Introduction

Although Johannes Kepler was introduced to the Copernican system by his teacher Michael Maestlin at the Lutheran university of Tübingen, the greatest contemporary astronomer, Tycho Brahe, had rejected this system, while the followers of the influential Lutheran theologian Philipp Melanchthon (apart from Rheticus, the disciple of Copernicus) had adapted the Copernican models to a geostatic framework. Within certain limits, astronomers were free to invent mathematical circles, such as epicycles and eccentrics, in order to describe the apparent motions of the planets in the heavens. Hartmann Bayer,[1] one of Melanchthon's followers, explained the permitted combination of realism and instrumentalism in his commentary on the *Sphere* of Sacrobosco. The concentric planetary spheres (or shells) and the sphere of fixed stars (which he claimed were revealed to our eyes), together with the ninth and tenth spheres introduced by more recent astronomers, he regarded as real. Each simple body, he remarked, could only have one motion *per se* but this could be brought about *per accidens* by many diverse motions. It was therefore permissible to invent circles within the boundaries of each concentric spherical shell in order to describe the apparent motions.

Jean Pena,[2] one of the collaborators of the humanist educational reformer Pierre de la Ramée (Ramus), was the first to reject the concentric planetary spheres on the basis of empirical evidence. For in the preface to his edition of Euclid's *Optics* (1557), he claimed that the existence of such spheres was inconsistent with optical theory. By measurements with his *radius astronomicus*, Gemma Frisius had shown the separation of two neighboring stars, one slightly higher in the sky than the other, was the same in all altitudes. The absence of refraction, Pena claimed, proved that there was a single medium extending to the fixed stars and this had to be air. Although Gemma had failed to detect the differences that Tycho Brahe's observations later revealed, Pena deserves credit for rejecting the spheres on the basis of optical theory and the best observational data available to him. Firm empirical evidence against the Aristotelian concept of the spheres was provided by Tycho Brahe's observation of the supernova of 1572, which revealed no detectable parallax.

In 1563 Ramus[3] himself wrote to Georg Joachim Rheticus asking for an astronomy without hypotheses, by which he meant concentric spheres, epicycles, eccentrics, and equants. Believing Rheticus to be the author of the anonymous preface to Copernicus' *De revolutionibus*, in fact written by Andreas Osiander, in which it is agreed that the epicycle of Venus is

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[1] Aiton (1981), 100. Bayer wrote under the pseudonym Ariel Bicard.

[2] Ibid., 101.

[3] Aiton (1975).

a fiction, Ramus hoped for a favorable response. At the time, Rheticus in fact held the epicycles and eccentrics to be real, but he later changed his mind and in 1568 accepted Ramus' challenge, promising to "undertake the work, which had been present in your mind also, of freeing astronomy from hypotheses by restricting myself to the observations alone."[4] In 1570, while in Augsburg, Ramus met Tycho Brahe and discussed with him the idea of an astronomy based only on the observations. He must have been disappointed to hear from Tycho that this was not possible.

Ramus made his views generally known in his *Scholae mathematicae* (1569), where he described all hypotheses as absurd fabrications. He then expressed the hope that one of the celebrated schools of Germany (where mathematics was cultivated) would produce a philosopher and mathematician capable of responding to his challenge. As an inducement, he offered his Royal Chair in Paris to the one who would construct an astronomy without hypotheses.

Soon after the publication of his first major astronomical work, the *Mysterium cosmographicum* in 1596, Kepler[5] wrote to Maestlin claiming that he (and Copernicus also) had answered the challenge of Ramus. For he supposed that Ramus required only the rejection of fictitious hypotheses or hypotheses that could not be demonstrated. If Ramus intended the rejection of all hypotheses, both fictitious and true, then, in Kepler's view, Ramus was a fool, and he explained to Maestlin that he would rather claim the Royal Chair than call Ramus a fool. Misunderstanding the joke, Maestlin supposed that Kepler had actually been offered a chair in Paris.

Kepler repeated his claim to have answered Ramus' challenge in his *Astronomia nova* (1609) and again in his *Rudolphine Tables* (1627), where he mentioned among the causes for the long delay in publication the "transfer of the whole of astronomy from fictitious circles to natural causes."[6] Thus Kepler saw his response to Ramus' challenge as a central pillar of his new astronomy.

Kepler's views on hypotheses were set out in his *Apologia Tychonis contra Ursum*,[7] composed in 1600–1601 at the request of Tycho Brahe to counter the claim of Raimarus Ursus to priority in the formulation of the Tychonic system. For Kepler, the distinction between true and fictitious hypotheses corresponds to that between astronomical and geometrical hypotheses. If an astronomer says that the path of the moon is an oval, this is an astronomical hypothesis representing the true motion. When, however, he proposes a combination of circular motions by which the oval orbit may be described, he is proposing a geometrical or fictitious hypothesis. For example, Ptolemy proposed an astronomical or true hypothesis when he said that the motion of the planets slowed down at apogee and acceler-

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[4] Burmeister (1967–1968), vol. 3, p. 188.
[5] Duncan (1981), 29.
[6] Aiton (1975), 57.
[7] Jardine (1984).

ated at perigee, but he introduced a geometrical or fictitious hypothesis with his equant point or circle.

According to Kepler, true hypotheses, besides describing physical reality, must also explain the causes of planetary motion. In the *Apologia*, a polemic written in support of Tycho where dynamical arguments would have been out of place, there is just an implied allusion to such causes in a reference to William Gilbert's work on the magnet. However, the primacy assigned by Kepler to what he called physical or metaphysical causes is made quite explicit in the *Mysterium cosmographicum*,[8] where he recalls that, in a student dissertation, he had ascribed the earth's motion to the sun on physical, or if the reader would prefer, metaphysical grounds, as Copernicus had done on mathematical grounds. In this context he seems to have regarded the terms physical and metaphysical as equivalent. Besides mechanical causes, the terms included also archetypal causes, for Kepler habitually refers to his appeals to the latter as physical reasoning. He appears to be using the word φύσικος in something close to its etymological meaning; that is, pertaining to nature in the sense that a "physical" reason is describing the way things work in the natural world, taking the natural world to include celestial as well as terrestrial phenomena.[9] Although Kepler's physical or metaphysical causes were regarded as true hypotheses, he emphasized nevertheless, in a letter to David Fabricius[10] of 4 July 1603, that such hypotheses must be built upon and confirmed by observations.

Kepler was imbued with the spirit of Platonism and in a marginal note to a passage from Proclus quoted in his *Harmonice mundi*, he described the *Timaeus* as a commentary on the book of Genesis, transforming it into Pythagorean philosophy.[11] The general idea of the world as the visible image of God, which we find at the end of the *Timaeus*,[12] was one that Kepler made his own. Having raised the question why God had first created bodies, he found the key to the solution in the comparison of God with the "curved" and created nature with the "straight," a comparison that had been made by Nikolaus von Kues (Cusanus) and others.[13] Kepler saw the harmony between the things at rest, in the order sun, sphere of fixed stars, and intervening space, as a symbol of the Three Persons of the Trinity. It was God's intention, Kepler believed, that we should discover the plan of creation by sharing in His thoughts. It seemed to Kepler that the distinction between the curved and the straight was such a useful idea, that it could not have arisen by accident but must have been contrived in the beginning by God. Then in order that the world should be the best and most beautiful and reveal His image, Kepler supposed that God had created

[8] Duncan (1981), 63.

[9] Field (1988), 53. Cf. Jardine (1979), 165.

[10] KGW 14, p. 412.

[11] KGW 6, p. 221.

[12] Plato, *Timaeus*, 92C.

[13] Duncan (1981), 93. Kepler amplified his description of the symbolism in the *Epitome* (KGW 7, p. 47, p. 51 and p. 258). See also Petri (1971).

magnitudes and designed quantities whose nature was locked in the distinction between the curved and the straight, and to bring these quantities into being, He created bodies before all other things. Finally God provided us with a mind or intellect, which, as Kepler wrote to Maestlin on 9 April 1597,[14] was an instrument for the knowledge of quantity like the eye for colors and the ear for sounds. Quite clearly the quantities that Kepler had in mind were not the abstract numerical ratios of the Pythagoreans but concrete ratios embodied in real bodies; that is, for a Platonist, quantities embodied in geometrical objects such as the regular polygons and the Platonic and Archimedean polyhedra. For Kepler, as for Plato, God was a geometer.

The details of Kepler's geometry were, of course, mainly derived from Euclid, though he appealed to the philosophy of mathematics developed by Proclus as a basis. Following Proclus, he believed that the principal aim of Euclid's *Elements* was to establish the properties and existence of the five regular polyhedra, which were in some sense "World Figures."

In the *Timaeus*,[15] Plato constructed a theory of matter in which regular polyhedra were assigned to the elements. The theory is described entirely in terms of geometrical properties; that is, in terms applicable only to mathematical entities regarded by Plato as belonging to the realm of forms. By this means, he succeeded in giving a fairly detailed qualitative account of sublunary nature. Kepler seems to have rejected this theory, for in his *Strena seu de nive sexangula* (1611), where he could have used it to explain the action of cold air on water to form snow, he does not even mention the polyhedral forms of the elementary particles. Concerning sublunary nature, Kepler was content to give a sketchy account in terms of the response of the World-Soul to the aspects or effective astrological configurations.

The theory of the elements is the only more or less complete scientific theory in the *Timaeus*. By contrast, Plato's description of the heavens is nothing more than a sketch, perhaps conveying the impression of an animated armillary sphere rather than the real cosmos. For Kepler, the regular polyhedra or Platonic solids provided a key to the structure of the planetary system. In developing this application, he used mathematics in a way that was very similar to that of Plato in the *Timaeus*, but whereas Plato only produced a vague qualitative theory, Kepler succeeded in devising a testable mathematical model of the cosmos. Although this model, as described in the *Mysterium cosmographicum*, was only partially successful, Kepler incorporated it in a modified form into his definitive account of the cosmos set out in the *Harmonice mundi*. There he reiterates his faith in what he had shown "in my *Mysterium cosmographicum*, published twenty-two years ago, that the number of planets, or spheres, surrounding the

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[14] KGW 13, p. 113.
[15] Field (1988), 1–16.

sun was taken by the most wise Creator from the five regular solids on which Euclid wrote a book many centuries ago."[16]

In the preface to the *Mysterium cosmographicum*,[17] the youthful work that marked the beginning of his vocation as an astronomer, Kepler relates that, in his student days, he compiled a list, based partly on Maestlin's lectures and partly on his own reflections, of the advantages of the system of Copernicus over that of Ptolemy from the mathematical point of view. He had been attracted to the Copernican system because each motion attributed to the earth in this system explained some irregularity or apparent coincidence in the motions of the other planets, which had remained inexplicable in the geometric systems of Ptolemy and Tycho Brahe. Whereas Copernicus, however, had recognized the wonderful arrangement of the world *a posteriori* from the observations, Kepler claimed that this could have been proved *a priori* from the idea of creation, or God's purpose to create the most beautiful and perfect world that would reflect the divine image.

There were above all three things, Kepler explained, whose causes he sought; namely, the number, magnitudes, and motions of the planetary spheres.[18] These questions were answered at least in part by his polyhedral hypothesis. Because the regular polyhedra or Platonic solids were the most perfect bodies constituted from straight quantities, he supposed that, in a nest of the regular polyhedra separated by spherical shells, he had found the divine blueprint or *a priori* reason that could explain the number and arrangement of the planets. For the five bodies could be interpolated between the six known planets and the agreement with observation, though not perfect, was sufficient to satisfy Kepler that he was on the right track. To account for the motions of the planets, he postulated an "anima motrix" (moving soul) in the sun, thus introducing the idea of an efficient cause. By combining formal causes like the polyhedral hypothesis and efficient causes such as the anima motrix in his search for explanations of the cosmos, Kepler was following the authentic teaching of Plato, who emphasizes in the *Timaeus*[19] that, in explaining the origins of things, both mechanical causes and divine purposes should be considered, and moreover, that if we wish to attain a true scientific explanation that satisfies the human reason, we must be primarily concerned with the causes that lie outside the material in the realm of the spiritual.

Owing to the discrepancy between the predictions of the polyhedral hypothesis and those of the theory derived from the motions,[20] Kepler

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[16] KGW 6, p. 298.

[17] Duncan (1981), 63.

[18] Kepler, like Tycho Brahe, rejected the concept of solid (i.e. material) spheres. Ibid., p. 167.

[19] Plato, *Timaeus*, 46D–E.

[20] Kepler's theory of the effect of an anima motrix led to an erroneous relation between the mean distances and the periodic times. See Duncan (1981), 197–207 and 249–250.

recognized that further researches would be needed to perfect his explanations. Yet he was confident that, some day, the two theories would be reconciled, and indeed, in the second edition of the *Mysterium cosmographicum*,[21] he was able to comment that this reconciliation had been effected twenty-two years later. In particular, he was conscious of the fact that, at the time of writing the *Mysterium cosmographicum*, he did not yet know the *a priori* or formal cause of the eccentricities of the planetary orbits.[22] This cause he later located in the cosmic musical harmony, a concept which, strangely perhaps, had no place in the *Mysterium cosmographicum*.[23]

On 26 March 1598 in a letter to Herwart von Hohenburg, the Bavarian chancellor, Kepler explained that the *Mysterium cosmographicum* or *Prodromus* (forerunner) would serve as an introduction to a series of cosmographical treatises dealing more fully with the subjects of Aristotle's *De caelo* and *De generatione*.[24] As a consequence of his meeting with Tycho Brahe, his plans were changed, so that these works in fact were never written, but twenty-two years later, in his notes for the second edition of the *Mysterium cosmographicum*, Kepler remarked that he regarded the *Harmonice mundi* as "the authentic and appropriate successor" of his *Prodromus*.[25]

Kepler worked out the principal ideas of the *Harmonice mundi* in the summer months of 1599, at a time of personal tragedy, with the death of his young daughter and insecurity as the Counter-Reformation came to Graz. These ideas were described in his letters to Edmund Bruce in Padua,[26] Herwart von Hohenburg in Munich, and Michael Maestlin in Tübingen, especially in the detailed writings of 6 August and 14 September to Herwart[27] and 29 August to Maestlin.[28] Then on 14 December 1599 he communicated to Herwart[29] his intention to write a cosmographic dissertation, evidently based on the quadrivium, with the title *De harmonice mundi*, which would consist of five parts:

1. A geometrical part on the constructible figures.
2. An arithmetical part on the solid relations.
3. A musical part on the origins of the harmonies.
4. An astrological part on the origins of the aspects.
5. An astronomical part on the origins of the periodic motions of the planets.

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[21] Ibid., 215.

[22] Ibid., 211.

[23] In the *Mysterium cosmographicum*, chapter 12, Kepler correlated the Platonic solids with the musical harmonies, as Plato had done, and also correlated musical harmonies and astrological aspects, but there is no suggestion of a "harmony of the spheres." See Duncan (1981), 131–147 and 240–243.

[24] KGW 13, pp. 190–191.

[25] Duncan (1981), 51.

[26] KGW 14, pp. 7–16.

[27] KGW 14, pp. 21–41 and 62–76.

[28] KGW 14, pp. 43–59.

[29] KGW 14, p. 100.

Although this division was essentially retained in the *Harmonice mundi*, the first part was divided into two books and the subject of the second part was taken up at the beginning of the third book.

Three questions concerning harmony were considered by Kepler in the letters to Bruce, Herwart, and Maestlin in 1599.[30] The first question related to the origin of the musical harmonies, briefly touched upon in the *Mysterium cosmographicum*.[31] By tradition, explanations of the origin of the musical ratios appealed to the supposed special property of the first few numbers. For Pythagoras, the set of numbers 1, 2, 3, 4, to which he gave the name tetractys, was special on account of the fact that 1 + 2 + 3 + 4 = 10, which he regarded as the perfect number. The musical theory which appears in Plato's *Timaeus* and is attributed to Pythagoras, is based on the tetractys, so that only the fourth, fifth, and octave, corresponding to the ratios 3:4, 2:3 and 1:2, are recognized as consonances. In the resulting scale, using only the major tone (8:9)—the interval between the fourth and the fifth—and a narrow semitone (243:256), which combines with two tones to make up a fourth, thirds, and sixths are treated as dissonant. Although the Pythagorean system was defended by Vincenzo Galilei in his *Dialogo della musica antica et della moderna* (1581), by the second half of the sixteenth century, Pythagorean intonation had really given way to a system of just intonation. This was a scale that contained the maximum number of just or exact consonances, including the major and minor thirds (3:5 and 5:8) and sixths (4:5 and 5:6) that were needed for the polyphonic music of composers such as Orlande de Lassus. It was of course impossible to devise any scale in which all the consonances were just. In this one, only the minor third and the fifth starting on the second note of the scale had to be slightly narrow but the scale employed both major and minor tones (8:9 and 9:10) and also wide semitones (15:16).

The system of just intonation was described by Gioseffo Zarlino in his *Istitutioni harmoniche* (1558) but it is in fact substantially equivalent to that described by Ptolemy in his *Harmonica*. At this time Kepler had not yet read Ptolemy's work,[32] and as his letters[33] indicate, his principal source for the theory of harmony of the Greeks was Boethius' *De institutione musica*, Latin editions of which had been published in Venice in 1492 and in Basel in 1546 and 1570.[34]

Kepler accepted the system of just intonation, which he generally re-

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[30] It was at this time that Kepler discovered the star polyhedra. See KGW 14, p. 34.

[31] Duncan (1981), 131–133.

[32] At the end of chapter 12 of the *Mysterium cosmographicum*, Kepler surmised that Ptolemy's *Harmonica* and the commentary of Porphyry, which Regiomontanus proposed to publish, no doubt treated the relation between the consonances and the aspects. These were just two of a collection of books that Regiomontanus intended to publish but his early death prevented him from carrying out the scheme. The prospectus is reproduced in Zinner (1938), Tafel 26.

[33] See, for example, KGW 14, pp. 60 and 64; 15, pp. 238, 389, and 449; 16, pp. 86 and 141.

[34] There is a German translation by Oscar Paul (1872).

ferred to as Ptolemy's system, on the grounds that observation showed the thirds and sixths to be consonances.[35] By following Zarlino and Ptolemy, however, Kepler took the side of orthodoxy in music theory rather than that of the more progressive composers of his day, who considered the system of just intonation to be inadequate. Claudio Monteverdi, for example, in reply to accusations of using dissonances improperly, promised a work on the different system that he employed, but unfortunately this never appeared in print.

While he accepted Ptolemy's system, Kepler rejected the explanation in terms of abstract numbers, seeking instead the origins of the musical harmonies in the archetypal forms of geometry; in fact, in the division of the circle by the vertices of regular polygons. The circle is imagined to be opened out into a string. In order to account for no more than seven consonances, a number of restrictions had to be imposed. After some unsuccessful attempts, he finally arrived at a successful formula. First, the sources were restricted to polygons that could be constructed with ruler and compasses. Then a harmonic proportion was held to be produced by a division of the circle if, and only if, the parts formed ratios with the whole and with each other that belonged to a constructible polygon. With these conditions Kepler was able to account for the exact number of seven consonances not greater than an octave. Consonances with intervals greater than an octave could be regarded as identical with one or other of these seven. The principal difficulty had arisen with the octagon, which not only generated the minor sixth 5:8 but also the dissonance 7:8. Since the two parts 1 and 7 did not belong as part and whole to a constructible polygon, however, the ratio 1:7 and hence also 7:8 were excluded. Another difficulty was presented by the 15-sided polygon. Kepler excluded this on the grounds that it did not have its own construction but depended on that of the pentagon.[36]

From the differences of the consonances he obtained the melodic intervals of the major and minor tones (8:9 and 9:10), the semitone (15:16), and the diesis (24:25). With these he built up the diatonic and chromatic scales on a given base note.

The second question raised by Kepler in the letters of 1599 concerned the relation of the harmonies with the aspects.[37] Kepler worked in astrology not only because it was part of his duties as District Mathematician to prepare an annual calendar containing information about the weather and important events, but also because he believed in it. For him it was a fact of experience that the aspects had an influence on the weather and

[35] Accounts of Kepler's music theory may be found in Walker (1978) and Dickreiter (1973).

[36] Kepler returned to this problem in a letter to Herwart in 1607, KGW 15, pp. 395–396.

[37] Scattered throughout the letters are also references to the relation of harmonies to the meters of poets, dance rhythms, refraction of colors in the rainbow, smells and tastes, parts of the body, and architecture.

also on the human soul.[38] Until 1608 Kepler believed in the correspondence of the aspects with the musical harmonies, so that the ground for the influence of the aspects was the same as that for the origin of the musical harmonies. In this he was in agreement with Ptolemy, who used the musical harmonies to explain the aspects in his *Harmonica*, Book III, chapter 9.[39] Kepler did not ascribe any direct physical influence to the celestial bodies but supposed the astrological effects to be the result of instinctive responses of individual souls to the harmonies of certain configurations or aspects. The soul of man, which carried the geometrical archetypes, responded both to music and the aspects. A soul was also ascribed to the earth, whose response to the aspects explained their influence on the weather. Kepler expounded this theory in his Calendar for 1599. He recognized eight aspects (or influential configurations), corresponding to the eight musical harmonies (including unison). These were conjunction, sextile, quadrature, trine, opposition, quintile, trioctile, and biquintile. Ptolemy recognized only the first five.[40]

The third question considered by Kepler in the letters of 1599 concerned the speeds of the planets in their orbits. Here for the first time he introduced the Pythagorean idea of the harmony of the spheres.[41] If the heavens were filled with air, he believed, audible music would be produced. But in the absence of air, an intellectual harmony was present, in which even God, in a certain sense, found no less pleasure than that afforded to man by the sound of musical consonances. Kepler assigned to the single planets speeds whose ratios were in agreement with the musical consonances. To Saturn he assigned 3, Jupiter 4, Mars 8, Earth 10, Venus 12, and Mercury 16. Consequently Jupiter and Mars produced the octave, Saturn and the Earth a major sixth plus an octave, the Earth and Mercury a minor sixth, Mars and Venus a fifth, Saturn and Jupiter a fourth, Mars and the Earth a major third, and the Earth and Venus a minor third. All the basic harmonies were thus used and Kepler had a medley of reasons why exactly these and no other intervals appertained between two planets.

Ptolemy[42] assigned tones (or notes) to the heavenly bodies according to their distances from the Earth, giving the lowest note to the Moon and the highest note to Saturn. The Moon, Venus, and Mars were represented by two conjoint tetrachords and the Sun (which he placed above Venus), together with Jupiter and Saturn, by a similar pair of tetrachords overlapping the first pair. According to this scheme, the pairs Moon-Venus,

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[38] See Kepler's letter to Herwart of 14 September 1599, KGW 14, p. 74.

[39] The most comprehensive account of Greek astrology is that of A. Bouché-Leclercq (1899). On the aspects, see pp. 165–179.

[40] See KOF, vol. 5, pp. 371–378.

[41] The source of inspiration for schemes of celestial harmony seems to have been Plato, *Republic*, 617B. Kepler's scheme may be compared with that of Plato, *Timaeus*, 36D–E.

[42] Ptolemy, *Harmonica*, Book III, chapter 16. There is a German translation by I. Düring (1934). The note assigned to the moon on p. 136 should be Hypate meson.

Venus-Mars, Sun-Jupiter, and Jupiter-Saturn each produced a fourth, while the moon and Jupiter produced an octave.

In reply to Herwart, who objected that the whole theory of cosmic harmony was grounded in conjecture,[43] Kepler explained[44] that not every conjecture was false. For man is an image of God, and it is easily possible, that in things appertaining to the decoration of the world, he thinks like God. The world participates in quantity and the human soul grasps nothing so well as quantity, for the knowledge of which it was evidently created.

The relative speeds assigned to the planets in accordance with the harmonic theory, together with the periodic times, permitted Kepler to calculate the relative distances of the planets from the Sun. He was satisfied that these distances were in better agreement with those of Copernicus than were the distances he had deduced from the motions in chapter 20 of the *Mysterium cosmographicum*. From the interpolation of the polyhedra, he knew the interval between the least distance of any planet and the greatest distance of the planet immediately below. Using the mean distances obtained from the harmonic theory, it would have been possible to deduce the eccentricities *a priori*. Yet Kepler did not make such a test, although he recognized that it was possible. For he was reluctant to proceed until he had the more exact support from experience that only Tycho Brahe could provide. Then, he remarked to Herwart, he would erect a grand structure.[45]

Early in 1600 Kepler visited Tycho Brahe in Prague, later in the year becoming his assistant and finally succeeding to the position of Imperial Mathematician on Tycho's death in October 1601. The move to Prague changed the course of Kepler's researches. Finding that Tycho was secretive about his observations and recognizing that the immense observational material needed analysis, Kepler set to work on the orbit of Mars, which had presented difficulties to Tycho's assistant Longomontanus. The harmonic studies[46] were pushed into the background while the work on Mars proceeded. This extended over five years (though one of these was devoted mainly to the study of optics) and resulted in the *Astronomia nova*, completed in 1605 but published only in 1609.

A few weeks after he discovered the orbit of Mars to be an ellipse, Kepler expressed to Christopher Heydon[47] the hope that God would release him from astronomy so that he could turn his attention again to the work on harmony. Even during the time that he was preoccupied with the orbit of Mars, the subjects of his projected work on harmony had not been entirely neglected. A special study of Book X of Euclid's *Elements* on the theory of irrationals helped Kepler to elaborate the more system-

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[43] KGW 14, p. 59.

[44] KGW 14, p. 71.

[45] KGW 14, p. 29.

[46] These *a priori* speculations, he wrote to Herwart, should not be in conflict with known experience. Rather they must be brought into agreement with it. KGW 14, p. 130.

[47] KGW 15, p. 233.

atic foundation for his theory that was to make up Book I of the *Harmonice mundi.* By means of the concept of knowability, a term first used in his letter of 1 October 1602 to David Fabricius,[48] he ordered the regular polygons according to degrees of irrationality. Polygons that could not be constructed with ruler and compasses, such as the regular heptagon, were completely unknowable, even for God, so that they played no part in the construction of the world. When Kepler met Jost Bürgi in Prague in 1603 and learned from him how an algebraic relation could be formed between the side of the regular heptagon and the diameter of the circumscribing circle, Kepler objected that this was of no use since it could not provide an exact geometrical construction. Besides knowability, another property had already been introduced by Kepler for limiting the polygons giving rise to harmonic ratios in his letter to Herwart of 6 August 1599. This property, which he called congruence and which relates to the possibility of fitting together to form tessellations in the plane or polyhedra in the solid, became the basis of Book II of the *Harmonice mundi.* This property was mentioned again in connection with the musical harmonies in 1605 in a letter to Christopher Heydon.[49] By 1608, however, observations had led Kepler to the conclusion that the musical harmonies and the aspects did not correspond.[50] In consequence of this recognition, which disproved the teaching of Ptolemy, he sought to locate the origin of the musical consonances in the constructible polygons and that of the aspects in the congruent polygons.

During his time in Prague, Kepler published several works on astrology. The first appeared in 1602 with the title *De fundamentis astrologiae certioribus.*[51] Written hastily in the last months of 1601, following Tycho's death, it seems likely that Kepler's intention, at least partly, was to help him secure the first payment of his salary as Imperial Mathematician, for which he had in fact to wait for five months. On the other hand, the composition of the work may be regarded as a response to a remark of Maestlin, to whom Kepler in 1598 had expressed his intention to reform astrology. Those who defended all the nonsense, he explained, were like the Jesuits, but then he added, "I am a Lutheran astrologer, throwing out the chaff and keeping the grain."[52] Although Kepler was not exceptional in proposing reform, for astrologers had always differed on minor points and defended their favorite systems, his reform of astrology does seem to have been as radical as Luther's reform of the Church. A theory of astrology freed from superstition was important to Kepler because of the way in which astrology fitted into his conception of the world as a whole and the part that it played in his cosmology. Maestlin[53] advised him, however,

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[48] KGW 14, p. 266.
[49] KGW 15, p. 236.
[50] KGW 6, p. 258.
[51] For English translations, see Meywald (1941), Rossi (1979), and Field (1984a).
[52] KGW 13, p. 184.
[53] KGW 13, p. 210.

that the kind of basic criticism of traditional astrology that he had introduced into his Calendars for 1598 and 1599 should be reserved for the learned.

In October 1604 the appearance of a new star close to the return of a Great Conjunction of Saturn and Jupiter to the Fiery Trigon—the triplicity of signs Aries, Leo, and Sagittarius—was an event of some astronomical significance,[54] for such a return occurred only once in eight hundred years. Two years later, Kepler published an account of the event with the title *De stella nova*. In this book, he took the opportunity to expound his ideas on astrology. For example, he produced arguments for the cultural origin of the division of the zodiac, so that there was no reason to expect natural events from this division.[55] Indeed Kepler declared that the aspects were almost the only items of traditional astrology worthy to be retained.[56]

Kepler did believe in the exceptional possibility of supernatural appearances. Having made a detailed distinction between the natural and supernatural, he claimed to show that the new star of 1604 appearing at a return to the Fiery Trigon—itself a natural event—was due to Divine Providence, like the Star of Bethlehem.[57] To support his argument that the formation of the new star at the time of the return to the Fiery Trigon could not be attributed to chance, he cited his wife's reply when he asked her if the salad she had prepared for dinner could have come together of its own accord.[58] But he criticized the astrologers for making predictions from such signs, for in his view only a prophet could grasp the meaning of God's message.

Another opportunity for Kepler to expound his ideas on astrology was provided by the appearance of the works of two physicians, Helisaeus Röslin, a firm believer in the whole of traditional astrology, and Philip Feselius, who opposed astrology altogether. Kepler responded in two German works, *Antwort auf Röslini Diskurs* (1609), in which he opposed the author's superstitions, and *Tertius interveniens* (1610), in which he suggested to Feselius that one should not throw out the baby with the bath water. It was in the *Tertius interveniens* that Kepler announced his conclusion, based on over sixteen years of daily weather observations, that there was not an exact correspondence between the aspects and the musical consonances.[60] In the same work, he explained more clearly the basis of horoscopes. When at birth man begins his independent life, his (instinctive) perception of the constellations combines with the ideas and emotions

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[54] Mars was also nearby. For a detailed account, see Field (1984a), 199–201.
[55] KGW 1, pp. 168–177.
[56] KGW 1, p. 166.
[57] KGW 1, pp. 275–292.
[58] KGW 1, p. 285.
[59] KGW 1, pp. 335–356.
[60] KGW 4, p. 205.

he receives from his mother to form his character.[61] Although the birth configurations could thus give an indication of a person's disposition, Kepler opposed the idea that it could determine his future. Commenting on his own horoscope, he explained that the astrologers would look in vain to find in it the causes that led him to discover the polyhedral hypothesis in 1595, for his stars were not those in the sky but Copernicus and Tycho Brahe.[62]

Although Kepler made no further progress on the subject of the harmony of the planetary motions during his time in Prague, he was able to make a more penetrating study of Ptolemy's *Harmonica.* As he believed the Latin translation of Antonio Gogava, published in Venice in 1562, to be inaccurate (it was in fact based on a corrupt Greek version), he had asked Herwart to arrange the loan of a copy of the Greek manuscript for him. This was received together with the commentary of Porphyry in 1607[63] and in their correspondence at this time Herwart and Kepler discussed the possibility of publishing the text along with a new Latin translation.

Ptolemy's description of musical consonances, astrological aspects, and the structure of the planetary system as manifestations of a universal harmony expressible in mathematical terms corresponded in general with Kepler's own vision of a cosmic harmony. While he could adopt a sympathetic approach to Ptolemy's work and model his own version of the cosmic harmony on it, Kepler had some basic disagreements with Ptolemy. First, he rejected the numerological origin of the musical consonances that Ptolemy shared with the Pythagoreans in favor of a geometrical basis. Second, having originally shared Ptolemy's belief in an exact analogy between the musical consonances and the astrological aspects, he changed his mind, on the basis of observations, attributing these two manifestations of harmony to different geometrical causes. Third, he regarded Ptolemy's planetary harmony as only poetic or rhetorical.

Owing to political events in Prague, Kepler took up a new office in Linz in the spring of 1612. There in October 1613 he started on the calculations of the elements of the orbits of the other planets and the Moon that would be needed in the preparation of the *Rudolphine Tables*, a burdensome task he had inherited from Tycho Brahe. As the first fruit of this research he offered his *Ephemerides novae* for 1617. In the dedication to Emperor Matthias, written on 1 November 1616, he explained that he wished to test the calculations of the Ephemerides following his new theory against the observations, before proceeding to the final composition of the Tables. According to his own account, it was at this time that he had the first dawn of insight into the relationship of the periodic times and the mean distances of the planets that, eighteen months later, developed

[61] KGW 4, pp. 209–210.

[62] Hammer (1971), 16–30. An account of Kepler's theory of personality is given by Sticker (1973).

[63] KGW 15, p. 415. On Kepler's reading of Ptolemy's *Harmonica*, see Klein (1971).

into the full light of day with the discovery of the third law. The actual discovery followed another personal tragedy. On 9 February 1618, Kepler's daughter Katharina died. He laid the Tables aside, for they required tranquillity, and turned his mind to the completion of the Harmony. An error of calculation prevented success on 8 March but having corrected this, he discovered the law on 15 May, thus finding the missing piece that would enable him to complete the puzzle.

According to his own account, Kepler completed the *Harmonice mundi* on 27 May 1618. The printing took more than a year, beginning with Book III, then Books IV and V, and finally Books I and II. Wilhelm Schickard, whom Kepler had met on a visit to Württemberg in 1617, produced the tables and diagrams. Kepler dedicated the work to King James I of England, in accordance with a decision he had taken several years earlier. James, he believed, was a peacemaker who could reunify the Protestants and Catholics, and when his *De stella nova* was published in 1606, Kepler had presented a copy to him which is now in the British Library. In the political circumstances of 1620, when James's son-in-law, the Elector of the Palatinate, set himself up against the Emperor, this dedication could, however, be seen as having been foolhardy, and indeed it is lacking in some copies.

In the summer of 1619, Kepler received from Johannes Remus news that the first part of his *Epitome astronomiae Copernicanae*, published in 1618, had been placed on the Church's Index of prohibited books. Although he was assured by friends that books by German authors would be bought and read secretly in Italy, even if they had been prohibited, he was apprehensive about the circulation of the *Harmonice mundi*, so he directed an open letter to foreign book dealers, especially those in Italy, in which he asked the censors to examine the new evidence he had produced in favor of the Copernican system.[64]

The principal argument of the *Harmonice mundi* begins with the introduction of the harmonic proportions in the first two chapters of Book III. In Books I and II, Kepler provides the geometrical foundation of these proportions by revealing and expounding the geometrical figures from which they are derived. With the introduction of algebraic calculation in the sixteenth century, of which the educational reformer Pierre de la Ramée (Ramus) was an influential advocate, the old opposition between realism and nominalism came to the fore in the philosophy of mathematics. For the nominalist Ramus, there was no logical objection to the representation of irrationals by rational approximations and Euclid's discussion of irrationals in Book X of the *Elements* could be dismissed as obscure and worthless, having no application. Although Kepler himself used approximations in calculations of planetary positions, he followed the Greek ideal in the foundations of mathematics. In this part of mathe-

[64] The letter is published in KOF, vol. 5, pp. 8–9. There is a German translation in Caspar (1939), 384.

matics, he explained, he wished to appear as a philosopher. Thus Kepler was a realist in the sense of Plato and Proclus. For him the geometrical archetypes were first in the mind of God and then in those of creatures, while geometrical figures were first in an archetype and then in the world.[65]

In opposition to Ramus, Kepler declared that, for one who seeks the causes of things, Book X of Euclid's *Elements* was most important. He added that Ramus' student, Lazarus Schöner, a schoolteacher in Kornbach, recognized the utility of the five Platonic bodies on reading the *Mysterium cosmographicum.* Kepler extends the analysis of Book X of Euclid's *Elements* to things particularly important for his project. First, he introduces the concept of geometrical quantities that are knowable.[66] In effect, these are quantities that can be constructed with ruler and compasses. A quantity is knowable if it can be deduced, through some chain of operations, either from the diameter of a circle, if it is a line, or from the square of the diameter, if it is a surface. In order to rank the relative complexities of the constructions of different quantities, Kepler introduces the concept of degrees of knowability. The first and most immediate degree of knowability occurs when a line is equal to the diameter or an area is equal to the square of the diameter.[67] Slightly less immediate is the second degree of knowability,[68] when the line or area is equal to some number of parts of the diameter or its square. In this case, the line is called expressible in length and the area is simply called expressible. The third degree of knowability[69] occurs when the line is inexpressible in length but its square is expressible. Such a line is called expressible in square. All the remaining degrees of knowability involve quantities that are inexpressible. For example, in the case of the fourth degree of knowability, neither the line nor its square is expressible but the square can be transformed into a rectangle whose sides are expressible in square.[70]

In the second part of Book I, Kepler identifies the constructible regular polygons and orders them according to the degrees of knowability of their sides and areas. Along with the constructible polygons, he sometimes introduces the corresponding star polygons as independent, though secondary figures. Finally, he considers the inconstructible polygons, such as the regular heptagon. The sides of such polygons are unknowable and for this reason God was not able to use them for the ornament of the world.[71] For being unknowable, they must remain outside God's mind and hence cannot contribute to the divine archetype.

Another property for classifying the regular polygons, that which he

[65] KGW 6, p. 15.
[66] Definition 8.
[67] Definition 12.
[68] Definition 13.
[69] Definition 14.
[70] Definition 15.
[71] Proposition 45.

called congruence, is introduced by Kepler in Book II. It concerns the capacity of a regular polygon, either with its own kind or other regular polygons, to tessellate in a plane or form solid figures. Apart from these two types, which may be described as congruence in the plane and in space respectively, Kepler mentions a third type, namely space filling congruence, and he notes that only the cube and the rhombic dodecahedron can form such a congruence.[72]

Each type of congruence possesses degrees of perfection. For example, congruence in the plane is perfect when the angles of the figure come together in the same way at each meeting point so that the pattern can be continued to infinity, while it is most perfect if, in addition, the figures are of the same kind.[73] An example of an imperfect congruence would be the case of a larger figure surrounded by similar meeting points but in which the congruence could not be continued to infinity or could only be continued by the introduction of different kinds of meeting points.[74]

Congruence in space is most perfect when the plane figures are all of the same shape. In these cases, the congruence gives rise to the most perfect solid figures. These are principally the five Platonic bodies[75] and also the star polyhedra that Kepler had discovered.[76] He evidently regarded the star polyhedra as of relatively minor importance, since they could be derived from the dodecahedron and icosahedron. This could be done either by the addition of pentagonal and triangular pyramids respectively to the faces of these Platonic solids, or by converting the pentagons found in the dodecahedron and icosahedron into the corresponding star pentagons. Kepler does not in fact explain how he discovered the star polyhedra. They were significant for Kepler mainly because they affected the status of the pentagram in connection with the astrological aspects, though he pointed out that the twelve pointed star polyhedron could also be fitted into the scheme of nested polyhedra and planetary spheres described in the *Mysterium cosmographicum.*

Congruence which is still perfect but of lower degree occurs when all the angles lie on the same spherical surface, though the plane figures are of different kinds. Such congruences give rise to the thirteen Archimedean solids.[77] Imperfect congruences arise when the larger plane figure does not occur more than once or twice. Examples are a pyramid on a square base and prisms of various kinds.

Kepler finally classifies the regular polygons and their stars according to the degree of congruence. For example, the triangle and square are of the first degree because they form congruences in the plane and in space, both by themselves and when combined with other figures. At the

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[72] Definition 5.

[73] Definitions 2 and 3.

[74] Definition 4.

[75] Proposition 25.

[76] Proposition 26. On these polyhedra, see Field (1979a).

[77] Proposition 28.

other end of the scale is the icosigon, which will only form congruences in the plane and then only when combined with other figures; moreover, these congruences are imperfect.

The order of the regular polygons derived from the property of congruence does not coincide exactly with that based on degrees of knowability. A notable difference is that the number of constructible regular polygons is infinite, while the number having the property of congruence is limited to eight basic figures and four stars. However, all the polygons having the property of congruence are also constructible.

Although the primary aim of Books I and II was to provide the geometrical foundation for the world harmony, they also contain some of Kepler's chief original contributions to pure mathematics. Even the geometry of Book I amounts to more than a commentary on Book X of Euclid's *Elements.* For the classification of regular polygons according to the degree of commensurability of the sides with the diameter of the circle in which they are inscribed, though making use of Euclid's results, is original with Kepler. No more than a hint of its possibility may be found in Euclid. Another of Kepler's innovations is the inclusion of the diameter as the first regular polygon. He probably introduced the idea simply because it would be needed to deduce the harmonic ratios but it was far from being a trivial result, and has found significant applications in our own century.

Kepler's demonstration of the inconstructibility of the regular heptagon was a novel type of argument. Other mathematicians, such as Clavius and Cardano, seem to have taken it for granted that the regular heptagon could be constructed in the same way as the regular pentagon. Of particular interest is Kepler's linking of the heptagon problem with the classical problem of the trisection of an angle, also insoluble by the prescribed geometrical means. Although Kepler concluded that Jost Bürgi's method made no contribution to the problem with which he was concerned, he was nevertheless aware that the technique had something to offer to mathematics, in the sense that it was a general method of calculating approximations. He had in fact edited the introduction to Bürgi's lost sine tables.[78]

In Book II, Kepler gave the first systematic treatment of the problem of constructing all the tessellations formed by regular polygons, a topic that is still of interest to mathematicians of the present day. From flat patterns he turned to polyhedra. Besides discovering the two new regular polyhedra to which reference has been made, he also gave the earliest known proof that there are exactly thirteen semi-regular polyhedra or Archimedean solids. Kepler seems to have attached no particular importance to his significant discoveries in pure mathematics. Evidently he regarded these as incidental to his primary purpose of providing a sound geometrical basis for his world harmony.

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[78] List and Bialas (1973).

In the first chapter of Book III, Kepler seeks the origins of the harmonic proportions. After rejecting the number speculations of the Pythagoreans, which had led them, against the judgement of the ear, to reject thirds and sixths as consonances, he introduces the principles on which his own musical theory is based. First, an interval can only be accepted as harmonic if it satisfies the judgement of the ear. Second, it is not the sense of hearing but the soul or intellect that distinguishes the consonant intervals from the dissonant, so that the boundaries of the consonant intervals are knowable and those of the dissonant intervals are unknowable.

Although Kepler sets out the theory rigorously in a series of definitions, axioms, and propositions, the conclusions can be given in a few statements. First, a distinction is drawn between a part of a circle cut off in a division and the remainder. Any expressible fraction is called a part if it is less than a semicircle and a remainder if it is greater than a semicircle. Then the essence of the theory is expressed in the first three axioms. Axiom 1 states that the diameter of a circle, and the sides of the fundamental figures expounded in Book I that have their own construction, mark off a part of the circle which is consonant with the whole. Although the stars are included among the fundamental figures, a restriction is made for the purpose of excluding unwanted ratios. Axiom 2 correlates the quality of the consonance with the degree of knowability of the side of the figure that gives rise to it. Axiom 3 states that the sides of regular and star figures which are inconstructible mark off a part of the circle which is dissonant with the whole circle. The same applies to the side of a figure which is in fact constructible but not in its own right nor by a proper construction.

In general the axioms refer chiefly to constructibility rather than to congruence, because the motions in which harmonic proportions occur concern figures extended in straight lines, whereas congruence is a property of figures as a whole. Congruence, however, does play a part. For example, although the constructible 15-sided regular polygon is excluded from giving rise to a consonance because it does not have its own construction (the construction in fact depending on a combination of pentagon and triangle), Kepler offers the lack of congruence as an alternative reason for exclusion.

As doubling the number of sides of a constructible polygon leads always to another constructible polygon, the ratios $1{:}2^n$ represent a special class of consonances, which will later be called octaves. These are described by Kepler as identical consonances but in more remote degrees as n increases. Apart from unison, the consonance represented by the ratio 1:2 is the only one which is identical and perfect, since the diameter of the circle giving rise to it possesses the first degree of knowability.

Apart from the identical consonances, the parts of a circle or the remainders that Kepler proves to be consonant with the whole may be represented by the formula m/n, where n is the number of sides of a constructible regular polygon, m and n are co-prime and m is not the number of sides of an inconstructible regular polygon.

Having established the origin of the harmonic proportions, Kepler turns his attention to the harmonic divisions of a string. Such a division occurs when the whole string is divided into two parts such that they are consonant with one another and each with the whole. He finds that there are seven of these divisions, the same number as that of the consonances not greater than an octave that he had discovered in the first place with hearing as a guide.

Kepler classifies the seven consonances into three perfect consonances and two pairs of imperfect consonances.[79] Besides the octave, arising from the division of a circle by a diameter (a line having the first degree of knowability), the fifth (2:3) and the fourth (3:4) belong to the class of perfect consonances on account of the high degrees of knowability associated with their related figures, the triangle and the square respectively. For the sides of these figures are expressible in square and therefore have the third degree of knowability. On the other hand, the side of the pentagon, which gives rise to the major sixth (3:5) and the major third (4:5), is inexpressible, from which Kepler concludes that these consonances are imperfect. Although the minor sixth (5:8) can also be seen to be imperfect (for the reason that the side of the octagon giving rise to it is inexpressible), the reasoning by which Kepler concludes that the minor third (5:6) is imperfect seems contrived to obtain a desired result in face of the strongest indications of the opposite. For the minor third arises from the hexagon, whose side is expressible in length and therefore has the second degree of knowability. According to Kepler's principles, it would appear that this consonance should be classified as perfect, along with the fourth and fifth. However, he classifies the minor sixth and minor third as imperfect because they "bring something from the nature of the pentagon," though he also argues that they are less imperfect than the corresponding major intervals. When the imperfection is less, he adds, the intervals sound softer and smoother to the ear. For this reason he refers to the minor intervals as the soft third and sixth. By contrast the more imperfect major intervals sound hard and harsh, so that he calls them the hard third and sixth.

According to Kepler, the ancients were wrong in supposing the melodic intervals, such as tones and semitones, to be the basic elements out of which the consonances were built. On the contrary, he supposes that the consonances are prior by nature and that the melodic intervals, discordant in themselves but suitable for the flow of melody, arise as differences of the consonances.[80] Of the four melodic intervals, Kepler regards the progeny of the two perfect consonances of the fifth and the fourth, namely the major tone (8:9), as the only one that is perfect. The minor tone (9:10), born either of the fifth and major sixth or the fourth and minor third, inherits the imperfection of its imperfect parent. Likewise the semitone (15:16), child of the fifth and minor sixth or the fourth and

[79] Book III, chapter 5.
[80] Book III, chapter 4.

major third, inherits the imperfection of the imperfect parent. Last, the diesis (24:25), having two imperfect parents, either the major and minor thirds or the major and minor sixths, is so imperfect that it almost ceases to be melodic.

The diesis appears again as the difference between a minor tone and a semitone, one of three third-intervals that are sometimes needed in modulation. The others are the limma (128:135), arising from the major tone and semitone, which Kepler notes is scarcely distinguishable from a semitone, and the comma (80:81) arising from the major and minor tone.

Having derived the various kinds of interval, Kepler next explains the origin of the two kinds of scale, which he calls soft and hard.[81] The difference arises from the properties of the regular polygons with which the divisions of the strings are associated. Thus the division in the proportion of continuous doubling, and the triangular divisions and the continuous doubling of that (that is, divisions into 2, 4, 8, 3, 6) have a relation to the square and triangle, which have expressible sides, whereas the pentagonal division (into 5 parts) involves an inexpressible line.[82] The divisions which exclude the pentagon give the soft scale and those which include the pentagon give the hard scale. It follows that the scale containing the minor third and sixth is soft, while that containing the major third and sixth is hard. Kepler remarks that this distinction between the two kinds of harmony has been expressed by God himself in the motions of the planets.

After showing how the harmonic intervals are divided into melodic intervals so that the octave consists of twelve notes,[83] in the following chapters Kepler expounds all aspects of musical theory, including the modes, extension of the system over two (or more) octaves, and the principles of composition. Finally, at the end of the Book, he appends a digression on the political and judicial applications of harmony.[84]

In Book IV Kepler turns to the applications of harmony in the works of nature, reserving for Book V, which will form the crown of the *Harmonice mundi*, consideration of the role of harmony in the work of creation itself.[85] At this point Kepler distinguished between sensible harmony and pure harmony. Sensible harmony involves two sensible things, such as two musical sounds or two light rays from planets, that can be compared and ordered according to quantity. It is, however, the soul that compares them and creates the sensible harmony. The perception of such harmony is partly passive and partly active. In respect of the passive com-

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[81] The theoretical distinction of the modern major and minor scales was only just beginning to emerge at the time of composition of the *Harmonice mundi*. Although in Book III, Kepler's soft and hard seem to be identical to the modern minor and major, the characterization in Book V of certain harmonies as soft and hard seems to indicate that he was using the terms in their original sense, according to which a scale or chord was soft if it contained a B flat and hard if it contained a B natural. See Walker (1978), 57–58.

[82] Book III, chapter 6.

[83] Book III, chapters 7 and 8.

[84] See Nitschke (1973).

[85] Book IV, chapter 1.

ponent, Kepler follows the scholastic doctrine of immaterial species, according to which there is an emanation (a form in the Aristotelian sense) from the sensible things, which is received by the senses as the servants of the soul. To understand the active component of the perception of sensible harmonies, it is necessary to consider the nature of pure harmony. According to Kepler, this is the archetypal harmony that comes from mathematical categories and is innate in the soul. Following Proclus especially, he accepts Plato's recollection theory of learning. Discursive thought, however, is not necessary to perceive the sensible harmonies, for the soul has an instinct that enables it to compare the terms received through the senses with the corresponding archetypal terms present within itself, that is, the circle and a knowable part of it. Nevertheless, besides the instinct or lower faculty, which is shared by the souls of animals and sublunary nature, the soul of man also possesses a higher faculty of reflection and reasoning that operates in conformity with the will when, for example, we fit the voice to intelligible harmonies.[86] Thus the soul, in an active response to the external stimuli, brings to light the similarity of proportion in the sensible things to some particular archetype of the pure harmony that is within itself. In a sense, Kepler identifies the terms of the archetypal harmonies, that is, the circle and its knowable parts, with the soul. For, by extension of the symbolism of the Trinity, in which God is represented by the sphere, he takes the circle to be the symbol of created mind.

A configuration is influential (or constitutes an aspect), Kepler explains,[87] when the rays from a pair of planets make an angle that is apt to stimulate sublunary nature and the lower faculties of animate beings to be more active at the time of the aspect. When the sublunary soul is thus moved, it stirs itself to draw out from the bowels of the earth material for every kind of weather.

As in the case of the musical harmonies, Kepler sets out his theory of the aspects in a series of definitions, axioms, and propositions.[88] The origins or causes of the aspects are stated in two principal axioms. According to the first, an arc of the zodiac cut off by the side of a regular polygon that is congruent and knowable defines the angle of an aspect. In this case, the figure is placed at the circumference, or in other words, is inscribed in the zodiacal circle. The second axiom makes use of a figure placed at the center, so that two adjacent sides show the directions of the light rays from the planets. According to the axiom, an aspect arises whenever the polygon or star polygon concerned is knowable and congruent.

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[86] Book IV, chapter 2.

[87] Book IV, chapter 5, Definition 1. The aspect has nothing to do with the planets themselves except their position as seen from the earth. In opposition to traditional astrology, Kepler held that the planets themselves had no effect.

[88] Until 1608, as he explains in chapter 6, Kepler had supposed the aspects to correspond exactly with the musical harmonies and to have the same origins. Since then he had decided that the aspects do not depend on music but that both are derived from geometry, music by one set of laws and the aspects by another.

The two axioms always apply together, for in every case there is a figure at the circumference and a corresponding figure at the center. Both figures have responsibility for the influence of the aspect but not equally. First, Kepler argues that congruence is better suited to giving rise to a finite number of aspects than it had been to explaining an infinity of consonances. Second, congruence is more influential than knowability, because the sublunary soul and the faculty of the human soul that perceives the aspects have closer affinity with instinct than with reason. Third, knowability takes precedence over the congruence in the figure at the center, since only one vertex of this figure is involved, whereas congruence is a property of the whole figure. From these arguments he concludes that the figure at the circumference, in which congruence has precedence over knowability, has the greater responsibility for the effectiveness of the aspect. By consideration of the division of this responsibility and the relative importance of the properties of congruence and knowability, Kepler develops a series of propositions which establish the existence of twelve aspects (thus adding seven to the traditional five) and also ranks them in order of their degree of effectiveness. In order of decreasing degrees of effectiveness, they are:

1. conjunction and opposition (0, 180°)
2. quadrature (90°)
3. trine, sextile, semi-sextile (120°, 60°, 30°)
4. quintile, biquintile, quincunx (72°, 144°, 150°)
5. decile, tridecile, octile, trioctile (36°, 108°, 45°, 135°).

To some celebrated professors, as Kepler explains in the last chapter of Book IV,[89] he had seemed to be founding an *Astrologia nova.* For this reason, he had decided that he should explain the main features in more detail, especially those concerning sublunary nature and the inferior faculties of the soul on which astrology depended. First he emphasizes that his belief in the existence of the earth-soul was formed not by reading books nor as a result of his admiration of Plato, but only by observation of the weather and study of the aspects by which it was excited. He then seeks to show that the earth is animate—for example, by comparing the tides to breathing—so that the earth has a soul bearing the impress of the geometrical archetypes and containing an image of the sensible circle of the zodiac that enables it to respond to the aspects. The soul is reminded of itself by the aspects and enters into its operations, which are perpetual, with more vigor.

A particular problem is posed by the question as to how the sensible aspect is received into the earth-soul. In one sense, the problem also applies to human vision, for it is something of a mystery how the image on the retina is received into the soul. However, Kepler argues that there is a more obscure way of perception, not involving an organ of sight, which

[89] Chapter 7.

is in fact common to both sublunary nature and the human soul, by means of which they are able to recognize the aspects. For the soul extends, by emanation, towards the outer parts of the body, so that it has knowledge of all the members of its body and the changes that occur in them at a given time. In the case of the Earth-soul the emanation is always along radii from the center; that is, according to the same laws as the rays of the planets proceed towards it. Kepler supposes that perception of the rays of one planet occurs when the entry of the ray and the exit of the emanation are in the same general straight line, on a pattern very similar to vision, which is perfect and accurate on a single unique perpendicular; that is, the central ray of the whole eye. In this way, the Earth-soul perceives the aspects. By means of the same obscure way of perception, the human soul, when the vital faculty[90] is freshly kindled at birth, perceives the aspects, like a spur to a horse. In this way, Kepler justifies the casting of horoscopes but he also points to their limitations, emphasizing the influences of many factors other than the birth configurations of the heavens in the determination of character and achievement. In particular he notes again that his own stars were not those in the sky at the time of his birth but Copernicus and Tycho Brahe.

Kepler's celestial harmony, the subject of Book V, is based on: (1) the regular polyhedra, through which the number and distances of the planets from the Sun is determined, and (2) the basic harmonies, derived from the regular polygons, through which the eccentricities of the orbits and the periodic times of the planets are explained. However, Kepler recognized that the role of the regular polyhedra was more complicated than he had supposed in the *Mysterium cosmographicum*, for the discrepancies between the observed distances and those predicted by the polyhedral hypothesis had convinced him that these distances were not taken from the regular polyhedra alone but depended also on the requirements of the harmonic principle.

In his investigation of the celestial harmony, which begins in chapter 4, Kepler employs the results of his astronomical theory,[91] including the three laws of planetary motion that he had discovered. Following his usual methodological principles, he first searches for harmonic relations in the data, such as the periodic times and the perihelion and aphelion distances of single planets and of pairs. Having failed to find harmonic relations in these quantities, he advances reasons why they are not appropriate. For example, he is not surprised that the distances fail, because harmonies are more intimately connected with motions. This thought leads him to consider the true daily paths of the planets and when these reveal no harmonies, he is able to explain that they are inappropriate because such paths cannot be observed. Finally, he locates the harmonies in the

[90] The vital faculty, comparable to a flame, is lit at birth and extinguished at death, while the soul itself is immortal.

[91] This is summarized in chapter 3.

angular velocities of the planets in their orbits at perihelion and aphelion. Indeed this is just where one should expect to find them. For the fitness of the angular velocities is immediately recognizable. They represent the apparent daily paths as seen from the Sun; that is, from a uniquely prominent place in the world. Although it would seem to follow that the celestial harmonies were considered by Kepler to be intelligible rather than sensible, so that they are perceived by the intellect and not by the senses, he did in fact suggest they were recognized instinctively, just like the aspects.

Two kinds of harmonies can be distinguished. First, there are harmonic relations between the angular velocities of individual planets at perihelion and aphelion. Second, there are harmonic relations between the convergent and divergent angular velocities of neighboring pairs of planets.[92] In the first type of harmony, that of the individual planets, the two terms cannot exist at the same time but in the second type, that of pairs of neighboring planets, they can. For this reason, Kepler compares the harmonies of the single planets to the music of the ancients, but that of the planets in combination to the more recent polyphonic music of his time.

The harmonies of the single planets and of pairs of planets are set out in Tables I and II respectively. All the consonances are represented except the fourth. This, however, is represented by the angular velocities of the Moon at apogee and perigee as seen from the Earth. Kepler was aware that the numbers do not agree exactly and, as we shall see, in later chapters, he shows that, for *a priori* reasons, the small deviations are for the most part necessary.

Taking the aphelion speed of Saturn to represent the lowest note G, the notes for all the harmonies of the planets are obtained by comparing their various aphelion and perihelion speeds with this speed of Saturn, which is the slowest of all. Division of these speeds by suitable powers of two brings all the notes within a single octave. In this case Kepler finds that the aphelion and perihelion speeds of the planets correspond to the notes of the hard scale. Alternatively, by taking the perihelion speed of Saturn to represent the lowest note G, the planetary harmonies are found to build the soft scale.[93] Kepler explains further that the extreme motions of the individual planets express melodic modes but only in a certain way, for the intermediate notes are not explicitly expressed. There is a glissando between the extremes, which of course passes through the intermediate notes. Only Mercury, however, has a sufficiently large compass to express all the Church modes.[94]

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[92] The convergent interval is formed from the aphelion angular velocity of the lower planet and the perihelion angular velocity of the higher planet. The divergent interval is formed from the perihelion angular velocity of the lower planet and the aphelion angular velocity of the higher planet.

[93] Book V, chapter 5.

[94] Book V, chapter 6.

Kepler next considers the harmonies created by the planets in combination.[95] While harmonies of three planets are fairly frequent, those of four begin to be scattered over centuries and those of five over myriads of years. Indeed Kepler believes that a complete harmony of all the planets may have occurred only once at the Creation. Searching for possible harmonies, he begins with Venus and the Earth, as these planets can only make two harmonies, namely the major and minor sixths. In each case, there is a latitude of tuning, since the motions do not correspond exactly to these intervals. Without this latitude of tuning, he explains, the harmonies of several planets would occur very infrequently if at all. Taking first the major sixth, the aphelion motion of the Earth is chosen for the lower tuning and the perihelion motion of Venus for the upper tuning. In each case, the chosen tuning is taken as a base from which the motions of the other planets required for the formation of harmonies can be calculated and tested to decide whether they lie within the permitted ranges. The result is two possible hard chords of all the planets,[96] namely E minor and C major. Similarly, in the case when Venus and the Earth produce a minor sixth, there are two possible soft chords of all the planets, namely E flat major and C minor.

So far Kepler has explained the origin of the harmonies in the regular polygons and, using the empirical data as a guide, he has constructed his hypothesis of the celestial harmony. Next he sets out to give an *a priori* demonstration,[97] explaining why the planets produce the harmonies they do and why there are small discrepancies. Indeed the identification of the theoretical nature of these discrepancies enables him to refine the hypothesis and in particular, to predict the eccentricities of the orbits. First, he declares that the universal harmonies of all six planets, especially in the extremities of their motions, and the representation of the two scales, cannot be accidental. Then adopting the premise that it was God's intention to link the harmonic proportions to the five regular polyhedra and to use both types of entity in shaping the one most perfect archetype of the heavens, he develops his *a priori* demonstration in a series of 48 axioms and propositions.

As an example of Kepler's line of demonstration we consider the case of the Earth and Venus. First he shows that there must be a pair of planets whose harmonies are the major and minor sixths.[98] For these harmonies are needed in the building of the hard and soft scales, which requires two harmonies differing by a diesis. Although the major and minor thirds also differ by a diesis, these could not be applied as he has already shown, from the properties of the interpolated regular polyhedra, that no pair

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[95] Book V, chapter 7.

[96] This shows that Kepler's hard and soft scales do not correspond to the modern major and minor.

[97] Book V, chapter 9.

[98] Proposition 23.

of planets can have thirds as harmonies.[99] Next he shows that the two planets must form the major sixth from their aphelion motions and the minor sixth from their perihelion motions, for if the consonances were formed from the divergent and convergent motions, each planet would vary its own motion corresponding to half a diesis, which of course is not a melodic interval and therefore unacceptable.[100] Then he shows that the Earth and Venus are the only pair of planets that can produce these consonances.[101] In fact, the proportion of the aphelion motions is 0.602 (major sixth 0.600) and the proportion of the perihelion motions is 0.628 (minor sixth 0.625).

Turning now to the proportions of the individual planets, Kepler argues that these must be smaller than a minor tone and a semitone respectively,[102] otherwise the convergent and divergent motions would also produce consonances, a possibility that has already been excluded.[103] Now there are only two melodic intervals smaller than the minor tone and semitone respectively: these are the semitone and diesis. In order to build the hard and soft scales, however, the difference of the proportions for the two planets must be exactly a diesis. This could be achieved by simply substituting a double diesis for the semitone but in this case the two planets would not have been treated equally. Kepler's solution is to take for the interval of the Earth a comma less than the double diesis and for that of Venus a comma less than a diesis. In this case the deviation is approximately the same for both planets. For the *double diesis—comma* exceeds a semitone by about the same amount that the *diesis—comma* is deficient. Thus the deviation from the exact interval is in each case a comma, which is musically acceptable.[104] In this way Kepler finds for the Earth the proportion 2916/3125 and for Venus the proportion 243/250. Hence for the Earth the theoretical proportion is approximately 0.933, compared with an observed value of 0.931, and for Venus the theoretical proportion is 0.972, compared with an observed value of 0.971.

To complete the application of his methodological principle that hypotheses must be built upon and confirmed by observations,[105] Kepler still needed to test an observable prediction of the harmonic theory involving all the planets. This was made possible by means of the third law of planetary motion, whose discovery on 15 May 1618 Kepler had greeted with unbounded enthusiasm. Having compared his discovery to the golden vessels of the Egyptians plundered by the Israelites to build a tabernacle for their God far from the borders of Egypt, he declared: "See, I cast the

[99] Proposition 6.
[100] Proposition 24.
[101] Proposition 27.
[102] Proposition 25.
[103] Proposition 23.
[104] Proposition 26.
[105] Kepler stated this principle explicitly in a letter to David Fabricius of 4 July 1603. KGW 14, p. 412.

die and write the book. Whether it is to be read by the people of the present or of the future makes no difference; let it await its reader for a hundred years, if God Himself has stood ready for six thousand years for one to study Him."[106]

Taking the harmonic proportion for each planet to represent v_a/v_p, where v_a and v_p are the angular velocities at aphelion and perihelion, and employing the rule $v_a/v_p = r_p^2/r_a^2$, (which is an easy consequence of the area law) where r_p and r_a are the perihelion and aphelion distances, Kepler finds the ratio of these distances and from this deduces the eccentricity. Using the formula $v = G - \frac{1}{2}(A - G)$, where G and A are the geometric and arithmetic means of the extreme motions, he calculates the mean motions. Application of the harmonic law T^2 is proportional to r^3, where T is the periodic time and r the mean distance, then gives the mean distances. Finally, using the eccentricities deduced from the harmonies, he derives the predicted absolute distances in a common scale. These are shown in Table III, with the empirically based distances, using Tycho's observations, in brackets. The agreement between theory and observation is quite impressive.

Concerning the role of the regular polyhedra in the determination of the distances of the planets, Kepler explains that the interpolation between the planetary spheres had to be modified to satisfy the requirements of the harmonic theory. Characteristically, he offers *a priori* reasons for the discrepancies and in particular makes use of the twelve pointed star polyhedron (which he calls a hedgehog) to explain the proportion of the spheres of Mars and Venus, which its associated figures, the dodecahedron and the icosahedron, had failed to do. Evidently, the polyhedral hypothesis had come to be seen by Kepler as a preliminary sketch of the cosmos, which God had then refined in accordance with the harmonic theory. In relation to the harmonic theory itself, Kepler acknowledged that he was unable to account for the very small discrepancy between the predicted and observed distances of Mercury.[107] Nevertheless, he did not consider that this could invalidate a theory that agreed so closely with the observed phenomena in general.

As late as February 1619, when he wrote the introduction to Book V, indicating the contents, Kepler still intended to append a translation of the part of Book III of Ptolemy's *Harmonica* that was most closely related to his own work. But in an appendix to Book V, he explains the reasons for his change of plan. Dissatisfaction with the poetic symbolism of Ptolemy, which he compares unfavorably with his own demonstrations, seems to be the real reason. However, Kepler declares his preference for a complete publication at a later time rather than publication of the extracts he had already prepared.[108] At this stage, he is content to summarize the

106 Preface to Book V of the *Harmonice mundi*.

107 Proposition 48.

108 These were published in KOF, vol. 5, pp. 335–412.

contents of Book III of Ptolemy's work and compare them with the corresponding parts of his own.

It is easy to understand why Kepler may have come to regard Ptolemy's work as having only historical interest. Its purely musical content had been competently developed by Zarlino, while the application of the musical results to astrology and astronomy had been completely replaced by Kepler's own theories of harmony as expounded in the *Harmonice mundi.*

In his appendix, Kepler also criticized the views of Robert Fludd, who had introduced a theory of cosmic harmony in a recent book about the macrocosm and the microcosm. According to Kepler, Fludd's work was imbued with the spirit of symbolism; his treatment was poetic or rhetorical rather than philosophical or mathematical. A controversy ensued, leading to the publication in 1622 of Kepler's *Pro suo opere Harmonices mundi apologia.*[109]

Both Fludd and Kepler, of course, were concerned with a classical theme, attributed to the Pythagoreans and developed by Plato and Ptolemy. But while Fludd's treatment remained at the poetic and rhetorical level of the Greek writers, Kepler had perfected the idea to provide a detailed and coherent explanation of the structure of the cosmos in terms of a divine harmony based on geometry. Moreover, he had used for this purpose the same methodology as he had employed in his search for the true nature of the planetary orbits, for the speculative hypotheses were based on observation, while the conclusions arrived at by mathematical demonstration were then subjected to empirical test. Kepler's *Astronomia nova*, replacing Ptolemy's *Almagest*, has been recognized as one of the great books in the history of astronomy, while his *Harmonice Mundi*, replacing Ptolemy's *Harmonica*, has been generally dismissed as unimportant, except as the source of the third law of planetary motion. Yet Kepler regarded both books as accounts of major discoveries stemming from the same enterprise, namely the exercise of his vocation, adopted following the discovery of the polyhedral hypothesis, to reveal, through his work in astronomy, the wisdom of the Creator.

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109 KGW 6, pp. 379–457. This drew a further response from Fludd, *Monochordum mundi replicatio R. F . . . ad apologiam J. Kepleri* (Frankfurt, 1622), which is taken up mostly with an attack on Copernicanism. Kepler just ignored it. See Field (1984b), 285.