

Table of Contents

Preface	v
-------------------	---

Gunnar Fløystad

Boij–Söderberg Theory: Introduction and Survey

1 The Boij–Söderberg Conjectures	5
1.1 Resolutions and Betti Diagrams	5
1.2 The Positive Cone of Betti Diagrams	6
1.3 Herzog–Kühl Equations	7
1.4 Pure resolutions	8
1.5 Linear Combinations of Pure Diagrams	9
1.6 The Boij–Söderberg Conjectures	11
1.7 Algorithmic Interpretation	12
1.8 Geometric Interpretation	12
2 The Exterior Facets of the Boij–Söderberg Fan and Their Supporting Hyper- planes	13
2.1 The Exterior Facets	13
2.2 The Supporting Hyperplanes	15
2.3 Pairings of Vector Bundles and Resolutions	21
3 The Existence of Pure Free Resolutions and of Vector Bundles with Supernat- ural Cohomology	24
3.1 The Equivariant Pure Free Resolution	24
3.2 Equivariant Supernatural Bundles	29
3.3 Characteristic Free Supernatural Bundles	29
3.4 The Characteristic Free Pure Resolutions	30
3.5 Pure Resolutions Constructed from Generic Matrices	33
4 Cohomology of Vector Bundles on Projective Spaces	35
4.1 Cohomology Tables	35
4.2 The Fan of Cohomology Tables of Vector Bundles	37
4.3 Facet Equations	38
5 Extensions to Non-Cohen–Macaulay Modules and to Coherent Sheaves	41
5.1 Betti Diagrams of Graded Modules in General	42
5.2 Cohomology of Coherent Sheaves	43
6 Further Topics	45
6.1 The Semigroup of Betti Diagrams of Modules	45
6.2 Variations on the Grading	49

6.3	Poset Structures	50
6.4	Computer Packages	50
6.5	Three Basic Problems	51

Anthony V. Geramita, Brian Harbourne and Juan C. Migliore

Hilbert Functions of Fat Point Subschemes of the Plane: the Two-fold Way

1	Introduction	55
2	Approach I: Nine Double Points	58
3	Approach I: Points on Cubics	67
4	Approach II: Points on Cubics	74

Susan Morey and Rafael H. Villarreal

Edge Ideals: Algebraic and Combinatorial Properties

1	Introduction	85
2	Algebraic and Combinatorial Properties of Edge Ideals	86
3	Invariants of Edge Ideals: Regularity, Projective Dimension, Depth	90
4	Stability of Associated Primes	104

Jeff Mermin

Three Simplicial Resolutions

1	Introduction	127
2	Background and Notation	128
	2.1 Algebra	128
	2.2 Combinatorics	129
3	The Taylor Resolution	130
4	Simplicial Resolutions	132
5	The Scarf Complex	135
6	The Lyubeznik Resolutions	136
7	Intersections	138
8	Questions	139

Timothy B. P. Clark

A Minimal Poset Resolution of Stable Ideals

1	Introduction	143
2	Poset Resolutions and Stable Ideals	147
3	The Shellability of P_N	150
4	The topology of P_N and properties of $\mathcal{D}(P_N)$	155
5	Proof of Theorem 2.4	160
6	A Minimal Cellular Resolution of R/N	163

Susan M. Cooper

Subsets of Complete Intersections and the EGH Conjecture

1	Introduction	167
---	------------------------	-----

2	Preliminary Definitions and Results	168
2.1	The Eisenbud–Green–Harris Conjecture and Complete Intersections . .	168
2.2	Some Enumeration	171
3	Rectangular Complete Intersections	173
4	Some Key Tools	177
4.1	Pairs of Hilbert Functions and Maximal Growth	177
4.2	Ideals Containing Regular Sequences	178
5	Subsets of Complete Intersections in $\mathbb{P}^2$	179
6	Subsets of C.I.(2, d_2 , d_3) with $d_2 = d_3$	181
7	Subsets of C.I.(3, d_2 , d_3) with $d_3 = d_2$	186
8	An Application: The Cayley–Bacharach Property	194

Paul C. Roberts

The Homological Conjectures

1	Introduction	199
2	The Serre Multiplicity Conjectures	201
2.1	The Vanishing Conjecture	204
2.2	Gabber’s Proof of the Nonnegativity Conjecture	205
2.3	The Positivity Conjecture	206
3	The Peskine–Szpiro Intersection Conjecture	207
3.1	Hochster’s Metatheorem	209
4	Generalizations of the Multiplicity Conjectures	211
4.1	The Graded Case	211
4.2	The Generalized Rigidity Conjecture	214
5	The Monomial, Direct Summand, and Canonical Element Conjectures	215
6	Cohen–Macaulay Modules and Algebras	217
6.1	Weakly Functorial Big Cohen–Macaulay Algebras	218
7	The Syzygy Conjecture and the Improved New Intersection Conjecture . . .	219
8	Tight Closure Theory	220
9	The Strong Direct Summand Conjecture	222
10	Almost Cohen–Macaulay Algebras	223
11	A Summary of Open Questions	225
11.1	The Serre Positivity Conjecture	225
11.2	Partial Euler Characteristics	225
11.3	Strong Multiplicity Conjectures	226
11.4	Cohen–Macaulay Modules and Related Conjectures	226
11.5	Almost Cohen–Macaulay Algebras	227

Yongwei Yao

The Compatibility, Independence, and Linear Growth Properties

1	Introduction	231
2	Primary decomposition	232

3	Compatibility of Primary Components	236
4	Maximal Primary Components, Independence	237
5	Linear growth of primary components	239
6	Linear Growth of $\{\mathrm{Tor}_c^R(\frac{M}{I^m M}, \frac{N}{J^n N})\}$	241
7	Secondary Representation	247
8	Compatibility of Secondary Components	250
9	Applying a Result of Sharp on Artinian Modules	251
10	Independence	259
11	Minimal Secondary Components	262
12	Linear Growth of Secondary Components	264

Livia Hummel

Recent Progress in Coherent Rings: a Homological Perspective

1	Introduction	271
2	Coherent Rings and Grade	272
	2.1 Coherent Rings and $(\mathrm{FP})_\infty^R$ Modules	272
	2.2 Non-Noetherian Grade	272
3	Cohen–Macaulay Rings	274
4	Gorenstein Dimensions and the Auslander–Bridger Property	277
	4.1 Gorenstein Dimensions	277
	4.2 The Auslander–Bridger Formula	280
5	Gorenstein Rings and Injective Dimensions	281
6	Foundations for Coherent Complete Intersections	288

Graham J. Leuschke

Non-commutative Crepant Resolutions: Scenes from Categorical Geometry

1	Introduction	293
2	Morita Equivalence	296
3	(Quasi)coherent Sheaves	298
4	Derived Categories of Modules	299
5	Derived Categories of Sheaves	301
6	Example: Tilting on Projective Space	303
7	The Non-existence of Non-commutative Spaces	306
8	Resolutions of Singularities	310
9	The Minimal Model Program	313
10	Categorical Desingularizations	316
11	Example: the McKay Correspondence	318
12	Non-commutative Crepant Resolutions	323
13	Example: Normalization	328
14	MCM Endomorphism Rings	329
15	Global Dimension of Endomorphism Rings	331
16	Rational Singularities	334

17 Examples: Finite Representation Type	336
18 Example: the Generic Determinant	338
19 Non-commutative Blowups	343
20 Omissions and Open Questions	345