Chapter 12
Inventory Control Models

Have you ever hosted a Super Bowl party for, say, 10 people? If so, how many beverages did you purchase? If you expected the average attendee to have three drinks, did you purchase 30 drinks? Or did you purchase 40, 50, or 60 “just in case?” If your favorite team wins, people might drink more to celebrate. If your favorite team loses, people might drown their sorrows with more drinks. And what if Fred unexpectedly brings three friends without telling you? If you ran out of drinks, either everyone would be mad at you, or you would have to miss part of the game to go get some more. Your drink purchase decision was a decision about *inventory*. You probably ordered more than the expected demand, but how much more should you have purchased?

Organizations constantly make inventory decisions about how much to order and when to order. Even service organizations deal with inventory regularly. Restaurants need food, banks need sufficient cash on hand, and quick oil change shops need various grades of oil, along with air filters and other auto supplies. The primary trade-offs involve the cost of placing orders versus the cost of holding inventory and the cost of stockouts.

We begin this chapter by delineating the many functions that inventory serves. We then describe the specific cost factors involved in inventory decisions. Following that, we present the *economic order quantity* (EOQ) model, which is the foundation of all inventory models. We next explain how to determine when it is time to place a new order. This is followed by a description of two extensions of the EOQ model: (1) determining lot size when it takes time to build up inventory, and (2) determining order size when presented with quantity discount opportunities. These models all assume that demand is known with certainty. We also describe how to determine how much *safety stock* to hold when future demand is uncertain. Finally, we end the chapter with a discussion of *ABC Analysis*, which is a simple method for classifying items according to one of three categories. An item’s category determines how much attention that management should pay to it with respect to inventory control. As with Chapters 9 and 11, all of our quantitative models for this chapter are coded into the ExcelModules program that is available from the companion website.

Chapter Objectives

After completing this chapter, students will be able to:

1. Understand the importance of inventory control.
2. Use inventory control models to determine how much to order or produce and when to order or produce.
3. Understand inventory models that allow quantity discounts.
4. Understand the use of safety stock with known and unknown stockout costs.
5. Understand the importance of ABC inventory analysis.
6. Use Excel to analyze a variety of inventory control models.

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12.1 Three Phases of Inventory Planning and Control

Inventory is one of the most expensive and important assets of many companies, representing as much as 50% of total invested capital. Managers have long recognized that good inventory control is crucial. On one hand, a firm can try to reduce costs by reducing on-hand inventory levels. On the other hand, customers become dissatisfied when frequent inventory outages, called stockouts, occur. Thus, companies must find the proper balance between low and high inventory levels. As you would expect, cost minimization is the major factor in obtaining this delicate balance.

Inventory is any stored resource that is used to satisfy a current or future need. Raw materials, work-in-process, and finished goods are examples of inventory. Inventory levels for finished goods, such as clothes dryers, are a direct function of market demand. By using this demand information, it is possible to determine how much raw materials (e.g., sheet metal, paint, and electric motors in the case of clothes dryers) and work-in-process are needed to produce the finished product.

Every organization has some type of inventory planning and control system. A bank has methods to control its inventory of cash. A hospital has methods to control blood supplies and other important items. State and federal governments, schools, and virtually every manufacturing and production organization are concerned with inventory planning and control. Studying how organizations control their inventory is equivalent to studying how they achieve their objectives by supplying goods and services to their customers. Inventory is the common thread that ties all the functions and departments of the organization together.

Figure 12.1 illustrates the basic components of an inventory planning and control system. The *planning* phase involves primarily what inventory is to be stocked and how it is to be acquired (whether it is to be manufactured or purchased). This information is then used in *forecasting* demand for the inventory and in *controlling* inventory levels. The feedback loop in Figure 12.1 provides a way of revising the plan and forecast based on experiences and observation.

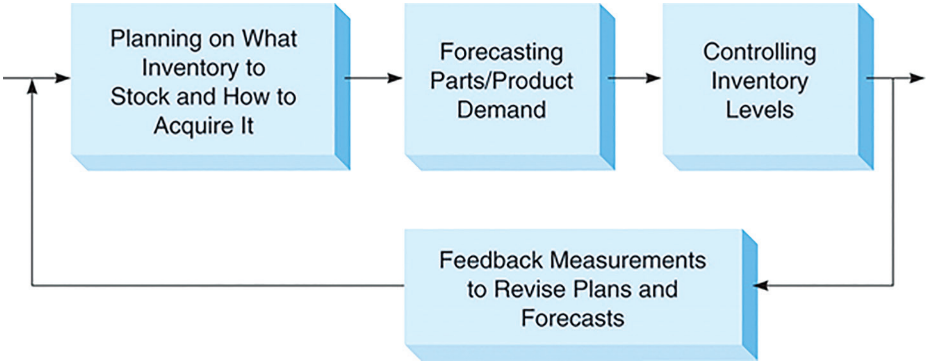


Figure 12.1: Inventory Planning and Control

Through inventory planning, an organization determines what goods and/or services are to be produced. In cases of physical products, the organization must also determine whether to produce these goods or to purchase them from another manufacturer. When this has been determined, the next step is to forecast the demand. As discussed in Chapter 11, many mathematical techniques can be used in forecasting demand for a particular product. The emphasis in this chapter is on inventory control—that is, how to maintain adequate inventory levels within an organization to support a production or procurement plan that will satisfy the forecasted demand.

In this chapter, we discuss several different inventory control models that are commonly used in practice. For each model, we provide examples of how they are analyzed. Although we show the equations needed to compute the relevant parameters for each model, we use Excel worksheets (included in ExcelModules) to actually calculate these values.

12.2 Importance of Inventory Control

Inventory control serves several important functions and adds a great deal of flexibility to the operation of a firm. Five main uses of inventory are as follows:
1. The decoupling function
2. Storing resources
3. Managing irregular supply and demand
4. Quantity discounts
5. Avoiding stockouts and shortages

Decoupling Function

One of the major functions of inventory is to decouple manufacturing processes within the organization. If a company did not store inventory, there could be many delays and inefficiencies. For example, when one manufacturing activity has to be completed before a second activity can be started, it could stop the entire process. However, stored inventory between processes could act as a buffer.

Storing Resources

Agricultural and seafood products often have definite seasons over which they can be harvested or caught, but the demand for these products is somewhat constant during the year. In these and similar cases, inventory can be used to store these resources.

In a manufacturing process, raw materials can be stored by themselves, as work-in-process, or as finished products. Thus, if your company makes lawn mowers, you might obtain lawn mower tires from another manufacturer. If you have 400 finished lawn mowers and 300 tires in inventory, you actually have 1,900 tires stored in inventory. Three hundred tires are stored by themselves, and 1,600 (= 4 tires per lawn mower × 400 lawn mowers) tires are stored on the finished lawn mowers. In the same sense, labor can be stored in inventory. If you have 500 subassemblies and it takes 50 hours of labor to produce each assembly, you actually have 25,000 labor hours stored in inventory in the subassemblies. In general, any resource, physical or otherwise, can be stored in inventory.

Managing Irregular Supply and Demand

When the supply or demand for an inventory item is irregular, storing certain amounts in inventory can be important. If the greatest demand for Diet-Delight beverage is during the summer, the Diet-Delight company will have to make sure there is enough supply to meet this irregular demand. This might require that the company produce more of the soft drink in the winter than is actually needed in order to meet the winter demand. The inventory levels of Diet-Delight will gradually build up over the winter, but this inventory will be needed in the summer. The same is true for irregular *supplies*.

Quantity Discounts

Another use of inventory is to take advantage of quantity discounts. Many suppliers offer discounts for large orders. For example, an electric jigsaw might normally cost $10 per unit. If you order 300 or more saws at one time, your supplier may lower the cost to $8.75. Purchasing in larger quantities can substantially reduce the cost of products. There are, however, some disadvantages of buying in larger quantities. You will have higher storage costs and higher costs due to spoilage, damaged stock, theft, insurance, and so on. Furthermore, if you invest in more inventory, you will have less cash to invest elsewhere.

Avoiding Stockouts and Shortages

Another important function of inventory is to avoid shortages or stockouts. If a company is repeatedly out of stock, customers are likely to go elsewhere to satisfy their needs. Lost goodwill can be an expensive price to pay for not having the right item at the right time.

12.3 Inventory Control Decisions

Even though there are literally millions of different types of products manufactured in our society, there are only two fundamental decisions that you have to make when controlling inventory:

1. How much to order
2. When to order

The purpose of all inventory models is to determine how much to order and when to order. As you know, inventory fulfills many important functions in an organization. But as the inventory levels go up to provide these functions, the cost of storing and holding inventory also increases. Thus, we must reach a fine balance in establishing

inventory levels. A major objective in controlling inventory is to minimize total inventory costs. Some of the most significant inventory costs are as follows:
1. Cost of the items
2. Cost of ordering
3. Cost of carrying, or holding, inventory
4. Cost of stockouts
5. Cost of safety stock, the additional inventory that may be held to help avoid stockouts

The inventory models discussed in the first part of this chapter assume that demand and the time it takes to receive an order are known and constant, and that no quantity discounts are given. When this is the case, the most significant costs dependent on order size are the cost of placing an order and the cost of holding inventory items over a period of time. Table 12.1 provides a list of important factors that make up these costs. Later in this chapter we discuss several more sophisticated inventory models.

Table 12.1: Inventory Cost Factors

Ordering Cost Factors	Carrying Cost Factors
Developing and sending purchase orders	Cost of capital
Processing and inspecting incoming inventory	Taxes
Bill paying	Insurance
Inventory inquiries	Spoilage
Utilities, phone bills, and so on for the purchasing department	Theft
Salaries and wages for purchasing department employees	Obsolescence
Supplies such as forms and paper for the purchasing department	Salaries and wages for warehouse employees
	Utilities and building costs for the warehouse
	Supplies such as forms and papers for the warehouse

12.4 Economic Order Quantity: Determining How Much to Order

The economic order quantity (EOQ) model is one of the oldest and most commonly known inventory control techniques. Research on its use dates back to a 1915 publication by Ford W. Harris. This model is still used by a large number of organizations today. This technique is relatively easy to use, but it makes a number of assumptions. Some of the more important assumptions follow:
1. Demand is known and constant.
2. The lead time—that is, the time between the placement of the order and the receipt of the order—is known and constant.

3. The receipt of inventory is instantaneous. In other words, the inventory from an order arrives in one batch, at one point in time.
4. Quantity discounts are not possible.
5. The only variable costs are the cost of placing an order, ordering cost, and the cost of holding or storing inventory over time, carrying, or holding, cost.
6. If orders are placed at the right time, stockouts and shortages can be avoided completely.

With these assumptions, inventory usage has a sawtooth shape, as in Figure 12.2. Here, Q represents the amount that is ordered. If this amount is 500 units, all 500 units arrive at one time when an order is received. Thus, the inventory level jumps from 0 to 500 units. In general, the inventory level increases from 0 to Q units when an order arrives.

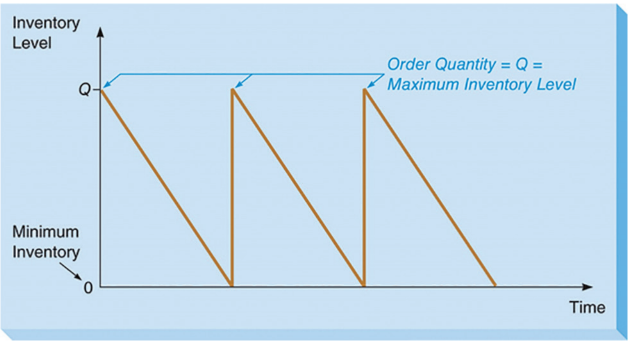


Figure 12.2: Inventory Usage over Time

Because demand is constant over time, inventory drops at a uniform rate over time. (Refer to the sloped line in Figure 12.2.) Another order is placed such that when the inventory level reaches 0, the new order is received and the inventory level again jumps to Q units, represented by the vertical lines. This process continues indefinitely over time.

Ordering and Inventory Costs

The objective of most inventory models is to minimize the total cost. With the assumptions just given, the significant costs are the ordering cost and the inventory carrying cost. All other costs, such as the cost of the inventory itself, are constant. Thus, if we minimize the sum of the ordering and carrying costs, we also minimize the total cost.

To help visualize this, Figure 12.3 graphs total cost as a function of the order quantity, Q. As the value of Q increases, the total number of orders placed per year decreases. Hence, the total ordering cost decreases. However, as the value of Q increases, the carrying cost increases because the firm has to maintain larger average inventories.

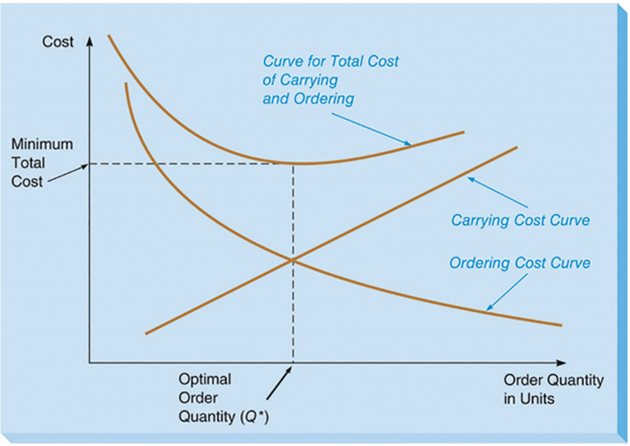


Figure 12.3: Total Cost as a Function of Order Quantity

The optimal order size, Q^*, is the quantity that minimizes the total cost. Note in Figure 12.3 that Q^* occurs at the point where the ordering cost curve and the carrying cost curve intersect. This is not by chance. With this particular type of cost function, the optimal quantity always occurs at a point where the ordering cost is equal to the carrying cost.

Decision Modeling In Action

Optimizing Inventory at Procter & Gamble

Procter & Gamble (P&G) is a world leader in consumer products with annual sales of over $76 billion. Managing inventory in such a large and complex organization requires making effective use of the right people, organizational structure, and tools. P&G's logistics planning personnel coordinate material flow, capacity, inventory, and logistics for the firm's extensive supply chain network, which comprises 145 P&G-owned manufacturing facilities, 300 contract manufacturers, and 6,900 unique product-category market combinations. Each supply chain requires effective management based on the latest available information, communication, and planning tools to handle complex challenges and trade-offs on issues such as production batch sizes, order policies, replenishment timing, new-product introductions, and assortment management.

Through the effective use of inventory optimization models, P&G has reduced its total inventory investment significantly. Spreadsheet-based inventory models that locally optimize different portions of the supply chain drive nearly 60 percent of P&G's business. For more complex supply chain networks (which drive about 30 percent of P&G's business), advanced multi-stage models yield additional average inventory reductions of 7 percent. P&G estimates that the use of these tools was instrumental in driving $1.5 billion in cash savings in 2009, while maintaining or increasing service levels.

Source: Based on I. Farasyn et al. "Inventory Optimization at Procter & Gamble: Achieving Real Benefits Through User Adoption of Inventory Tools," *Interfaces* 41, 1 (January–February 2011): 66–78.

Now that we have a better understanding of inventory costs, let us see how we can determine the value of Q^* that minimizes the total cost. In determining the *annual* carrying cost, it is convenient to use the average inventory. Referring to Figure 12.2, we see that the on-hand inventory ranges from a high of Q units to a low of zero units, with a uniform rate of decrease between these levels. Thus, the average inventory can be calculated as the average of the minimum and maximum inventory levels. That is,

$$\text{Average inventory level} = (0 + Q)/2 = Q/2 \tag{12–1}$$

We multiply this average inventory by a factor called the *annual inventory carrying cost per unit* to determine the annual inventory cost.

Finding the Economic Order Quantity

We pointed out that the optimal order quantity, Q^*, is the point that minimizes the total cost, where total cost is the sum of ordering cost and carrying cost. We also indicated graphically that the optimal order quantity was at the point where the ordering cost was equal to the carrying cost. Let us now define the following parameters:

$$Q^* = \text{Optimal order quantity (i.e., the EOQ)}$$

$$D = \text{Annual demand, in units, for the inventory item}$$

$$C_o = \text{Ordering cost per order}$$

$$C_h = \text{Carrying or holding cost per unit per year}$$

$$P = \text{Purchase cost per unit of the inventory item}$$

The unit carrying cost, C_h, is usually expressed in one of two ways, as follows:

1. As a fixed cost. For example, C_h is \$0.50 per unit per year.
2. As a percentage (typically denoted by I) of the item's unit purchase cost or price. For example, C_h is 20% of the item's unit cost. In general,

$$C_h = I \times P \tag{12–2}$$

For a given order quantity Q, the ordering, holding, and total costs can be computed using the following formulas:[1]

$$\text{Total ordering cost} = (D/Q) \times C_o \tag{12–3}$$

$$\text{Total carrying cost} = (Q/2) \times C_h \tag{12–4}$$

$$\begin{aligned} \text{Total cost} &= \text{Total ordering cost} + \text{Total carrying cost} + \text{Total purchase cost} \\ &= (D/Q) \times C_o + (Q/2) \times C_h + P \times D \end{aligned} \tag{12–5}$$

1 See a recent operations management book such as J. Heizer, B. Render, and C. Munson. *Operations Management: Sustainability and Supply Chain Management*, 12th ed. Boston: Pearson, 2017, for more details of these formulas (and other formulas in this chapter).

Observe that the total purchase cost (i.e., $P \times D$) does not depend on the value of Q because, regardless of how many orders we place each year, or how many units we order each time, we will still incur the same annual total purchase cost.

The presence of Q in the denominator of the first term makes Equation 12–5 a *nonlinear* equation with respect to Q. Nevertheless, because the total ordering cost is equal to the total carrying cost at the optimal value of Q, we can set the terms in Equations 12–3 and 12–4 equal to each other and calculate the EOQ as

$$Q^{\star} = \sqrt{(2DC_o / C_h)} \qquad \textbf{(12–6)}$$

Sumco Pump Company Example

Let us now apply these formulas to the case of Sumco, a company that buys pump housings from a manufacturer and distributes to retailers. Sumco would like to reduce its inventory cost by determining the optimal number of pump housings to obtain per order. The annual demand is 1,000 units, the ordering cost is $10 per order, and the carrying cost is $0.50 per unit per year. Each pump housing has a purchase cost of $5. How many housings should Sumco order each time? To answer these and other questions, we use the ExcelModules program.

EXCEL NOTES

- The Companion Website for this book, at *degruyter.com/view/product/486941*, contains a set of Excel worksheets, bundled together in a software package called ExcelModules. Appendix B describes the procedure for installing and running this program, and it gives a brief description of its contents.
- The Companion Website also provides the Excel file for each sample problem discussed here. The relevant file/sheet name is shown below the title of the corresponding figure in this book.
- For clarity, all worksheets for inventory models in ExcelModules are color coded as follows:
  - *Input cells,* where we enter the problem data, are shaded yellow.
  - *Output cells,* which show results, are shaded green.

Using ExcelModules for Inventory Model Computations

When we run the ExcelModules program, we see a new tab titled ExcelModules in Excel’s Ribbon. We select this tab and then click the Modules icon followed by the Inventory Models menu. The choices shown in Figure 12.4 are displayed. From these choices, we select the appropriate model.

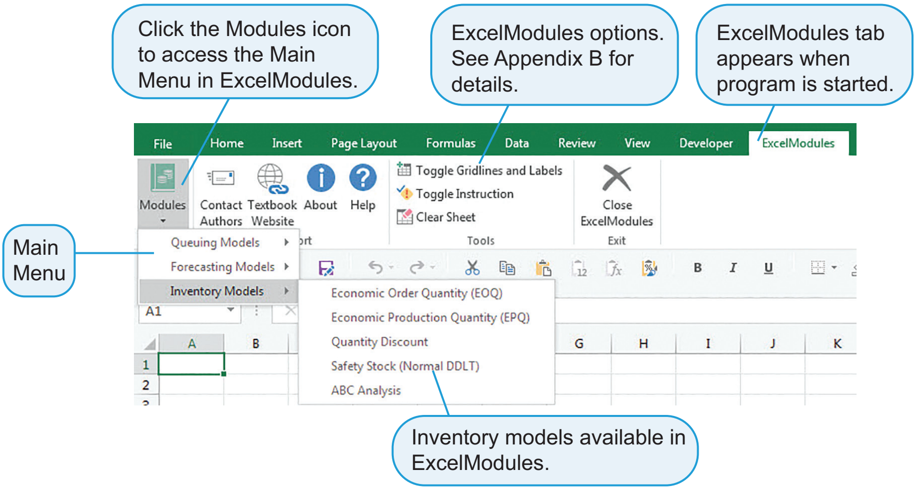


Figure 12.4: Inventory Models Menu in ExcelModules

When we select any of the inventory models in ExcelModules, we are first presented with a window that allows us to specify several options. Some of these options are common for all models, whereas others are specific to the inventory model selected. For example, Figure 12.5 shows the options window that appears when we select the Economic Order Quantity (EOQ) model.

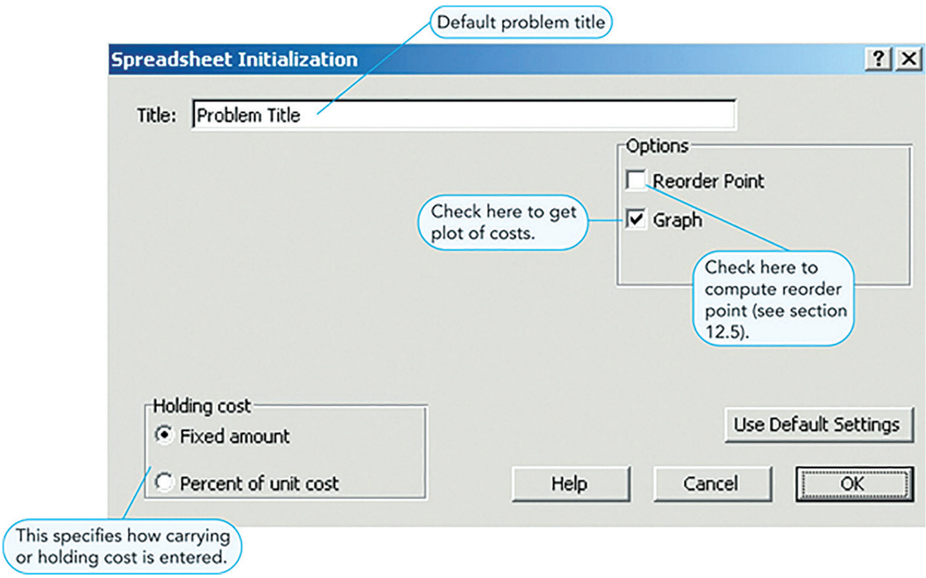


Figure 12.5: Sample Options Window for Inventory Models in ExcelModules

The options in Figure 12.5 include the following:

1. *Title*. The default value is Problem Title.
2. *Graph*. Checking this box results in a graph of ordering, carrying, and total costs versus order quantity.
3. *Holding cost*. This is either a fixed amount or a percentage of unit purchase cost.
4. *Reorder Point*. Checking this box results in the calculation of the reorder point, for a given lead time between placement of the order and receipt of the order. We discuss the reorder point in section 12.5. This option is available only for the EOQ model.

Using ExcelModules for the EOQ Model Figure 12.6 shows the options we select for the Sumco Pump Company example.

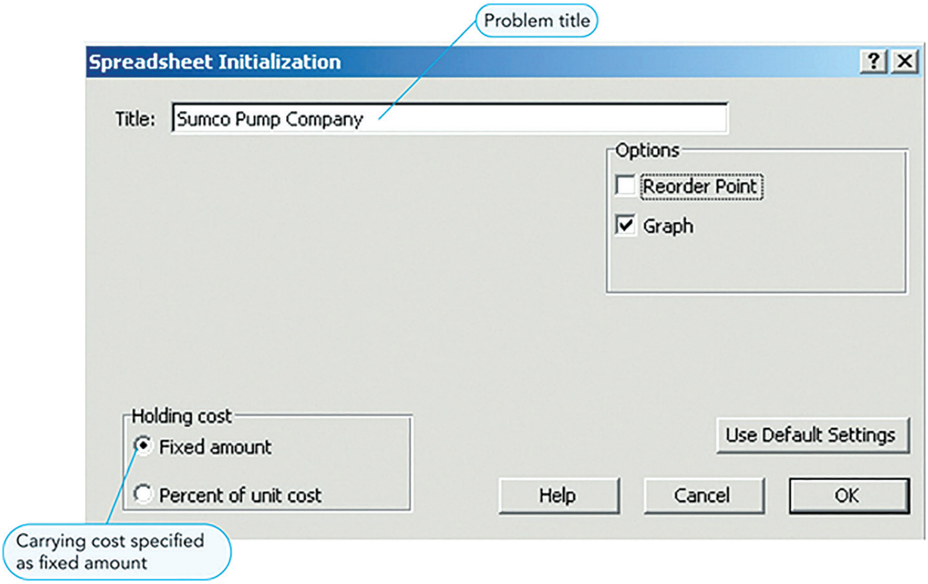
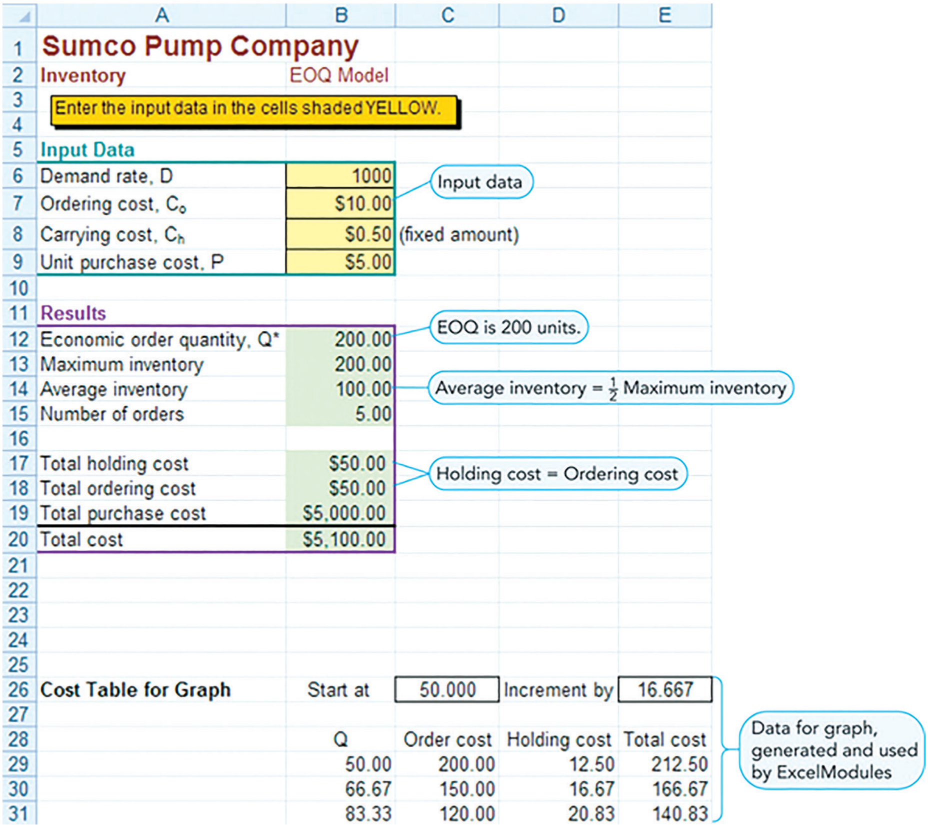


Figure 12.6: Options Window for EOQ Model in ExcelModules

When we click OK on this screen, we get the worksheet shown in Figure 12.7. We now enter the values for the annual demand, D, ordering cost, C_o, carrying cost, C_h, and unit purchase cost, P, in cells B6 to B9, respectively.



	A	B	C	D	E
1	Sumco Pump Company				
2	Inventory	EOQ Model			
3	Enter the input data in the cells shaded YELLOW.				
4					
5	Input Data				
6	Demand rate, D	1000			
7	Ordering cost, C_o	$10.00			
8	Carrying cost, C_h	$0.50	(fixed amount)		
9	Unit purchase cost, P	$5.00			
10					
11	Results				
12	Economic order quantity, Q*	200.00			
13	Maximum inventory	200.00			
14	Average inventory	100.00			
15	Number of orders	5.00			
16					
17	Total holding cost	$50.00			
18	Total ordering cost	$50.00			
19	Total purchase cost	$5,000.00			
20	Total cost	$5,100.00			
21					
22					
23					
24					
25					
26	Cost Table for Graph	Start at	50.000	Increment by	16.667
27					
28		Q	Order cost	Holding cost	Total cost
29		50.00	200.00	12.50	212.50
30		66.67	150.00	16.67	166.67
31		83.33	120.00	20.83	140.83

Figure 12.7: EOQ Model for Sumco Pump

File: Figure 12.7.xlsx, Sheet: Figure 12.7

EXCEL NOTES

– The worksheets in ExcelModules contain formulas to compute the results for different inventory models. The default value of zero for the input data causes the results of these formulas to initially appear as #N/A, #VALUE!, or #DIV/0!. However, as soon as we enter valid values for these input data, the worksheets display the formula results.

– Once ExcelModules has been used to create the Excel worksheet for a particular inventory model (e.g., EOQ), the resulting worksheet can be used to compute the results with several different input data. For example, we can enter different input data in cells B6:B9 of Figure 12.7 and compute the results without having to create a new EOQ worksheet each time.

The worksheet calculates the EOQ (shown in cell B12 of Figure 12.7). In addition, the following output measures are calculated and reported:

– Maximum inventory $(= Q^*)$, in cell B13
– Average inventory $(= Q^*/2)$, in cell B14
– Number of orders $(= D/Q^*)$, in cell B15

- Total holding cost $(= C_h \times Q^*/2)$, in cell B17
- Total ordering cost $(= C_o \times D/Q^*)$, in cell B18
- Total purchase cost $(= P \times D)$, in cell B19
- Total cost $(= C_h \times Q^*/2 + C_o \times D/Q^* + P \times D)$, in cell B20

As you might expect, the total ordering cost of $50 is equal to the total carrying cost. (Refer to Figure 12.3 again to see why.) You may wish to try different values for the order quantity *Q*, such as 100 or 300 pump housings. (Plug in these values one at a time in cell B12.) You will find that the total cost (in cell B20) has the lowest value when *Q* is 200 units. That is, the EOQ, *Q**, for Sumco is 200 pump housings. The total cost, including the purchase cost of $5,000, is $5,100.

If requested, a plot of the total ordering cost, total holding cost and total cost for different values of *Q* is drawn by ExcelModules. The graph, shown in Figure 12.8, is drawn on a separate worksheet.

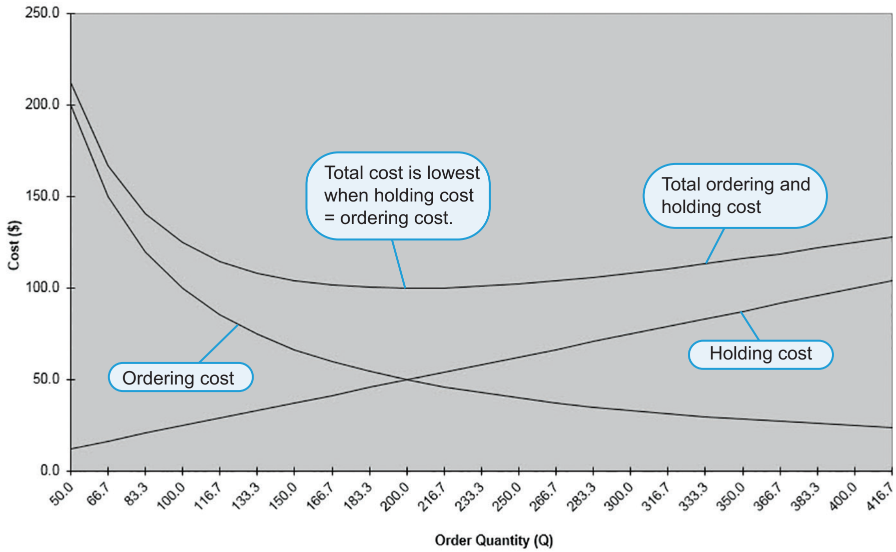


Figure 12.8: Plot of Costs versus Order Quantity for Sumco Pump

File: Figure 12.7.xlsx, Sheet: Figure 12.8

Purchase Cost of Inventory Items

It is often useful to know the value of the average inventory level in dollar terms. We know from Equation 12–1 that the average inventory level is *Q/2*, where *Q* is the order quantity. If we order *Q** (the EOQ) units each time, the value of the average inventory can be computed by multiplying the average inventory by the unit purchase cost, *P*. That is,

$$\text{Average dollar value of inventory} = P \times (Q^*/2) \qquad \textbf{(12–7)}$$

Calculating the Ordering and Carrying Costs for a Given Value of *Q*

Recall that the EOQ formula is given by Equation 12–6 as

$$Q^* = \sqrt{(2DC_0 / C_h)}$$

In using this formula, we assumed that the values of the ordering cost C_o and carrying cost C_h are *known* constants. In some situations, however, these costs may be difficult to estimate precisely. For example, if the firm orders several items from a supplier simultaneously, it may be difficult to identify the ordering cost separately for each item. In such cases, we can use the EOQ formula to compute the value of C_o or C_h that would make a given order quantity the optimal order quantity.

To compute these C_o or C_h values, we can manipulate the EOQ formula algebraically and rewrite it as follows:

$$C_o = Q^2 \times C_h / (2D) \tag{12–8}$$

and

$$C_h = 2DC_o / Q^2 \tag{12–9}$$

where Q is the given order quantity. We illustrate the use of these formulas in Solved Problem 12–1 at the end of this chapter.

Sensitivity of the EOQ Formula

The EOQ formula in Equation 12–6 assumes that all input data are known with certainty. What happens if one of the input values is incorrect? If any of the values used in the formula changes, the optimal value of Q^* also changes. Determining the magnitude and effect of these changes on Q^* is called *sensitivity analysis*. This type of analysis is important in practice because the input values for the EOQ model are usually *estimated* and hence subject to error or change.

Let us use the Sumco example again to illustrate this issue. Suppose the ordering cost, C_o, is actually \$15, instead of \$10. Let us assume that the annual demand for pump housings is still the same, namely, $D = 1{,}000$ units, and that the carrying cost, C_h, is \$0.50 per unit per year.

If we use these new values in the EOQ worksheet (as in Figure 12.7), the revised EOQ turns out to be 245 units. (See if you can verify this for yourself.) That is, when the ordering cost increases by 50% (from \$10 to \$15), the optimal order quantity increases only by 22.5% (from 200 to 245). This is because the EOQ formula involves a square root and is, therefore, *nonlinear*.

We observe a similar occurrence when the carrying cost, C_h, changes. Let us suppose that Sumco's annual carrying cost is \$0.80 per unit instead of \$0.50. Let us also assume that the annual demand is still 1,000 units, and the ordering cost is \$10 per order. Using the EOQ worksheet in ExcelModules, we can calculate the revised EOQ as 158 units. That is, when the carrying cost increases by 60% (from \$0.50 to

$0.80), the EOQ decreases by only 21%. Note that the order quantity decreases here because a higher carrying cost makes holding inventory more expensive.

12.5 Reorder Point: Determining When to Order

Now that we have decided how much to order, we look at the second inventory question: when to order. In most simple inventory models, it is assumed that we have instantaneous inventory receipt. That is, we assume that a firm waits until its inventory level for a particular item reaches zero, places an order, and receives the items in stock immediately.

In many cases, however, the time between the placing and receipt of an order, called the *lead time*, or delivery time, is often a few days or even a few weeks. Thus, the when to order decision is usually expressed in terms of a reorder point (ROP), the inventory level at which an order should be placed. The ROP is given as

$$\begin{aligned} \text{ROP} &= (\text{Demand per day}) \times (\text{Lead time, in days}) \\ &= d \times L \end{aligned} \qquad \textbf{(12–10)}$$

Figure 12.9 shows the reorder point graphically. The slope of the graph is the daily inventory usage. This is expressed in units demanded per day, *d*. The *lead time*, *L*, is the time that it takes to receive an order. Thus, if an order is placed when the inventory level reaches the ROP, the new inventory arrives at the same instant the inventory is reaching zero. Let's look at an example.

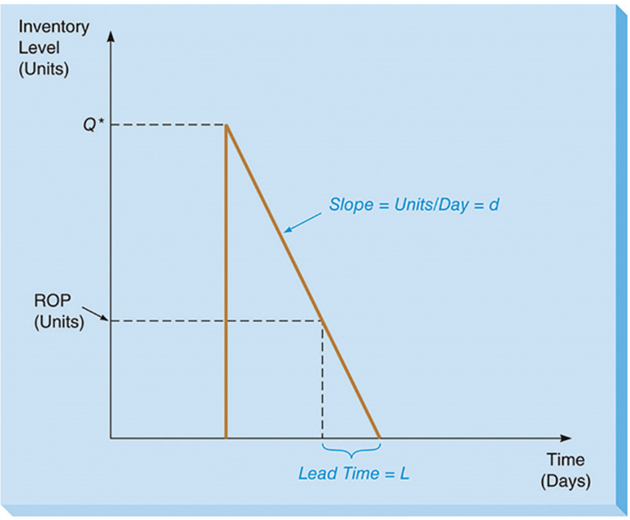


Figure 12.9: Reorder Point (ROP) Curve

Sumco Pump Company Example Revisited

Recall that we calculated an EOQ value of 200 and a total cost of $5,100 for Sumco (see Figure 12.7). These calculations were based on an annual demand of 1,000 units, an ordering cost of $10 per order, an annual carrying cost of $0.50 per unit, and a purchase cost of $5 per pump housing.

Now let us assume that there is a lead time of 3 business days between the time Sumco places an order and the time the order is received. Further, let us assume there are 250 business days in a year.

To calculate the ROP, we must first determine the daily demand rate, d. In Sumco's case, because there are 250 business days in a year and the annual demand is 1,000, the daily demand rate is $4\ (=1{,}000/250)$ pump housings.

Using ExcelModules to Compute the ROP We can include the ROP computation in the EOQ worksheet provided in ExcelModules. To do so for Sumco's problem, we once again select Economic Order Quantity (EOQ) from the Inventory Models menu in ExcelModules (refer to Figure 12.4). The only change in the options window (see Figure 12.6) is that we now check the box labeled Reorder Point. The worksheet shown in Figure 12.10 is now displayed. We enter the input data as before (see Figure 12.7). Note the additional input entries for the daily demand rate in cell B10 and the lead time in cell B11. In addition to all the computations shown in Figure 12.7, the worksheet now calculates and reports the ROP of 12 units (shown in cell B24).

	A	B	C
1	Sumco Pump Company (Revisited)		
2	Inventory	EOQ Model	
3	Enter the input data in the cells shaded YELLOW.		
4			
5	Input Data		
6	Demand rate, D	1000	
7	Ordering cost, C_s	$10.00	
8	Carrying cost, C_h	$0.50	(fixed amount)
9	Unit purchase cost, P	$5.00	
10	Daily demand rate, d	4	Input data for computing ROP
11	Lead time (in days), L	3	
12			
13	Results		
14	Economic order quantity, Q*	200.00	EOQ is 200 units.
15	Maximum inventory	200.00	
16	Average inventory	100.00	
17	Number of orders	5.00	
18			
19	Total holding cost	$50.00	
20	Total ordering cost	$50.00	
21	Total purchase cost	$5,000.00	
22	Total cost	$5,100.00	
23			
24	Reorder point, ROP	12	ROP is 12 units.

Figure 12.10: EOQ Model with ROP for Sumco Pump

File: Figure 12.10.xlsx

Hence, when the inventory stock of pump housings drops to 12, an order should be placed. The order will arrive three days later, just as the firm's stock is depleted to zero. It should be mentioned that this calculation assumes that all the assumptions listed earlier for EOQ are valid. When demand is not known with complete certainty, these calculations must be modified. This is discussed later in this chapter.

Decision Modeling In Action

Dell Uses Inventory Optimization Models in Its Supply Chain

Dell was the leader in global market share in the computer-systems industry during the early 2000s. It was also the fastest-growing company in this industry, competing in multiple market segments. Dell was founded on the concept of selling computer systems directly to customers without a retail middleman, thereby reducing delays and costs due to the elimination of this stage in the supply chain. Dell's superior financial performance during this period can be attributed in large measure to its successful implementation of this direct-sales model.

In the rapidly evolving computer systems industry, holding inventory is a huge liability. It is common for components to lose 0.5 to 2.0 percent of their value each week, rendering a supply chain filled with old technology obsolete in a short time. Within Dell, the focus was on speeding components and products through its supply chain, while carrying very little inventory. Dell's management was, however, keen that its suppliers also hold just the right inventory to ensure a high level of customer service while reducing total costs. Working with a team from the University of Michigan, Dell identified a sustainable process and decision-support tools for determining optimal levels of component inventory at different stages of the supply chain to support the final assembly process.

The tools allowed Dell to change how inventory was being pulled from supplier logistics centers into Dell's assembly facilities, resulting in a more linear, predictable product pull for suppliers. The direct benefit for Dell is that there is more robustness in supply continuity and suppliers are better able to handle unexpected demand variations.

Source: Based on R. Kapuscinski et al. "Inventory Decisions in Dell's Supply Chain," *Interfaces* 34, 3 (May–June 2004): 191–205.

12.6 Economic Production Quantity: Determining How Much to Produce

In the EOQ model, we assumed that the receipt of inventory is instantaneous. In other words, the entire order arrives in one batch, at a single point in time. In many cases, however, a firm may build up its inventory gradually over a period of time. For example, a firm may receive shipments from its supplier uniformly over a period of time. Or, a firm may be producing at a rate of p per day *and* simultaneously selling at a rate of d per day (where $p > d$). Figure 12.11 shows inventory levels as a function of time in these situations.

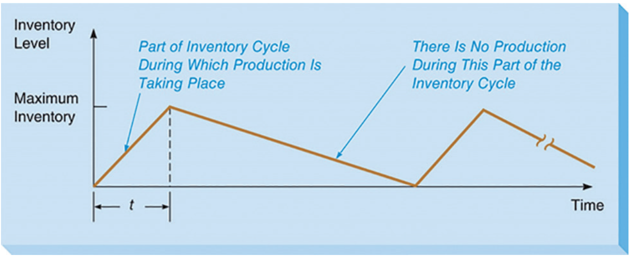


Figure 12.11: Inventory Control and the Production Process

Clearly, the EOQ model is no longer applicable here, and we need a new model to calculate the optimal order (or production) quantity. Because this model is especially suited to the production environment, it is also commonly known as the *production lot size* model or the economic production quantity (EPQ) model. We refer to this model as the EPQ model in the remainder of this chapter.

In a production process, instead of having an ordering cost, there will be a setup cost. This is the cost of setting up the production facility to manufacture the desired product. It normally includes the salaries and wages of employees who are responsible for setting up the equipment, engineering and design costs of making the setup, and the costs of paperwork, supplies, utilities, and so on. The carrying cost per unit is composed of the same factors as the traditional EOQ model, although the equation to compute the annual carrying cost changes.

In determining the annual carrying cost for the EPQ model, it is again convenient to use the *average* on-hand inventory. Referring to Figure 12.11, we can show that the maximum on-hand inventory is $Q \times (1 - d/p)$ units, where d is the daily demand rate and p is the daily production rate. (The maximum inventory level does not simply equal Q—as in the EOQ model—because every day some of the units that are produced are passed on to customers. Thus, total inventory will never equal the full lot-size quantity.) The minimum on-hand inventory is again zero units, and the inventory decreases at a uniform rate between the maximum and minimum levels. Thus, the average inventory can be calculated as the average of the minimum and maximum inventory levels. That is,

$$\text{Average inventory level} = \left[0 + Q \times (1 - d/p)\right]/2 = Q \times (1 - d/p)/2 \qquad \textbf{(12–11)}$$

Analogous to the EOQ model, it turns out that the optimal order quantity in the EPQ model also occurs when the total setup cost equals the total carrying cost. We should note, however, that making the total setup cost equal to the total carrying cost does not always guarantee optimal solutions for models more complex than the EPQ model.

Finding the Economic Production Quantity

Let us first define the following additional parameters:

$$Q^{*} = \text{Optimal order or production quantity (i.e., the EPQ)}$$

$$C_s = \text{Setup cost per setup}$$

For a given order quantity, Q, the setup, holding, and total costs can now be computed using the following formulas:

$$\text{Total setup cost} = (D/Q) \times C_s \tag{12–12}$$

$$\text{Total carrying cost} = [Q(1 - d/p)/2] \times C_h \tag{12–13}$$

$$\begin{aligned} \text{Total cost} &= \text{Total setup cost} &&+ \text{Total carrying cost} &&+ \text{Total production cost} \\ &= (D/Q) \times C_s &&+ [Q(1 - d/p)/2] \times C_h &&+ P \times D \end{aligned} \tag{12–14}$$

As in the EOQ model, the total production (or purchase, if the item is purchased) cost does not depend on the value of Q. Further, the presence of Q in the denominator of the first term makes the total cost function nonlinear. Nevertheless, because the total setup cost should equal the total ordering cost at the optimal value of Q, we can set the terms in Equations 12–12 and 12–13 equal to each other and calculate the EPQ as

$$Q^{*} = \sqrt{2DC_s / [C_h(1 - d/p)]} \tag{12–15}$$

Brown Manufacturing Example

Brown Manufacturing produces mini-sized refrigeration packs in batches. The firm's estimated demand for the year is 10,000 units. Because Brown operates for 167 business days each year, this annual demand translates to a daily demand rate of about 60 units per day. It costs about \$100 to set up the manufacturing process, and the carrying cost is about \$0.50 per unit per year. When the production process has been set up, 80 refrigeration packs can be manufactured daily. Each pack costs \$5 to produce. How many packs should Brown produce in each batch? As discussed next, we determine this value, as well as values for the associated costs, by using ExcelModules.

Using ExcelModules for the EPQ Model We select the choice titled Economic Production Quantity (EPQ) from the Inventory Models menu in ExcelModules (refer to Figure 12.4). The options for this procedure are similar to those for the EOQ model (see Figure 12.6). The only change is that the ROP option is not available here. After we enter the title and other options for this problem, we get the worksheet shown in Figure 12.12. We now enter the values for the annual demand, D, setup cost, C_s, carrying cost, C_h, daily production rate, p, daily demand rate, d, and unit production (or purchase) cost, P, in cells B7 to B12, respectively.

	A	B	C	D	E	
1	**Brown Manufacturing**					
2	Inventory	Economic Production Quantity Model				
3	Enter the data in the cells shaded YELLOW. You may have to do some					
4	work to compute the daily production and demand rates.					
5						
6	Input Data					
7	Demand rate, D	10000				
8	Setup cost, C_s	$100.00				
9	Carrying cost, C_h	$0.50	(fixed amount)			
10	Daily production rate, p	80				
11	Daily demand rate, d	60	Carrying cost is specified as a fixed amount.			
12	Unit purchase cost, P	$5.00				
13						
14	Results					
15	Economic production quantity, Q*	4000.00	EPQ is 4,000 units.			
16	Maximum inventory	1000.00				
17	Average inventory	500.00				
18	Number of setups	2.50				
19						
20	Total holding cost	$250.00	Holding cost = Setup cost			
21	Total setup cost	$250.00				
22	Total production cost	$50,000.00				
23	Total cost, TC	$50,500.00				
24						
25						
26	**Cost Table for Graph**	Start at	1000.00	Increment by	333.33	
27						
28		Q	Setup cost	Holding cost	Total cost	Data for graph, generated and used by ExcelModules
29		1000.00	1000.00	62.50	1062.50	
30		1333.33	750.00	83.33	833.33	
31		1666.67	600.00	104.17	704.17	

Figure 12.12: EPQ Model for Brown Manufacturing

File: Figure 12.12.xlsx, Sheet: Figure 12.12

The worksheet calculates and reports the EPQ (shown in cell B15), as well as the following output measures:

– Maximum inventory $(=Q^*[1-d/p])$, in cell B16
– Average inventory $(=Q^*[1-d/p]/2)$, in cell B17
– Number of setups $(=D/Q^*)$, in cell B18
– Total holding cost $(=C_h \times Q^*[1-d/p]/2)$, in cell B20
– Total setup cost $(=C_s \times D/Q^*)$, in cell B21
– Total purchase cost $(=P \times D)$, in cell B22
– Total cost Total cost $(=C_h \times Q^*[1-d/p]/2 + C_s \times D/Q^* + P \times D)$, in cell B23

Here again, as you might expect, the total setup cost is equal to the total carrying cost ($250 each). You may wish to try different values for Q, such as 3,000 or 5,000 pumps. (Plug these values, one at a time, into cell B15.) You will find that the minimum total cost occurs when Q is 4,000 units. That is, the EPQ, Q^*, for Brown is 4,000 units. The total cost, including the production cost of $50,000, is $50,500.

If requested, a plot of the total setup cost, holding cost, and total cost for different values of Q is drawn by ExcelModules. This graph, shown in Figure 12.13, is drawn on a separate worksheet.

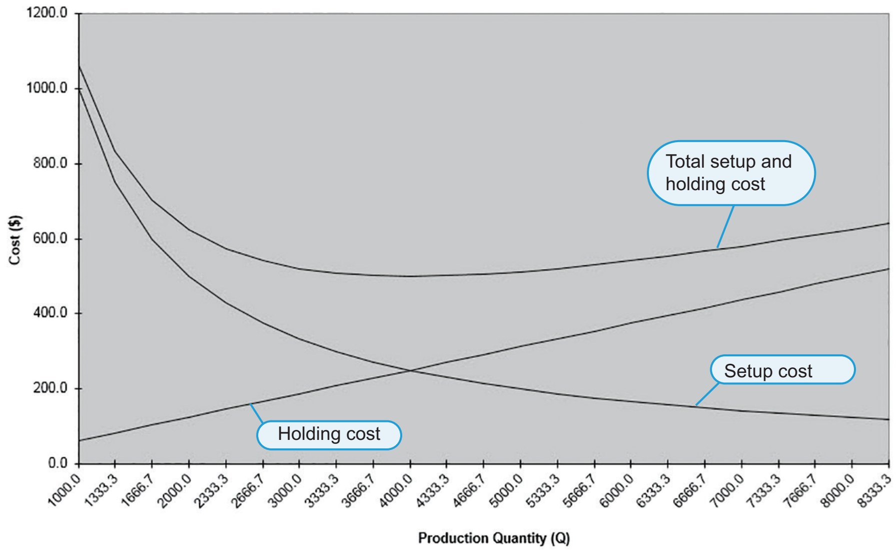


Figure 12.13: Plot of Costs versus Order Quantity for Brown Manufacturing

File: Figure 12.12.xlsx, Sheet: Figure 12.13

Length of the Production Cycle

Referring to Figure 12.11, we see that the inventory buildup occurs over a period t during which Brown is both producing and selling refrigeration packs. We refer to this period t as the *production cycle*. In Brown's case, if $Q^* = 4{,}000$ units and we know that 80 units can be produced daily, the length of each production cycle will be $Q^*/p = 4{,}000/80 = 50$ days. Thus, when Brown decides to produce refrigeration packs, the equipment will be set up to manufacture the units for a 50-day time span.

12.7 Quantity Discount Models

To increase sales, many companies offer quantity discounts to their customers. A quantity discount is simply a decreased unit cost for an item when it is purchased in larger quantities. It is not uncommon to have a discount schedule with several discounts for large orders. A typical quantity discount schedule is shown in Table 12.2.

Table 12.2: Quantity Discount Schedule

Discount Number	Discount Quantity	Discount	Discount Cost
1	0 to 999	0%	$5.00
2	1,000 to 1,999	4%	$4.80
3	2,000 and over	5%	$4.75

As can be seen in Table 12.2, the normal cost for the item in this example is $5. When 1,000 to 1,999 units are ordered at one time, the cost per unit drops to $4.80, and when the quantity ordered at one time is 2,000 units or more, the cost is $4.75 per unit. As always, management must decide when and how much to order. But with quantity discounts, how does a manager make these decisions?

As with other inventory models discussed so far, the overall objective is to minimize the total cost. Because the unit cost for the third discount in Table 12.2 is lowest, you might be tempted to order 2,000 units or more to take advantage of this discount. Placing an order for that many units, however, might not minimize the total inventory cost. As the discount quantity goes up, the item cost goes down, but the carrying cost increases because the order sizes are large. (Think of your own living arrangements. Would you purchase and store a full year's worth of toilet paper just because it was on sale?) Thus, the major trade-off when considering quantity discounts is between the reduced item cost and the increased carrying cost.

Recall that we computed the total cost (including the total purchase cost) for the EOQ model as follows (see Equation 12–5):

$$\begin{aligned}\text{Total cost} &= \text{Total ordering cost} &&+ \text{Total carrying cost} &&+ \text{Total purchase cost}\\ &= (\mathrm{D}/\mathrm{Q}) \times C_o &&+ (Q/2) \times C_h &&+ P \times D\end{aligned}$$

Next, we illustrate the four-step process to determine the quantity that minimizes the total cost. However, we use a worksheet included in ExcelModules to actually compute the optimal order quantity and associated costs in our example.

Four Steps to Analyze Quantity Discount Models

1. For each discount price, calculate a Q^* value, using the EOQ formula (see Equation 12–6). In quantity discount EOQ models, the unit carrying cost, C_h, is typically expressed as a percentage (I) of the unit purchase cost (P). That is, $C_h = I \times P$, as discussed in Equation 12–2. As a result, the value of Q^* will be different for each discounted price.
2. For any discount level, if the Q^* computed in step 1 is too low to qualify for the discount, adjust Q^* upward to the *lowest* quantity that qualifies for the discount. For example, if Q^* for discount 2 in Table 12.2 turns out to be 500 units, adjust this value up to 1,000 units. The reason for this step is illustrated in Figure 12.14.

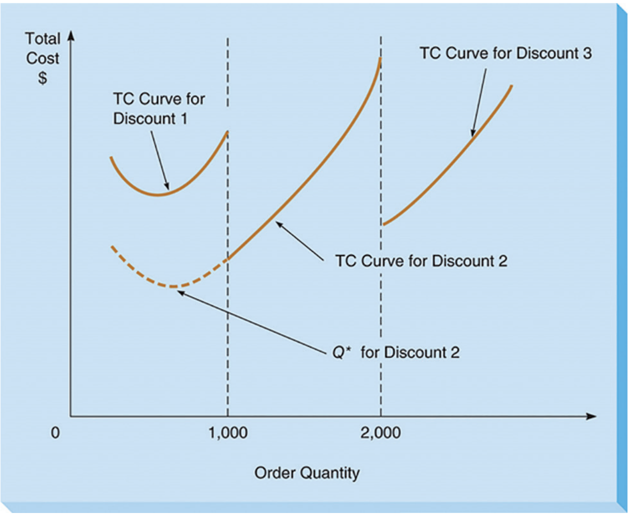


Figure 12.14: Total Cost Curve for the Quantity Discount Model

As seen in Figure 12.14, the total cost curve for the discounts shown in Table 12.2 is broken into three different curves. There are separate cost curves for the first $(0 \le Q \le 999)$, second $(1{,}000 \le Q \le 1{,}999)$, and third $(Q \ge 2{,}000)$ discounts. Look at the total cost curve for discount 2. The Q^* for discount 2 is less than the allowable discount range of 1,000 to 1,999 units. However, the total cost at 1,000 units (which is the minimum quantity needed to get this discount) is still less than the lowest total cost for discount 1. Thus, step 2 is needed to ensure that we do not discard any discount level that may indeed produce the minimum total cost. Note that an order quantity computed in step 1 that is *greater* than the range that would qualify it for a discount may be discarded.

3. Using the total cost equation (Equation 12–5), compute a total cost for every Q^* determined in steps 1 and 2. If a Q^* had to be adjusted upward because it was below the allowable quantity range, be sure to use the adjusted Q^* value.
4. Select the Q^* that has the lowest total cost, as computed in step 3. It will be the order quantity that minimizes the total cost.

Brass Department Store Example

Brass Department Store stocks toy cars. Recently, the store was given a quantity discount schedule for the cars, as shown in Table 12.2. Thus, the normal cost for the cars is \$5. For orders between 1,000 and 1,999 units, the unit cost is \$4.80, and for orders of 2,000 or more units, the unit cost is \$4.75. Furthermore, the ordering cost is \$49 per order, the annual demand is 5,000 race cars, and the inventory carrying charge as a percentage of cost, I, is 20%, or 0.2. What order quantity will minimize the total cost? We use the ExcelModules program to answer this question.

Using ExcelModules for the Quantity Discount Model We select the choice titled Quantity Discount from the Inventory Models menu in ExcelModules (refer back to Figure 12.4). The window shown in Figure 12.15 is displayed. The option entries in this window are similar to those for the EOQ model (see Figure 12.6). The only additional choice is the box labeled Number of price ranges. The specific entries for Brass Department Store's problem are shown in Figure 12.15.

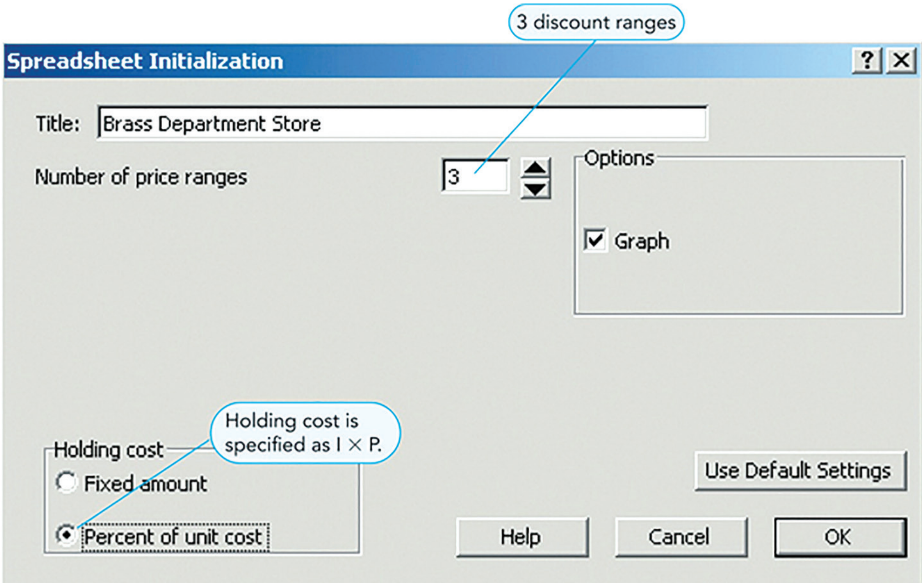
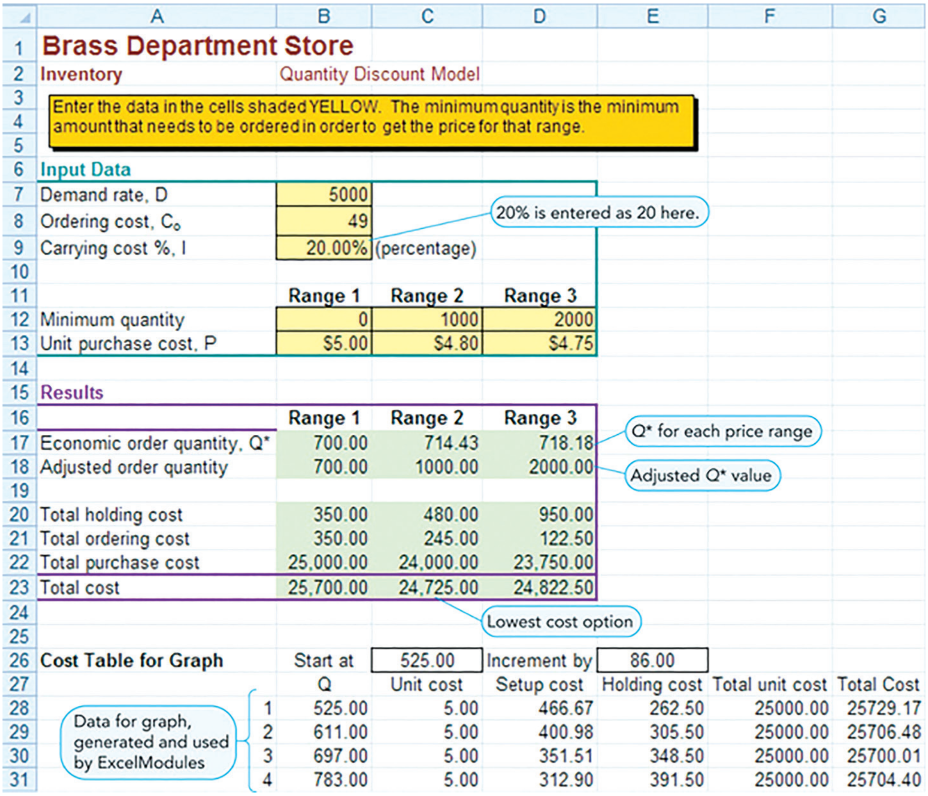


Figure 12.15: Options Window for Quantity Discount Model in ExcelModules

When we click OK on this screen, we get the worksheet shown in Figure 12.16. We now enter the values for the annual demand, D, ordering cost, C_o, and holding cost percentage, I, in cells B7 to B9, respectively. Note that I is entered as a percentage value (e.g., enter 20 for the Brass Department Store example). Then, for each of the three discount ranges, we enter the minimum quantity needed to get the discount and the discounted unit price, P. These entries are shown in cells B12:D13 of Figure 12.16.



	A	B	C	D	E	F	G
1	Brass Department Store						
2	Inventory	Quantity Discount Model					
3	Enter the data in the cells shaded YELLOW. The minimum quantity is the minimum						
4	amount that needs to be ordered in order to get the price for that range.						
5							
6	Input Data						
7	Demand rate, D	5000					
8	Ordering cost, C_o	49					
9	Carrying cost %, I	20.00%	(percentage)				
10							
11		Range 1	Range 2	Range 3			
12	Minimum quantity	0	1000	2000			
13	Unit purchase cost, P	$5.00	$4.80	$4.75			
14							
15	Results						
16		Range 1	Range 2	Range 3			
17	Economic order quantity, Q*	700.00	714.43	718.18			
18	Adjusted order quantity	700.00	1000.00	2000.00			
19							
20	Total holding cost	350.00	480.00	950.00			
21	Total ordering cost	350.00	245.00	122.50			
22	Total purchase cost	25,000.00	24,000.00	23,750.00			
23	Total cost	25,700.00	24,725.00	24,822.50			
24							
25							
26	Cost Table for Graph	Start at	525.00	Increment by	86.00		
27		Q	Unit cost	Setup cost	Holding cost	Total unit cost	Total Cost
28	1	525.00	5.00	466.67	262.50	25000.00	25729.17
29	2	611.00	5.00	400.98	305.50	25000.00	25706.48
30	3	697.00	5.00	351.51	348.50	25000.00	25700.01
31	4	783.00	5.00	312.90	391.50	25000.00	25704.40

Figure 12.16: Quantity Discount Model for Brass Department Store

File: Figure 12.16.xlsx, Sheet: Figure 12.16

The worksheet works through the four-step process and reports the following output measures for each discount range:

– EOQ value (shown in cells B17:D17), computed using Equation 12–6
– Adjusted EOQ value (shown in cells B18:D18), as discussed in step 2 of the four-step process
– Total holding cost, total ordering cost, total purchase cost, and overall total cost, shown in cells B20:D23

In the Brass Department Store example, observe that the Q^* values for discounts 2 and 3 are too low to be eligible for the discounted prices. They are, therefore, adjusted upward to 1,000 and 2,000, respectively. With these adjusted Q^* values, we find that the lowest total cost of $24,725 results when we use an order quantity of 1,000 units.

If requested, ExcelModules will also draw a plot of the total cost for different values of Q. This graph, shown in Figure 12.17, is drawn on a separate worksheet.

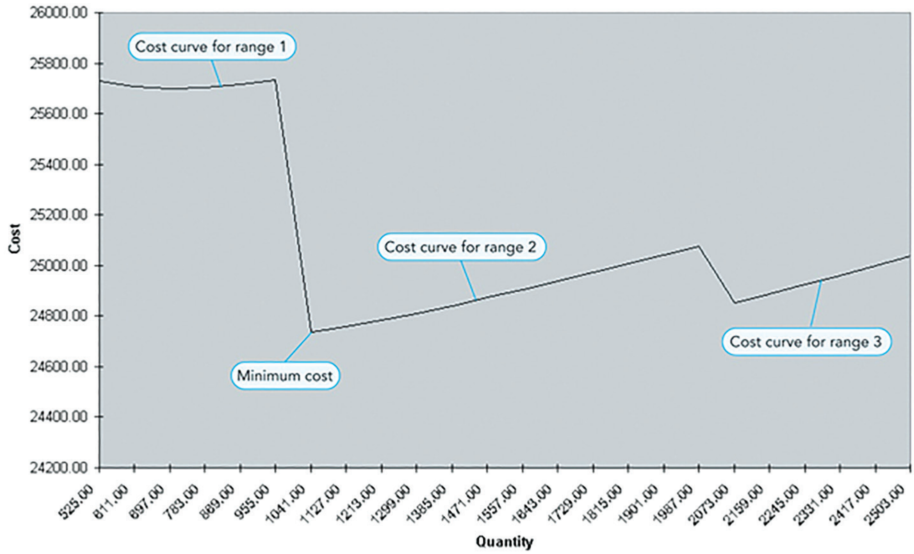


Figure 12.17: Plot of Total Cost versus Order Quantity for Brass Department Store

File: Figure 12.16.xlsx, Sheet: Figure 12.17

12.8 Use of Safety Stock

Safety stock is additional stock that is kept on hand.[2] If, for example, the safety stock for an item is 50 units, you are carrying an average of 50 units more of inventory during the year. When demand is unusually high, you dip into the safety stock instead of encountering a stockout. Thus, the main purpose of safety stock is to avoid stockouts when the demand is higher than expected. Its use is shown in Figure 12.18. Note that although stockouts can often be avoided by using safety stock, there is still a chance that they may occur. The demand may be so high that all the safety stock is used up, and thus there is still a stockout.

2 Safety stock is needed only when demand is uncertain, and models under uncertainty are generally much harder to deal with than models under certainty.

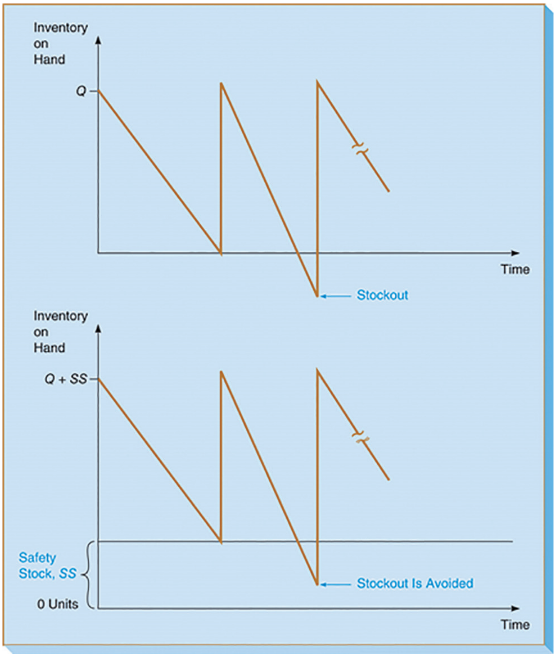


Figure 12.18: Use of Safety Stock

One of the best ways of maintaining a safety stock level is to use the ROP. This can be accomplished by adding the number of units of safety stock as a buffer to the reorder point. Recall from Equation 12–10 that

$$\text{ROP} = d \times L$$

where d is the daily demand rate and L is the order lead time. With the inclusion of safety stock (SS), the reorder point becomes

$$\text{ROP} = d \times L + SS \quad \textbf{(12–16)}$$

How to determine the correct amount of safety stock is the only remaining question. The answer to this question depends on whether we know the cost of a stockout. We discuss both of these situations next.

Safety Stock with Known Stockout Costs

When the EOQ is fixed and the ROP is used to place orders, the only time a stockout can occur is during the lead time. Recall that the lead time is the time between when an order is placed and when it is received. In the procedure discussed here, it is necessary to know the probability distribution of demand during lead time (DDLT) and the cost of a stockout. In the following pages, we assume that DDLT follows a discrete probability distribution. This approach, however, can be easily modified when DDLT follows a continuous probability distribution.

Decision Modeling in Action

3M Uses Inventory Models to Reduce Inventory Costs

3M, a diverse company that manufactures tens of thousands of products, has operations in 60 countries with annual sales of over $16 billion. Supply-chain structures at 3M vary as much as its product offerings. While many of 3M's products are made entirely within individual facilities, others are manufactured and distributed using multiple steps across several facilities. 3M's finished-goods inventory just in the United States exceeds $400 million, and therefore reducing inventory-related costs in the supply chain is of vital importance.

Historically, most 3M supply chains have used ad hoc inventory-control policies that do not account for differences in factors such as setup costs, demand and supply variability, lead time, or site- or product-specific issues. As a result, lot sizes, safety stocks, and service levels can often be severely misestimated. To address these limitations, 3M extended the classical reorder-point/order-quantity approach to develop an integrated inventory-management system that determines optimal lot sizes and safety stocks based on the characteristics of each individual stock-keeping-unit (SKU) in the supply chains.

Between 2003 and 2004, the new system was implemented at just 22 of 3M's supply chains, which varied from a few dozen SKUs to over 10,000 SKUs. The system reduced inventory by over $17 million and annual operating expenses by over $1.4 million, leading 3M to implement it at several other of its supply chains. Although this initiative began as a grassroots effort, 3M's management has fully embraced it and made it the standard for managing inventory.

Source: Based on D. M. Strike and A. Benjaafar. "Practice Abstracts: Optimizing Inventory Management at 3M," *Interfaces* 34, 2 (March–April 2004): 113–116.

What factors should we include in computing the stockout cost per unit? In general, we should include all costs that are a direct or indirect result of a stockout. For example, let us assume that if a stockout occurs, we lose that specific sale forever. Thus, if there is a profit margin of $1 per unit, we have lost this amount. Furthermore, we may end up losing future business from customers who are upset about the stockout. An estimate of this cost must also be included in the stockout cost.

When we know the probability distribution of DDLT and the cost of a stockout, we can determine the safety stock level that minimizes the total cost. We illustrate this computation using an example.

ABCO Example ABCO, Inc., uses the EOQ model and ROP analysis (which we saw in sections 12.4 and 12.5, respectively) to set its inventory policy. The company has determined that its optimal ROP is $50\ (= d \times L)$ units, and the optimal number of orders per year is 6. ABCO's DDLT is, however, not a constant. Instead, it follows the probability distribution shown in Table 12.3.[3]

3 Note that we have assumed that we already know the values of Q^* and ROP. If this is not true, the values of Q^*, ROP, and safety stock would have to be determined simultaneously. This requires a more complex solution.

Table 12.3: Probability of Demand During Lead Time for ABCO, Inc.

Number of Units	Probability
30	0.2
40	0.2
ROP $\rightarrow$ 50	0.3
60	0.2
70	0.1
	1.0

Because DDLT is uncertain, ABCO would like to find the revised ROP, including safety stock, which will minimize total expected cost. The total expected cost is the sum of expected stockout cost and the expected carrying cost of the *additional* inventory.

When we know the unit stockout cost and the probability distribution of DDLT, the inventory problem becomes a decision making under risk problem. (Refer to section 8.5 in Chapter 8 for a discussion of such problems, if necessary.) For ABCO, the decision alternatives are to use an ROP of 30 (alternative 1), 40 (alternative 2), 50 (alternative 3), 60 (alternative 4), or 70 (alternative 5) units. The outcomes are DDLT values of 30 (outcome 1), 40 (outcome 2), 50 (outcome 3), 60 (outcome 4), or 70 (outcome 5) units.

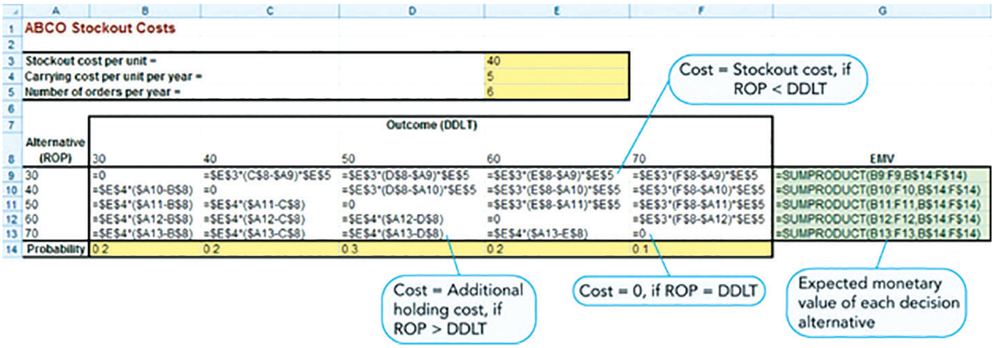
Determining the economic payoffs for any decision alternative and outcome combination involves a careful analysis of the stockout and additional carrying costs. Consider a situation in which the ROP equals the DDLT (say, 30 units each). This means that there will be no stockouts and no extra units on hand when the new order arrives. Thus, stockouts and additional carrying costs will be zero. In general, when the ROP equals the DDLT, total cost will be zero.

Now consider what happens when the ROP is less than the DDLT. For example, say that ROP is 30 units and DDLT is 40 units. In this case we will be 10 units short. The cost of this stockout situation is $\$2{,}400$ ($=10$ units short $\times\$40$ per stockout $\times\,6$ orders per year). Note that we have to multiply the stockout cost per unit and the number of units short by the number of orders per year (6, in this case) to determine annual expected stockout cost. Likewise, if the ROP is 30 units and the DDLT is 50 units, the stockout cost will be $\$4{,}800$ $(=20\times\$40\times6)$, and so on. In general, when the ROP is less than the DDLT, the total cost is equal to the total stockout cost.

Finally, consider what happens when the ROP exceeds the DDLT. For example, say that the ROP is 70 units and the DDLT is 60 units. In this case, we will have 10 additional units on hand when the new inventory is received. If this situation continues during the year, we will have 10 additional units on hand, on average. The additional carrying cost is \$50 (= 10 additional units × \$5 carrying cost per unit per year). Likewise, if the DDLT is 50 units, we will have 20 additional units on hand when the new inventory arrives, and the additional carrying cost will be $\$100$ $(=20\times\$5)$.

In general, when the ROP is greater than the DDLT, total cost will be equal to the total additional carrying cost.

Using the procedures described previously, we can easily set up a spreadsheet to compute the total cost for every alternative and state of nature combination. The formula view for this spreadsheet is shown in Figure 12.19.

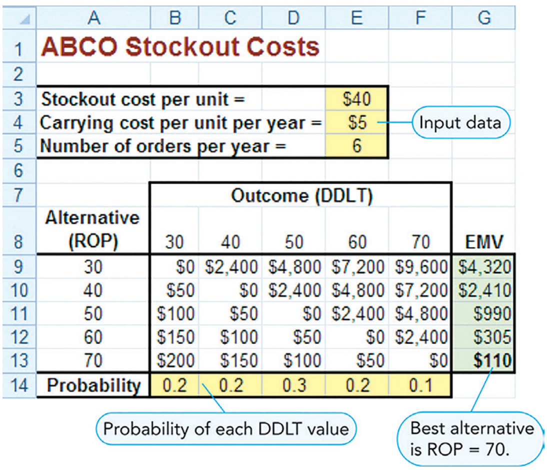


	A	B	C	D	E	F	G
1	ABCO Stockout Costs						
2							
3	Stockout cost per unit =				40		
4	Carrying cost per unit per year =				5		
5	Number of orders per year =				6		
6							
7				Outcome (DDLT)			
8	Alternative (ROP)	30	40	50	60	70	EMV
9	30	=0	=$E$3*(C$8-$A9)*$E$5	=$E$3*(D$8-$A9)*$E$5	=$E$3*(E$8-$A9)*$E$5	=$E$3*(F$8-$A9)*$E$5	=SUMPRODUCT(B9:F9,B$14:F$14)
10	40	=$E$4*($A10-B$8)	=0	=$E$3*(D$8-$A10)*$E$5	=$E$3*(E$8-$A10)*$E$5	=$E$3*(F$8-$A10)*$E$5	=SUMPRODUCT(B10:F10,B$14:F$14)
11	50	=$E$4*($A11-B$8)	=$E$4*($A11-C$8)	=0	=$E$3*(E$8-$A11)*$E$5	=$E$3*(F$8-$A11)*$E$5	=SUMPRODUCT(B11:F11,B$14:F$14)
12	60	=$E$4*($A12-B$8)	=$E$4*($A12-C$8)	=$E$4*($A12-D$8)	=0	=$E$3*(F$8-$A12)*$E$5	=SUMPRODUCT(B12:F12,B$14:F$14)
13	70	=$E$4*($A13-B$8)	=$E$4*($A13-C$8)	=$E$4*($A13-D$8)	=$E$4*($A13-E$8)	=0	=SUMPRODUCT(B13:F13,B$14:F$14)
14	Probability	0.2	0.2	0.3	0.2	0.1	

Figure 12.19: Formula View of Safety Stock Computation for ABCO, Inc.

File: Figure 12.19.xlsx, Sheet: Figure 12.19

The results of the analysis are shown in Figure 12.20. The expected monetary values (EMV) in column G show that the best reorder point for ABCO is 70 units, with an expected total cost of $110. Recall that ABCO had determined its optimal ROP to be 50 units if DDLT was a constant. Hence, the results in Figure 12.20 imply that due to the uncertain nature of DDLT, ABCO should carry a safety stock of $20\ (=70-50)$ units.



	A	B	C	D	E	F	G
1	ABCO Stockout Costs						
2							
3	Stockout cost per unit =				$40		
4	Carrying cost per unit per year =				$5		
5	Number of orders per year =				6		
6							
7				Outcome (DDLT)			
8	Alternative (ROP)	30	40	50	60	70	EMV
9	30	$0	$2,400	$4,800	$7,200	$9,600	$4,320
10	40	$50	$0	$2,400	$4,800	$7,200	$2,410
11	50	$100	$50	$0	$2,400	$4,800	$990
12	60	$150	$100	$50	$0	$2,400	$305
13	70	$200	$150	$100	$50	$0	**$110**
14	Probability	0.2	0.2	0.3	0.2	0.1	

Figure 12.20: Safety Stock Computation for ABCO, Inc.

File: Figure 12.19.xlsx, Sheet: Figure 12.20

EXCEL EXTRA

Form Controls: Scroll Bars

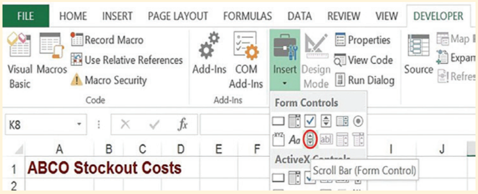
You can add a touch of professionalism to an Excel model while simultaneously restricting certain cell entries by making use of Excel's Form Controls. These are special mechanisms for users to enter data into Excel. In each case, the form controls "float" above the actual spreadsheet as opposed to actually being imbedded into certain cells.

Some of the Form Control options include list boxes, option buttons, check boxes, scroll bars, and spin buttons.

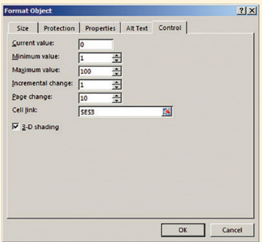
You can insert any of these Form Controls from the Developer ribbon. If you don't see Developer as one of your choices in the ribbon at the top of Excel, add it by selecting:

File|Options|Customize Ribbon|Main Tabs: Developer|OK

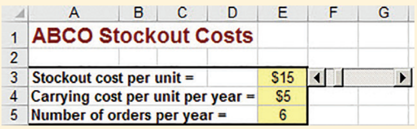
To illustrate one of the Form Controls, let's add a scroll bar for the stockout cost per unit in Figure 12.20. To insert a scroll bar, select Developer|Insert, then click the scroll bar icon:



Next, drag and shape the bar over cells F3:G3. (You can create as small/large of a bar as you wish. But it is good to have it spaced over at least a couple of cells so that using it becomes easier to use.) Right-click on the scroll bar and select Format Control.... Under the Control tab, set the Minimum value: to 1, set the Maximum value: to 100, set the Incremental change: to 1, set the Page change: to 10, and set the Cell link to E3.



Now when you click the right (left) arrow on the scroll bar, the value in Cell E3 will increase (decrease) by 1. If you click on the bar between the center "scroll box" and the right (left) arrow, the value in Cell E3 will increase (decrease) by 10 (this is the Page change). You can also drag the center scroll box right and left to move the value in Cell E3. Note that by establishing a Minimum value, a Maximum value, and an Incremental change of 1, we have restricted the values that users can enter into Cell E3 to be an integer between 1 and 100 (as long as they're using the scroll bar).



Try dragging the scroll box left and right to observe how the numbers in the table quickly change.

Safety Stock with Unknown Stockout Costs

When stockout costs are not available or if they are not relevant, the preceding type of analysis cannot be used. Actually, there are many situations in which stockout costs are unknown or extremely difficult to determine. For example, let's assume that you run a small bicycle shop that sells mopeds and bicycles with a one-year service warranty. Any adjustments made within the year are done at no charge to the customer. If the customer comes in for maintenance under the warranty, and you do not have the necessary part, what is the stockout cost? It cannot be lost profit because the maintenance is done free of charge. Thus, the major stockout cost is the loss of goodwill. The customer may not buy another bicycle from your shop if you have a poor service record. In this situation, it could be very difficult to determine the stockout cost. In other cases, a stockout cost may simply not apply. What is the stockout cost for life-saving drugs in a hospital? The drugs may cost only $10 per bottle. Is the stockout cost $10? Is it $100 or $10,000? Perhaps the stockout cost should be $1 million. What is the cost when a life may be lost as a result of not having the drug?

In such cases, an alternative approach to determining safety stock levels is to use a service level. In general, a service level is the percentage of the time that you will have the item in stock. In other words, the probability of having a stockout is 1 minus the service level. That is,

$$\text{Service level} = 1 - \text{Probability of a stockout}$$

or

$$\text{Probability of a stockout} = 1 - \text{Service level}$$

To determine the safety stock level, it is only necessary to know the probability of the DDLT and the *desired* service level. Here is an example of how the safety stock level can be determined when the DDLT follows a normal probability distribution.

Hinsdale Company Example Hinsdale Company carries an item whose DDLT follows a normal distribution, with a mean of 350 units and a standard deviation of 10 units. Hinsdale wants to follow a policy that results in a service level of 95%. How much safety stock should Hinsdale maintain for this item?

Figure 12.21 may help you to visualize the example. We use the properties of a standardized normal curve to get a Z value for an area under the normal curve of 0.95 = (1 – 0.05). Using the normal table in Appendix C or the Excel formula =NORMSINV(0.95), we find this Z value to be 1.645.

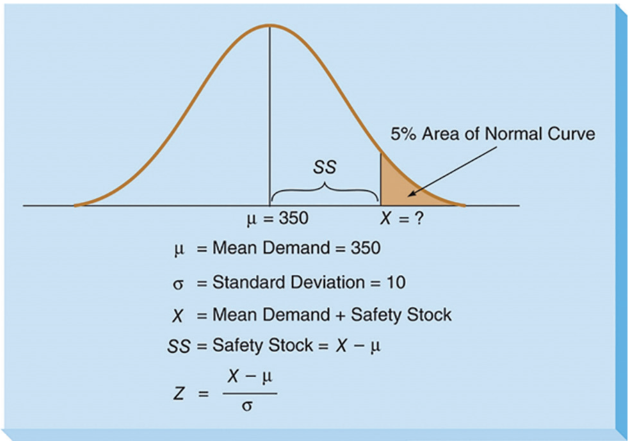


Figure 12.21: Safety Stock and the Normal Distribution

As shown in Figure 12.21, Z is equal to $(X-\mu)/\sigma$, or SS/σ. Hence, SS is equal to $Z \times \sigma$. That is, Hinsdale's safety stock for a service level of 95% is $(1.645 \times 10) = 16.45$ units (which can be rounded up to 17 units, if necessary). We can calculate the safety stocks for different service levels in a similar fashion.

Let's assume that Hinsdale has a carrying cost of $1 per unit per year. What is the carrying cost for service levels that range from 90% to 99.99%? To compute this cost, we first compute the safety stock for each service level (as discussed earlier) and then multiply the safety stock by the unit carrying cost. The Z value, safety stock, and total carrying cost for different service levels for Hinsdale are summarized in Table 12.4.

Table 12.4: Cost of Different Service Levels

Service Level	*Z* Value from Normal Table	Safety Stock (units)	Carrying Cost
90%	1.28	12.8	$12.80
91%	1.34	13.4	$13.40
92%	1.41	14.1	$14.10
93%	1.48	14.8	$14.80
94%	1.55	15.5	$15.50
95%	1.65	16.5	$16.50
96%	1.75	17.5	$17.50
97%	1.88	18.8	$18.80
98%	2.05	20.5	$20.50
99%	2.33	23.3	$23.20
99.99%	3.72	37.2	$37.20

A graph of the total carrying cost as a function of the service level is given in Figure 12.22. Note from Figure 12.22 that the relationship between service level and carrying cost is nonlinear. As the service level increases, the carrying cost increases at an increasing rate. Indeed, at very high service levels, the carrying cost becomes very large. Therefore, as you are setting service levels, you should be aware of the additional carrying cost that you will encounter. Although Figure 12.22 was developed for a specific case, the general shape of the curve is the same for all service-level problems.

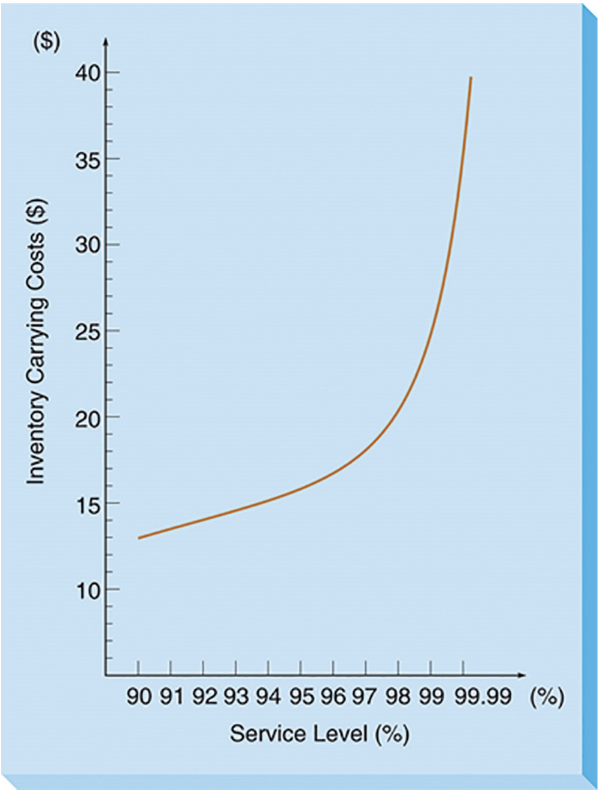
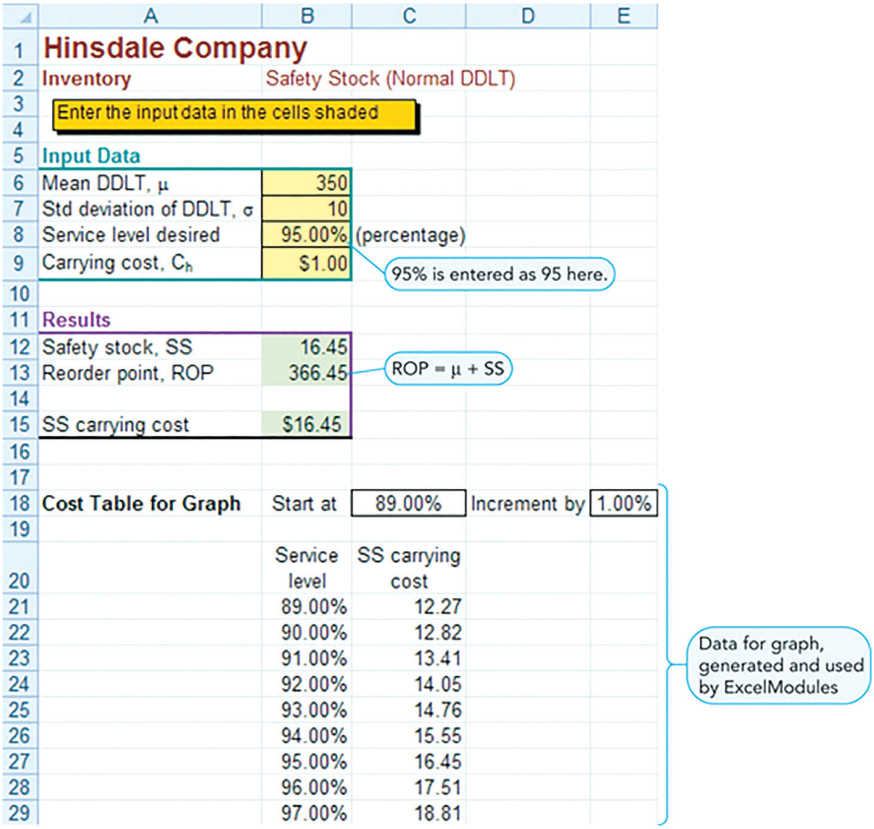


Figure 12.22: Service Level versus Annual Carrying Costs

Using ExcelModules to Compute the Safety Stock We select the choice titled Safety Stock (Normal DDLT) from the Inventory Models menu in ExcelModules (refer to Figure 12.4). The options for this procedure include the problem title and a box to specify whether we want a graph of carrying cost versus service level. After we specify these options, we get the worksheet shown in Figure 12.23. We now enter values for the mean DDLT (μ), standard deviation of DDLT (σ),service level desired, and carrying cost, C_h, in cells B6 to B9, respectively.



	A	B	C	D	E
1	Hinsdale Company				
2	Inventory	Safety Stock (Normal DDLT)			
3	Enter the input data in the cells shaded				
4					
5	Input Data				
6	Mean DDLT, μ	350			
7	Std deviation of DDLT, σ	10			
8	Service level desired	95.00%	(percentage)		
9	Carrying cost, C_h	$1.00			
10					
11	Results				
12	Safety stock, SS	16.45			
13	Reorder point, ROP	366.45			
14					
15	SS carrying cost	$16.45			
16					
17					
18	Cost Table for Graph	Start at	89.00%	Increment by	1.00%
19					
20		Service level	SS carrying cost		
21		89.00%	12.27		
22		90.00%	12.82		
23		91.00%	13.41		
24		92.00%	14.05		
25		93.00%	14.76		
26		94.00%	15.55		
27		95.00%	16.45		
28		96.00%	17.51		
29		97.00%	18.81		

Figure 12.23: Safety Stock (Normal DDLT) Model for Hinsdale

File: Figure 12.23.xlsx, Sheet: Figure 12.23

The worksheet calculates and displays the following output measures:

– Safety stock, SS (= $Z \times \sigma$) in cell B12
– Reorder point (= $\mu + Z \times \sigma$), in cell B13
– Safety stock carrying cost (= $C_h \times Z \times \sigma$) in cell B15

If requested, ExcelModules will draw a plot of the safety stock carrying cost for different values of the service level. This graph, shown in Figure 12.24, is drawn on a separate worksheet. As expected, the shape of this graph is the same as that shown in Figure 12.22.

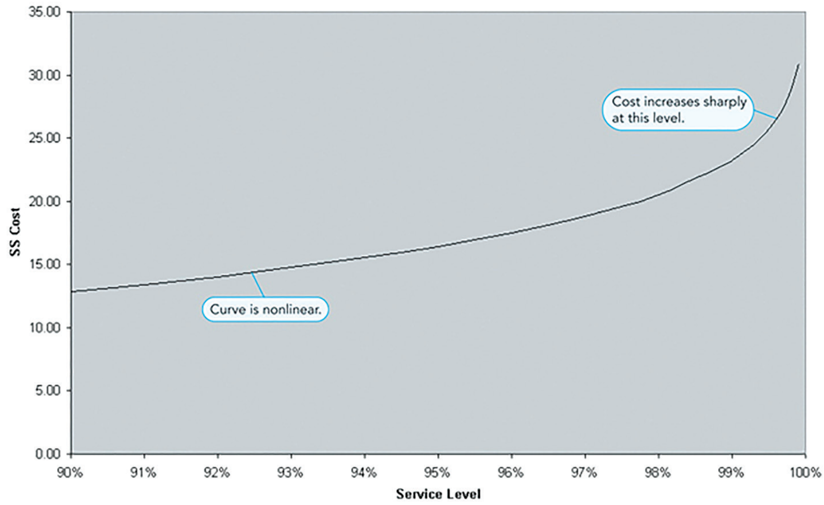


Figure 12.24: Plot of Safety Stock Cost versus Service Level for Hinsdale

File: Figure 12.23.xlsx, Sheet: Figure 12.24

12.9 ABC Analysis

So far, we have shown how to develop inventory policies using quantitative decision models. There are, however, some very practical issues, such as ABC analysis, that should be incorporated into inventory decisions. ABC analysis recognizes the fact that some inventory items are more important than others. The purpose of this analysis is to divide all of a company's inventory items into three groups: A, B, and C. Then, depending on the group, we decide how the inventory levels should be controlled. A brief description of each group follows, with general guidelines as to which items are A, B, and C.

The inventory items in the A group are critical to the functioning of the company. As a result, their inventory levels must be closely monitored. These items typically make up more than 70% of the company's business in monetary value but only about 10–15% of all inventory items. That is, a few inventory items are very important to the company. As a result, the inventory control techniques discussed in this chapter should be used where appropriate for every item in the A group (see Table 12.5).

Table 12.5: Summary of ABC Analysis

Inventory Group	Dollar Usage	Inventory Items	Are Quantitative Inventory Control Techniques Used?
A	70–80%	10–15%	Yes
B	15–25%	20–30%	In some cases
C	5–10%	55–70%	No

The items in the B group are important to the firm but not critical. Thus, it may not be necessary to monitor all these items closely. These items typically represent about 20% of the company's business in monetary value and constitute about 20–30% of the items in inventory. Quantitative inventory models should be used only on some of the B items. The cost of implementing and using these models must be carefully balanced with the benefits of better inventory control. Usually, less than half of the B group items are controlled through the use of inventory control models.

The items in the C group are not as important to the operation of the company. These items typically represent only about 5–10% of the company's business in monetary value but may constitute as much as 70% of the items in inventory. Group C could include inexpensive items such as bolts, washers, screws, and so on. They are usually not controlled using inventory control models because the cost of implementing and using such models would far exceed the value gained.

We illustrate the use of ABC analysis using the example of Silicon Chips, Inc.

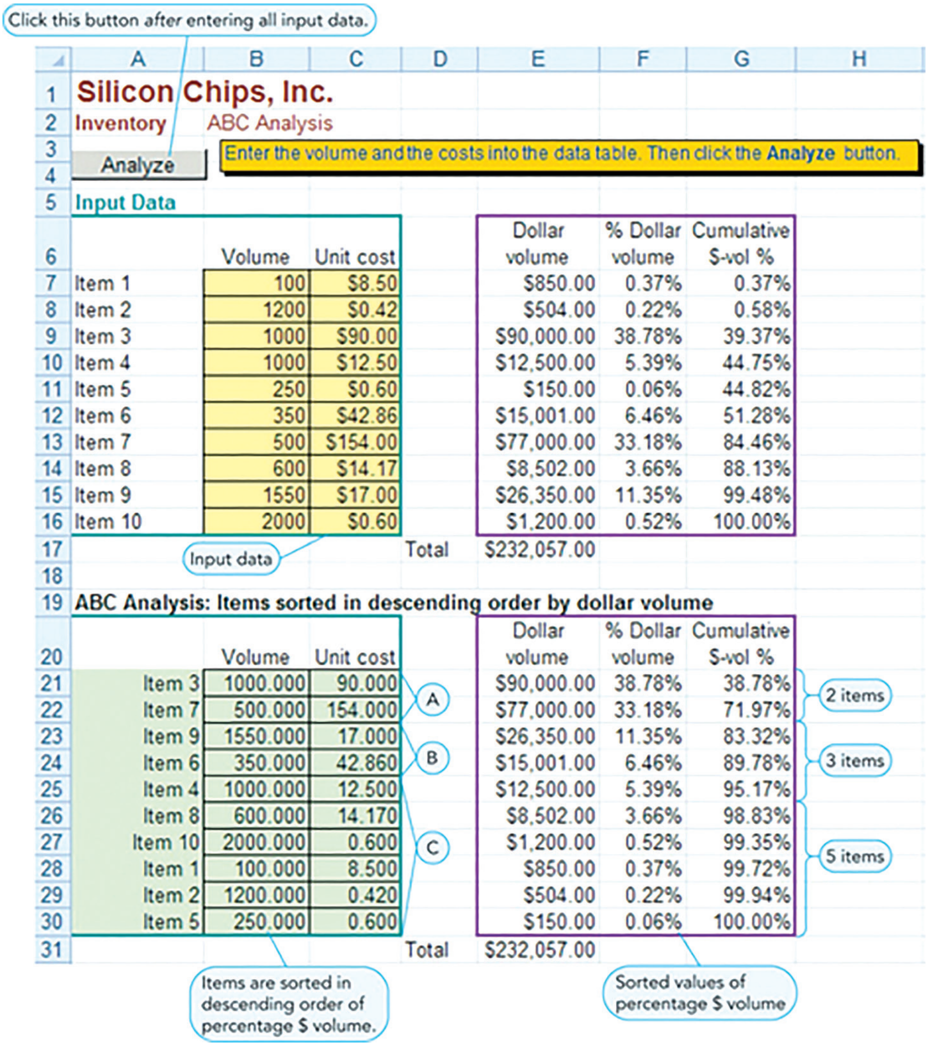
Silicon Chips, Inc., Example

Silicon Chips, Inc., maker of super-fast DRAM chips, has organized its 10 inventory items on an annual dollar-volume basis. Table 12.6 shows the items (identified by item number and part number), their annual demands, and unit costs. How should the company classify these items into groups A, B, and C? As discussed next, we use the worksheet provided in ExcelModules to answer this question.

Table 12.6: Inventory Data for Silicon Chips, Inc.

Item Number	Part Number	Annual Volume (units)	Unit Cost
Item 1	01036	100	$ 8.50
Item 2	01307	1,200	$ 0.42
Item 3	10286	1,000	$ 90.00
Item 4	10500	1,000	$ 12.50
Item 5	10572	250	$ 0.60
Item 6	10867	350	$ 42.86
Item 7	11526	500	$154.00
Item 8	12572	600	$ 14.17
Item 9	12760	1,550	$ 17.00
Item 10	14075	2,000	$ 0.60

Using ExcelModules for ABC Analysis We select the choice titled ABC Analysis from the Inventory Models menu in ExcelModules (refer to Figure 12.4). The options for this procedure include the problem title and boxes to specify the number and names of the items we want to classify. After we specify these options for the Silicon Chips example, we get the worksheet shown in Figure 12.25. We now enter the volume and unit cost for each item in cells B7:C16 of this worksheet.



Silicon Chips, Inc.

Inventory ABC Analysis

Analyze

Enter the volume and the costs into the data table. Then click the **Analyze** button.

Input Data

	Volume	Unit cost		Dollar volume	% Dollar volume	Cumulative $-vol %
Item 1	100	$8.50		$850.00	0.37%	0.37%
Item 2	1200	$0.42		$504.00	0.22%	0.58%
Item 3	1000	$90.00		$90,000.00	38.78%	39.37%
Item 4	1000	$12.50		$12,500.00	5.39%	44.75%
Item 5	250	$0.60		$150.00	0.06%	44.82%
Item 6	350	$42.86		$15,001.00	6.46%	51.28%
Item 7	500	$154.00		$77,000.00	33.18%	84.46%
Item 8	600	$14.17		$8,502.00	3.66%	88.13%
Item 9	1550	$17.00		$26,350.00	11.35%	99.48%
Item 10	2000	$0.60		$1,200.00	0.52%	100.00%
			Total	$232,057.00		

ABC Analysis: Items sorted in descending order by dollar volume

	Volume	Unit cost		Dollar volume	% Dollar volume	Cumulative $-vol %
Item 3	1000.000	90.000		$90,000.00	38.78%	38.78%
Item 7	500.000	154.000		$77,000.00	33.18%	71.97%
Item 9	1550.000	17.000		$26,350.00	11.35%	83.32%
Item 6	350.000	42.860		$15,001.00	6.46%	89.78%
Item 4	1000.000	12.500		$12,500.00	5.39%	95.17%
Item 8	600.000	14.170		$8,502.00	3.66%	98.83%
Item 10	2000.000	0.600		$1,200.00	0.52%	99.35%
Item 1	100.000	8.500		$850.00	0.37%	99.72%
Item 2	1200.000	0.420		$504.00	0.22%	99.94%
Item 5	250.000	0.600		$150.00	0.06%	100.00%
			Total	$232,057.00		

Figure 12.25: ABC Analysis for Silicon Chips, Inc.

File: Figure 12.25.xlsx

When we enter the input data, the worksheet computes the dollar volume and percentage dollar volume (based on total dollar volume) for each item. These values are shown in cells E7:F16 of Figure 12.25. After entering the data for *all* items, we click the Analyze button. The worksheet now sorts the items, in descending order of percentage dollar volume. These values are shown in descending order in cells F21:F30 of Figure 12.25.

The sorted results for the Silicon Chips, Inc., problem are shown in cells A21:C30 of Figure 12.25. Items 3 and 7, which constitute only 20% (= 2 / 10) of the total number of items, account for 71.97% of the total dollar volume of all items. These two items should therefore be classified as group A items.

Items 9, 6, and 4, which constitute 30% (= 3 / 10) of the total number of items, account for 23.20% (= 95.17 – 71.97) of the total dollar volume of all items. These three items should therefore be classified as group B items.

The remaining items constitute 50% (= 5 / 10) of the total number of items. However, they account for only $4.83\% \ (= 100 - 95.17)$ of the total dollar volume of all items. These five items should therefore be classified as group C items.

12.10 Summary

This chapter presents several inventory models and discusses how we can use Excel-Modules to analyze these models. The focus of all models is to answer the same two primary questions in inventory planning: (1) how much to order and (2) when to order. The basic EOQ inventory model makes a number of assumptions: (1) known and constant demand and lead times, (2) instantaneous receipt of inventory, (3) no quantity discounts, (4) no stockouts or shortages, and (5) the only variable costs are ordering costs and carrying costs. If these assumptions are valid, the EOQ inventory model provides optimal solutions. If these assumptions do not hold, more complex models are needed. For such cases, the economic production quantity and quantity discount models are necessary. We also discuss the computation of safety stocks when demand during lead time is unknown for two cases: (1) cost of stockout is known and (2) cost of stockout is unknown. Finally, we present ABC analysis to determine how inventory items should be classified based on their importance and value. For all models discussed in this chapter, we show how Excel worksheets can be used to perform the computations.

Key Points

- Inventory is any stored resource that is used to satisfy a current or future need.
- There are five main uses of inventory: (1) inventory can act as a buffer, (2) resources can be stored in work-in-process, (3) inventory helps when there is irregular supply or demand, (4) purchasing in large quantities may lower unit costs, and (5) inventory can help avoid stockouts.
- The purpose of all inventory models is to minimize inventory costs.

- Components of total cost include the cost of: (1) the items, (2) ordering (or setup), (3) carrying (or holding) inventory, (4) stockouts, and (5) safety stock.
- Assumptions of the EOQ model include: (1) known and constant demand, (2) known and constant lead time, (3) instantaneous receipt of inventory, (4) no quantity discounts, (5) ordering cost and carrying cost comprise the only variable costs, and (6) no stockouts.
- The inventory usage curve has a sawtooth shape in the EOQ model.
- The objective of the simple EOQ model is to minimize ordering and carrying costs.
- The average inventory level is one-half the maximum level in the EOQ model.
- I is the annual carrying cost expressed as a percentage of the unit cost of the item.
- Total cost is a *nonlinear* function of Q in the EOQ model.
- In the EOQ model, we determine Q^* by setting ordering cost equal to carrying cost.
- We can calculate the average inventory value in dollar terms.
- In the EOQ model, for a given Q, we compute a C_o or C_h that makes Q optimal.
- If any of the input data values change, the EOQ also changes.
- Due to the nonlinear formula for EOQ, changes in Q^* are *less* severe than changes in input data values.
- The ROP determines when to order inventory.
- To compute the ROP, we need to know the demand rate per period.
- The EPQ model eliminates the instantaneous receipt assumption.
- Production cycle is the length of each manufacturing run.
- A quantity discount is a reduced price for an item when it is purchased in large quantities.
- Quantity discount models are analyzed in four steps: (1) calculate the EOQ for each discount price, (2) for each price, if necessary, adjust the EOQ value upward to the level that would qualify for that price, (3) compute the total cost for each quantity from steps 1 and 2, and (4) select the quantity leading to the lowest cost.
- Safety stock helps in avoiding stockouts. It is extra stock kept on hand.
- Safety stock is included in the ROP.
- Loss of goodwill must be included in stockout costs.
- We use a decision making under risk approach when we know the unit stockout cost and the probability distribution of DDLT.
- Stockout and additional carrying costs will be zero when ROP = demand during lead time.
- If ROP < DDLT, total cost = total stockout cost.
- If ROP > DDLT, total cost = total additional inventory carrying cost.
- Determining stockout costs may be difficult or impossible.
- An alternative to determining safety stock with stockout costs is to use service level and the normal distribution.
- We find the Z value for the desired service level, and then safety stock = Z times the standard deviation of demand during lead time.
- The relationship between service level and carrying cost is nonlinear.

- For ABC analysis, divide the organization's inventory into three groups: A (critical), B (important but not critical), and C (less important than A and B items in terms of annual dollar value).
- In ABC analysis, items are sorted in descending order of percentage dollar volume.

Glossary

ABC Analysis An analysis that divides inventory into three groups: Group A is more important than group B, which is more important than group C.

Average Inventory The average inventory on hand. Computed as (Maximum inventory + Minimum inventory)/2.

Carrying, or Holding, Cost The cost of holding one unit of an item in inventory for one period (typically a year).

Demand during Lead Time (DDLT) The demand for an item during the lead time between order placement and order receipt.

Economic Order Quantity (EOQ) The amount of inventory ordered that will minimize the total inventory cost. It is also called the optimal order quantity, or Q^*.

Economic Production Quantity (EPQ) Model An inventory model in which the instantaneous receipt assumption has been eliminated. The inventory build-up, therefore, occurs over a period of time.

Instantaneous Inventory Receipt A system in which inventory is received or obtained at one point in time and not over a period of time.

Lead Time The time it takes to receive an order after it is placed.

Ordering Cost The cost of placing an order.

Purchase Cost The cost of purchasing one unit of an inventory item.

Quantity Discount The cost per unit when large orders of an inventory item are placed.

Reorder Point (ROP) The number of units on hand when an order for more inventory is placed.

Safety Stock Extra inventory that is used to help avoid stockouts.

Service Level The chance, expressed as a percentage, that there will not be a stockout. Service level = 1 – Probability of a stockout.

Setup Cost The cost to set up the manufacturing or production process.

Stockout A situation that occurs when there is no inventory on hand.

Total Cost The sum of the ordering, carrying, and purchasing costs.

12.11 Exercises

Solved Problems

Solved Problem 12–1 Patterson Electronics supplies microcomputer circuitry to a company that incorporates microprocessors into refrigerators and other home appliances. Currently, Patterson orders a particular component in batches of 300 units from one of its suppliers. The annual demand for this component is 2,000.

a. If the carrying cost is estimated at $1 per unit per year, what would the ordering cost have to be to make the order quantity optimal?
b. If the ordering cost is estimated to be $50 per order, what would the carrying cost have to be to make the order quantity optimal?

Solution

a. Recall from Equation 12–8 that the ordering cost can be computed as

$$C_o = Q^2 \times C_h / (2D)$$

In Patterson's case, $D = 2{,}000$, $Q = 300$, and $C_h = \$1$. Substituting these values, we get an ordering cost, C_o, of $22.50 per order.

b. Recall from Equation 12–9 that the carrying cost can be computed as

$$C_h = 2DC_o / Q^2$$

In Patterson's case, $D = 2{,}000$, $Q = 300$, and $C_o = \$50$. Substituting these values, we get a carrying cost, C_h, of $2.22 per unit per year.

Solved Problem 12–2 Flemming Accessories produces paper slicers used in offices and in art stores. The minislicer has been one of its most popular items: annual demand is 6,750 units. Kristen Flemming, owner of the firm, produces the minislicers in batches. On average, Kristen can manufacture 125 minislicers per day. Demand for these slicers during the production process is 30 per day. The setup cost for the equipment necessary to produce the minislicers is $150. Carrying costs are $1 per minislicer per year. How many minislicers should Kristen manufacture in each batch? Assume that minislicers cost $10 each to produce.

Solution To solve this problem, we select Economic Production Quantity (EPQ) from the Inventory Models menu in ExcelModules. The input entries, as well as the resulting computations, are shown in Figure 12.26.

	A	B	C	D
1	**Solved Problem 12-2**			
2	**Inventory**	Economic Production Quantity Model		
3	Enter the data in the cells shaded YELLOW. You may have to do some			
4	work to compute the daily production and demand rates.			
5				
6	**Input Data**			
7	Demand rate, D	6750		
8	Setup cost, C_s	$150.00	Input data	
9	Carrying cost, C_h	$1.00	(fixed amount)	
10	Daily production rate, p	125		
11	Daily demand rate, d	30		
12	Unit purchase cost, P	$10.00		
13				
14	**Results**			
15	Economic production quantity, Q*	1632.32	EPQ is 1632 units.	
16	Maximum inventory	1240.56		
17	Average inventory	620.28		
18	Number of setups	4.14		
19				
20	Total holding cost	$620.28		
21	Total setup cost	$620.28		
22	Total production cost	$67,500.00		
23	Total cost, TC	$68,740.56	Total cost is $68,740.56.	

Figure 12.26: EPQ Model for Flemming Accessories

File: Figure 12.26.xlsx

The results show that Flemming Accessories has an EPQ of 1,632 units. The annual total setup and carrying costs are $620.28 each. The annual total cost, including the cost of production, is $68,740.56.

Solved Problem 12–3 Dorsey Distributors has an annual demand for a metal detector of 1,400. The cost of a typical detector to Dorsey is $400. Carrying cost is estimated to be 20% of the unit cost, and the ordering cost is $25 per order. If Dorsey orders in quantities of 300 or more, it can get a 5% discount on the cost of the detectors. Should Dorsey take the quantity discount?

Solution To solve this problem, we select the Quantity Discount option from the Inventory Models menu in ExcelModules. There are two discount levels in this case. The input entries and the resulting computations are shown in Figure 12.27. The results show that Dorsey should order 300 units each time, at a discounted unit cost of $380. The annual total ordering cost is $116.67, and the annual total carrying cost is $11,400.00. The annual total cost, including the total purchase cost, is $543,516.57.

	A	B	C	D	E
1	**Solved Problem 12-3**				
2	Inventory	Quantity Discount Model			
3	Enter the data in the cells shaded YELLOW. The minimum quantity is the minimum amount that needs to be ordered in order to get the price for that range.				
4					
5					
6	Input Data				
7	Demand rate, D	1400			
8	Ordering cost, C_o	25			
9	Carrying cost %, I	20.00%	(percentage)	Carrying cost entered as a percentage of unit price	
10					
11		**Range 1**	**Range 2**		
12	Minimum quantity	0	300		
13	Unit purchase cost, P	$400.00	$380.00		
14					
15	Results				
16		**Range 1**	**Range 2**		
17	Economic order quantity, Q*	29.58	30.35		
18	Adjusted order quantity	29.58	300.00	Adjusted Q* values	
19					
20	Total holding cost	1,183.22	11,400.00		
21	Total ordering cost	1,183.22	116.67		
22	Total purchase cost	560,000.00	532,000.00		
23	Total cost	562,366.43	543,516.67	Minimum cost option	

Figure 12.27: Quantity Discount Model for Dorsey Distributors

File: Figure 12.27.xlsx

Discussion Questions

12–1. Why is inventory an important consideration for managers?

12–2. What is the purpose of inventory control?

12–3. Why wouldn't a company always store large quantities of inventory to eliminate shortages and stockouts?

12–4. Describe the major decisions that must be made in inventory control.

12–5. What are some of the assumptions made in using the EOQ?

12–6. Discuss the major inventory costs that are used in determining the EOQ.

12–7. What is the ROP? How is it determined?

12–8. What is the purpose of sensitivity analysis?

12–9. What assumptions are made in the EPQ model?

12–10. What happens to the EPQ model when the daily production rate becomes very large?

12–11. In the quantity discount model, why is the carrying cost expressed as a percentage of the unit cost, I, instead of the cost per unit per year, C_h?

12–12. Briefly describe what is involved in solving a quantity discount model.

12–13. Discuss the methods that are used in determining safety stock when the stockout cost is known and when the stockout cost is unknown.

12–14. Briefly describe what is meant by ABC analysis. What is the purpose of this inventory technique?

Problems

12–15. Shakina Harris, who works in her brother's hardware store, is in charge of purchasing. Shakina has determined that the annual demand for #6 screws is 150,000 and is fairly constant over the 200 days that the store is open each year. She estimates that it costs $30 every time an order is placed. This cost includes her wages, the cost of the forms used in placing the order, and so on. Furthermore, she estimates that the cost of carrying one screw in inventory for a year is 0.6 cents.

(a) How many #6 screws should Shakina order at a time?

(b) It takes 8 working days for an order of #6 screws to arrive once the order has been placed. Because the demand is fairly constant, Shakina believes that she can avoid stockouts completely if she orders the screws only when necessary. What is the ROP?

(c) Shakina's brother believes that she is placing too many orders for screws each year. He believes that orders should be placed only twice per year. If Shakina follows her brother's policy, how much more would this cost every year over the ordering policy that she developed in part (a)? If only two orders are placed each year, what effect would this have on the ROP?

(d) Shakina now believes that her estimate of an ordering cost of $30 per order is too low. Although she does not know the exact cost, she believes that it could be as high as $60 per order. How would the optimal order quantity in part (a) change if the ordering cost were $40, $50, or $60?

12–16. Neha Shah is the purchasing agent for a firm that sells industrial valves and fluid control devices. One of the most popular valves is the KA1, which has an annual demand of 6,000 units. The cost of each valve is $120, and the inventory carrying cost is estimated to be 8% of the cost of each valve. Neha has made a study of the costs involved in placing an order for any of the valves that the firm stocks, and she has concluded that the average ordering cost is $45 per order. Furthermore, it takes about two weeks for an order to arrive from the supplier, and during this time the demand per week for KA1 valves is approximately 120. Compute the EOQ, ROP, optimal number of orders per year, and total annual cost for KA1 valves.

12–17. Keith Smart has been in the building business for most of his life. Keith's biggest competitor is Delta Birch. Through many years of experience, Keith knows that the ordering cost for an order of plywood is $150 and that the carrying cost is 25% of the unit cost. Both Keith and Delta receive plywood in loads that cost $600 per load. Furthermore, Keith and Delta use the same supplier of plywood, and Keith was able to find out that Delta orders in quantities of 300 loads at a time. Ken also knows that 300 loads is the EOQ for Delta. What is Delta's annual demand, in loads of plywood?

12–18. Shoes R Us is a local shoe store located in Camden. Annual demand for a popular sandal is 1,000 pairs of sandals, and Gary Cole, the owner of Shoes R Us, has been in the habit of ordering 200 pairs of sandals at a time. Gary estimates that the ordering cost is $20 per order. The cost of a pair of sandals is $10.

(a) For Gary's ordering policy to be correct, what would the carrying cost have to be as a percentage of the unit cost?

(b) If the carrying cost were 20% of the unit cost, what would be the optimal order quantity?

12–19. Annual demand for the Dobbs model airplane kit is 80,000 units. Albert Dobbs, president of Dobbs' Terrific Toys, controls one of the largest toy companies in Nevada. He estimates that the ordering cost is $40 per order. The carrying cost is $7 per unit per year. It takes 25 days between the time that Albert places an order for the model airplane kit and the time when they are received at his warehouse. During this time, the daily demand is estimated to be 450 units.

(a) Compute the EOQ, ROP, and optimal number of orders per year.

(b) Albert Dobbs now believes that the carrying cost may be as high as $14 per unit per year. Furthermore, Albert estimates that the lead time may be 35 days instead of 25 days. Redo part (a), using these revised estimates.

12–20. Floral Beauty, Inc., is a large floral arrangements store located in Eastwood Mall. Bridal Lilies, which are a specially created bunch of lilies for bridal bouquets, cost Floral Beauty $17 each. There is an annual demand for 24,000 Bridal Lilies. The manager of Floral Beauty has determined that the ordering cost is $120 per order, and the carrying cost, as a percentage of the unit cost, is 18%. Floral Beauty is now considering a new supplier of Bridal Lilies. Each lily would cost only $16.50, but to get this discount, Floral Beauty would have to buy shipments of 3,000 Bridal Lilies at a time. Should Floral Beauty use the new supplier and take this discount for quantity buying?

12–21. Cameron Boats, a supplier of boating equipment, sells 5,000 WM-4 diesel engines every year. These engines are shipped to Cameron in a shipping container of 100 cubic feet, and Cameron Boats keeps the warehouse full of these WM-4 motors. The warehouse can hold 5,000 cubic feet of boating supplies. Cameron estimates that the ordering cost is $400 per order and that the carrying cost is $100 per motor per year. Cameron Boats is considering the possibility of expanding the warehouse for the WM-4 motors. How much should Cameron Boats expand, and how much would it be worth it for the company to make the expansion?

12–22. Tarbutton Lawn Distributors is a wholesale organization that supplies retail stores with lawn care and household products. One building is used to store FirstClass lawn mowers. The building is 50 feet wide by 40 feet deep by 8 feet high. Andrea Dormer, manager of the warehouse, estimates that only 90% of the warehouse can be used to store the FirstClass lawn mowers. The remaining 10% is used for walkways and a small office. Each FirstClass lawn mower comes in a box that is 5 feet by 4 feet by 2 feet high. The annual demand for these lawn mowers is 48,000, and the ordering cost for Tarbutton Lawn Distributors is $75 per order. It is estimated that it costs Tarbutton Lawn $45 per lawn mower per year for storage. Tarbutton Lawn Distributors is thinking about increasing the size of the warehouse. The company can do this only by making the warehouse deeper. At the present time, the warehouse is 40 feet deep. How many feet of depth should be added on to the warehouse if they wish to minimize the total annual costs? How much should the company be willing to pay for this addition? Remember that only 90% of the total space can be used to store FirstClass lawn mowers.

12–23. Morgan Arthur has spent the past few weeks determining inventory costs for Armstrong, a toy manufacturer located near Cincinnati, Ohio. She knows that annual demand will be 30,000 units per year and that carrying cost will be $1.50 per unit per year. Ordering cost, on the other hand, can vary from $45 per order to $50 per order. During the past 450 working days, Morgan has observed the following frequency distribution for the ordering cost:

Ordering Cost	Frequency
$45	85
$46	95
$47	90
$48	80
$49	55
$50	45

Morgan's boss would like Morgan to determine an EOQ value for each possible ordering cost and to determine an EOQ value for the expected ordering cost.

12–24. Blaine Abrams is the owner of a small company that produces electric scissors used to cut fabric. The annual demand is for 75,000 scissors, and Blaine produces the scissors in batches. On average, Blaine can produce 1,000 pairs of scissors per day during the production process. Demand for scissors has been about 250 pairs of scissors per day. The cost to set up the production process is $800, and it costs Blaine $0.90 to carry 1 pair of scissors for one year. How many scissors should Blaine produce in each batch?

12–25. Carter Cohen, inventory control manager for Raydex, receives wheel bearings from Wheel & Gears, a small producer of metal parts. Unfortunately, Wheel & Gears can produce only 5,000 wheel bearings per day. Raydex receives

100,000 wheel bearings from Wheel & Gears each year. Because Raydex operates 200 working days each year, the average daily demand of wheel bearings by Raydex is 500. The ordering cost for Raydex is $400 per order, and the carrying cost is $6 per wheel bearing per year. How many wheel bearings should Raydex order from Wheel & Gears at one time? Wheel & Gears has agreed to ship the maximum number of wheel bearings that it produces each day to Raydex when an order has been received.

12–26. Chandler Manufacturing has a demand for 1,000 pumps each year. The cost of a pump is $50. It costs Chandler Manufacturing $40 to place an order, and the carrying cost is 25% of the unit cost. If pumps are ordered in quantities of 200, Chandler Manufacturing can get a 3% discount on the cost of the pumps. Should Chandler Manufacturing order 200 pumps at a time and take the 3% discount?

12–27. Arms of Steel is an organization that sells weight training sets. The carrying cost for the AS1 model is $5 per set per year. To meet demand, Arms of Steel orders large quantities of AS1 seven times a year. The stockout cost for AS1 is estimated to be $50 per set. Over the past several years, Arms of Steel has observed the following demand during the lead time for AS1:

Demand During Lead Time	Probability
40	0.1
50	0.2
60	0.2
70	0.2
80	0.2
90	0.1

The reorder point for AS1 is 60 units. What level of safety stock should be maintained for the AS1 model?

12–28. Victoria Blunt is in charge of maintaining hospital supplies at Mercy Hospital. During the past year, the mean lead time demand for bandage BX-5 was 600. Furthermore, the standard deviation for BX-5 was 70. Ms. Blunt would like to maintain a 95% service level.

(a) What safety stock level do you recommend for BX-5?

(b) Victoria has just been severely chastised for her inventory policy. Dwight Seymour, her boss, believes that the service level should be 99%. Compute the safety stock levels for a 99% service level.

(c) Victoria knows that the carrying cost of BX-5 is $0.50 per unit per year. Compute the carrying cost associated with 95% and 99% service levels.

12–29. Finn simply does not have time to analyze all the items in his company's inventory. As a young manager, he has more important things to do. The following is a table of six items in inventory, along with the unit cost and the demand, in units:

Identification Code	Unit Cost	Demand (units)
U1	$ 2.00	1,110
V2	$ 5.40	1,110
W3	$ 2.08	961
X4	$74.54	1,104
Y5	$ 5.84	1,200
Z6	$ 1.12	896

Which item(s) should be carefully controlled using a quantitative inventory technique, and which item(s) should not be closely controlled?

12–30. The demand for barbeque grills has been fairly large in the past several years, and Estate Supplies, Inc., usually orders new barbeque grills five times a year. It is estimated that the ordering cost is $60 per order. The carrying cost is $10 per grill per year. Furthermore, Estate Supplies, Inc., has estimated that the stockout cost is $50 per unit. The reorder point is 650 units. Although the demand each year is high, it varies considerably. The demand during the lead time is as follows (see the table below).

Demand During Lead Time	Probability
600	0.25
650	0.23
700	0.12
750	0.10
800	0.08
850	0.05
900	0.05
950	0.04
1,000	0.03
1,050	0.03
1,100	0.02

The lead time is 12 working days. How much safety stock should Estate Supplies, Inc., maintain?

12–31. Omar Nagoon receives 5,000 tripods annually from Top-Grade Suppliers to meet his annual demand. Omar runs a large photographic outlet, and the tripods are used primarily with 35mm cameras. The ordering cost is $15 per order, and the carrying cost is $0.50 per unit per year. Weekly demand is 100 tripods.

(a) What is the optimal order quantity?

(b) Top-Grade is offering a new shipping option. When an order is placed, Top-Grade will ship one-third of the order every week for three weeks instead of shipping the entire order at one time. What is the order quantity if Omar chooses to use this option? To simplify your calculations, assume that the average inventory is equal to one-half of the maximum inventory level for Top-Grade's new option.

(c) Suppose Top-Grade Suppliers offers to ship one-fifth of the order every week for five weeks. What is the order quantity under this option? Make the same assumption as in part (b).

(d) Calculate the total cost for each option. What do you recommend?

12–32. Amco Convenience Store purchases 500 hammers a year for its inventory from its supplier, who offers pricing at quantity discounts. The quantities and pricing from this supplier are shown in the following table:

Order Quantity	**Unit Price**
0–149	\$12.00
150–349	\$11.50
350–599	\$10.50
600 or more	\$ 9.80

The cost for Amco to place an order is \$60, and the cost to store a hammer in inventory for a year is \$2. What quantity should Amco order?

12–33. Asbury Products offers the following discount schedule for its 4- by 8-foot sheets of good-quality plywood:

Order Quantity	**Unit Price**
99 sheets or less	\$18.00
100 to 500 sheets	\$17.70
More than 500 sheets	\$17.45

Home Sweet Home Company orders plywood from Asbury Products. Home Sweet Home has an ordering cost of \$55. The carrying cost is 25%, and the annual demand is 1,000 sheets. What do you recommend?

12–34. Tropic Citrus Products produces orange juice, grapefruit juice, and other citrus-related items. Tropic obtains fruit concentrate from a cooperative in Orlando that consists of approximately 50 citrus growers. The cooperative will sell a minimum of 100 cans of fruit concentrate to citrus processors such as Tropic. The cost per can is \$9.90.

Last year, a cooperative developed an incentive bonus program (IBP) to give an incentive to its large customers to buy in quantity. Here is how it works: If 200 cans of concentrate are purchased, 10 cans of free concentrate are included in the deal. In addition, the names of the companies purchasing the concentrate are added to a drawing for a new personal computer. The personal computer has a value of about \$3,000, and currently about 1,000 companies are eligible for this drawing. At 300 cans of concentrate, the cooperative will give away 30 free cans and will also place the company name in the drawing for the personal computer. When the quantity goes up to 400 cans of concentrate, 40 cans of concentrate will be given away free with the order. In addition, the company is also placed in a drawing for the personal computer and a free trip for two. The value of the trip for two is approximately $5,000. About 800 companies are expected to qualify and to be in the running for this trip.

Tropic estimates that its annual demand for fruit concentrate is 1,000 cans. In addition, the ordering cost is estimated to be $10.00, and the carrying cost is estimated to be 10%, or about $1.00 per unit. The firm is intrigued with the IBP. If the company decides that it will keep the trip or the computer if they are won, what should it do?

12–35. Lindsay Hawkes sells discs that contain 25 software packages that perform a variety of financial functions typically used by business students. Depending on the quantity ordered, Lindsay offers the following price discounts:

Order Quantity	**Price**
1 to 600	$10.00
601 to 1,100	$ 9.92
1,101 to 1,550	$ 9.86
1,551 and up	$ 9.80

The annual demand is 3,000 units on average. Lindsay's setup cost to produce the discs is $350. She estimates holding costs to be 10% of the price, or about $1 per unit per year. What is the optimal number of discs to produce at a time?

12–36. Demand during lead time for one brand of TV is normally distributed with a mean of 56 TVs and a standard deviation of 18 TVs. What safety stock should be carried for a 95% service level? What is the appropriate ROP?

12–37. Based on available information, lead time demand for CD-ROM drives averages 250 units (normally distributed), with a standard deviation of 25 drives. Management wants a 98% service level. How many drives should be carried as safety stock? What is the appropriate ROP?

12–38. A product is delivered to Monica's company once a year. The ROP, without safety stock, is 300 units. Carrying cost is $25 per unit per year, and the cost of a stockout is $80 per unit per year. Given the following demand probabilities during the reorder period, how much safety stock should be carried?

Demand during Reorder Period	**Probability**
100	0.3
200	0.1
300	0.3
400	0.2
500	0.1

12–39. Barb's company has compiled the following data on a small set of products:

Item	Annual Demand	Unit Cost
1	410	$ 0.75
2	330	$ 17.00
3	300	$ 3.00
4	200	$ 0.90
5	240	$110.00
6	625	$ 0.75
7	85	$ 25.00
8	75	$ 2.00
9	100	$125.00
10	125	$ 1.50

Perform an ABC analysis on her data.

12–40. The Century One Store has 10 items in inventory, as shown in the following table. The manager wants to divide these items into ABC classifications. What would you recommend?

Item	Annual Demand	Unit Cost
A	1,750	$ 10
B	300	$1,500
C	500	$ 500
D	6,000	$ 10
E	1,000	$ 20
F	1,500	$ 45
G	4,000	$ 12
H	600	$ 20
I	3,000	$ 50
J	2,500	$ 5

12–41. Lea Ash opens a new cosmetics store. There are numerous items in inventory, and Lea knows that there are costs associated with inventory. Lea wants to classify the items according to dollars invested in them. The following table provides information about the 10 items that she carries:

Item Number	Unit Cost	Demand (units)
A	$4	1,500
B	$1	1,500
C	$3	700

Item Number	Unit Cost	Demand (units)
D	$1	1,200
E	$8	200
F	$6	500
G	$2	1,000
H	$7	800
I	$8	1,200
J	$4	800

Use ABC analysis to classify these items into categories A, B, and C.

12–42. The following table shows the inventory data for the six items stocked by Apex Enterprises:

Item Code	Unit Cost	Annual Demand (units)	Ordering Cost	Carrying Cost as a Percentage of Unit Cost
1	$150.00	560	$40	15
2	$ 4.10	490	$40	17
3	$ 2.25	500	$50	15
4	$ 10.60	600	$40	20
5	$ 4.00	540	$35	16
6	$ 11.00	450	$30	25

Lynn Robinson, Apex's inventory manager, does not feel that all the items can be controlled. What ordered quantities do you recommend for which inventory product(s)?

12–43. Consider a product with a *daily* demand of 400 units, a setup cost per production run of $100, a *monthly* holding cost per unit of $2.00, and an *annual* production rate of 292,000 units. The firm operates and experiences demand 365 days per year.

Suppose that Trevor mistakenly used the basic EOQ model to calculate the batch size instead of using the EPQ model. How much money per year will that mistake cost the company?

12–44. In the EOQ model, the total annual ordering and carrying cost (TC) equals the first two terms of equation (12–5). It turns out that if the EOQ is ordered, TC can be expressed as a single formula that is independent of Q.

(a) Plug the EOQ formula from equation (12–6) in for Q into the total cost formula and combine terms to determine the formula for TC whenever the EOQ is purchased.

(b) Based your answer to part (a), what happens to TC when demand doubles? What happens when the ordering cost per order is cut in half? What happens when the holding cost per unit per year doubles?

(c) Suppose that your company decides to store its inventory in N warehouses instead of 1, and suppose that C_o is the same at each warehouse and C_h is the same at each warehouse. Based on your answer to part (a), if total company demand is D, what is the general formula for the *total company* annual ordering and carrying cost of using N warehouses instead of one (if the demand is spread evenly over those warehouses)?

12–45. A special type of quantity discount is a *one-time sale*, in which the regular sales price is discounted by an amount Δ dollars per unit but only for units purchased immediately. This often results in order sizes that *significantly* exceed the standard EOQ (much more than occurs for traditional quantity discounts that can be received at each purchase point over time). The formula for the optimal order size with a one-time sale is:

$$Q_1^* = \frac{D\Delta}{(I)(P-\Delta)} + \frac{P}{(P-\Delta)} EOQ,$$

where P is regular purchase cost per unit, and EOQ is the standard EOQ order size when no discounts are offered.

Consider a problem where $D = 20{,}000$ units, $C_o = \$80$, $I = 30\%$, and $P = \$10$.

(a) What is the EOQ? What proportion of annual demand does this represent?

(b) How many units should be ordered if a one-time discount of \$2 per unit is offered? What proportion of annual demand does this represent?

12–46. Create analogous versions of equations (12–8) and (12–9) for the EPQ problem.

12–47. An important assumption of the EOQ model is that demand is smooth and continuous. Effectively, that implies that customers are constantly ordering infinitesimally small quantities. In reality, of course, customer orders vary in size, particularly for business-to-business (B2B) transactions. To the extent that there are many customers and there's little seasonality in demand, the EOQ assumption may be a good one. However, what happens if a large industrial customer orders its own EOQ from your company? For example, if you produce a customer-specific item that is being ordered, say, once per month at 10,000 units per order, what should your lot size be?

It turns out that given a steady, lumpy demand pattern, the supplier should produce in an *integer multiple K* of the customer's incoming order size Q. For example, if your single customer periodically orders its EOQ of 2000 units from you, then your lot sizes should be either 2000 ($K = 1$), or 4000 ($K = 2$), or 6000 ($K = 3$), etc. So from the supplier's perspective, the decision variable is the integer multiplier K of the (now fixed) incoming order size Q. The formula for the optimal value of K is:

$$K^* = \left\lfloor \frac{1}{2}\left(1 + \sqrt{1 + \frac{8DC_o}{C_h Q^2}}\right) \right\rfloor,$$

where $\lfloor x \rfloor$ is a function that returns the greatest integer $\le x$. The lot size for the supplier therefore equals K^*Q.

Consider Vicky Luo's Spy Planes, which supplies drones to a single large retailer who periodically orders its own EOQ of 900 units. Vicky's annual demand is the same as the retailer's at 25,000 units, her ordering cost is $200 per order, and her holding cost per unit per year is $2.00. How many units should Vicky order at a time?

12–48. Use Excel to replicate Table 12.4 using formulas. Note that the *Z* value from the normal table can be computed with the NORMSINV function. For example, =NORMSINV(.90) will return 1.28.

12–49. Consider a production environment with a setup cost of $100, an annual holding cost per unit of $15, an annual demand of 24,000 units, a monthly production rate of 4000 units, and monthly demand of 2000 units. How much space (in units) needs to be available in the factory to hold finished products of this item?

12–50. In this chapter, we characterize *service level* as the probability of not running out of stock during lead time. However, this can be a very strict measure, and many firms would be more interested in defining service as a *fill rate*, which is the proportion of demand satisfied from stock. After all, if only 1 unit of demand out of 10,000 were missed, that would be considered to be "good" service in most instances even though the firm "ran out of stock."

The service level measure is easier to calculate. Fill rate calculations, on the other hand, require a numerical search. Fortunately, we can use Excel's Goal Seek application to perform that search for us. The idea is to equate the actual *Expected Shortage per Cycle* (*ESC*) based on the safety stock (*SS*) chosen with the *target ESC*, which is based on the desired fill rate (*fr*). The Excel formula for *ESC* based on safety stock is: *ESC* = *SS**(1–NORMDIST(*SS*/σ,0,1,1))+σ *NORMDIST(*SS*/σ,0,1,0). The target *ESC* can be derived from the definition of the fill rate: $fr = 1 - (ESC/Q)$.

Create an Excel spreadsheet application to determine safety stock under the fill rate criterion. The inputs will be fill rate, σ, and the order quantity *Q*. The *target ESC* will be based on a rearrangement of the *fr* formula above (i.e., solve that formula for ESC). The *actual ESC* uses the first formula. The decision to be made is the amount of safety stock *SS* to have on hand. Notice that because the formula uses the normal distribution, we cannot isolate *SS* to solve for it directly. However, Goal Seek can do the work for us. The Set cell should be the *actual ESC*, the To value should be the number that appears in the *target ESC* cell, and the By changing cell would be the cell containing the decision (*SS*).

To validate your model, try a fill rate of 97.5%, σ of 707, and *Q* of 10,000 units. You should find that you need 66.6 units of safety stock to guarantee your desired fill rate.

12–51. What service level is provided if σ of DDLT = 40 units and 60 units of safety stock are held?