

Contents

Preface	v
Preface to the English edition	vi
Acknowledgements	vii
Introduction	1
1 Interpolation and Hörmander spaces	9
1.1 Interpolation with function parameter	9
1.1.1 Definition of interpolation	9
1.1.2 Embeddings of spaces	11
1.1.3 Reiteration property	13
1.1.4 Interpolation of dual spaces	15
1.1.5 Interpolation of orthogonal sums of spaces	18
1.1.6 Interpolation of subspaces and factor spaces	20
1.1.7 Interpolation of Fredholm operators	21
1.1.8 Estimate of the operator norm in interpolation spaces	23
1.1.9 Criterion for a function to be an interpolation parameter	25
1.2 Regularly varying functions and their generalization	29
1.2.1 Regularly varying functions	29
1.2.2 Quasiregularly varying functions	31
1.2.3 Auxiliary results	36
1.3 Hörmander spaces and the refined Sobolev scale	38
1.3.1 Preliminary information and notation	38
1.3.2 Hörmander spaces	40
1.3.3 Refined Sobolev scale	42
1.3.4 Properties of the refined scale	44
1.4 Uniformly elliptic operators on the refined scale	47
1.4.1 Pseudodifferential operators	47
1.4.2 <i>A priori</i> estimate of the solutions	50
1.4.3 Smoothness of the solutions	51
1.5 Remarks and comments	55
2 Hörmander spaces on closed manifolds and their applications	59
2.1 Hörmander spaces on closed manifolds	59
2.1.1 Equivalent definitions	59

2.1.2	Interpolation properties	61
2.1.3	Equivalent norms	68
2.1.4	Embedding theorem	77
2.2	Elliptic operators on closed manifolds	78
2.2.1	Pseudodifferential operators on closed manifolds	79
2.2.2	Elliptic operators on the refined scale	81
2.2.3	Smoothness of solutions to the elliptic equation	84
2.2.4	Parameter-elliptic operators	86
2.3	Convergence of spectral expansions	93
2.3.1	Convergence almost everywhere for general orthogonal series	93
2.3.2	Convergence almost everywhere for spectral expansions .	95
2.3.3	Convergence of spectral expansions in the metric of the space C^k	97
2.4	RO-varying functions and Hörmander spaces	98
2.4.1	RO-varying functions in the sense of Avakumović	98
2.4.2	Interpolation spaces for a pair of Sobolev spaces	100
2.4.3	Applications to elliptic operators	107
2.5	Remarks and comments	108
3	Semihomogeneous elliptic boundary-value problems	111
3.1	Regular elliptic boundary-value problems	111
3.1.1	Definition of the problem	111
3.1.2	Formally adjoint problem	113
3.2	Hörmander spaces for Euclidean domains	114
3.2.1	Spaces for open domains	115
3.2.2	Spaces for closed domains	120
3.2.3	Rigging of $L_2(\Omega)$ with Hörmander spaces	123
3.3	Boundary-value problems for homogeneous elliptic equations ...	126
3.3.1	Main result: boundedness and Fredholm property of the operator	126
3.3.2	A theorem on interpolation of subspaces	127
3.3.3	Elliptic boundary-value problem in Sobolev spaces	131
3.3.4	Proof of the main result	134
3.3.5	Properties of solutions to the homogeneous elliptic equation	139
3.4	Elliptic problems with homogeneous boundary conditions	142
3.4.1	Theorem on isomorphisms for elliptic operators	142
3.4.2	Interpolation and homogeneous boundary conditions	146

3.4.3	Proofs of theorems on isomorphisms and the Fredholm property	152
3.4.4	Local increase in smoothness of solutions up to the boundary	156
3.5	Some properties of Hörmander spaces	158
3.5.1	Space $H_0^{s,\varphi}(\Omega)$ and its properties	158
3.5.2	Equivalent description of $H^{s,\varphi}(\Omega)$	160
3.6	Remarks and comments	162
4	Inhomogeneous elliptic boundary-value problems	165
4.1	Elliptic boundary-value problems in the positive one-sided scale	165
4.1.1	Theorems on Fredholm property and isomorphisms	165
4.1.2	Smoothness of the solutions up to the boundary	169
4.1.3	Nonregular elliptic boundary-value problems	173
4.1.4	Parameter-elliptic boundary-value problems	175
4.1.5	Formally mixed elliptic boundary-value problem	186
4.2	Elliptic boundary-value problems in the two-sided scale	188
4.2.1	Preliminary remarks	188
4.2.2	The refined scale modified in the sense of Roitberg	189
4.2.3	Roitberg-type theorems on solvability. The complete collection of isomorphisms	199
4.2.4	Smoothness of generalized solutions up to the boundary	204
4.2.5	Interpolation in the modified refined scale	207
4.3	Some properties of the modified refined scale	210
4.3.1	Statement of results	210
4.3.2	Proof of results	212
4.4	Generalization of the Lions–Magenes theorems	226
4.4.1	Lions–Magenes theorems	227
4.4.2	Key individual theorem	230
4.4.3	Individual theorem for Sobolev spaces	236
4.4.4	Individual theorem for weight spaces	238
4.5	Hörmander spaces and individual theorems on solvability	243
4.5.1	Key individual theorem for the refined scale	243
4.5.2	Other individual theorems	244
4.6	Remarks and Comments	247
5	Elliptic systems	251
5.1	Uniformly elliptic systems in the refined Sobolev scale	251
5.1.1	Uniformly elliptic systems	251
5.1.2	<i>A priori</i> estimate for the solutions of the system	252
5.1.3	Smoothness of solutions	253

5.2	Elliptic systems on a closed manifold	257
5.2.1	Elliptic Systems	257
5.2.2	Operator of the elliptic system on the refined scale	258
5.2.3	Local smoothness of solutions	262
5.2.4	Parameter-elliptic systems	264
5.3	Elliptic boundary-value problems for systems of equations	268
5.3.1	Statement of the problem	269
5.3.2	Theorem on solvability	271
5.4	Remarks and comments	272
	Bibliography	275
	Index	291