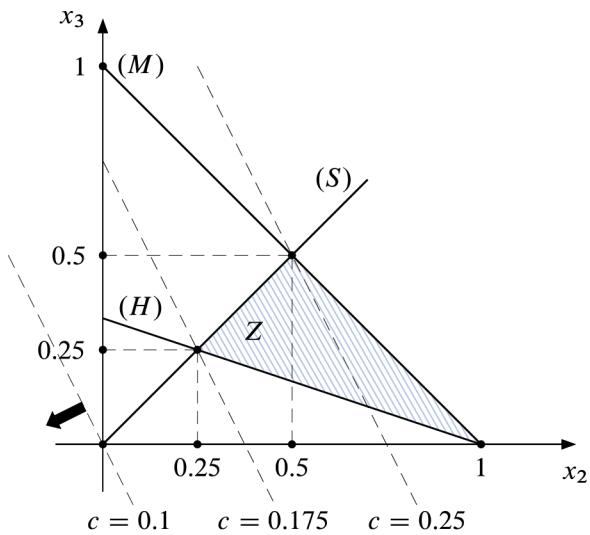
Figur 15.1: Herstellung von Produkten P_1 , P_2 mit Hilfe der Produktionsfaktoren F_1 , F_2 , F_3



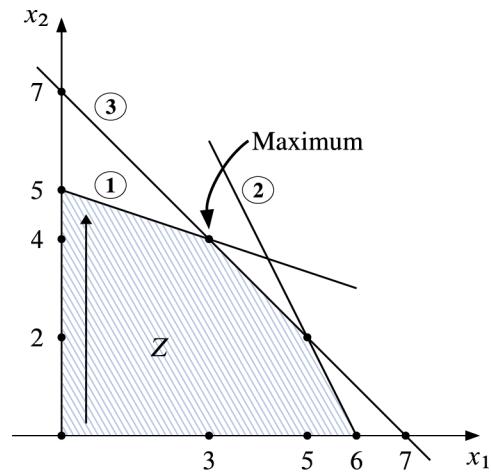
Figur 15.2: Lieferströme zwischen produzierenden Sektoren S_1, S_2, S_3 und Endverbrauch EV



Figur 16.1: Graphische Darstellung des Zulässigkeitsbereichs Z und der Deckungsbeiträge $g(x) = c$ mit $c = 0, 12, 20, 32, 40$



Figur 16.2: Graphische Darstellung des Mischungsproblems
mit $\hat{k}(x_2, x_3) = c$ für $c = 0.1, 0.175, 0.25$



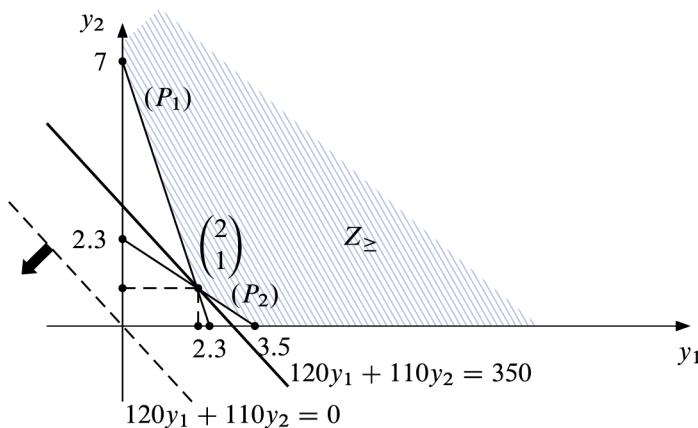
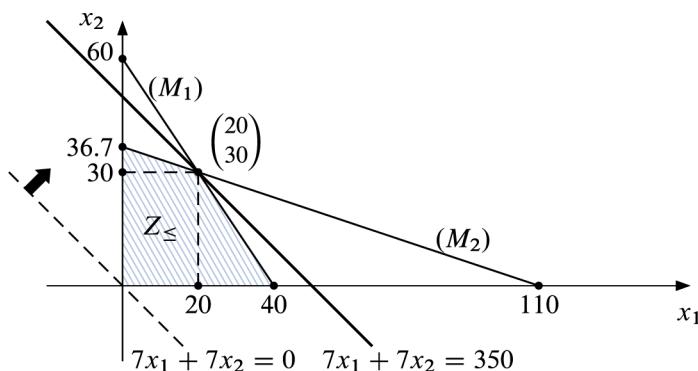
Figur 16.3: Basistransformation mit dem Simplexalgorithmus

Zeile	Basis	$a^1 \dots a^j \dots a^n$	$e_1 \dots e_i \dots e_m$	b
(1)	e_1	$a_{11} \dots a_{1j} \dots a_{1n}$	1 ... 0 ... 0	b_1
:	:	:	:	:
(i)	e_i	$a_{i1} \dots a_{ij} \dots a_{in}$	0 ... 1 ... 0	b_i
:	:	:	:	:
(m)	e_m	$a_{m1} \dots a_{mj} \dots a_{mn}$	0 ... 0 ... 1	b_m
(m')		$-c_1 \dots -c_j \dots -c_n$	0 ... 0 ... 0	0

Figur 16.4: Starttableau zum Standardmaximumproblem mit dem Beschränkungsvektor $b \geq \mathbf{0}$

Zeile	Basis	a^1	\dots	a^j	\dots	a^n	e_1	\dots	e_i	\dots	e_m	b	Pivot-spalte	Pivot-zeile
(1)	e_1	a_{11}	\dots	a_{1j}	\dots	a_{1n}	1	\dots	0	\dots	0	b_1	j	i
\vdots	\vdots	\vdots	\dots	\vdots	\dots	\vdots	\vdots	\dots	\vdots	\dots	\vdots	\vdots	\vdots	
(i)	e_i	a_{i1}	\dots	a_{ij}	\dots	a_{in}	0	\dots	1	\dots	0	b_i	$-c_j$	$\frac{b_i}{a_{ij}}$
\vdots	\vdots	\vdots	\dots	\vdots	\dots	\vdots	\vdots	\dots	\vdots	\dots	\vdots	\vdots	\vdots	
(m)	e_m	a_{m1}	\dots	a_{mj}	\dots	a_{mn}	0	\dots	0	\dots	1	b_m	minimal	minimal
(m')		$-c_1$	\dots	$-c_j$	\dots	$-c_n$	0	\dots	0	\dots	0	0		
(m+1)	e_1	$a_{11} - a_{1j} \frac{a_{i1}}{a_{ij}}$	\dots	0	\dots	$a_{1n} - a_{1j} \frac{a_{in}}{a_{ij}}$	1	\dots	$-a_{1j} \frac{1}{a_{ij}}$	\dots	0	$b_1 - a_{1j} \frac{b_i}{a_{ij}}$		
\vdots	\vdots	\vdots	\dots	\vdots	\dots	\vdots	\vdots	\dots	\vdots	\dots	\vdots	\vdots	\vdots	
(m+i)	a^j	$\frac{a_{i1}}{a_{ij}}$	\dots	1	\dots	$\frac{a_{in}}{a_{ij}}$	0	\dots	$\frac{1}{a_{ij}}$	\dots	0	$\frac{b_i}{a_{ij}}$		
\vdots	\vdots	\vdots	\dots	\vdots	\dots	\vdots	\vdots	\dots	\vdots	\dots	\vdots	\vdots	\vdots	
(2m)	e_m	$a_{m1} - a_{mj} \frac{a_{i1}}{a_{ij}}$	\dots	0	\dots	$a_{mn} - a_{mj} \frac{a_{in}}{a_{ij}}$	0	\dots	$-a_{mj} \frac{1}{a_{ij}}$	\dots	1	$b_m - a_{mj} \frac{b_i}{a_{ij}}$		
(2m')		$-c_1 + c_j \frac{a_{i1}}{a_{ij}}$	\dots	0	\dots	$-c_n + c_j \frac{a_{in}}{a_{ij}}$	0	\dots	$c_j \frac{1}{a_{ij}}$	\dots	0	$c_j \frac{b_i}{a_{ij}}$		

Figur 16.5: Basistransformation für das Standardmaximumproblem mit $b \geq 0$



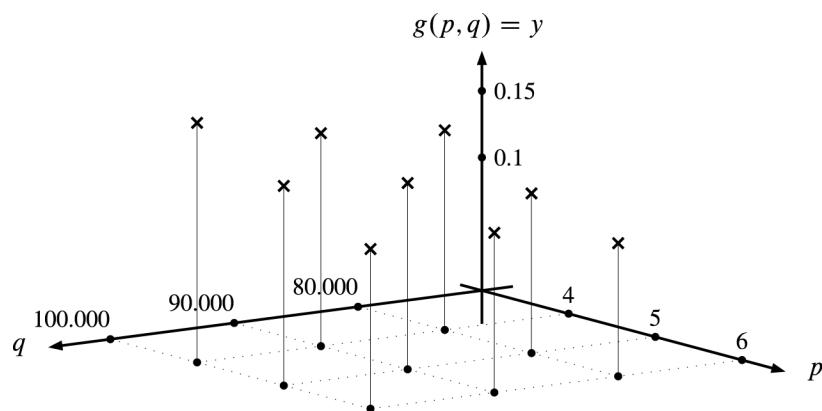
Figur 16.6: Graphische Lösung dualer linearer Optimierungsprobleme

$$\det A = \begin{pmatrix} a_{11} & a'_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

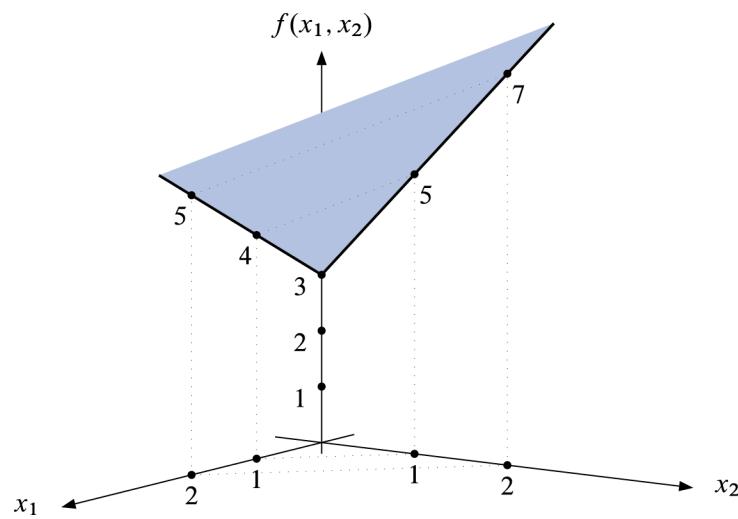
Figur 17.1: Sarrus-Regel für 2×2 -Matrizen

$$\det A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{pmatrix}$$

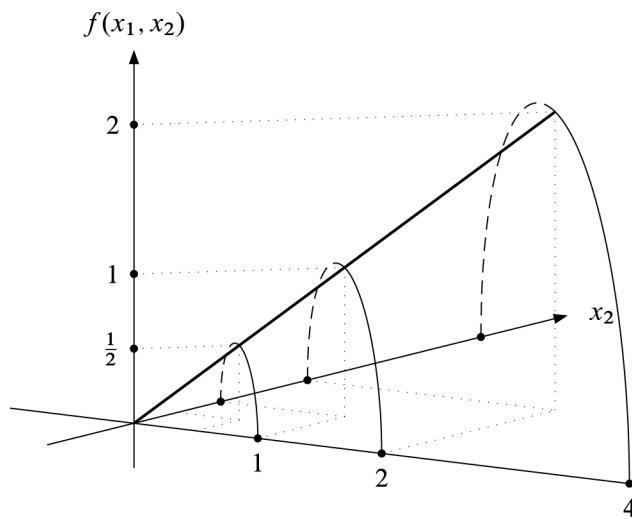
Figur 17.2: Sarrus-Regel für 3×3 -Matrizen



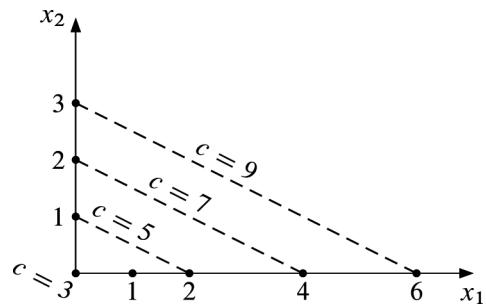
Figur 19.1: Graph der Funktion $g: D_1 \times D_2 \rightarrow \mathbb{R}$



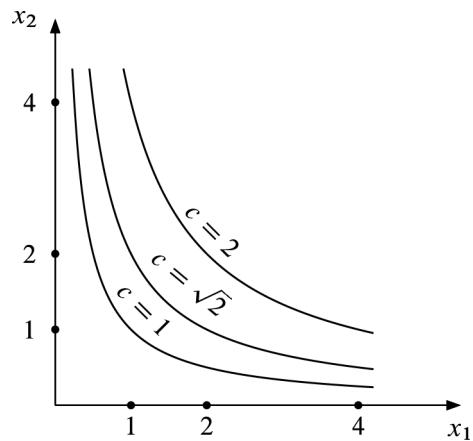
Figur 19.2: Graph der Funktion k mit $k(x_1, x_2) = 3 + x_1 + 2x_2$



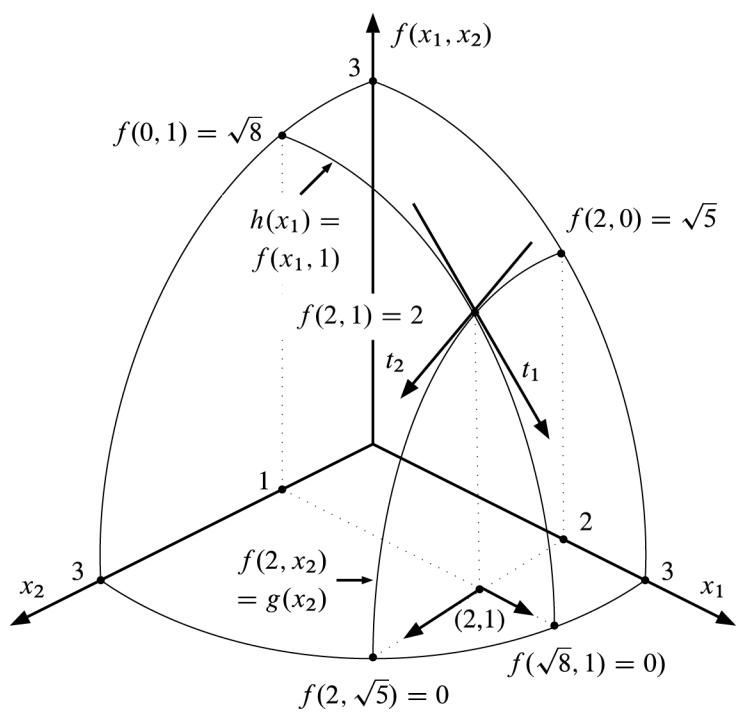
Figur 19.3: Graph der Funktion f mit $f(x_1, x_2) = \sqrt{x_1 x_2}$



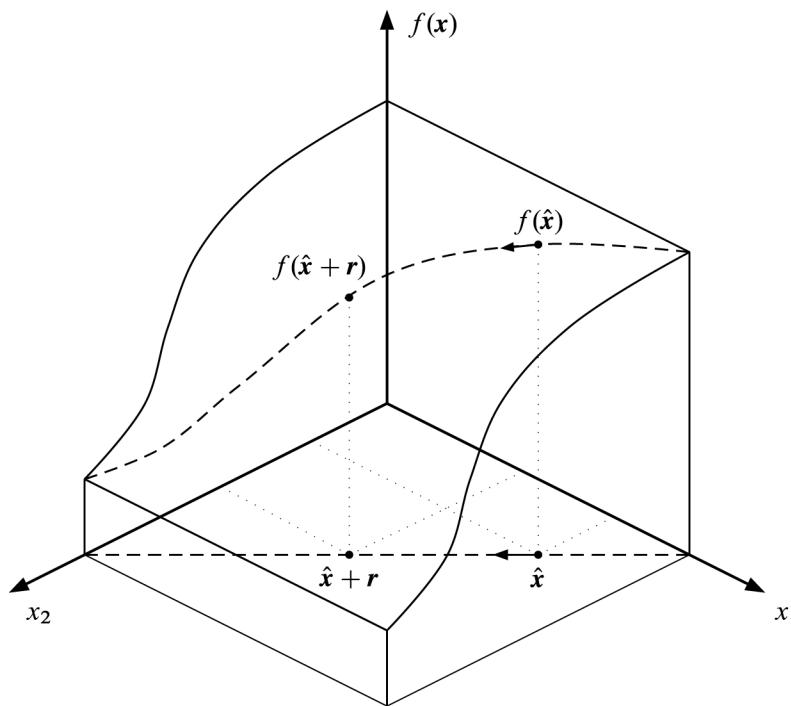
Figur 19.4: Niveaulinien der Funktion k mit $k(x_1, x_2) = 3 + x_1 + 2x_2$



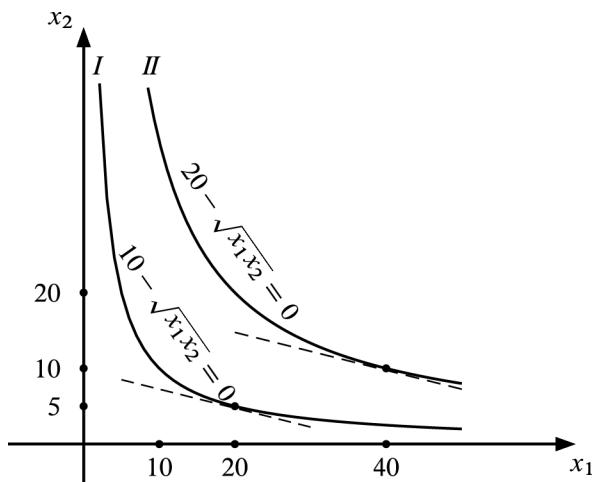
Figur 19.5: Niveaulinien der Funktion f mit $f(x_1, x_2) = \sqrt{x_1 x_2}$



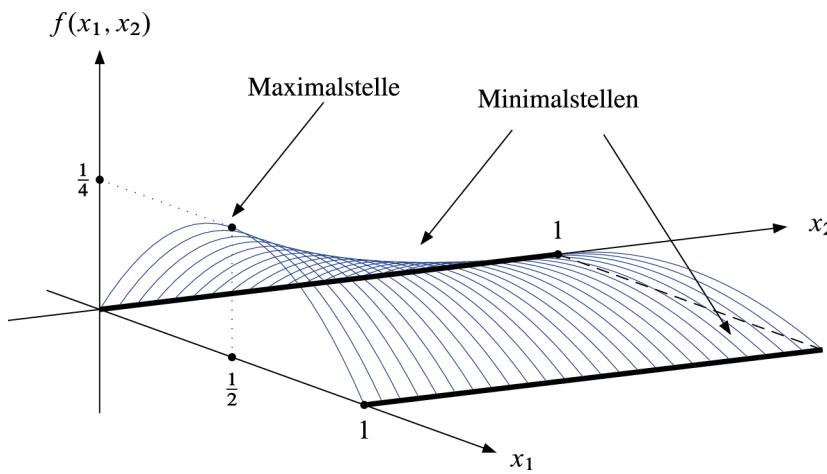
Figur 19.6: Differentiation einer reellen Funktion mit zwei Variablen x_1, x_2



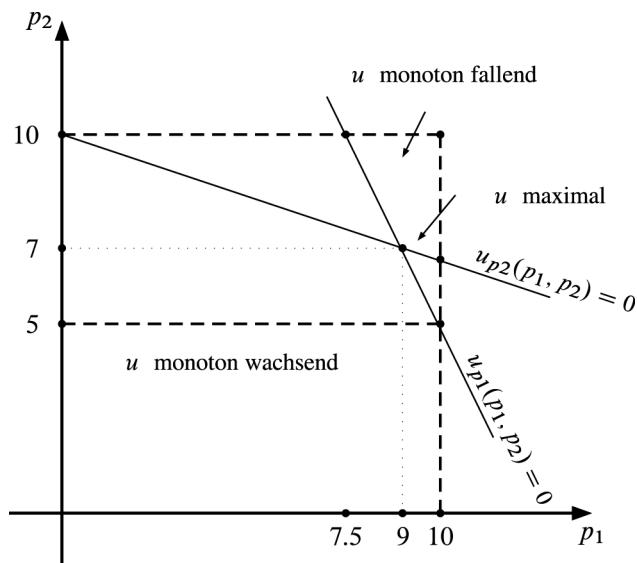
Figur 19.7: Zur Bestimmung der Richtungsableitung einer Funktion f
an der Stelle \hat{x} in Richtung r



Figur 19.8: Graphen der Produktionsgleichungen $10 = \sqrt{x_1 x_2}$ und $20 = \sqrt{x_1 x_2}$

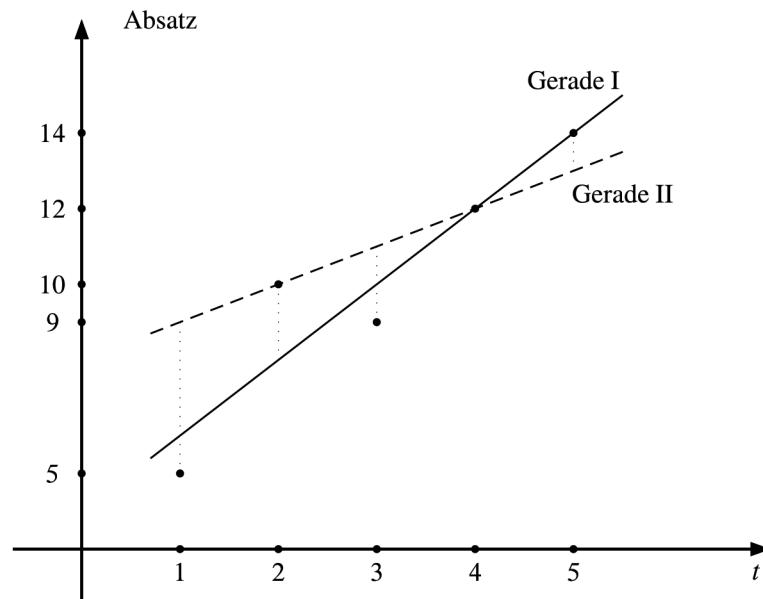


Figur 20.1: Extremalstellen der Funktion f mit $f(x_1, x_2) = \frac{x_1(1-x_1)}{\sqrt{x_2+1}}$

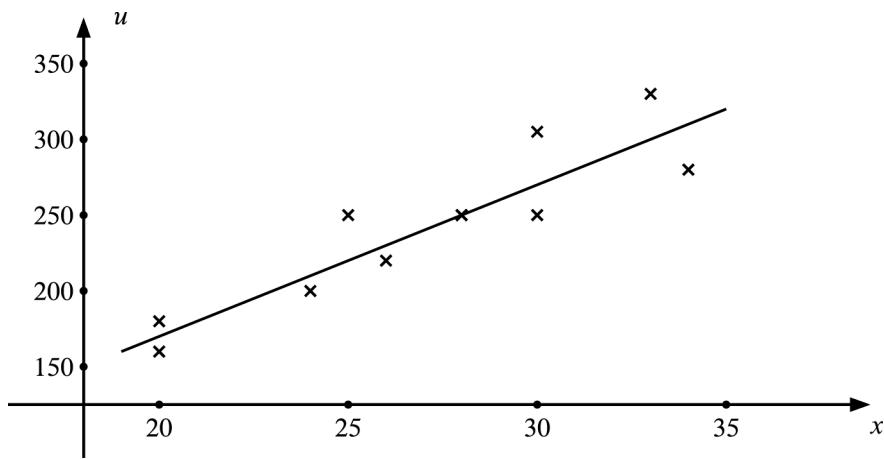


Figur 20.2: Zur Kurvendiskussion der Funktion u

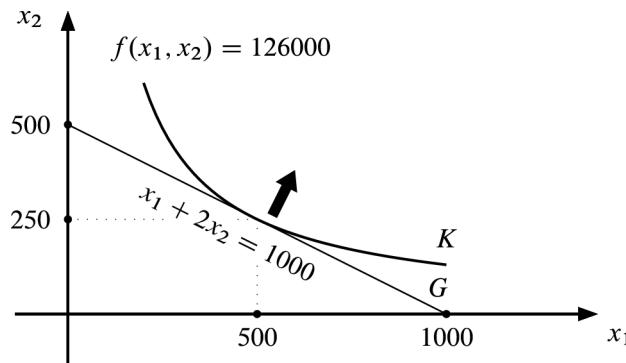
t	1	2	3	4	5
Absatz	5	10	9	12	14



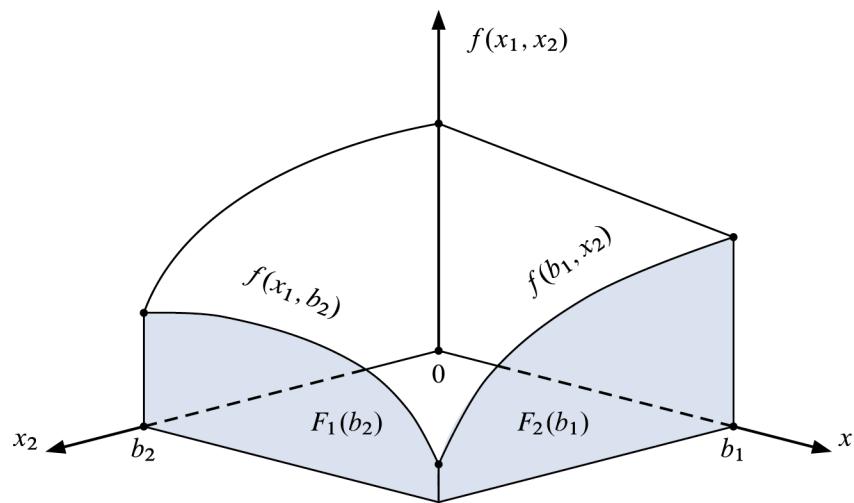
Figur 20.3: Beispiel einer Zeitreihe



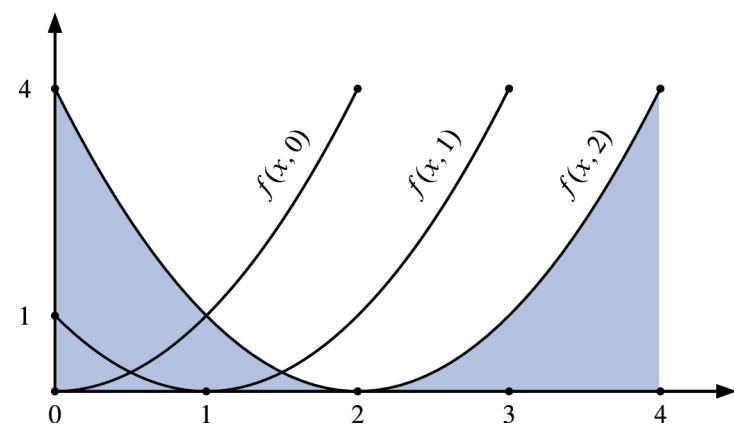
Figur 20.4: Einfache lineare Regression



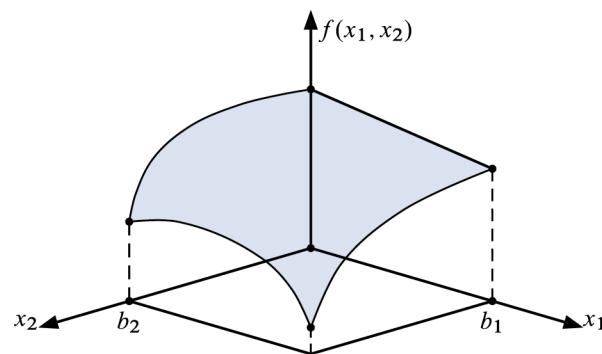
Figur 20.5: Maximum einer nichtlinearen Zielfunktion mit einer linearen Gleichung als Nebenbedingung



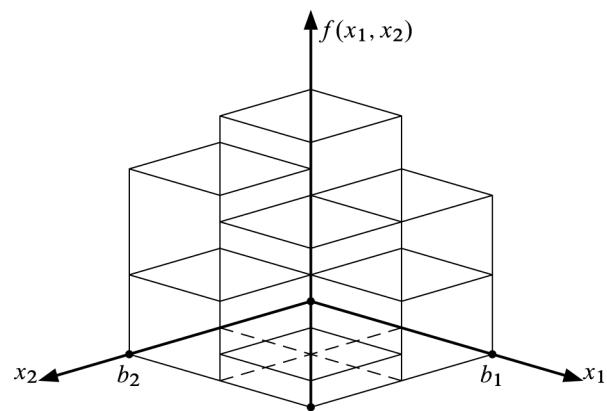
Figur 21.1: Graph einer Funktion f und der Parameterintegrale $F_1(x_2)$, $F_2(x_1)$ für $x_2 = b_2$, $x_1 = b_1$ und $a_1 = a_2 = 0$



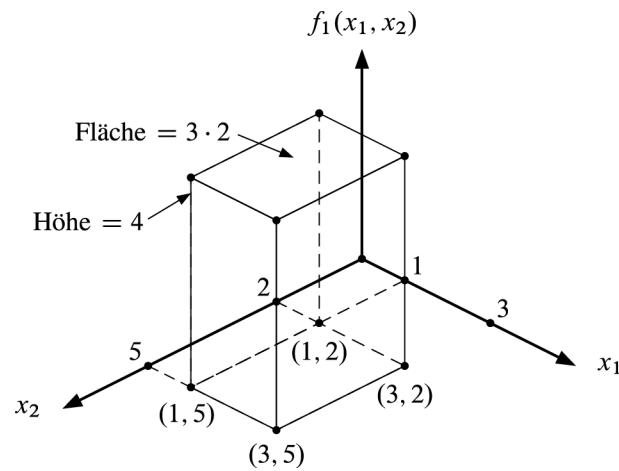
Figur 21.2: Graph der Funktion f für $t = 0, 1, 2$



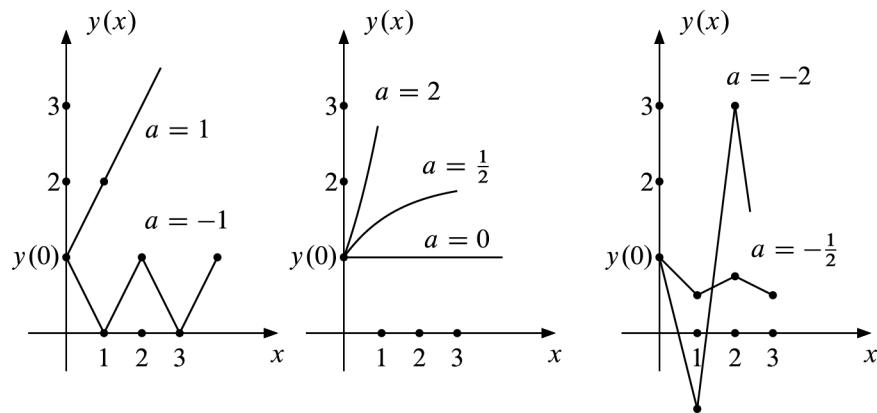
Figur 21.3: Rauminhalt zwischen dem Graphen von f und dem Rechteck $[a_1, b_1] \times [a_2, b_2]$ der $x_1 - x_2$ -Ebene mit $a_1 = a_2 = 0$



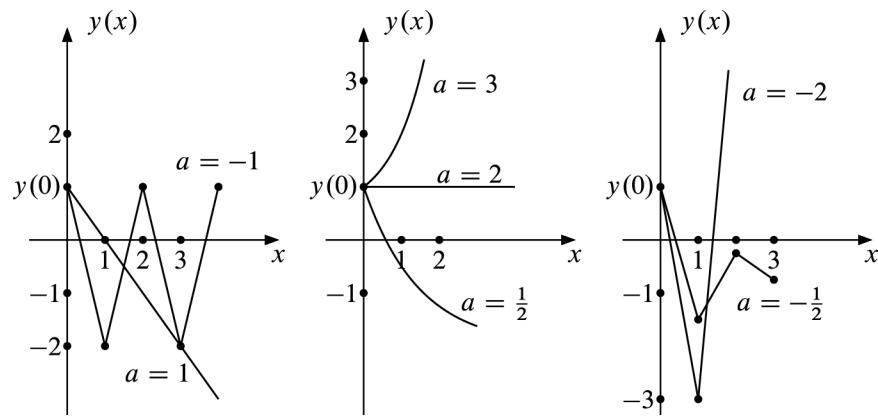
Figur 21.4: Unter- und Oberschranken des Rauminhalts für $n = 2$



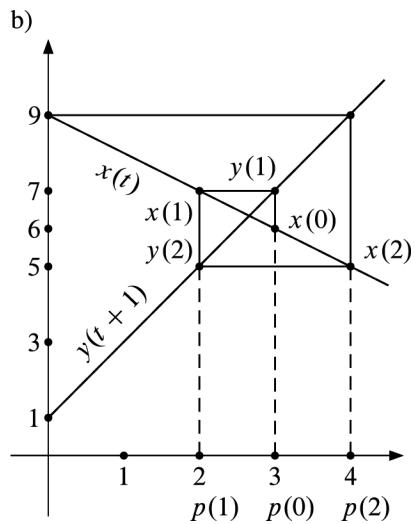
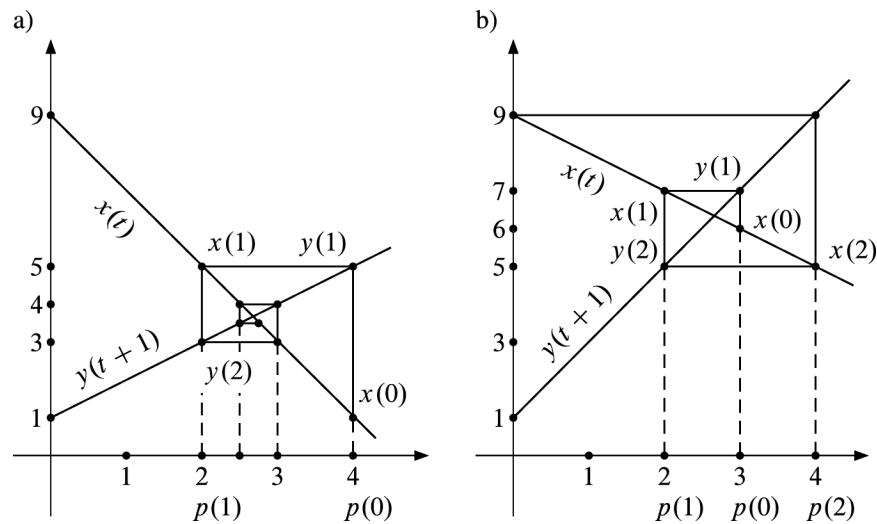
Figur 21.5: Gesuchtes Volumen = $(5 - 2)(3 - 1)4 = 24$



Figur 22.1: Graphen der Lösung von $y(x+1) = ay(x) + 1$ für $y(0) = 1$

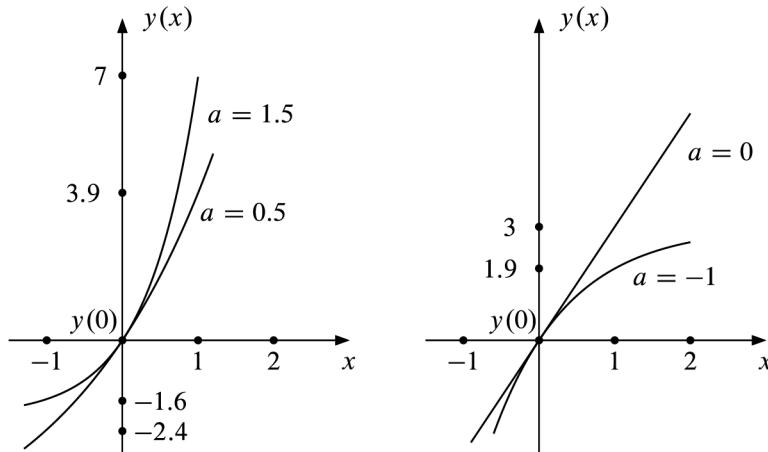


Figur 22.2: Graphen der Lösung von $y(x+1) = ay(x) - 1$ für $y(0) = 1$



Figur 22.3: Konvergenz und Divergenz beim Cobwebmodell (Spinnwebmodell)

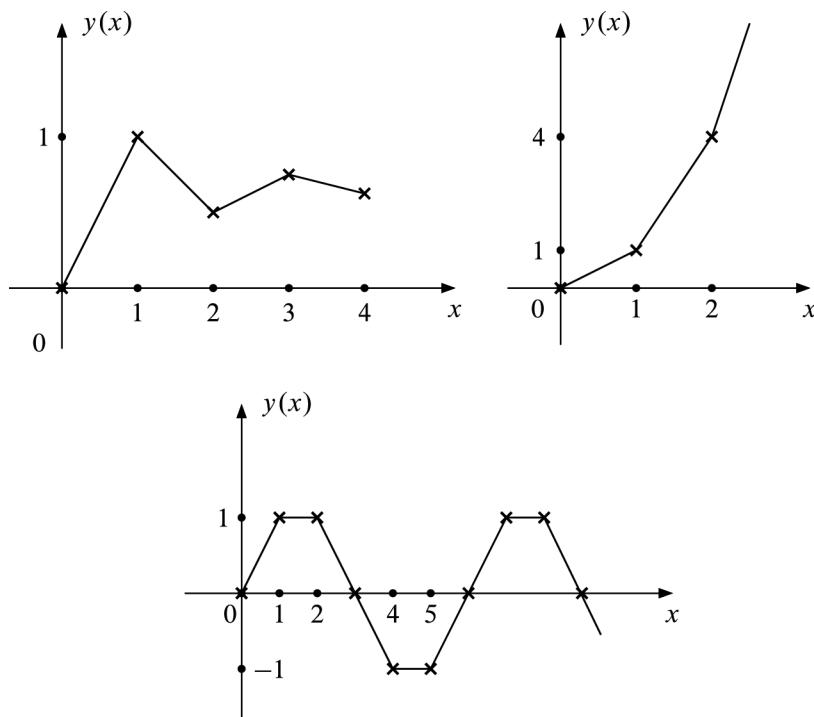
$$y(x) = \begin{cases} \frac{3}{a}(e^{ax} - 1) & \text{für } a \neq 0 \\ 3x & \text{für } a = 0 \end{cases}.$$



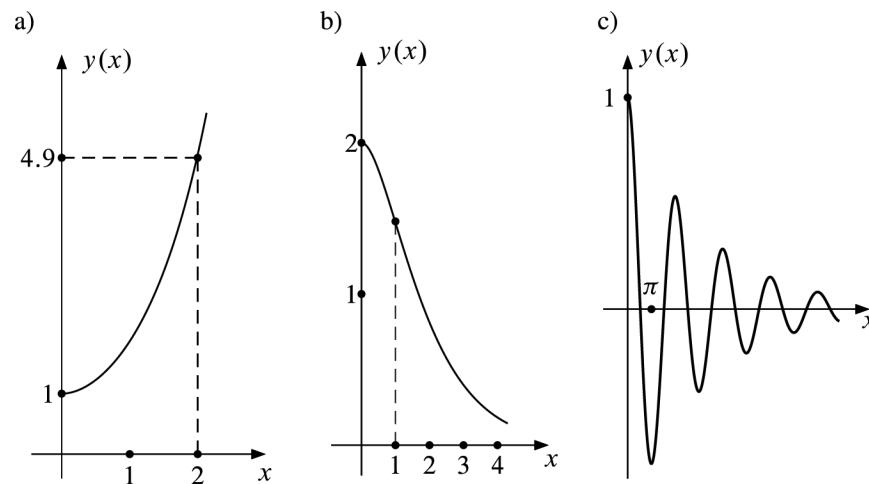
Figur 22.4: Graphen der Lösung von $y'(x) = ay(x) + 3$ für $y(0) = 0$

Differenzengleichung	Differentialgleichung
$y(x+1) - y(x) = ay(x) + b$	$y'(x) = ay(x) + b$
bzw. $y(x+1) = (1+a)y(x) + b$	
Lösung (Satz 22.6):	Lösung (Satz 22.12):
$y(x) = y(0) + bx \quad \text{für } a = 0$	$y(x) = y(0) + bx \quad \text{für } a = 0$
$y(x) = -\frac{b}{a} + \left(y(0) + \frac{b}{a}\right)(1+a)^x$ für $a \neq 0$	$y(x) = -\frac{b}{a} + \left(y(0) + \frac{b}{a}\right)e^{ax}$ für $a \neq 0$

Figur 22.5: Lineare Differenzen- und Differentialgleichungen erster Ordnung und ihre Lösung für den Fall, dass a und b konstant sind



Figur 23.1: Graphen der speziellen Lösungen aus Beispiel 23.5



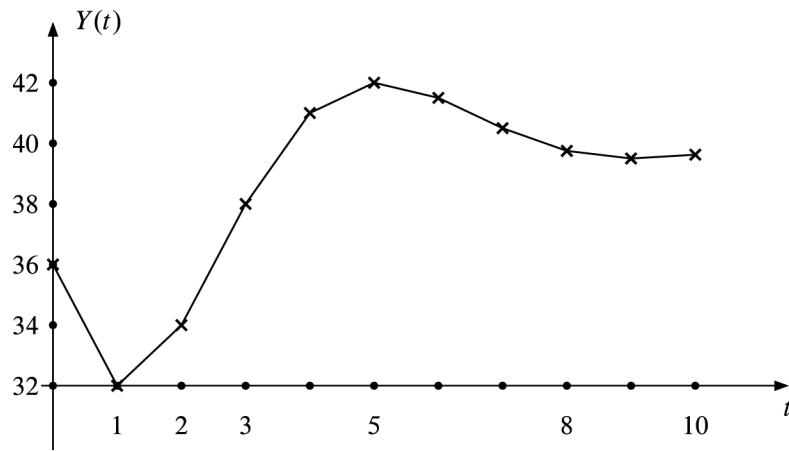
Figur 23.2: Graphen der speziellen Lösungen aus Beispiel 23.6

Nullstellen der charakteristischen Gleichung	unabhängige spezielle Lösungen der homogenen Differenzengleichung	unabhängige spezielle Lösungen der homogenen Differentialgleichung
Nullstelle λ_0 einfach, reell	$y(x) = \lambda_0^x$	$y(x) = e^{\lambda_0 x}$
Nullstelle λ_0 k -fach, reell	$y_j(x) = x^{j-1} \lambda_0^x$ ($j = 1, \dots, k$)	$y_j(x) = x^{j-1} e^{\lambda_0 x}$ ($j = 1, \dots, k$)
Nullstellen λ_1, λ_2 einfach, konjugiert komplex mit $\lambda_1 = \alpha + i\beta$ $\lambda_2 = \alpha - i\beta$	$y_1(x) = r^x \cos x\varphi$ $y_2(x) = r^x \sin x\varphi$ $r = \sqrt{\alpha^2 + \beta^2}$ $\cos \varphi = \frac{\alpha}{r}, \sin \varphi = \frac{\beta}{r}$	$y_1(x) = e^{\alpha x} \cos \beta x$ $y_2(x) = e^{\alpha x} \sin \beta x$
Nullstellen λ_1, λ_2 k -fach, konjugiert komplex mit $\lambda_1 = \alpha + i\beta$ $\lambda_2 = \alpha - i\beta$	$y_{1j}(x) = x^{j-1} r^x \cos x\varphi$ $y_{2j}(x) = x^{j-1} r^x \sin x\varphi$ ($j = 1, \dots, k$) $r = \sqrt{\alpha^2 + \beta^2}$ $\cos \varphi = \frac{\alpha}{r}, \sin \varphi = \frac{\beta}{r}$	$y_{1j}(x) = x^{j-1} e^{\alpha x} \cos \beta x$ $y_{2j}(x) = x^{j-1} e^{\alpha x} \sin \beta x$ ($j = 1, \dots, k$)

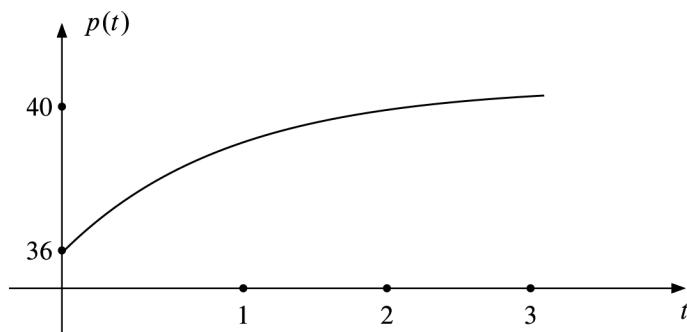
Figur 23.3: Spezielle Lösungen homogener linearer Differenzen- und Differentialgleichungen mit konstanten Koeffizienten

Form von $b(x)$	Ansatz für inhomogene Lösung $y_I(x)$
ba^x	za^x bei Form (A)
be^{ax}	ze^{ax} bei Form (B)
$b_1 \sin ax + b_2 \cos ax$	$z_1 \sin ax + z_2 \cos ax$ bei (A) und (B)
$b_0 + b_1 x + \dots + b_m x^m$ bei (A) und (B) mit $a, b, b_0, b_1, \dots, b_m$ gegeben	$z_0 + z_1 x + \dots + z_m x^m$ bei (A) und (B) mit a gegeben, z, z_0, \dots, z_m unbekannt

Figur 23.4: Spezialfälle für einen Störgliedansatz



Figur 23.5: Graph der Lösung von $Y(t) - Y(t - 1) + \frac{1}{2}Y(t) = 20$
mit $Y(0) = 36, Y(1) = 32$



Figur 23.6: Graph der Lösung von $p'' + 1.5p' + 0.5p = 20$ mit $p(0) = 36$, $p'(0) = 4$