Upper-Convected Maxwell Fluid Flow over an Unsteady Stretching Surface Embedded in Porous Medium Subjected to Suction/Blowing

Swati Mukhopadhyay

Department of Mathematics, The University of Burdwan, Burdwan-713104, W. B., India

Reprint requests to S. M.; E-mail: swati_bumath@yahoo.co.in

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Unsteady two-dimensional flow of a Maxwell fluid over a stretching surface in a porous medium subjected to suction/blowing is investigated. The upper-convected Maxwell fluid model is used to characterize the non-Newtonian fluid behaviour. With the help of similarity transformations, the boundary layer equation corresponding to the momentum equation is transformed to an ordinary one and then solved numerically by the shooting method. The flow characteristics for different values of the governing parameters are analyzed and discussed in detail. The fluid velocity initially decreases with increasing unsteadiness parameter. Also, it is found that the fluid velocity decreases with increasing permeability parameter. The effect of increasing values of the Maxwell parameter is to suppress the velocity field. Due to suction, the fluid velocity is found to decrease in the boundary layer region.

Key words: Unsteady Flow; Maxwell Fluid; Porous Medium; Stretching Surface; Suction/Blowing.

1. Introduction

During the past few decades, the flows of non-Newtonian fluids have been analyzed and studied by numerous investigators [1 – 6]. There is not a single constitutive equation which exhibits all properties of such non-Newtonian fluids due to its complex nature. The vast majority of non-Newtonian fluid models in literature are concerned with simple models viz. the power law and second-grade or third-grade models [7 – 12]. These simple fluid models have some drawbacks. The power-law model is used to model fluids with shear-dependent viscosity but it cannot predict the effects of elasticity. On the other hand, the second-grade or third-grade fluid models can calculate the effects of elasticity but the viscosity in these models is not shear dependent, and they are unable to predict the effects of stress relaxation. The Maxwell model, a subclass of rate type fluids, can predict the stress relaxation and therefore, have become more popular. This model excludes the complicated effects of shear-dependent viscosity and thus enables one to focus solely on the effects of fluid's elasticity on the characteristics of its boundary layer [13]. The Maxwell model is one of the simplest models to account for fluid rheological effects. But an inadequacy of 'Upper-convected Maxwell model' which is mostly used is that it does not properly describe the typical relation between shear rate and shear stress in a simple shear flow of a real fluid: it predicts only a linear relation. Moreover, a singularity at a critical strain rate in purely extensional flow is in the model [14].

The study of hydrodynamic flow in porous medium becomes much more interesting due to its vast applications on the boundary layer flow control [15]. It is well known that Darcy's law is an empirical formula relating the pressure gradient, the bulk viscous fluid resistance, and the gravitational force for a forced convective flow in a porous medium. Deviations from Darcy's law occur when the Reynolds number based on the pore diameter is within the range of 1 to 10 [16]. Representative studies dealing with flow through porous medium can be found from Pal and Mondal [17], Mukhopadhyay [18], Mukhopadhyay et al. [19], Ishak et al. [20], and Hayat et al. [21].

The separation of the boundary layer is associated with large energy losses and in most applications adversely affects the aerodynamic loads in the form of lift loss and drag increase. Therefore there is a strong tendency to delay or manipulate the occurrence of flow separation. To control the flow, passive or active devices are used. Passive control devices are those which are not energy consumptive. They mainly affect the flow by the geometry of the airfoil. The process of suction and injection (blowing) has also its importance in many engineering applications such as in the design of thrust bearing and radial diffusers, and thermal oil recovery. Suction is applied to chemical processes to remove reactants. Blowing is used to add reactants, prevent corrosion or scaling, and reduce the drag [22]. Injection or withdrawal of fluid through a porous bounding heated or cooled wall is of general interest in practical problems involving control of boundary layers, and so forth. This can help to delay the transition from a laminar flow [23].

In all of the above mentioned studies, the flow and temperature fields are considered to be at steady state. However, in some cases the flow field can be unsteady due to a sudden stretching of the flat sheet. Several researchers [24–28] studied the problem for unsteady stretching surface by using a similarity method to transform the governing time-dependent boundary layer equations into a set of ordinary differential equations. Elbashbeshy and Bazid [29] have presented similarity solutions of the boundary layer equations, that describe the unsteady flow and heat transfer over an unsteady stretching sheet. Sharidan et al. [30] analysed the unsteady flow and heat transfer over a stretching sheet in a viscous and incompressible fluid. Recently, Tsai et al. [31], Mukhopadhyay [32], Chamkha et al. [33] discussed the unsteady flow and heat transfer over a stretching sheet under different conditions. Of late, Yacob et al. [34] investigated the unsteady flow of a power-law fluid and mass transfer past a shrinking sheet.

No attempt has been made so far to analyze the effects of suction/blowing on a Maxwell fluid flow past an unsteady permeable stretching surface embedded in a porous medium. Motivated by this, an attempt is made in this paper to extend the work of Andersson et al. [24] for an upper-convected Maxwell fluid in a porous medium subjected to suction/blowing. The corresponding boundary layer equation is transformed into an ordinary differential equation by means of similarity transformations and then solved numerically using the shooting method. Comparisons are made with the available results from the literature and found in excellent agreement. The effects of unsteadiness parameter, Maxwell parameter, permeability parameter, suction/blowing parameter on velocity fields are investigated and analysed with the help of their graphical representations.

2. Equations of Motion

We consider the laminar boundary-layer two-dimensional flow of an incompressible, upper-convected Maxwell fluid over an unsteady permeable stretching sheet embedded in a porous medium with permeability $k(t) = k_0(1-\alpha t)$, where k_0 is the initial permeability. $k(t)$ is assumed to vary as a linear function of time. The unsteady fluid flow starts at $t = 0$. The sheet emerges out of a slit at origin $(x = 0, y = 0)$ and moves with non-uniform velocity $U(x,t) = \frac{cx}{1-\alpha t}$, where $c > 0$ and $\alpha \geq 0$ are constants with dimensions $(\text{time})^{-1}$, c is the initial stretching rate, and $\frac{c}{1-\alpha t}$ is the effective stretching rate which is increasing with time. In case of polymer extrusion, the material properties of the extruded sheet may vary with time. It is also assumed that the stretching surface is subjected to such amount of tension which does not alter the structure of the permeable material.

The equation of continuity and the equation of motion of such type of flow are, in the usual notation,

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0\,, \tag{1}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y}\right) = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} - \frac{\mu}{k}u\,. \tag{2}$$

Here, the linear Darcy term representing distributed body force due to porous media is retained while the nonlinear Forchheimer term is neglected. Equation (2) is true in the absence of a pressure gradient in x-direction. u and υ are the components of the velocity respectively in the x- and y-direction, μ is the coefficient of fluid viscosity, ρ the fluid density, and τ_{xx} , τ_{xy} are the components of the extra stress tensor.

The fluid of interest obeys the upper-convected Maxwell model in which the components τ_{ij} of the extra stress tensor can be related to the components d_{ij} of the deformation rate tensor by the following relation: $\tau_{ij} + \lambda \frac{\Delta}{\Delta t}\tau_{ij} = 2\delta d_{ij}$, where δ is the coefficient of viscosity and $\lambda = \lambda_0(1-\alpha t)$ is the relaxation time of the fluid, λ_0 is a constant. The time derivative $\frac{\Delta}{\Delta t}$ appearing in the above equation is the upper-convected time derivative devised to satisfy the requirements of the continuum mechanics, i.e the material objectivity and frame difference. This time derivative when applied to the stress tensor reads as $\frac{\Delta}{\Delta t}\tau_{ij} = \frac{D}{Dt}\tau_{ij} - L_{jk}\tau_{ik} - L_{ik}\tau_{kj}$ where L_{ij} are the components of the velocity gradient tensor L. For an incompressible fluid obeying the upper-convected Maxwell model, the momentum equation can be simplified using the boundary layer theory as [19, 35]

$$\begin{aligned}&\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y}\\ &+\lambda\left(u^2\frac{\partial^2 u}{\partial x^2} + \upsilon^2\frac{\partial^2 u}{\partial y^2} + 2u\upsilon\frac{\partial^2 u}{\partial x \partial y}\right)\\ &= \nu\frac{\partial^2 u}{\partial y^2} - \frac{\nu}{k}u,\end{aligned} \tag{3}$$

where ν is the kinematic viscosity of the fluid.

For the upper-convected maxwell (UCM) fluid model used in this paper, no unsteady term for the shear stress need to be included as mentioned by Alizadeh-Pahlavan and Sadeghy [35]. Therefore no unsteady term appears to the coefficient of λ.

The appropriate boundary conditions for the problem are given by

$$u = U(x,t), \quad \upsilon = \upsilon_w(t) \quad \text{at} \quad y = 0, \tag{4}$$
$$u \to 0 \quad \text{as} \quad y \to \infty, \tag{5}$$

where $\upsilon_w(t) = -\frac{\upsilon_0}{\sqrt{1-\alpha t}}$ is the velocity of suction ($\upsilon_0 > 0$) and blowing ($\upsilon_0 < 0$). The expressions for $U(x,t)$, $\upsilon_w(t)$, $k(t)$, and $\lambda(t)$ are valid for time $t < \alpha^{-1}$ (except for $\alpha = 0$). $\alpha = 0$ stands for the steady case.

We now introduce new relations for u and υ as

$$u = \frac{\partial \psi}{\partial y}, \quad \upsilon = -\frac{\partial \psi}{\partial x}, \tag{6}$$

where ψ is the stream function.

We introduce $\eta = \sqrt{\frac{c}{\nu(1-\alpha t)}}y$, $\psi = \sqrt{\frac{\nu c}{(1-\alpha t)}}xf(\eta)$.

With the help of the above relations, the governing equations finally reduce to

$$\begin{aligned}&M\left(\frac{\eta}{2}f'' + f'\right) + f'^2 - ff''\\ &+\beta\left(f^2 f''' - 2ff'f''\right)\\ &= f''' - k_1 f',\end{aligned} \tag{7}$$

where $M = \frac{\alpha}{c}$ is the unsteadiness parameter, $\beta = c\lambda_0$ the Maxwell parameter, and $k_1 = \frac{\nu}{k_0 c}$ the permeability parameter [33].

Table 1. Values of $f''(0)$ for various values of unsteadiness parameter M with $\beta = 0$, $k_1 = 0$, and $S = 0$ compared with the results of Sharidan et al. [30] and Chamkha et al. [33] .

M	[30]	[33]	Present study
0.8	−1.261042	−1.261512	−1.261479
1.2	−1.377722	−1.378052	−1.377850

The boundary conditions (4) and (5) then become

$$f' = 1, \quad f = S \quad \text{at} \quad \eta = 0 \tag{8}$$
$$\text{and} \quad f' \to 0 \quad \text{as} \quad \eta \to \infty. \tag{9}$$

Here $S = \frac{\upsilon_0}{\sqrt{\nu c}}$ is the suction/blowing parameter: $S > 0$ corresponds to suction and $S < 0$ corresponds to blowing.

The above equation (4) along with boundary conditions (8)–(9) is solved numerically with the help of the shooting method (see [32]).

3. Results and Discussions

In order to validate the method used in this study and to judge the accuracy of the present analysis, a comparison with available results corresponding to the skin-friction coefficient $f''(0)$ for the unsteady flow of a viscous incompressible Newtonian fluid ($\beta = 0$) in a non-porous medium and in the absence of suction/blowing is made. We compared the available results of Sharidan et al. [30] and Chamkha et al. [33] in Table 1 and found that the results agree well.

In order to study the behaviour of velocity fields for an upper-convected Maxwell fluid, a comprehensive numerical computation is carried out for various values of the parameters that describe the flow characteristics, and the results are reported in terms of graphs as shown in Figures 1–5.

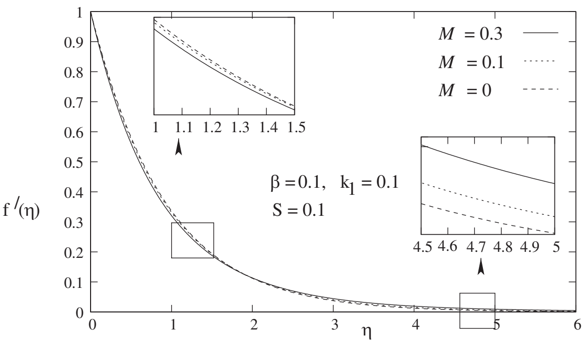

Fig. 1. Velocity profiles for variable unsteadiness parameter M.

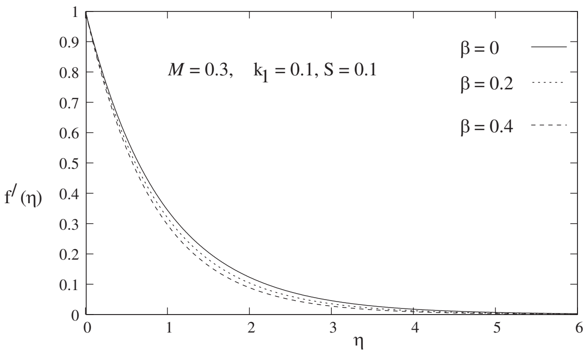

Fig. 2. Velocity profiles for variable Maxwell parameter β.

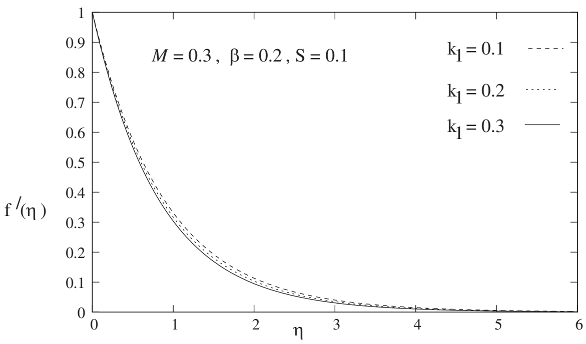

Fig. 3. Velocity profiles for variable permeability parameter k_1.

Figure 1 exhibits the velocity profiles for several values of unsteadiness parameter M. It is seen that the velocity along the sheet decreases initially with the increase of M, and this implies an accompanying reduction of the thickness of the momentum boundary layer near the wall. But away from the wall, the fluid velocity increases with increasing unsteadiness. $M = 0$ indicates the steady case. Since the fluid flow is caused solely by the stretching sheet, the velocity decreases with increasing η.

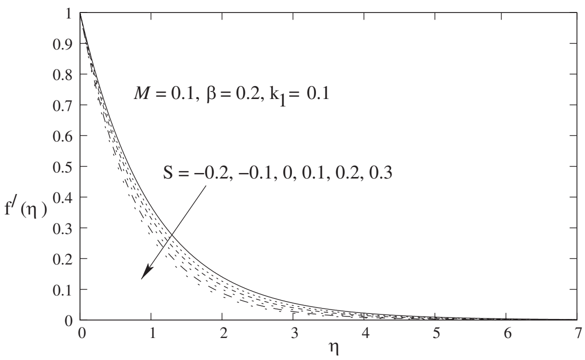

Fig. 4. Velocity profiles for variable suction/blowing parameter S.

Effects of Maxwell parameter β on velocity profiles are clearly exhibited in Figure 2. Here $\beta = 0$ gives the result for a viscous incompressible Newtonian fluid. The effect of increasing values of β is to reduce the velocity and hence the boundary layer thickness decreases. The velocity curves in Figure 2 show that the rate of transport is considerably reduced with the increase of β. It can also be seen from Figure 2 that the momentum boundary layer thickness decreases as β increases, and hence induces an increase in the absolute value of the velocity gradient at the surface.

The effect of permeability parameter k_1 on the velocity field is exhibited in Figure 3. As the permeability of the medium increases, the value of k_1 decreases (as $k_1 = \frac{\nu}{k_0 c}$). For large permeability, i.e. for decreasing k_1,

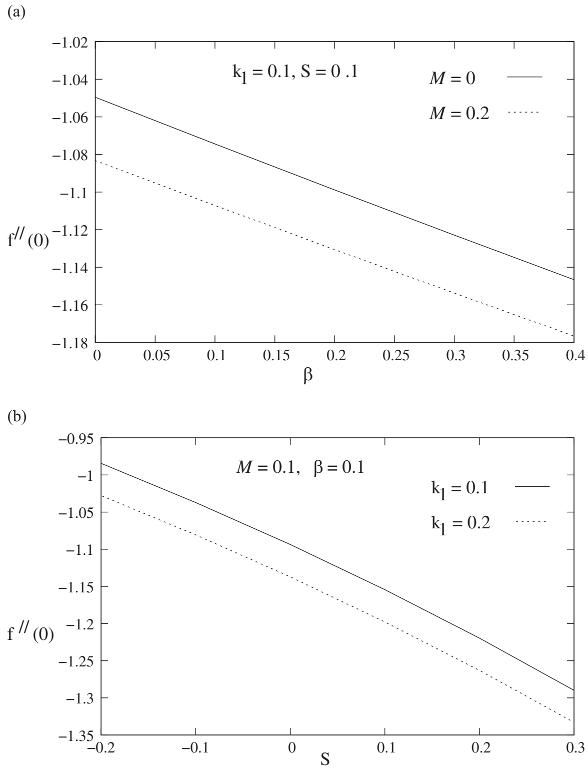

Fig. 5. (a) Variation of $f''(0)$ related to skin-friction coefficient with Maxwell parameter β for two values of unsteadiness parameter M. (b) Variation of $f''(0)$ related to skin-friction coefficient with suction/blowing parameter S for two values of permeability parameter k_1.

the fluid gets more space to flow and as a consequence its velocity increases. However, this change in the velocity has a maximum near the surface and far away from the surface, this change is small and finally approaches to zero. The presence of the porous medium causes higher restriction to the fluid, which reduces the fluid velocity. The boundary layer thickness decreases when k_1 increases.

Figure 4 displays the effects of suction/blowing parameter S on the velocity profiles. The fluid velocity is found to decrease with increasing values of S. With increasing suction ($S > 0$), the fluid velocity is found to decrease, i.e. suction causes to decrease the velocity of the fluid in the boundary layer region. This effect acts to decrease the wall shear stress. Increase in suction causes progressive thinning of the boundary layer.

Furthermore, the effects of unsteadiness parameter M and Maxwell parameter β on $f''(0)$ related to the skin-friction coefficient are presented in Figure 5a. The value of $f''(0)$ related to the skin-friction coefficient decreases with increasing unsteadiness parameter M and also with Maxwell parameter β. A drop in skin friction as investigated in this paper has the important implication that in free coating operations, elastic properties of the coating formulations may be beneficial for the whole process. This means that less force may be needed to pull a moving sheet at a given withdrawal velocity or equivalently higher withdrawal speeds can be achieved for a given driving force resulting in an increase in the rate of production [36]. The effects of suction/blowing and permeability parameter on $f''(0)$ related to the skin-friction coefficient is exhibited in Figure 5b. It is very clear that the value of $f''(0)$ related to the skin-friction coefficient decreases with increasing permeability parameter and also with the increasing suction/blowing parameter.

4. Conclusions

It can be concluded that the present study provides the numerical solutions for unsteady boundary layer flow of a Maxwell fluid over a permeable stretching surface in porous medium in the presence of suction/blowing. The fluid velocity decreases initially due to an increase in the unsteadiness parameter. The effect of increasing values of the Maxwell parameter as well as permeability parameter is to suppress the velocity field. A decrease in fluid velocity is observed with increasing suction.

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