

Contents

Preface	vii
Notations	xi
1 Topological structure of fixed point sets	1
1.1 Case of single-valued mappings	1
1.1.1 Fundamental fixed point theorems	1
1.1.1.1 Banach's fixed point theorem	1
1.1.1.2 Brouwer's fixed point theorem	5
1.1.1.3 Schauder's fixed point theorem	7
1.1.2 Approximation theorems	11
1.1.3 Browder–Gupta theorems	13
1.1.4 Acyclicity of the solution sets of operator equations	20
1.1.5 Nonexpansive maps	23
1.1.5.1 Existence theory	23
1.1.5.2 Solution sets	26
1.2 The case of multi-valued mappings	27
1.2.1 Approximation of multi-valued maps	27
1.2.2 Fixed point theorems	30
1.2.3 Multi-valued contractions	33
1.2.4 Fixed point sets of multi-valued contractions	35
1.2.5 Fixed point sets of multi-valued nonexpansive maps	38
1.2.6 Fixed point sets of multi-valued condensing maps	39
1.2.6.1 Measure of noncompactness	39
1.2.6.2 Condensing maps	43
1.3 Admissible maps	44
1.3.1 Generalities	44
1.3.2 Fixed point theorems for admissible multi-valued maps	53
1.3.3 The general Brouwer fixed point theorem	58
1.3.4 Browder–Gupta type results for admissible mappings	60
1.3.5 Topological dimensions of solution sets	62

1.4	Topological structure of fixed point sets of inverse limit maps	65
1.4.1	Definition	65
1.4.2	Basic properties	66
1.4.3	Multi-maps of inverse systems	67
2	Existence theory for differential equations and inclusions	72
2.1	Fundamental theorems	72
2.1.1	Existence and uniqueness results	72
2.1.2	Picard–Lindelöf theorem	73
2.1.2.1	Maximal solutions	75
2.1.3	Peano and Carathéodory theorems	77
2.1.3.1	Peano theorem	77
2.2	The extendability problem	79
2.2.1	Global existence theorems	79
2.2.2	Existence results on noncompact intervals	82
2.2.2.1	The Lipschitz case	82
2.2.2.2	The Lipschitz–Nagumo case	83
2.2.2.3	The Nagumo case	86
2.2.3	A boundary value problem on the half-line	88
2.3	The case of differential inclusions	94
2.3.1	Initial value problems	94
2.3.1.1	A Nagumo type nonlinearity	94
2.3.1.2	A Lipschitz nonconvex nonlinearity	97
2.3.2	Boundary value problems	99
2.3.2.1	The convex case	100
2.3.2.2	The nonconvex case	103
3	Solution sets for differential equations and inclusions	105
3.1	General results	105
3.1.1	Kneser–Hukuhara theorem	105
3.1.2	Problems on bounded intervals	108
3.1.3	Problems on unbounded intervals	109
3.1.4	Second-order differential equations	111
3.1.5	Abstract Volterra equations	113
3.1.6	Aronszajn type results for differential inclusions	114
3.2	Second-order differential inclusions	122
3.2.1	The convex case	122
3.2.2	The nonconvex case	127
3.2.3	Solution sets	130

3.3	Higher-order differential inclusions	134
3.4	Neutral differential inclusions	135
3.4.1	The convex case	136
3.4.2	The nonconvex case	142
3.4.3	Solutions sets	146
3.5	Nonlocal problems	146
3.5.1	Main results	147
3.5.2	A viability problem	149
3.6	Hyperbolic differential inclusions	154
3.6.1	Existence results	155
3.6.1.1	The convex case	155
3.6.1.2	The nonconvex case	159
3.6.2	Solution sets	160
4	Impulsive differential inclusions: existence and solution sets	163
4.1	Motivation	163
4.1.1	Ecological model with impulsive control strategy	163
4.1.2	Leslie predator-prey system	164
4.1.3	Pulse vaccination model	165
4.2	Semi-linear impulsive differential inclusions	166
4.2.1	Existence results	166
4.2.1.1	The convex case	167
4.2.1.2	The nonconvex case	181
4.2.2	Structure of solution sets	186
4.3	A periodic problem	197
4.3.1	Existence results: $1 \in \rho(T(b))$	198
4.3.2	The convex case: a direct approach	199
4.3.3	The convex case: an MNC approach	207
4.3.4	The nonconvex case	212
4.3.5	The parameter-dependant case	215
4.3.5.1	The convex case	215
4.3.5.2	The nonconvex case	217
4.3.6	Filippov's Theorem	220
4.3.7	Existence of solutions: $1 \notin \rho(T(b))$	230
4.3.7.1	A nonlinear alternative	230
4.3.7.2	A Poincaré translation operator	233
4.3.7.3	The MNC approach	233

4.4	Impulsive functional differential inclusions	236
4.4.1	Introduction	236
4.4.2	Existence results	237
4.4.3	Structure of the solution set	246
4.5	Impulsive differential inclusions on the half-line	250
4.5.1	Existence results and compactness of solution sets	250
4.5.1.1	The convex <i>u.s.c.</i> case	251
4.5.1.2	The nonconvex Lipschitz case	258
4.5.1.3	The nonconvex <i>l.s.c.</i> case	262
4.5.2	Topological structure via the projective limit	265
4.5.2.1	The nonconvex case	266
4.5.2.2	The convex case	271
4.5.2.3	The terminal problem	274
4.5.3	Using solution sets to prove existence results	283
5	Preliminary notions of topology and homology	288
5.1	Retracts, extension and embedding properties	288
5.2	Absolute retracts	294
5.3	Homotopical properties of spaces	296
5.4	Čech homology (cohomology) functor	304
5.5	Maps of spaces of finite type	306
5.6	Čech homology functor with compact carriers	313
5.7	Acyclic sets and Vietoris maps	315
5.8	Homology of open subsets of Euclidean spaces	319
5.9	Lefschetz number	323
5.10	The coincidence problem	330
6	Background in multi-valued analysis	337
6.1	Continuity of multi-valued mappings	339
6.1.1	Basic notions	339
6.1.2	Upper semi-continuity	341
6.1.2.1	Generalities	341
6.1.2.2	$\varepsilon - \delta$ <i>u.s.c.</i> mappings	344
6.1.2.3	<i>U.s.c.</i> maps and closed graphs	345
6.1.3	Lower semi-continuity	346
6.1.3.1	Generalities	346
6.1.3.2	$\varepsilon - \delta$ <i>l.s.c.</i> mappings	349
6.1.4	Hausdorff continuity	350

6.2	The selection problem	354
6.2.1	Michael's selection theorem	355
6.2.2	Michael's family of subsets	358
6.2.3	σ -selectionable mappings	362
6.2.4	The Kuratowski–Ryll–Nardzewski selection theorem	366
6.2.5	Aumann and Filippov theorems	378
6.2.6	Hausdorff measurable multi-valued maps	382
6.2.7	Product-measurability and the Scorza–Dragoni property	383
6.3	Decomposable sets	390
6.3.1	The Bressan–Colombo–Fryszkowski selection theorem	390
6.3.2	Decomposability in $L^1(T, E)$	390
6.3.3	Integration of multi-valued maps	392
6.3.4	Nemytskiĭ operators	393
Appendix		399
A.1	Axioms of the Čech homology theory	399
A.2	The Bochner integral	400
A.3	Absolutely continuous functions	403
A.4	Compactness criteria in $C([a, b], E)$, $C_b([0, \infty), E)$, and $PC([a, b], E)$	405
A.5	Weak-compactness in L^1	408
A.6	Proper maps and vector fields	410
A.7	Fundamental theorems in functional analysis	411
A.8	C_0 -Semigroups	412
References		415
Index		451