

Contents

PREFACE	ix
INTRODUCTION	1

Part I: Carathéodory Dimension Characteristics

CHAPTER 1. GENERAL CARATHÉODORY CONSTRUCTION	11
1. Carathéodory Dimension of Sets	12
2. Carathéodory Capacity of Sets	16
3. Carathéodory Dimension and Capacity of Measures	21
4. Coincidence of Carathéodory Dimension and Carathéodory Capacity of Measures	28
5. Lower and Upper Bounds for Carathéodory Dimension of Sets; Carathéodory Dimension Spectrum	31
CHAPTER 2. C -STRUCTURES ASSOCIATED WITH METRICS: HAUSDORFF DIMENSION AND BOX DIMENSION	34
6. Hausdorff Dimension and Box Dimension of Sets	35
7. Hausdorff Dimension and Box Dimension of Measures; Pointwise Dimension; Mass Distribution Principle	41
CHAPTER 3. C -STRUCTURES ASSOCIATED WITH METRICS AND MEASURES: DIMENSION SPECTRA	48
8. q -Dimension and q -Box Dimension of Sets	48
9. q -Dimension and q -Box Dimension of Measures	57
APPENDIX I: HAUSDORFF (BOX) DIMENSION AND q -(BOX) DIMENSION OF SETS AND MEASURES IN GENERAL METRIC SPACES	61
CHAPTER 4. C -STRUCTURES ASSOCIATED WITH DYNAMICAL SYSTEMS: THERMODYNAMIC FORMALISM	64
10. A Modification of the General Carathéodory Construction	65
11. Dimensional Definition of Topological Pressure; Topological and Measure-Theoretic Entropies	68
12. Non-additive Thermodynamic Formalism	83
APPENDIX II: VARIATIONAL PRINCIPLE FOR TOPOLOGICAL PRESSURE; SYMBOLIC DYNAMICAL SYSTEMS; BOWEN'S EQUATION	87
APPENDIX III: AN EXAMPLE OF CARATHÉODORY STRUCTURE GENERATED BY DYNAMICAL SYSTEMS	110

Part II: Applications to Dimension Theory and Dynamical Systems

CHAPTER 5. DIMENSION OF CANTOR-LIKE SETS AND SYMBOLIC DYNAMICS	117
13. Moran-like Geometric Constructions with Stationary (Constant) Ratio Coefficients	123

14. Regular Geometric Constructions	133
15. Moran-like Geometric Constructions with Non-stationary Ratio Coefficients	140
16. Geometric Constructions with Rectangles; Non-coincidence of Box Dimension and Hausdorff Dimension of Sets	153
CHAPTER 6. MULTIFRACTAL FORMALISM	170
17. Correlation Dimension	174
18. Dimension Spectra: Hentschel–Procaccia, Rényi, and $f(\alpha)$ -Spectra; Information Dimension	182
19. Multifractal Analysis of Gibbs Measures on Limit Sets of Geometric Constructions	189
CHAPTER 7. DIMENSION OF SETS AND MEASURES INVARIANT UNDER HYPERBOLIC SYSTEMS	196
20. Hausdorff Dimension and Box Dimension of Conformal Repellers for Smooth Expanding Maps	197
21. Multifractal Analysis of Gibbs Measures for Smooth Conformal Expanding Maps	209
22. Hausdorff Dimension and Box Dimension of Basic Sets for Axiom A Diffeomorphisms	227
23. Hausdorff Dimension of Horseshoes and Solenoids	238
24. Multifractal Analysis of Equilibrium Measures on Basic Sets of Axiom A Diffeomorphisms	247
APPENDIX IV: A GENERAL CONCEPT OF MULTIFRACTAL SPECTRA; MULTIFRACTAL RIGIDITY	259
CHAPTER 8. RELATIONS BETWEEN DIMENSION, ENTROPY, AND LYAPUNOV EXPONENTS	270
25. Existence and Non-existence of Pointwise Dimension for Invariant Measures	271
26. Dimension of Measures with Non-zero Lyapunov Exponents; The Eckmann–Ruelle Conjecture	279
APPENDIX V: SOME USEFUL FACTS	293
BIBLIOGRAPHY	295
INDEX	301