Darboux Transformation and N-Soliton Solution for the Coupled Modified Nonlinear Schrödinger Equations

Hai-Qiang Zhang

College of Science, P. O. Box 253, University of Shanghai for Science and Technology, Shanghai 200093, China

Reprint requests to H. Z.; E-mail: zhanghqbupt@yahoo.com.cn

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The pulse propagation in the picosecond or femtosecond regime of birefringent optical fibers is governed by the coupled mixed derivative nonlinear Schrödinger (CMDNLS) equations. A new type of Lax pair associated with such coupled equations is derived from the Wadati–Konno–Ichikawa spectral problem. The Darboux transformation method is applied to this integrable model, and the N-times iteration formula of the Darboux transformation is presented in terms of the compact determinant representation. Starting from the zero potential, the bright vector N-soliton solution of CMDNLS equations is expressed as a compact determinant by N complex eigenvalues and N linearly independent eigenfunctions. The collision mechanisms in two components shows that bright vector solitons can exhibit the standard elastic and inelastic collisions. Such energy-exchange collision behaviours have potential applications in the construction of logical gates, the design of fiber directional couplers, and quantum information processors.

Key words: Coupled Mixed Derivative Nonlinear Schrödinger Equations; Vector Soliton; Soliton Collision; Darboux Transformation.
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1. Introduction

The coupled mixed derivative nonlinear Schrödinger equations in the dimensionless form

$$\begin{aligned} \mathrm{i}\, q_{jt} + q_{jxx} + \mu \left(\sum_{k=1}^{2} |q_k|^2 \right) q_j \\ + \mathrm{i}\gamma \left[\left(\sum_{k=1}^{2} |q_k|^2 \right) q_j \right]_x = 0\,, \quad (j=1,2) \end{aligned} \tag{1}$$

describe the propagation of short pulses in birefringent optical fibers both in picosecond and femtosecond regions [1 – 5], where $q_j = q_j(x,t)$ $(j=1,2)$ is the slowly varying complex envelope for polarizations, x and t appended to q_j denote partial differentiations, the parameters μ and γ are real constants as the nonlinearity and derivative nonlinearity coefficients. For (1), the case $\gamma = 0$ is known as the coupled nonlinear Schrödinger (CNLS) equations [6, 7], and $\mu = 0$ is the coupled derivative nonlinear Schrödinger (CDNLS) equations governing the polarized Alfvén waves in plasma physics [8]. Thus, (1) could be regarded as a hybrid of CNLS and CDNLS equations. From the integrable viewpoint for nonlinear evolution equations (NLEEs), (1) possess infinitely many local conservation laws [5, 9], Lax pairs [2, 5], and bilinear representations [3 – 5]. In addition, the exact bright N-soliton, dark and antidark soliton solutions have been presented by employing Hirota's bilinear method [3 – 5, 10].

In soliton theory, the Darboux transformation method has been a very effective tool to construct the exact analytical solutions of integrable NLEEs [11 – 16], especially the N-soliton solutions. The purpose of the transformation is to produce new solutions by solving the linear equations with a trivial solution. The efficiency of the Darboux transformation is due to the fact that the iterative algorithm is purely algebraic and can be implemented on the symbolic computation system. With a successive application of the Darboux transformation, the N-

soliton solutions of NLEEs can be presented in a simple and compact form, such as in the form of the Wronskian determinant or Vandermonde-like determinant. The N-soliton solutions in terms of the determinant representations have been revealed for some integrable multi-component equations, such as the multi-component modified Korteweg–de Vries equations [17], multi-component nonlinear Schrödinger (NLS) equations [18, 19], and two-component derivative NLS equations [20, 21]. However, when the Darboux transformation is applied to the multi-component systems, the reduction problem between or among original potentials remains yet as a matter of technical difficulties.

The aim of the present work is to construct the Darboux transformation for (1) and to present the N-soliton solution in terms of the determinant representation. On the basis of the N-soliton solution, we will find that the bright vector solitons in two components for (1) can exhibit the standard elastic and inelastic collisions. In inelastic collision, we will identify that each bright vector soliton can undergo the partial or complete energy switching.

The structure of this paper will be arranged as follows. In Section 2, we will derive a new type of the Lax pair of (1) by means of the Wadati–Konno–Ichikawa (WKI) system. Based on the new Lax pair, we will apply the Darboux transformation method to (1). In Section 3, we will present the N-times iterative potential formula of (1). In Section 4, starting the zero seed solution, we will obtain the bright vector N-soliton solution of (1). Moreover, we will discuss energy-exchange collision behaviours between or among bright vector solitons in two components. Our conclusions will be given in Section 5.

2. Lax Pair and Darboux Transformation

2.1. Lax Pair

The Lax pair associated with NLEEs is an essential prerequisite for the construction of the Darboux transformation. As we know, it is possible that an integrable NLEE can admit several Lax pairs. Authors of [2, 5] have derived two kinds of Lax pairs associated with (1). In this subsection, based on the WKI spectral problem [12], we will present another new type of the Lax pair for (1).

Following the procedure generalizing the 2×2 WKI scheme to the 3×3 case, the Lax pair associated with (1) can be written as

$$\Psi_x = U\Psi = \left(\lambda^2 U_0 + \lambda U_1 + U_2\right)\Psi\,, \tag{2}$$
$$\Psi_t = V\Psi = \left(\lambda^4 V_0 + \lambda^3 V_1 + \lambda^2 V_2 + \lambda V_3 + V_4\right)\Psi\,, \tag{3}$$

where $\Psi = (\psi_1, \psi_2, \psi_3)^{\mathrm{T}}$ (the superscript T denotes the vector transpose) is the vector eigenfunction, λ is the eigenvalue parameter, U_j and V_k $(0 \leqslant j \leqslant 2;\, 0 \leqslant k \leqslant 4)$ are all 3×3 matrices to be determined. It is straighforward matter to check that the integrability condition of (2) and (3),

$$U_t - V_x + [U, V] = 0\,, \tag{4}$$

is equivalent to (1) if the matrices U_j and V_k $(0 \leqslant j \leqslant 2;\, 0 \leqslant k \leqslant 4)$ take the form

$$U_0 = \frac{1}{6\gamma}\begin{pmatrix} -2\mathrm{i} & 0 & 0 \\ 0 & \mathrm{i} & 0 \\ 0 & 0 & \mathrm{i} \end{pmatrix},\quad U_1 = \frac{1}{2}\begin{pmatrix} 0 & q_1 & q_2 \\ -q_1^* & 0 & 0 \\ -q_2^* & 0 & 0 \end{pmatrix},\quad U_2 = \frac{\mu}{3\gamma}\begin{pmatrix} 2\mathrm{i} & 0 & 0 \\ 0 & -\mathrm{i} & 0 \\ 0 & 0 & -\mathrm{i} \end{pmatrix}, \tag{5}$$

$$V_0 = \frac{1}{12\gamma^2}\begin{pmatrix} -2\mathrm{i} & 0 & 0 \\ 0 & \mathrm{i} & 0 \\ 0 & 0 & \mathrm{i} \end{pmatrix},\quad V_1 = \frac{1}{4\gamma}\begin{pmatrix} 0 & q_1 & q_2 \\ -q_1^* & 0 & 0 \\ -q_2^* & 0 & 0 \end{pmatrix},\quad V_4 = \frac{\mu^2}{3\gamma^2}\begin{pmatrix} -2\mathrm{i} & 0 & 0 \\ 0 & \mathrm{i} & 0 \\ 0 & 0 & \mathrm{i} \end{pmatrix}, \tag{6}$$

$$V_2 = \frac{\mu}{3\gamma^2}\begin{pmatrix} 2\mathrm{i} & 0 & 0 \\ 0 & -\mathrm{i} & 0 \\ 0 & 0 & -\mathrm{i} \end{pmatrix} + \frac{\mathrm{i}}{4}\begin{pmatrix} |q_1|^2+|q_2|^2 & 0 & 0 \\ 0 & -|q_1|^2 & -q_1^* q_2 \\ 0 & -q_1 q_2^* & -|q_2|^2 \end{pmatrix}, \tag{7}$$

$$V_3 = \frac{\gamma}{2}\begin{pmatrix} 0 & -q_1(|q_1|^2+|q_2|^2) & -q_2(|q_1|^2+|q_2|^2) \\ q_1^*(|q_1|^2+|q_2|^2) & 0 & 0 \\ q_2^*(|q_1|^2+|q_2|^2) & 0 & 0 \end{pmatrix} - \frac{\mu}{2\gamma}\begin{pmatrix} 0 & q_1 & q_2 \\ -q_1^* & 0 & 0 \\ -q_2^* & 0 & 0 \end{pmatrix} + \frac{\mathrm{i}}{2}\begin{pmatrix} 0 & q_{1x} & q_{2x} \\ q_{1x}^* & 0 & 0 \\ q_{2x}^* & 0 & 0 \end{pmatrix}, \quad (8)$$

where the asterisk denotes the complex conjugate.

Remark. The above Lax pair (2) and (3) associated with (1) is different from those in [2, 5]. Compared with the results in [2, 5], the main difference is that the traces of matrices U_j and V_k $(0 \leqslant j \leqslant 2; 0 \leqslant k \leqslant 4)$ in Lax pair (2) and (3) are all zero. On the basis of this matrix property, we will construct the Darboux transformation for (1).

2.2. Darboux Transformation

The Darboux transformation is a special gauge transformation which keeps the Lax pair invariant. In view of the feature that the space part (2) is quadratic about the spectral parameter λ, we take the Darboux transformation for (2) and (3) as the form

$$\widehat{\Psi} = D\Psi = \sum_{j=0}^{2} \lambda^{2-j}\begin{pmatrix} a_{1,j} & b_{1,j} & b_{2,j} \\ c_{1,j} & a_{2,j} & d_{1,j} \\ c_{2,j} & d_{2,j} & a_{3,j} \end{pmatrix}\Psi, \quad (9)$$

which is in terms of the second-order polynomial of λ, where $a_{k,j}$, $b_{\ell,j}$, $c_{\ell,j}$, and $d_{\ell,j}$ $(k = 1,2,3; \ell = 1,2; 0 \leqslant j \leqslant 2)$ are all functions of x and t to be determined.

In order to keep the invariance of Lax pair (2) and (3) under the Darboux transformation (9), it requires that $\widehat{\Psi}$ also satisfies the same linear eigenvalue problem (2) and (3) with U_j $(0 \leqslant j \leqslant 2)$ and V_k $(0 \leqslant k \leqslant 4)$, respectively, replaced by $\widehat{U}_j$ and $\widehat{V}_k$. Thus, the original potentials q_1 and q_2 are transformed into new potentials $\hat{q}_1$ and $\hat{q}_2$, and the Darboux matrix D is required to satisfy

$$D_x = \widehat{U}D - DU, \quad D_t = \widehat{V}D - DV. \quad (10)$$

With use of the Darboux matrix (9), from (10) and by comparing the coefficients of λ, we can directly compute

$$b_{1,0} = b_{2,0} = c_{1,0} = c_{2,0} = 0. \quad (11)$$

After simplifying the rest equations about each coefficient of λ, we can take

$$b_{1,2} = b_{2,2} = c_{1,2} = c_{2,2} = d_{1,2} = d_{2,2} = 0, \quad (12)$$

$$a_{1,1} = a_{2,1} = d_{1,1} = d_{2,1} = d_{3,1} = 0, \quad (13)$$

$$a_{1,2} = a_{2,2} = a_{3,2} = -1, \quad (14)$$

$$\hat{q}_1 = q_1 + \frac{2\mathrm{i}\mu}{\gamma} b_{1,1} - 2b_{1,1x}, \quad \hat{q}_2 = q_2 + \frac{2\mathrm{i}\mu}{\gamma} b_{2,1} - 2b_{2,1x}, \quad (15)$$

$$\hat{q}_1^* = q_1^* + \frac{2\mathrm{i}\mu}{\gamma} c_{1,1} + 2c_{1,1x}, \quad \hat{q}_2^* = q_2^* + \frac{2\mathrm{i}\mu}{\gamma} c_{2,1} + 2c_{2,1x}. \quad (16)$$

Thus, the Darboux matrix D can be expressed as:

$$D = \lambda^2\begin{pmatrix} a_{1,0} & 0 & 0 \\ 0 & a_{2,0} & d_{1,0} \\ 0 & d_{2,0} & a_{3,0} \end{pmatrix} + \lambda\begin{pmatrix} 0 & b_{1,1} & b_{2,1} \\ c_{1,1} & 0 & 0 \\ c_{2,1} & 0 & 0 \end{pmatrix} + \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (17)$$

Next, our work is to determine the rest entries $a_{k,0}$, $b_{\ell,1}$, $c_{\ell,1}$, and $d_{\ell,0}$ $(k = 1,2,3; \ell = 1,2)$ in D. From the knowledge about the kernel of the Darboux transformation, with $\Psi_1 = (\psi_1, \psi_2, \psi_3)^{\mathrm{T}}$ as a solution of Lax pair (2) and (3) for $\lambda = \lambda_1$, the undermined entries can be expressed in terms of the eigenvalue and eigenfunction as

$$a_{1,0} = \frac{1}{\Delta_1}\left(\lambda_1^*|\psi_1|^2 + \lambda_1|\psi_2|^2 + \lambda_1|\psi_3|^2\right), \quad (18)$$

$$a_{2,0} = \frac{1}{\Delta_2}\left(|\lambda_1|^2|\psi_1|^2 + \lambda_1^{2*}|\psi_2|^2 + \lambda_1^2|\psi_3|^2\right), \quad (19)$$

$$a_{3,0} = \frac{1}{\Delta_2}\left(|\lambda_1|^2|\psi_1|^2 + \lambda_1^2|\psi_2|^2 + \lambda_1^{2*}|\psi_3|^2\right), \quad (20)$$

$$d_{1,0} = \frac{1}{\Delta_2}\left(\lambda_1^{2*} - \lambda_1^2\right)\psi_2\psi_3^*, \quad d_{2,0} = \frac{1}{\Delta_2}\left(\lambda_1^{2*} - \lambda_1^2\right)\psi_3\psi_2^*, \quad (21)$$

$$b_{1,1} = \frac{1}{\Delta_1}\left(\lambda_1^{2*} - \lambda_1^2\right)\psi_1\psi_2^*, \quad b_{2,1} = \frac{1}{\Delta_1}\left(\lambda_1^{2*} - \lambda_1^2\right)\psi_1\psi_3^*, \quad (22)$$

$$c_{1,1} = \frac{1}{\Delta_1}\left(\lambda_1^{2*} - \lambda_1^2\right)\psi_2\psi_1^*, \quad c_{2,1} = \frac{1}{\Delta_1}\left(\lambda_1^{2*} - \lambda_1^2\right)\psi_3\psi_1^*, \tag{23}$$

where

$$\Delta_1 = |\lambda_1|^2\left(\lambda_1|\psi_1|^2 + \lambda_1^*|\psi_2|^2 + \lambda_1^*|\psi_3|^2\right), \quad \Delta_2 = |\lambda_1|^2\left(\lambda_1^{2*}|\psi_1|^2 + |\lambda_1|^2|\psi_2|^2 + |\lambda_1|^2|\psi_3|^2\right).$$

It is easy to see that (15) and (16) are satisfied if

$$b_{1,1} = -c_{1,1}^*, \quad b_{2,1} = -c_{2,1}^*. \tag{24}$$

Making use of (22) and (23), we can directly proof the constraints (24).

To this state, we have finished the construction of the Darboux transformation for (1). Under the transformation (17), the linear spectral problem (2) and (3) is invariant. At the same time, the new solution $(\hat{q}_1, \hat{q}_2)$ can be obtained by choosing a special eigenvalue λ_1 and corresponding eigenfunctions (ψ_1, ψ_2, ψ_3) from a seed solution. Therefore, we come to the following conclusion:

Assume that $\Psi_1 = (\psi_1, \psi_2, \psi_3)^{\mathrm{T}}$ is a solution of Lax pair (2) and (3) with $\lambda = \lambda_1$. The Darboux transformation $(\Psi, q_1, q_2) \to (\widehat{\Psi}, \hat{q}_1, \hat{q}_2)$ of (1) is constructed by Transformation (17) with entries defined in (18)–(23). The potential transformation between the new and original ones is given by

$$\hat{q}_1 = q_1 + 2\left(\lambda_1^{2*} - \lambda_1^2\right) \cdot \left[\frac{\mathrm{i}\mu}{\gamma\Delta_1}\psi_1\psi_2^* - \left(\frac{1}{\Delta_1}\psi_1\psi_2^*\right)_x\right], \tag{25}$$

$$\hat{q}_2 = q_2 + 2\left(\lambda_1^{2*} - \lambda_1^2\right) \cdot \left[\frac{\mathrm{i}\mu}{\gamma\Delta_1}\psi_1\psi_3^* - \left(\frac{1}{\Delta_1}\psi_1\psi_3^*\right)_x\right]. \tag{26}$$

3. *N*-Times Iteration of the Darboux Transformation

The Darboux matrix D in (9) discussed above is a second-order polynomial in λ. By applying the Darboux transformation successively, we can construct the N-times iteration of the Darboux transformation. The N-times iteration of the Darboux matrix is a $2N$-order polynomial in λ:

$$D_n(x,t,\lambda) = \sum_{k=0}^{2N} \Gamma_k \lambda^k = \sum_{n=1}^{N} \begin{pmatrix} a_{1,2n}\lambda^{2n} & b_{1,2n-1}\lambda^{2n-1} & b_{2,2n-1}\lambda^{2n-1} \\ c_{1,2n-1}\lambda^{2n-1} & a_{2,2n}\lambda^{2n} & d_{1,2n}\lambda^{2n} \\ c_{2,2n-1}\lambda^{2n-1} & d_{2,2n}\lambda^{2n} & a_{3,2n}\lambda^{2n} \end{pmatrix} + (-1)^N I, \tag{27}$$

where I is the 3×3 identity matrix and Γ_k $(0 \leqslant k \leqslant 2N)$ the coefficient matrix of λ; Γ_{2n-1} and Γ_{2n} $(0 \leqslant n \leqslant N)$ have the following structures:

$$\Gamma_0 = (-1)^N I, \quad \Gamma_{2n-1} = \begin{pmatrix} 0 & b_{1,2n-1} & b_{2,2n-1} \\ c_{1,2n-1} & 0 & 0 \\ c_{2,2n-1} & 0 & 0 \end{pmatrix}, \quad \Gamma_{2n} = \begin{pmatrix} a_{1,2n} & 0 & 0 \\ 0 & a_{2,2n} & d_{1,2n} \\ 0 & d_{2,2n} & a_{3,2n} \end{pmatrix}. \tag{28}$$

Let us take a set of linearly independent solutions $\Psi_k = (\psi_{1,k}, \psi_{2,k}, \psi_{3,k})^{\mathrm{T}}$ of (2) and (3) with (q_1, q_2) for different spectral parameters $\lambda = \lambda_k$ $(1 \leqslant k \leqslant N)$, i. e., $\Psi_k = (\psi_{1,k}, \psi_{2,k}, \psi_{3,k})^{\mathrm{T}}$ $(1 \leqslant k \leqslant N)$ satisfy the linear equations (2) and (3):

$$[\partial_x - U(\lambda = \lambda_k)]\Psi_k = 0, \quad [\partial_t - V(\lambda = \lambda_k)]\Psi_k = 0. \tag{29}$$

We also introduce a set of orthogonal vectors

$$\Phi_k^{(1)} = \left(-\psi_{2,k}^*, \psi_{1,k}^*, 0\right)^{\mathrm{T}}, \quad \Phi_k^{(2)} = \left(-\psi_{3,k}^*, 0, \psi_{1,k}^*\right)^{\mathrm{T}}, \tag{30}$$

which satisfy the orthogonality condition

$$\left\langle \Psi_k | \Phi_k^{(\ell)} \right\rangle = \Psi_k^\dagger \Phi_k^{(\ell)} = 0, \quad (1 \leqslant k \leqslant N; \ell = 1,2), \tag{31}$$

where the dagger $\dagger$ signifies the Hermitian conjugate. From the knowledge of the Darboux transformation, the following relations hold:

$$D_n(x,t,\lambda)|_{\lambda=\lambda_k}\Psi_k = 0, \quad D_n(x,t,\lambda)|_{\lambda=\lambda_k^*}\Phi_k^{(\ell)} = 0, \quad (1 \leqslant k \leqslant N; \ell = 1,2), \tag{32}$$

which can be rewritten in a matrix form

$$(\mathit{\Gamma}_1,\mathit{\Gamma}_2,\cdots,\mathit{\Gamma}_{2n})\,W_n=-B, \tag{33}$$

where

$$B=\mathit{\Gamma}_0\left(\Psi_1,\Psi_2,\cdots,\Psi_N,\Phi_1^{(1)},\Phi_2^{(1)},\cdots,\Phi_N^{(1)},\Phi_1^{(2)},\Phi_2^{(2)},\cdots,\Phi_N^{(2)}\right),$$

$$W_n=\begin{pmatrix}\lambda_1\Psi_1 & \cdots & \lambda_N\Psi_N & \lambda_1^*\Phi_1^{(1)} & \cdots & \lambda_N^*\Phi_N^{(1)} & \lambda_1^*\Phi_1^{(2)} & \cdots & \lambda_N^*\Phi_N^{(2)}\\ \lambda_1^2\Psi_1 & \cdots & \lambda_N^2\Psi_N & \lambda_1^{2*}\Phi_1^{(1)} & \cdots & \lambda_N^{2*}\Phi_N^{(1)} & \lambda_1^{2*}\Phi_1^{(2)} & \cdots & \lambda_N^{2*}\Phi_N^{(2)}\\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots\\ \lambda_1^{2N}\Psi_1 & \cdots & \lambda_N^{2N}\Psi_N & \lambda_1^{2N*}\Phi_1^{(1)} & \cdots & \lambda_N^{2N*}\Phi_N^{(1)} & \lambda_1^{2N*}\Phi_1^{(2)} & \cdots & \lambda_N^{2N*}\Phi_N^{(2)}\end{pmatrix}.$$

Substituting (28) into linear algebraic (33), we can obtain the expressions of $a_{j,2n},d_{\ell,2n},b_{\ell,2n-1},c_{\ell,2n-1}$ $(j=1,2,3;\ \ell=1,2;\ 1\leqslant n\leqslant N)$ by use of Cramer's rule. Therefore, the potential formula of N-times iterative Darboux transformation for (1) is given by

$$\begin{aligned}q_1[N]&=q_1+(-1)^{N+1}\left[\frac{2\mathrm{i}\mu}{\gamma}b_{1,1}-2b_{1,1x}\right]\\&=q_1+(-1)^{N+1}\left[\frac{2\mathrm{i}\mu}{\gamma}\frac{G}{H}-2\left(\frac{G}{H}\right)_x\right],\end{aligned} \tag{34}$$

$$\begin{aligned}q_2[N]&=q_2+(-1)^{N+1}\left[\frac{2\mathrm{i}\mu}{\gamma}b_{2,1}-2b_{2,1x}\right]\\&=q_2+(-1)^{N+1}\left[\frac{2\mathrm{i}\mu}{\gamma}\frac{F}{H}-2\left(\frac{F}{H}\right)_x\right],\end{aligned} \tag{35}$$

with

$$H=\begin{vmatrix}\lambda_1^2\psi_{1,1} & \lambda_1\psi_{2,1} & \lambda_1\psi_{3,1} & \lambda_1^4\psi_{1,1} & \lambda_1^3\psi_{2,1} & \lambda_1^3\psi_{3,1} & \cdots & \lambda_1^{2N}\psi_{1,1} & \lambda_1^{2N-1}\psi_{2,1} & \lambda_1^{2N-1}\psi_{3,1}\\ -\lambda_1^{2*}\psi_{2,1}^* & \lambda_1^*\psi_{1,1}^* & 0 & -\lambda_1^{4*}\psi_{2,1}^* & \lambda_1^{3*}\psi_{1,1}^* & 0 & \cdots & -\lambda_1^{2N*}\psi_{2,1}^* & \lambda_1^{2N-1*}\psi_{1,1}^* & 0\\ -\lambda_1^{2*}\psi_{3,1}^* & 0 & \lambda_1^*\psi_{1,1}^* & -\lambda_1^{4*}\psi_{3,1}^* & 0 & \lambda_1^{3*}\psi_{1,1}^* & \cdots & -\lambda_1^{2N*}\psi_{3,1}^* & 0 & \lambda_1^{2N-1*}\psi_{1,1}^*\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ \lambda_N^2\psi_{1,N} & \lambda_N\psi_{2,N} & \lambda_N\psi_{3,N} & \lambda_N^4\psi_{1,N} & \lambda_N^3\psi_{2,N} & \lambda_N^3\psi_{3,N} & \cdots & \lambda_N^{2N}\psi_{1,N} & \lambda_N^{2N-1}\psi_{2,N} & \lambda_N^{2N-1}\psi_{3,N}\\ -\lambda_N^{2*}\psi_{2,N}^* & \lambda_N^*\psi_{1,N}^* & 0 & -\lambda_N^{4*}\psi_{2,N}^* & \lambda_N^{3*}\psi_{1,N}^* & 0 & \cdots & -\lambda_N^{2N*}\psi_{2,N}^* & \lambda_N^{2N-1*}\psi_{1,N}^* & 0\\ -\lambda_N^{2*}\psi_{3,N}^* & 0 & \lambda_N^*\psi_{1,N}^* & -\lambda_N^{4*}\psi_{3,N}^* & 0 & \lambda_N^{3*}\psi_{1,N}^* & \cdots & -\lambda_N^{2N*}\psi_{3,N}^* & 0 & \lambda_N^{2N-1*}\psi_{1,N}^*\end{vmatrix}_{3N\times 3N}$$

and G and F can be got by replacing the second and third columns of H by Θ, respectively,

$$\begin{aligned}\Theta=(-1)^{N+1}\big(&\psi_{1,1},-\psi_{2,1}^*,-\psi_{3,1}^*,\cdots\\&\psi_{1,N},-\psi_{2,N}^*,-\psi_{3,N}^*\big)^{\mathrm{T}}.\end{aligned} \tag{36}$$

4. Bright Vector Multi-Soliton Solutions

In this section, we will solve the Lax pair (2) and (3) with the zero seed potential and apply the N-times iteration of the Darboux transformation to yield the bright vector multi-soliton solution of (1).

4.1. Bright Vector One-Soliton Solution

With $q_1=q_2=0$ as the seed solution and $\lambda=\lambda_1$, we can get the following general solution of Lax pair (2) and (3):

$$\Psi_1=\begin{pmatrix}\psi_{1,1}\\ \psi_{2,1}\\ \psi_{3,1}\end{pmatrix}=\begin{pmatrix}\alpha_1\,\mathrm{e}^{\theta_1}\\ \beta_1\,\mathrm{e}^{-\frac{\theta_1}{2}}\\ \delta_1\,\mathrm{e}^{-\frac{\theta_1}{2}}\end{pmatrix}, \tag{37}$$

where the phase $\theta_1=\frac{\mathrm{i}}{6\gamma^2}\left(2\mu-\lambda_1^2\right)\left(2\gamma x+\lambda_1^2 t-2\mu t\right)$; α_1, β_1, and δ_1 are all arbitrary complex constants.

Substituting solution (37) into the N-times-iterated potential formula (34) and (35) for $N=1$, we can obtain the bright one-soliton solution of (1):

$$\begin{aligned}q_1[1]=2\sqrt{2}|\beta_1||\xi_1\eta_1|\,\mathrm{e}^{\mathrm{i}\varphi_1}&\\ \cdot\Bigg[\gamma\Bigg(\left(|\beta_1|^2+|\delta_1|^2\right)\Bigg\{\left(\xi_1^2+\eta_1^2\right)&\\ \cdot\cosh\left[\frac{3}{2}(\theta_1+\theta_1^*)-\ln\frac{|\beta_1|^2+|\delta_1|^2}{|\alpha_1|^2}\right]&\\ +(\xi_1^2-\eta_1^2)\Bigg\}\Bigg)^{\frac{1}{2}}\Bigg]^{-1}&,\end{aligned} \tag{38}$$

$$\begin{aligned}q_2[1]=2\sqrt{2}|\delta_1||\xi_1\eta_1|\,\mathrm{e}^{\mathrm{i}\zeta_1}&\\ \cdot\Bigg[\gamma\Bigg(\left(|\beta_1|^2+|\delta_1|^2\right)\Bigg\{\left(\xi_1^2+\eta_1^2\right)&\\ \cosh\left[\frac{3}{2}(\theta_1+\theta_1^*)-\ln\frac{|\beta_1|^2+|\delta_1|^2}{|\alpha_1|^2}\right]&\\ +(\xi_1^2-\eta_1^2)\Bigg\}\Bigg)^{\frac{1}{2}}\Bigg]^{-1}&,\end{aligned} \tag{39}$$

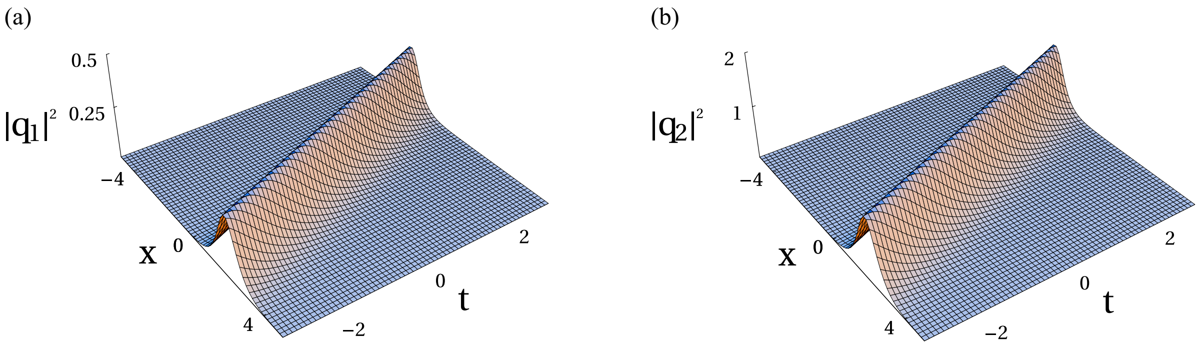

Fig. 1 (colour online). Bright vector solitons via solutions (38) and (39). The parameters of relevant physical quantities are $\alpha_1 = 2-\mathrm{i}$, $\beta_1 = 1$, $\delta_1 = -2$, $\lambda_1 = -2.2-1.2\mathrm{i}$, $\gamma = 2$, and $\mu = 1$.

where ξ_1 and η_1 are the real and imaginary part of λ_1, respectively, and

$$\begin{aligned}\varphi_1 = -\frac{\mathrm{i}}{2}\ln\Bigg\{ & \mathrm{e}^{\frac{3}{2}(\theta_1-\theta_1^*)}\alpha_1\beta_1^*\Big[\lambda_1(\beta_1\beta_1^*+\delta_1\delta_1^*) \\ & +\lambda_1^*\alpha_1\alpha_1^*\,\mathrm{e}^{\frac{3}{2}(\theta_1+\theta_1^*)}\Big]^3\Big\{\alpha_1^*\beta_1\Big[\lambda_1^*(\beta_1\beta_1^* \\ & +\delta_1\delta_1^*)+\lambda_1\alpha_1\alpha_1^*\,\mathrm{e}^{\frac{3}{2}(\theta_1+\theta_1^*)}\Big]^3\Big\}^{-1}\Bigg\},\\ \zeta_1 = -\frac{\mathrm{i}}{2}\ln\Bigg\{ & \mathrm{e}^{\frac{3}{2}(\theta_1-\theta_1^*)}\alpha_1\beta_1^*\Big[\lambda_1(\beta_1\delta_1^*+\delta_1\delta_1^*) \\ & +\lambda_1^*\alpha_1\alpha_1^*\,\mathrm{e}^{\frac{3}{2}(\theta_1+\theta_1^*)}\Big]^3\Big\{\alpha_1^*\delta_1\Big[\lambda_1^*(\beta_1\beta_1^* \\ & +\delta_1\delta_1^*)+\lambda_1\alpha_1\alpha_1^*\,\mathrm{e}^{\frac{3}{2}(\theta_1+\theta_1^*)}\Big]^3\Big\}^{-1}\Bigg\}.\end{aligned}$$

From the above solutions (38) and (39), the bright vector solitons are characterized by four arbitrary complex parameters α_1, β_1, δ_1, λ_1, and two real parameters μ and γ. The amplitudes of bright solitons in the first and second components are given as

$$A_1 = \frac{2|\eta_1\beta_1|}{\gamma\sqrt{|\beta_1|^2+|\delta_1|^2}}, \quad A_2 = \frac{2|\eta_1\delta_1|}{\gamma\sqrt{|\beta_1|^2+|\delta_1|^2}}. \tag{40}$$

The velocity, initial phase, and width of bright vector solitons in two components are, respectively,

$$v = \frac{1}{\gamma}(\eta_1^2-\xi_1^2+2\mu), \quad \varepsilon = \frac{\gamma}{\xi_1\eta_1}\ln\frac{|\beta_1|^2+|\delta_1|^2}{|\alpha_1|^2}, \quad W = \frac{2\xi_1\eta_1}{\gamma}.$$

The bell profiles of vector bright solitons in two components are plotted in Figure 1.

4.2. Bright Vector Two-Soliton Solution

In order to obtain the bright vector two-soliton solution, we adopt two sets of basic solutions of (2) and (3) with $q_1 = q_2 = 0$ for two different eigenvalues λ_1 and λ_2:

$$\Psi_k = \begin{pmatrix}\psi_{1,k}\\ \psi_{2,k}\\ \psi_{3,k}\end{pmatrix} = \begin{pmatrix}\alpha_k\,\mathrm{e}^{\theta_k}\\ \beta_k\,\mathrm{e}^{-\frac{\theta_k}{2}}\\ \delta_k\,\mathrm{e}^{-\frac{\theta_k}{2}}\end{pmatrix},\quad (k=1,2), \tag{41}$$

where the phase $\theta_k = \frac{\mathrm{i}}{6\gamma^2}(2\mu-\lambda_k^2)[2\gamma x + (\lambda_k^2-2\mu)t]$; α_k, β_k, and δ_k are all arbitrary complex constants. With the substitution of solutions (41) into (34) and (35) for $N = 2$, the bright vector two-soliton solution can be derived as

$$q_1[2] = -\left[\frac{2\mathrm{i}\mu}{\gamma}\frac{G_2}{H_2} - 2\left(\frac{G_2}{H_2}\right)_x\right], \tag{42}$$

$$q_2[2] = -\left[\frac{2\mathrm{i}\mu}{\gamma}\frac{F_2}{H_2} - 2\left(\frac{F_2}{H_2}\right)_x\right], \tag{43}$$

where

$$H_2 = \begin{vmatrix} \lambda_1^2\psi_{1,1} & \lambda_1\psi_{2,1} & \lambda_1\psi_{3,1} & \lambda_1^4\psi_{1,1} & \lambda_1^3\psi_{2,1} & \lambda_1^3\psi_{3,1} \\ -\lambda_1^{2*}\psi_{2,1}^* & \lambda_1^*\psi_{1,1}^* & 0 & -\lambda_1^{4*}\psi_{2,1}^* & \lambda_1^{3*}\psi_{1,1}^* & 0 \\ -\lambda_1^{2*}\psi_{3,1}^* & 0 & \lambda_1^*\psi_{1,1}^* & -\lambda_1^{4*}\psi_{3,1}^* & 0 & \lambda_1^{3*}\psi_{1,1}^* \\ \lambda_2^2\psi_{1,2} & \lambda_2\psi_{2,2} & \lambda_2\psi_{3,2} & \lambda_2^4\psi_{1,2} & \lambda_2^3\psi_{2,2} & \lambda_2^3\psi_{3,2} \\ -\lambda_2^{2*}\psi_{2,2}^* & \lambda_2^*\psi_{1,2}^* & 0 & -\lambda_2^{4*}\psi_{2,2}^* & \lambda_2^{3*}\psi_{1,2}^* & 0 \\ -\lambda_2^{2*}\psi_{3,2}^* & 0 & \lambda_2^*\psi_{1,2}^* & -\lambda_2^{4*}\psi_{3,2}^* & 0 & \lambda_2^{3*}\psi_{1,2}^* \end{vmatrix}_{6\times 6}$$

$$G_2 = \begin{vmatrix} \lambda_1^2\psi_{1,1} & -\psi_{1,1} & \lambda_1\psi_{3,1} & \lambda_1^4\psi_{1,1} & \lambda_1^3\psi_{2,1} & \lambda_1^3\psi_{3,1} \\ -\lambda_1^{2*}\psi_{2,1}^* & \psi_{2,1}^* & 0 & -\lambda_1^{4*}\psi_{2,1}^* & \lambda_1^{3*}\psi_{1,1}^* & 0 \\ -\lambda_1^{2*}\psi_{3,1}^* & \psi_{3,1}^* & \lambda_1^*\psi_{1,1}^* & -\lambda_1^{4*}\psi_{3,1}^* & 0 & \lambda_1^{3*}\psi_{1,1}^* \\ \lambda_2^2\psi_{1,2} & -\psi_{1,2} & \lambda_2\psi_{3,2} & \lambda_2^4\psi_{1,2} & \lambda_2^3\psi_{2,2} & \lambda_2^3\psi_{3,2} \\ -\lambda_2^{2*}\psi_{2,2}^* & \psi_{2,2}^* & 0 & -\lambda_2^{4*}\psi_{2,2}^* & \lambda_2^{3*}\psi_{1,2}^* & 0 \\ -\lambda_2^{2*}\psi_{3,2}^* & \psi_{3,2}^* & \lambda_2^*\psi_{1,2}^* & -\lambda_2^{4*}\psi_{3,2}^* & 0 & \lambda_2^{3*}\psi_{1,2}^* \end{vmatrix}_{6\times 6}$$

$$F_2 = \begin{vmatrix} \lambda_1^2\psi_{1,1} & \lambda_1\psi_{2,1} & -\psi_{1,1} & \lambda_1^4\psi_{1,1} & \lambda_1^3\psi_{2,1} & \lambda_1^3\psi_{3,1} \\ -\lambda_1^{2*}\psi_{2,1}^* & \lambda_1^*\psi_{1,1}^* & \psi_{2,1}^* & -\lambda_1^{4*}\psi_{2,1}^* & \lambda_1^{3*}\psi_{1,1}^* & 0 \\ -\lambda_1^{2*}\psi_{3,1}^* & 0 & \psi_{3,1}^* & -\lambda_1^{4*}\psi_{3,1}^* & 0 & \lambda_1^{3*}\psi_{1,1}^* \\ \lambda_2^2\psi_{1,2} & \lambda_2\psi_{2,2} & -\psi_{1,2} & \lambda_2^4\psi_{1,2} & \lambda_2^3\psi_{2,2} & \lambda_2^3\psi_{3,2} \\ -\lambda_2^{2*}\psi_{2,2}^* & \lambda_2^*\psi_{1,2}^* & \psi_{2,2}^* & -\lambda_2^{4*}\psi_{2,2}^* & \lambda_2^{3*}\psi_{1,2}^* & 0 \\ -\lambda_2^{2*}\psi_{3,2}^* & 0 & \psi_{3,2}^* & -\lambda_2^{4*}\psi_{3,2}^* & 0 & \lambda_2^{3*}\psi_{1,2}^* \end{vmatrix}_{6\times 6}.$$

Figure 2 displays the elastic collision of bright vector solitons S_1 and S_2. In this figure, like the scalar soliton, bright vector solitons S_1 and S_2 do not affect each other only by a phase shift along with the conserved energy. They collide with each other and exhibit the particle properties. In contrast, Figure 3 shows that the bright vector solitons S_1 and S_2 between two components undergo the partial energy exchange in inelastic collision. In fact, the complete energy exchange of two bright vector solitons also takes place between two components for suitable choice of parameters. In addition, with the vanishing boundary conditions $q_j|_{x\to\pm\infty} \to 0$ $(j = 1,2)$, one can observe that the total energy of (1) is conserved from the integral of motion for (1), that is,

$$\int_{-\infty}^{+\infty} \left(|q_1|^2 + |q_2|^2\right) \mathrm{d}x = \text{constant}.$$

As illustrated in Figures 2 and 3, such elastic and inelastic collision properties of bright vector solitons could be used in the construction of logic gates, the implementation of all-optical switching, and the design of quantum information processors.

4.3. Bright Vector N-Soliton Solution

Based on the above procedure, we can successively implement the Darboux transformation N times to generate the bright vector N-soliton solution of (1). By solving the linear equations (2) and (3) with the seed solution $q_1 = q_2 = 0$ for N different spectral parameters $\lambda = \lambda_k$ $(1 \leqslant k \leqslant N)$, we can take a set of linearly independent solutions

$$\Psi_k = \begin{pmatrix} \psi_{1,k} \\ \psi_{2,k} \\ \psi_{3,k} \end{pmatrix} = \begin{pmatrix} \alpha_k\,\mathrm{e}^{\theta_k} \\ \beta_k\,\mathrm{e}^{-\frac{\theta_k}{2}} \\ \delta_k\,\mathrm{e}^{-\frac{\theta_k}{2}} \end{pmatrix}, \quad (k = 1,2,\ldots,N), \tag{44}$$

where the phase $\theta_k = \frac{\mathrm{i}}{6\gamma^2}(2\mu - \lambda_k^2)[2\gamma x + (\lambda_k^2 - 2\mu)t]$; α_k, β_k, and δ_k are all arbitrary complex constants. With the substitution solutions (44) into (34) and (35), the bright vector N-soliton solution of (1) can be obtained by virtue of the symbolic computation platform. Using the bright vector N-soliton solution, the interaction behaviours among and more colliding bright vector solitons can be studied such as complete or partial energy switching between two components.

Taking $N = 3$ for example, we adopt three different spectral parameters λ_1, λ_2, and λ_3 and corresponding three linearly independent solutions Ψ_1, Ψ_2, and Ψ_3. We can study the collision behaviours among three bright vector solitons in two components. Figures 4 and 5 il-

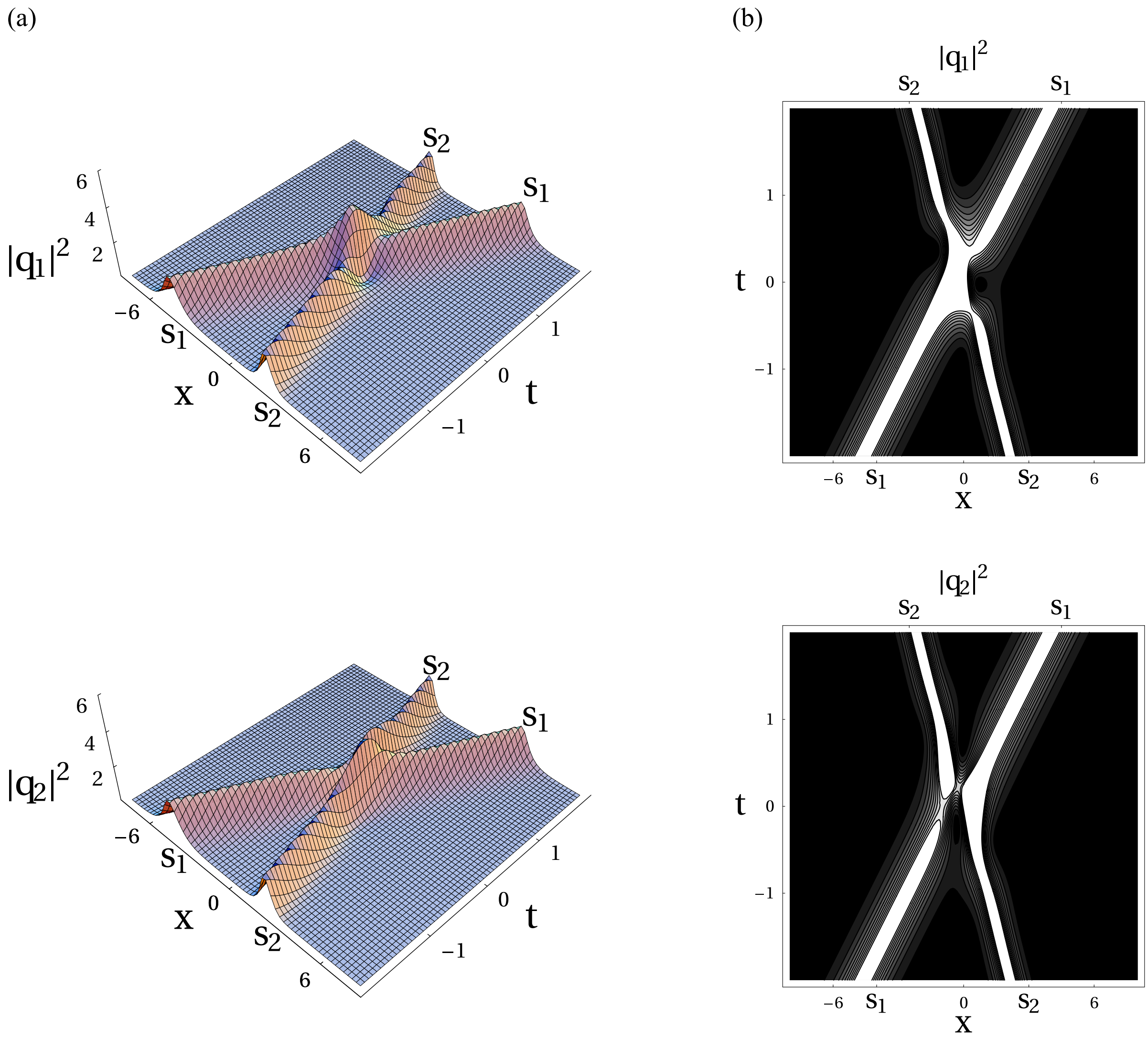

Fig. 2 (colour online). Elastic collision between two bright vector solitons. (a) Intensity profiles of the bright two-soliton solutions (42) and (43). (b) Contour plots of the intensity versus t and x. The parameters of relevant physical quantities are $\alpha_1 = 1 - \mathrm{i}$, $\beta_1 = 1$, $\delta_1 = \alpha_2 = \beta_2 = 1$, $\delta_2 = -1$, $\lambda_1 = 1 + \mathrm{i}$, $\lambda_2 = -2 + \mathrm{i}$, $\gamma = 1$, and $\mu = 1$.

lustrate the elastic and inelastic collisions among three bright vector solitons. In Figure 5, it can be easily observed that the intensity of bright vector soliton S_1 gets enhanced and those of other two bright vector solitons S_2 and S_3 get suppressed after the collision in the first component, while in the second component S_1' is suppressed and S_2' and S_3' are both enhanced after the collision. Furthermore, one can check easily that the intensity of each bright vector soliton S_j $(j = 1, 2, 3)$ is conserved in two components.

5. Conclusions

In this paper, we have investigated the coupled mixed derivative nonlinear Schrödinger equations (1), which describe the pulse propagation in the picosecond or femtosecond regime of birefringent optical fibers. A new type of Lax pair (2) and (3) associated with (1) has been derived from the Wadati–Konno–Ichikawa (WKI) spectral problem. In such a pair of linear equations (2) and (3), the traces of U_j and V_k are all

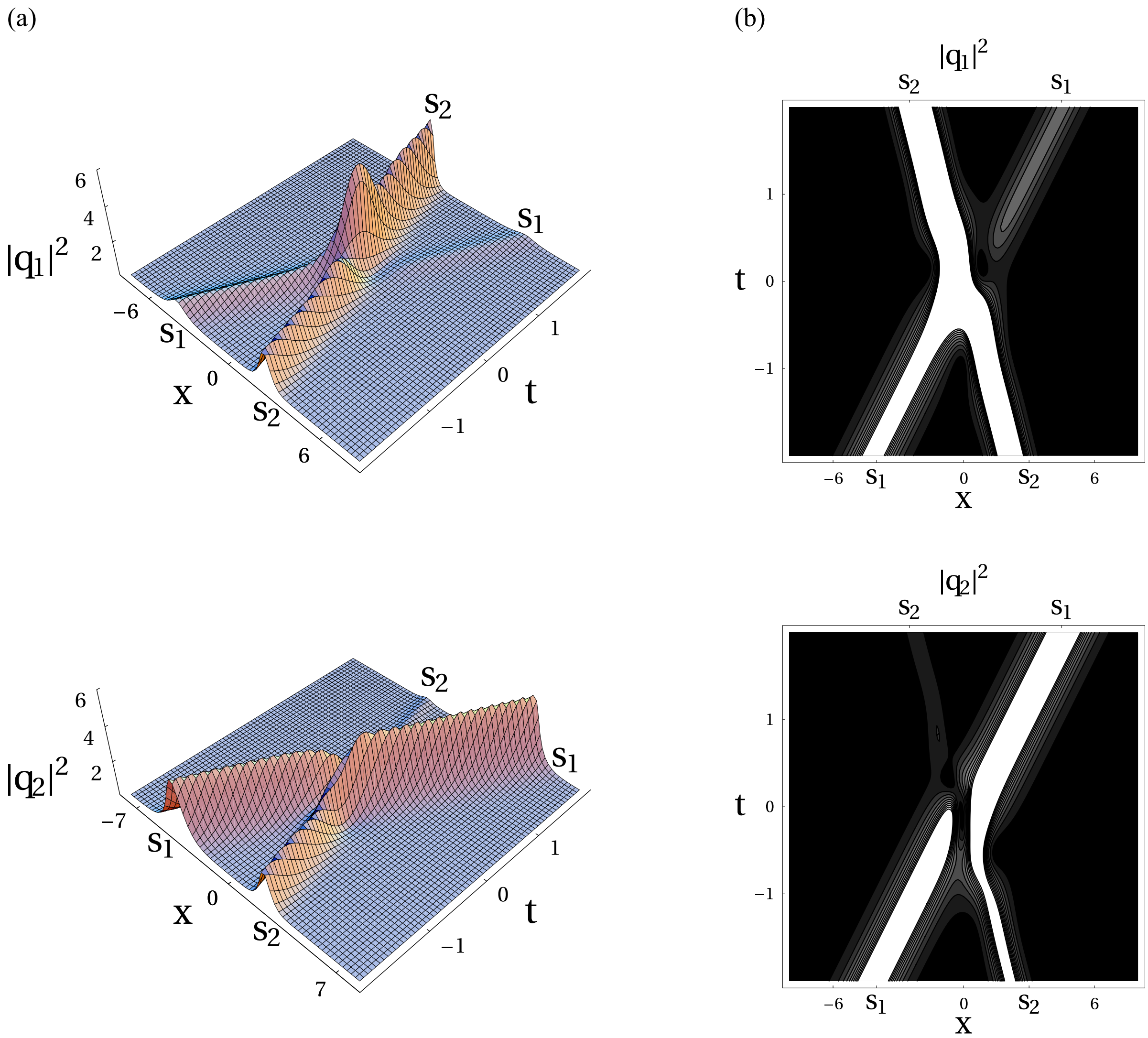

Fig. 3 (colour online). Inelastic collision between two bright vector solitons. (a) Intensity profiles of the bright two-soliton solutions (42) and (43). (b) Contour plots of the intensity versus t and x. The parameters of relevant physical quantities are $\alpha_1 = 1 - \mathrm{i}$, $\beta_1 = 1$, $\delta_1 = -2$, $\alpha_2 = \beta_2 = \delta_2 = 1$, $\lambda_1 = 1 + \mathrm{i}$, $\lambda_2 = -2 + \mathrm{i}$, $\gamma = 1$, and $\mu = 1$.

zero, which would be useful for keeping Darboux covariant reductions of (1). The Darboux transformation and iterative algorithm have been applied to this integrable coupled model, and the N-times iteration formulas (34) and (35) of the Darboux transformation have been presented in terms of the determinant representation. With the zero potential as seed solution and a given set of spectral parameters, the bright vector N-soliton solution has been expressed as a compact and transparent determinant by N linearly independent eigenfunctions of Lax pair (2) and (3). We have identified and discussed the interesting collision properties of bright vector solitons. It has been demonstrated that bright vector solitons can exhibit the standard elastic and inelastic collisions. We have shown that each vector soliton in two components can undergo the partial or complete energy switching in an inelastic collision.

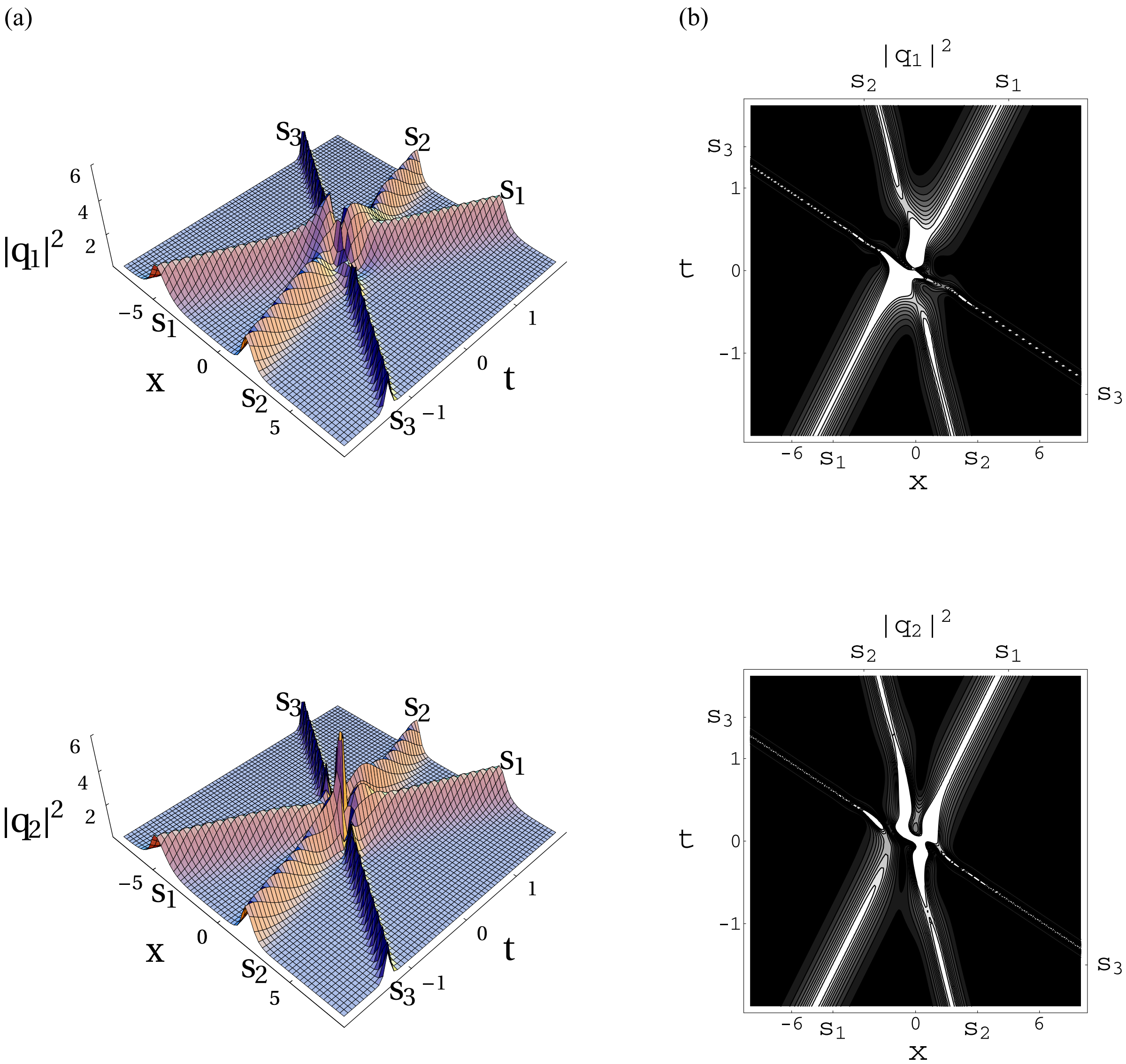

Fig. 4 (colour online). Elastic collision among three bright vector solitons. (a) Intensity profiles of the bright three-soliton solutions (34) and (35) for $N = 3$. (b) Contour plots of the intensity versus t and x. The parameters of relevant physical quantities are $\alpha_1 = 1 - \mathrm{i}$, $\beta_1 = 1$, $\delta_1 = \alpha_2 = \beta_2 = 1$, $\delta_2 = -1$, $\alpha_3 = \beta_3 = 1$, $\delta_3 = -1$, $\lambda_1 = 1 + \mathrm{i}$, $\lambda_2 = -2 + \mathrm{i}$, $\lambda_3 = -3 + \mathrm{i}$, $\gamma = 1$, and $\mu = 1$.

Based on the framework of the present paper, we have the following miscellaneous remarks and future works.

(i) With the bilinear method, the bright N-soliton solution of (1) has been obtained in terms of the determinants and involves $3N$ complex parameters in [10]. The determinants f and $g_i\,(i = 1,2)$ in the N-soliton solution have compact expressions, where f is a $2N \times 2N$ determinant and $g_i\,(i = 1,2)$ are both $(2N+1)\times(2N+1)$ determinants [10]. In the present paper, the N-soliton solution is expressed as a $3N \times 3N$ determinant by means of the Darboux transformation method. The determinant is generated by N complex eigenvalues and N linearly independent eigenfunctions, and involves $4N$ complex parameters λ_k, α_k, β_k, and δ_k. It seems that two types of the determinants are differ-

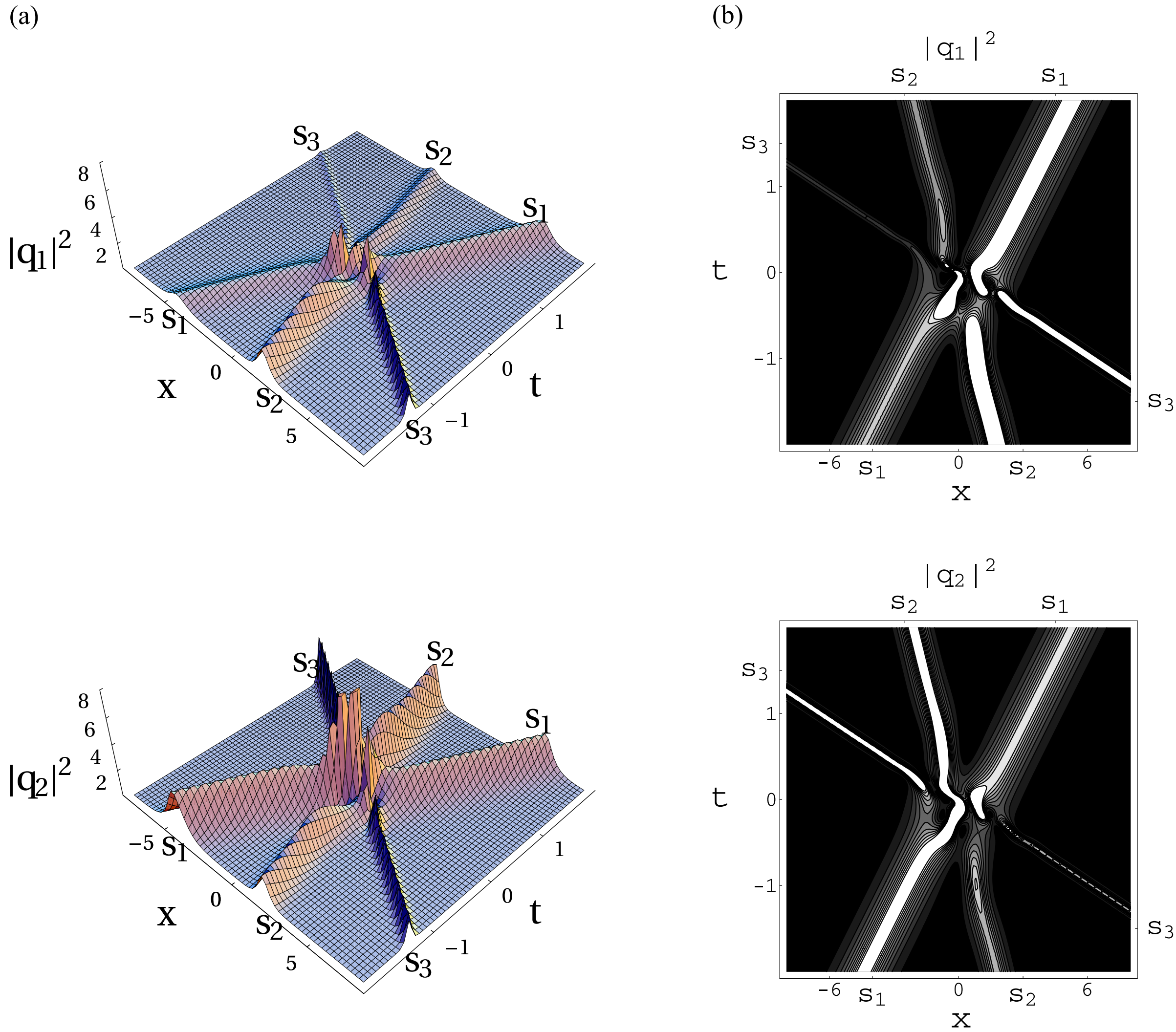

Fig. 5 (colour online). Inelastic collision among three bright vector solitons. (a) Intensity profiles of the bright three-soliton solutions (34) and (35) for $N = 3$. (b) Contour plots of the intensity versus t and x. The parameters of relevant physical quantities are $\alpha_1 = 1 - \mathrm{i}$, $\beta_1 = 1$, $\delta_1 = -2$, $\alpha_2 = \beta_2 = \delta_2 = 1$, $\alpha_3 = \beta_3 = 1$, $\delta_3 = -1$, $\lambda_1 = 1 + \mathrm{i}$, $\lambda_2 = -2 + \mathrm{i}$, $\lambda_3 = -3 + \mathrm{i}$, $\gamma = 1$, and $\mu = 1$.

ent. The relation between them seems to be a nontrivial problem and will be pursued in future works.

(ii) We have shown that the bright vector solitons display energy-exchange collision between two components in (1). Next, according to the determinant properties of N-times Darboux iteration formula (3), we will investigate the asymptotic behaviour of the bright vector solitons for arbitrary N colliding vector solitons. Such fascinating collision properties open possibilities for future applications in the design of logical gates, fiber directional couplers, and quantum information processors.

(iii) Our focus will be put on the m-component coupled mixed derivative nonlinear Schrödinger equations

$$\begin{aligned} &\mathrm{i}\, q_{jt} + q_{jxx} + \mu \left(\sum_{k=1}^{m} |q_k|^2 \right) q_j \\ &\quad + \mathrm{i}\gamma \left[\left(\sum_{k=1}^{m} |q_k|^2 \right) q_j \right]_x = 0, \quad (j = 1, 2, \ldots, m). \end{aligned} \tag{45}$$

With the direct method, the bright N-soliton solution of (45) has been obtained in the form of compact determinantal expressions [22]. Our Darboux transformation method in the present paper can be easily generalized to m-component (45), then the bright vector N-soliton solution of (45) can be also derived in terms of the determinant representation. Another interesting issue to be worth studying is the collision process of arbitrary N colliding bright vector solitons with complete or partial energy exchange among m components.

(iv) In the present paper, we take zero as the seed solution and obtain the bright N-soliton solution of (1) from the N-times iterative Darboux transformation. With nonvanishing background (e.g., monochromatic wave solution), the multi-periodic and breather solutions can be generated from the N-times-iterated potential formulas (34) and (35). Furthermore, we infer that the multi-rogue wave solution of (1) can also be derived with the N-times iterative Darboux transformation under the nonzero background. In future works, our focus will be put on the periodic, breather, and rogue wave solutions of (1) with nonvanishing background.

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