INTRODUCTION

In the following pages the reader will find tables giving the dates of all new and full moons during an historical era when these data were of considerable interest and importance. At present the scholar who needs such information must rather laboriously calculate the date and time using P. V. Neugebauer's tables or look them up in Ginzel's somewhat inconvenient ones.[1] As a result many scholars today have felt the need for a more convenient way to establish these chronological data. This has been frequently pointed out to me by Otto Neugebauer who has long urged the preparation of the present tables as a natural supplement to Bryant Tuckerman's very important numerical work on ancient chronology.[2]

It is therefore believed that these tables giving the dates and times of all lunar syzygies from 1001 B.C. through A.D. 1651 will be of real use to the scholars interested in this period. To make them more useful the longitudes of the moon at each of these times is also given, as is a consecutive enumeration of the conjunctions and a similar one of the oppositions.

All dates are reckoned in the Julian calendar and all times are given in hours and the nearest minute. These dates and times are calculated for an observer in Babylon, or equivalently Baghdad, since this location is fairly centrally located for the historians of the period. It is assumed by definition that the observer is exactly 3 hours east of Greenwich. Moreover, the time used is civil time and is based on a 24-hour clock with its origin at midnight; thus noon is 12 hours.

Since this volume may be considered as a supplement to Tuckerman's tables, I have taken all fundamental astronomical elements from them. So, for example, the formulas for the mean longitudes of the moon and sun, of the moon's ascending node and for the eccentricities of their orbits are taken from there.[3] However, for the completeness of this volume all formulas and elements utilized herein are summarized in §2 below.

In consonance with Tuckerman's approach the five most important perturbation terms affecting the moon's position have been applied. These include among others the evection, the variation, and the annual equation. However, since the perturbations applied by Tuckerman to the sun's position hardly influence the times of the moon's syzygies, they have not been included here. The lunar perturbations have been taken from P. V. Neugebauer's well-known astronomical chronologies and are those used by Tuckerman.[4]

While in a certain sense the present tables may be viewed as an extension and revision of both Ginzel's and P. V. Neugebauer's works on lunar syzygies, it is hoped that the data contained herein give somewhat better values for the times of the syzygies. The astronomical elements adopted here are improvements on those used by Ginzel. This matter is already discussed in Tuckerman's tables.

Inasmuch as Tuckerman in his tables has given a quite complete discussion of their accuracy, little more need be said here. Extensive spot checks have, however, been made to ensure the agreement between those and the present tables and the agreement is extremely close, the moon's positions in longitude agreeing exactly and the sun's differing occasionally by about .01 degree. This latter is explicable by the fact that the perturbations of the solar orbit mentioned above can in the worst possible case add up to about .016 degree. As still another check, we choose at random a half-dozen dates for new or full moons and used these data to interpolate for the sun's position in both volumes of Tuckerman's tables. Essentially complete agreement was reached in all cases. As a further and independent check the dates of the lunar syzygies for the year 1963 were calculated and compared to those in the *Nautical Almanac* for that year.[5] The greatest difference is about 10 minutes. As a further check the dates and times given in the

[1] P. V. Neugebauer, *Tafeln zur astronomischen Chronologie*, Vol. II; *Tafeln fur Sonne, Planeten und Mond*, Leipzig (1914) and F. K. Ginzel, *Handbuch der Mathematischen und Technischen Chronologie*, Vols. I and II (1906 and 1911, Leipzig). In the Ginzel tables new moons are given for the period 605 B.C. through 308 A.D. and full moons from 500 B.C. through 100 A.D.

[2] B. Tuckerman, *Planetary, Lunar, and Solar Positions 601 B.C. and A.D. 1*, Mem. Amer. Philos. Soc. 56 (1962) and *Planetary, Lunar and Solar Positions A.D. 2 to A.D. 1649*, Mem. Amer. Philos. Soc. 59 (1964). These will be referred to as Vols. I and II. In the Preface to Vol. I the reader will find an account by Neugebauer of some of the scholary usages for these tables.

[3] *Op. cit.*, Vol. II: p. 13.

[4] P. V. Neugebauer, *op. cit.* and *Astronomischen Chronologie* (Berlin and Leipzig, 1929).

[5] *The American Ephemeris and Nautical Almanac for the Year 1963* (Washington and London, 1963), p. 159.

Connaissance des Temps for 1953 for new and full moons were also used. The largest disagreement with those values was 19 minutes.[6] (This disagreement is in part explicable by the fact that the *Connaissance* includes the parallax of Paris.) The agreement of the present tables with those of Ginzel is not so close. In one sample of a year disagreements of slightly more than 20 minutes were occasionally noted. Comparable or even greater disagreements have also been noted in comparisons with P. V. Neugebauer's original—his 1914—chronology. This was before he modified his astronomical elements for the sun.

Since the moon moves in its orbit about 12 degrees per day, it moves nearly a hundredth of a degree each minute. For this reason lunar longitudes are here recorded to the nearest hundredth of a degree and the corresponding times to the nearest minute even though such precision is probably more nominal than real. It does, however, make possible the use of the results for further calculations prior to an ultimate rounding-off.

In closing this introduction I feel I would be unforgivably remiss were I not to acknowledge gratefully the constant encouragement and support given me by Otto Neugebauer; the excellent and essential programming help and enthusiastic cooperation of Roger C. Evans without which these tables would not have been possible; the many invaluable conversations on the subject I had with Bryant Tuckerman; and the noteworthy assistance given me by Marcus W. Hanlon, Robert K. McNeill, John H. Rooney, and Clifford C. van Fleet. I wish also to thank the International Business Machines Corporation and in particular T. Vincent Learson, Chairman of the Board, and Ralph E. Gomory, the Director of Research, for making it possible for me to undertake and carry to fruition this task.

Finally it should be said that the programming was first done in the well-known language, APL, and initial tests run on the IBM 360, Model 91, located in the Thomas J. Watson Research Center, Yorktown Heights, N. Y. For technical reasons connected with printing the final results, it was then reprogrammed in FORTRAN using double-precision arithmetic and then run on the same Model 91. The actual calculation of the entire table on that computer, exclusive of checking and printing, took, *mirabile dictu*, 132 seconds. This compares to about a quarter of an hour per syzygy for a human or about 17,500 hours for the entire table. In fairness it should be pointed out that various other clerical functions, such as checking and manipulating Julian day numbers, took an additional 300 seconds.

[6] *Connaissance des temps, ou des movements celestes, pour l'an 1953* (Paris, Bureau des longitudes).

1. UNDERLYING ASTRONOMICAL AND MATHEMATICAL CONCEPTS

The fundamental quantities that enter into the determination of the moon's syzygies are the longitudes of the sun and moon measured in the plane of the ecliptic by an observer at the earth's center. By definition a new moon occurs at the instant these are equal, and a full moon when they differ by 180°. Thus no account is taken of parallax.[7]

The calculation of these longitudes and the times at which they are equal modulo 180° are made possible by relatively simple formulas. There are several possible sets but we have used those of Tuckerman for consistency reasons. In these formulas quantities with "primes" on them refer to the sun and those without to the moon. The symbols used are L for mean longitude, P for the longitude of perihelion or perigee, e the eccentricity of the orbit, N the longitude of the moon's ascending node and i the inclination of the moon's orbit to the ecliptic. The symbol t is used to measure time in Julian years. In formulas (1)–(5) below this time is measured from the epoch Gregorian 1800 January 0, 12^h Greenwich civil time and in formula (7) it is Gregorian 1900, January 1, 12^h Greenwich civil time.

In terms of these the relevant formulas are the following:

(1) $$L = 355°43'24.''37 + 17325643.''945t + 1.''22 \times 10^{-3}t^2 + 6.''6 \times 10^{-9}t^3,$$

(2) $$L' = 279°54'49.''60 + 1296027.''6437t + 2.''6 \times 10^{-4}t^2 + 0.''0 \times t^3,$$

(3) $$P = 225°23'50.''37 + 146485.''937t - 3.''7033 \times 10^{-3}t^2 - 4.''4 \times 10^{-9}t^3,$$

(4) $$P' = 279°30'12.''30 + 61.''8026t + 1.''71 \times 10^{-4}t^2 + 1.''0 \times 10^{-9}t^3,$$

(5) $$N = 33°16'22.''57 - 69629.''209t + 7.''489 \times 10^{-4}t^2 + 7.''5 \times 10^{-9}t^3,$$

(6) $$e = 0.05490807,$$

(7) $$e' = 0.0167498 - 4.258 \times 10^{-7}t - 1.37 \times 10^{-11}t^2,$$

(8) $$i = 5°8'39.''96.$$[8]

To these formulas we must append the perturbations of the moon's orbit that have been taken into account. They are the same ones as used by Tucker-

[7] *The American Ephemeris*, *op. cit.*, pp. 491–492.

[8] Tuckerman, *op. cit.*, Vol. II: p. 13. (The value of e in formula (6) above is that adopted by Hansen. P. V. Neugebauer, followed by Tuckerman, used the value 0.05490897, which is probably a typographical error. This introduces no measurable changes in times or longitudes.)

man and P. V. Neugebauer and are those of Hansen. They include, *inter alia*, the important and well-known effects such as the evection, the variation and the annual equation.

$$A_1 = 4467'' \sin(L - 2L' + P),$$
$$A_2 = 2145'' \sin 2(L - L'),$$
$$(9) \qquad A_3 = 658'' \sin(L' - P' + 180°),$$
$$A_4 = 198'' \sin(L - 3L' + P + P'),$$
$$A_5 = 155'' \sin(2L - 3L' + P').$$

Then in the formulas (11) and (12) below L is replaced by

$$(10) \qquad L_M = L + A_1 + A_2 + A_3 + A_4 + A_5.$$

For notational simplicity we shall drop the subscript M on L and write simply L for the expression (10). This can cause no difficulties.

As mentioned above, in contradistinction to Tuckerman's work, we have introduced no perturbations in the sun's mean longitude.

Now given the corrected mean longitudes L and L' for the moon and sun we find the true longitudes λ and λ' of these bodies in their orbits. This is done with the help of a well-known approximation to the equation of center.[9] This formula is given below through terms of the order e^4.

$$(11) \qquad \lambda = L + \frac{180}{\pi} \times [(2e - \frac{1}{4} e^3) \sin (L - P) + (\frac{5}{4} e^2 - \frac{11}{24} e^4) \sin 2 (L - P) + \frac{13}{12} e^3 \sin 3 (L - P) + \frac{103}{96} e^4 \sin 4 (L - P)],$$

with an exactly similar formula for the primed variables, i.e., for the sun. In this formula (11) above and its counterpart with primes it should be understood that λ and L are considered to be expressed in degrees.

In the case of the sun, of course, the longitude λ' is already measured in the plane of the ecliptic whereas that for the moon is not. In fact the longitude in orbit of the moon is really the sum of two angles: one measured in the plane of the ecliptic from vernal equinox to the ascending node and the other measured in the plane of its orbit from the ascending node to the moon's true position. The latter plane is inclined slightly more than 5°—see formula (8) above—to the former one, the ecliptic. The final step then consists of projecting the moon's position onto the plane of the ecliptic. This replaces λ by a new angle ℓ measured entirely in the plane of the ecliptic. This can be done by a simple approximate formula known as the reduction to the ecliptic. It is this:

$$(12) \qquad \ell = L - \frac{180}{\pi} \times [p^2 \sin 2 (\lambda - N) - \frac{1}{2} p^4 \sin 4 (\lambda - N) + \frac{1}{3} p^6 \sin 6 (\lambda - N)],$$
$$p = \tan i/2,$$

where i is given in (8) and N in (5) above and where ℓ, L, and N are expressed in degrees.[10] This formula is correct through sixth powers of p.

Now it remains only to describe how the times of the syzygies are found. This is done inductively. Suppose we have found the date and time T_n of the nth new or full moon. Then to a first approximation the next one occurs at the time

$$T^{(0)}_{n+1} = T_n + 29.53/365.25$$

where the quantities are reckoned in Julian years. Then the equations

$$\ell(T) - \lambda'(T) = \begin{cases} 0 \\ \pi \end{cases}$$

are solved in the neighborhood of $T^{(0)}_{n+1}$ for T_{n+1}. The procedure is a modified Newton one. The value $T^{(k+1)}_{n+1}$ of the $(k+1)$-st approximant to T_{n+1} is calculated with the help of the relations

$$T^{(k+1)}_{n+1} = T^{(k)}_{n+1} - \frac{\ell\left(T^{(k)}_{n+1}\right) - \lambda'\left(T^{(k)}_{n+1}\right)}{D\left(T^{(k)}_{n+1}\right)},$$
$$D(T) = d[L(T) - L'(T)]/dT.$$

In these formulas the denominator, $D(T)$, is very close to the value of the derivative, $d[\ell(T) - \lambda'(T)]/dT$. When $T^{(k+1)}_{n+1} - T^{(k)}_{n+1}$ is an absolute value less than 2×10^{-7}, $T_{n+1} = T^{(k+1)}_{n+1}$ by definition.

2. USE OF THE TABLES AND CONVERSIONS TO OTHER MERIDIA

The tables themselves are virtually self-explanatory. Each page contains two pairs of columns headed NEW MOONS and FULL MOONS. In each column there are four sub-columns headed NUMBER, DATE, TIME, AND LONGITUDE. These give, respectively, consecutive enumerations, starting at zero, of the lunations; the days; the months in a two-character

[9] Cf., e.g., D. Brouwer and G. M. Clemence, *Methods of Celestial Mechanics* (New York, 1961), p. 77.

[10] *Ibid.*, p. 47.

form; the civil times of the syzygies for an observer in Babylon expressed by means of a 24-hour clock in hours and the nearest minute; and the longitudes of the moon at those times as calculated at the earth's center. The zero of the clock is midnight; times are recorded, for example, as 7;23 or 18;36 which mean, respectively, 7;23 A.M. and 6;36 P.M. The year is given in the blank rows over each pair of columns and is expressed as an astronomical date. (The conversion from astronomical to civil years is quite easy: for astronomical events bearing a minus sign add 1 to obtain the civil date B.C., and for those events without a sign simply append A.D. to the astronomical date. Thus $-100=101$ B.C., $0=1$ B.C. and $+1=$ 1 A.D.) On each page a given pair of columns will be found to contain the syzygies for six consecutive years. Thus each page contains the events of 12 years.

For an observer at any other location on the earth than Babylon, the little table below can be used to calculate when the syzygy will occur according to his clock. It is essentially an abridgement of one of O. Neugebauer.[11] The + sign is to be taken to mean the syzygy occurs later in local civil time than at Babylon and the − earlier. Thus a syzygy occurring at 11:20 in Babylon civil time occurs at 8:20 Greenwich civil time and 9:50 Samarkand civil time.

Cities	Δt Babylon	Cities	Δt Babylon
Toledo	-3^h14^m	Baghdad/Babylon	$0^h\ 0^m$
Greenwich	$-3^h\ 0^m$	Constantinople	$+1^h\ 2^m$
Hveen	$-2^h\ 7^m$	Samarkand	$+1^h30^m$
Prague	$-2^h\ 0^m$	Ujjain	$+2^h\ 6^m$
Alexandria	-0^h58^m	Peking	$+7^h45^m$
Damascus	-0^h32^m		

[11] *Cf.* Tuckerman, *op. cit.*, Vol. II: p. v.

NEW AND FULL MOONS

FROM 1001 B.C. TO A.D. 1651