

Contents

Preface	v
1 Characteristic functions	1
1.1 Basic properties	1
1.2 Differentiability	9
1.3 Inversion theorems	18
1.4 Basic properties of positive definite functions	25
1.5 Further properties of positive definite functions on $\mathbb{R}^d$	32
1.6 Lévy's continuity theorem	38
1.7 The theorems of Bochner and Herglotz	41
1.8 Fourier transformation on $\mathbb{R}^d$	47
1.9 Fourier transformation on discrete commutative groups	55
1.10 Basic properties of Gaussian distributions	57
1.11 Some inequalities	61
2 Correlation functions	67
2.1 Random fields	67
2.2 Correlation functions of second order random fields	70
2.3 Continuity and differentiability	75
2.4 Integration with respect to complex measures	77
2.5 The Karhunen–Loève decomposition	86
2.6 Integration with respect to orthogonal random measures	92
2.7 The theorem of Karhunen	98
2.8 Stationary fields	103
2.9 Spectral representation of stationary fields	109
2.10 Unitary representations	117
2.11 Unitary representations and positive definite functions	125
3 Special properties	132
3.1 Strict positive definiteness	132
3.2 Infinitely differentiable and rapidly decreasing functions	134

3.3	Analytic characteristic functions of one variable	139
3.4	Holomorphic L^2 Fourier transforms	148
3.5	Further properties of Gaussian distributions	154
3.6	Fourier transformation of radial measures and functions	160
3.7	Radial characteristic functions	165
3.8	Schoenberg's theorems on radial characteristic functions	172
3.9	Convex and completely monotone functions	175
3.10	Convolution roots with compact support	184
3.11	Infinitely divisible characteristic functions	187
3.12	Conditionally positive definite functions	189
4	The extension problem	200
4.1	General results	200
4.2	The cases $\mathbb{R}^d$ and $\mathbb{Z}^d$	208
4.3	Decomposition of locally defined positive definite functions	213
4.4	Extension of radial positive definite functions	221
5	Selected applications	224
5.1	Limit theorems	224
5.2	Sums of independent random vectors and the Jessen–Wintner purity law	226
5.3	Ergodic theorems for stationary fields	234
5.4	Filtration of discrete stationary fields	239
	Appendix	242
A	Basic notation	242
A.1	Standard notation	242
A.2	Multidimensional notation	243
B	Basic analysis	244
B.1	Miscellaneous results from classical analysis	244
B.2	Uniform convergence of continuous functions	256
B.3	Infinite products	261
B.4	Convex functions	264
B.5	The Riemann–Stieltjes integral	266
B.6	Multivariate calculus	267
B.7	The Lebesgue integral on $\mathbb{R}^d$	272

C	Advanced analysis	278
C.1	Functions of a complex variable	278
C.2	Almost periodic functions	285
C.3	Fourier series	287
C.4	The Gamma function and the formulae of Stirling and Binet	288
C.5	Bessel functions	296
C.6	The Mellin transform	300
C.7	The Laplace transform	302
C.8	Existence of continuous logarithms	304
C.9	Solutions of certain functional equations	306
C.10	Linear independence of exponential functions	310
D	Functional analysis	314
D.1	Inner product spaces	314
D.2	Matrices and kernels	315
D.3	Hilbert spaces and linear operators	327
D.4	Convex sets and the theorem of Kreĭn and Milman	332
D.5	Weak topologies	334
E	Measure theory	339
E.1	Borel measures, weak and vague convergence	339
E.2	Convolution of measures and functions	345
F	Probability	347
F.1	Basic notions	347
F.2	Convergence of random vectors	350
F.3	Products of probability spaces	352
F.4	Conditional expectation	354
	Bibliography	357
	Index	361