Exact Solutions for the Magnetohydrodynamic Flow of a Jeffrey Fluid with Convective Boundary Conditions and Chemical Reaction

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The two-dimensional magnetohydrodynamic (MHD) flow of a Jeffrey fluid is investigated in this paper. The characteristics of heat and mass transfer with chemical reaction have also been analyzed. Convective boundary conditions have been invoked for the thermal boundary layer problem. Exact similarity solutions for flow, temperature, and concentration are derived. Interpretation to the embedded parameters is assigned through graphical results for dimensionless velocity, temperature, concentration, skin friction coefficient, and surface heat and mass transfer. The results indicate an increase in the velocity and the boundary layer thickness by increasing the rheological parameter of the Jeffrey fluid. An intensification in the chemical reaction leads to a thinner concentration boundary layer.

Key words: Exact Solutions; Heat Transfer; Mass Transfer; Jeffrey Fluid; Chemical Reaction; Convective Boundary Condition; Numerical Solution.

1. Introduction

The study of boundary layer flows of non-Newtonian fluids has been the subject of great interest to investigators and researchers. Such flows widely occur in polymer and food processing, transpiration cooling, drag reduction, thermal oil recovery, and ice and magma flows. In spite of different physical structures of non-Newtonian fluids, the researchers have proposed a variety of these fluid models. Jeffrey fluid is a subclass of rate type fluids which can describe the characteristics of relaxation and retardation times. This subclass of fluid is not much addressed in the literature in view of the difficulties associated with the explicit expressions of stress components and velocity. Al-Nimr et al. [1] discussed the transient Couette flow, transient wind-driven flow over finite domains, and the transient Poiseuille flow in parallel-plates channels using the Jeffrey fluid model. Hayat and Mustafa [2], Hayat et al. [3], and Mustafa et al. [4] presented the series solutions for two-dimensional boundary layer flows of a Jeffrey fluid over moving extensible or inextensible surfaces. Kothandapani and Srinivas [5] discussed the peristaltic transport of the magnetohydrodynamic (MHD) Jeffrey fluid. Now a days, the Jeffrey fluid model is very popular amongst the researchers (see references [6 – 9] and many references there in) because its constitutive equations are simple in comparison to the other rate type fluids such as upper-convected Maxwell and Oldroyd-B fluids. Boundary layer flows of viscous and non-Newtonian fluids over the moving extensible surfaces have numerous industrial applications. In fact, various manufacturing processes involve the production of sheeting material which includes both metal and polymer sheets. Such flows have particular relevance in hot rolling, fibers spinning, manufacturing of plastic and rubber sheet, polymers continuous casting, and glass blowing. Blasius [10] initiated the boundary layer flow over a flat surface with uniform free stream. He provided an analytic solution of the arising nonlinear differential system. The numerical solution of the Blasius problem has been reported by Howarth [11]. Sakiadis [12] considered the boundary layer flow over a continuously moving flat plate. Crane [13] studied the flow configuration of references [12] over a stretching sheet. He has provided a closed form exact solution for the dimensionless momentum equation. In the past, several researchers have looked at Crane's problem under various configurations. The seminal work on the two-dimensional boundary layer flow of viscoelastic fluid over a stretching sheet has been conducted by Rajagopal et al. [14]. Sankara and Watson [15] and Andersson et al. [16] extended Crane's problem for micropolar and power-law fluids, respectively. Perturbation solutions for the boundary layer flow of a viscoelastic (Walters 'B' model) fluid near the stagnation point towards a stretching surface have been obtained by Mahapatra and Gupta [17]. Liu [18] derived the exact solutions for flow with heat and mass transfer of viscous fluid with internal heat generation and chemical reaction. Cortell [19] analyzed the influences of porosity, magnetohydrodynamics, and mass transfer on the flow of a viscoelastic fluid towards a stretching sheet. An exact solution for the dimensionless velocity has been provided in this study. However, the equation of mass transfer has been solved numerically by the Runge–Kutta method. An analytic solution for mass transfer in a viscous fluid over a stretching sheet with suction/injection has been presented by El-Arabawy et al. [20]. Recent contributions concerning the boundary layer flow analysis due to a stretching sheet include those of Mahapatra et al. [21], Nandeppanavar et al. [22], Ahmed and Asghar [23], and Hayat et al. [24].

Convective heat and mass transfer with chemical reaction has a pivotal role in design of chemical processing equipment, formation and dispersion of fog, damage of crops due to freezing, food processing and cooling towers, distribution of temperature and moisture over grove fields etc. Chemical reactions often escort a large amount of exothermic and endothermic reactions. Blasius and Sakiadis flows of viscous fluid with convective boundary conditions have been discussed by Cortell [25]. A shooting fourth-order Runge–Kutta procedure has been employed for the computation of numerical solutions of the developed differential system. Aziz [26] numerically investigated the Blasius flow and convective heat transfer using Runge–Kutta–Fehlberg fourth–fifth order (RFK45). An exact solution for the problem considered in ref. [26] has been obtained by Magyari [27]. Hydromagnetic mixed convection flow with convective boundary conditions has been addressed by Makinde and Aziz [28]. Yao et al. [29] obtained exact solutions for the fundamental stretching/shrinking wall problem with convective boundary conditions. The present work is undertaken to study the magnetohydrodynamic boundary layer flow of Jeffrey fluid over a stretching sheet. Further, heat transfer with convective boundary conditions and mass transfer with chemical reaction have been considered. Surprisingly, the exact solutions for the entire differential system have been obtained. Such exact solutions have never been reported before. Hence, this paper makes valuable contributions to the existing literature in view of pancity of exact solutions especially in two-dimensional flows of non-Newtonian fluids.

2. Basic Equations

We consider the steady incompressible flow with heat and mass transfer of a Jeffrey fluid over a stretching sheet situated at $y = 0$. The x- and y-axes are taken along and perpendicular to the sheet, respectively, and the flow is confined to $y \geq 0$. A uniform magnetic field of strength B_0 is applied perpendicular to the flow direction. Moreover, the induced magnetic field is neglected under the assumtion that the magnetic Reynolds number is very small. The velocity of the stretching sheet is $u_\mathrm{w}(x) = ax$, where a is a positive (stretching sheet) constant. T_f is the convective surface temperature below the moving sheet and $C_\mathrm{w}(x)$ is the concentration at the sheet. T_∞ and C_∞ are the ambient temperature and concentration, respectively. Thus the boundary layer equations governing the two-dimensional flow with heat and mass transfer of the magnetohydrodynamic Jeffrey fluid are (see [2–4, 18, 29])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\nu}{1+\lambda_2}\left[\frac{\partial^2 u}{\partial y^2} + \lambda_1\left(u\frac{\partial^3 u}{\partial x \partial y^2} - \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial^3 u}{\partial y^3}\right)\right] - \frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_\mathrm{p}}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - k_1(C - C_\infty). \tag{4}$$

The boundary conditions are (see [13, 18, 27])

$$\begin{aligned} &u = u_{\mathrm{w}}(x) = ax\,,\ \ v = 0\,,\ \ -k\frac{\partial T}{\partial y} = h_{\mathrm{f}}(T_{\mathrm{f}} - T)\,,\\ &C = C_{\mathrm{w}}(x) = C_\infty + bx \ \ \text{at} \ \ y = 0\,,\\ &u \to 0\,,\ \ T \to T_\infty\,,\ \ C \to C_\infty \ \ \text{as} \ \ y \to \infty\,, \end{aligned} \tag{5}$$

in which u and v are the velocity components along x- and y-directions, respectively, ρ is the fluid density, $\nu = \mu/\rho$ the kinematic viscosity, T the fluid temperature, h_{f} the heat transfer coefficient, c_{p} the specific heat, σ the electrical conductivity, C the concentration, D the effective diffusion coefficient, k_1 the first-order chemical reaction rate, k the thermal conductivity of the fluid, b the proportional constant, λ_2 the ratio of relaxation and retardation times, and λ_1 the relaxation time.

The boundary conditions (5) represent that the continuous sheet aligned with the x-axis at $y = 0$ moves in its own plane. In view of polymer extrusion, the material properties and in particular the elasticity of the extruded sheet is pulled out by a constant force. A stream of cold fluid at temperature T_∞ is moving over the sheet while the surface of the sheet is heated from below by convection from the hot fluid at temperature T_{f} which provides a heat transfer coefficient h_{f}. As a result the convective boundary conditions arise. Moreover, the equation for the concentration $C_{\mathrm{w}}(x)$ of the sheet represents a situation in which the sheet species increases if b is positive from b at the leading edge in proportion to x. It is worth mentioning that $k_1 > 0$ represents the constructive and $k_1 < 0$ denotes the destructive chemical reaction. In order to find exact solutions of the boundary layer equations (1–5), we introduce the following transformations:

$$\begin{aligned} &\eta = \sqrt{\frac{a}{\nu}}y\,,\ \ u = axf'(\eta)\,,\ \ v = -\sqrt{a\nu}f(\eta)\,,\\ &\theta(\eta) = \frac{T - T_\infty}{T_{\mathrm{f}} - T_\infty}\,,\ \ \phi(\eta) = \frac{C - C_\infty}{C_{\mathrm{w}} - C_\infty}\,. \end{aligned} \tag{6}$$

Equation (1) is automatically satisfied, and (2)–(4) become

$$\begin{aligned} &f''' + (1+\lambda_2)(-f'^2 + ff'')\\ &+\beta(f''^2 - ff'''') - (1+\lambda_2)M^2 f' = 0\,, \end{aligned} \tag{7}$$

$$\theta'' + \mathrm{Pr}\, f\theta' = 0\,, \tag{8}$$

$$\phi'' + \mathrm{Sc}(f\phi' - f'\phi) - L\mathrm{Sc}\phi = 0\,, \tag{9}$$

$$\begin{aligned} &f(0) = 0\,,\ \ f'(0) = 1\,,\\ &\theta'(0) = -\gamma(1-\theta(0))\,,\ \ \phi(0) = 1\,,\\ &f'(\infty) = 0\,,\ \ \theta(\infty) = 0\,,\ \ \phi(\infty) = 0\,, \end{aligned} \tag{10}$$

where Pr is the Prandtl number, M the Hartman number, L the chemical reaction parameter, Sc the Schmidt number, γ the Biot number, and β the Deborah number. These are defined as

$$\begin{aligned} &\mathrm{Pr} = \frac{\mu c_{\mathrm{p}}}{k}\,,\ \ \beta = a\lambda_1\,,\ \ L = \frac{k_1}{a}\,,\ \ M^2 = \frac{\sigma B_0^2}{\rho a}\,,\\ &\mathrm{Sc} = \frac{\nu}{D}\,,\ \ \gamma = \frac{h_{\mathrm{f}}}{k}\sqrt{\frac{\nu}{a}}\,. \end{aligned} \tag{11}$$

The skin friction coefficient C_{f}, the local Nusselt number Nu_x, and the local Sherwood numbers Sh_x are given by

$$\begin{aligned} &C_{\mathrm{f}} = \frac{\tau_{\mathrm{w}}}{\rho u_{\mathrm{w}}^2}\,,\ \ \mathrm{Nu}_x = \frac{xq_{\mathrm{w}}}{k(T_{\mathrm{w}} - T_\infty)}\,,\\ &\mathrm{Sh}_x = \frac{xc_{\mathrm{w}}}{D(C_{\mathrm{w}} - C_\infty)}\,, \end{aligned} \tag{12}$$

where the wall skin friction τ_{w}, heat transfer q_{w}, and mass transfer c_{w} from the plate are

$$\begin{aligned} &\tau_{\mathrm{w}} = \frac{\mu}{1+\lambda_2}\left[\frac{\partial u}{\partial y} + \lambda_1\left\{u\frac{\partial^2 u}{\partial y\partial x} + v\frac{\partial^2 u}{\partial y^2}\right\}\right]_{y=0}\,,\\ &q_{\mathrm{w}} = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}\,,\ \ c_{\mathrm{w}} = -D\left(\frac{\partial C}{\partial y}\right)_{y=0}\,. \end{aligned} \tag{13}$$

In view of (6), the above expressions in (12) and (13) provide

$$\begin{aligned} &(\mathrm{Re}_x)^{1/2}C_{\mathrm{f}} = \left(\frac{1+\beta}{1+\lambda_2}\right)f''(0)\,,\\ &(\mathrm{Re}_x)^{-1/2}\mathrm{Nu}_x = -\theta'(0)\,,\\ &(\mathrm{Re}_x)^{-1/2}\mathrm{Sh}_x = -\phi'(0)\,, \end{aligned} \tag{14}$$

where $\mathrm{Re}_x = ax^2/\nu$ is the local Reynolds number.

3. Exact Solutions

3.1. Exact Solutions for the Momentum Boundary Layer Problem

Now, we assume that the solution of the nonlinear differential equation (7) is

$$f(\eta) = \frac{1 - \mathrm{e}^{-m\eta}}{m}\,, \tag{15}$$

which satisfies the boundary conditions explained in (10). By substituting the above expression for f in (7), we obtain

$$m=\sqrt{\frac{(1+\lambda_2)(1+M^2)}{1+\beta}}\,. \tag{16}$$

Thus the exact solution is

$$f(\eta)=\frac{1-\mathrm{e}^{-m\eta}}{m}\,,\quad m=\sqrt{\frac{(1+\lambda_2)(1+M^2)}{1+\beta}}\,. \tag{17}$$

The skin friction at the wall (14) becomes

$$(\mathrm{Re}_x)^{1/2}C_\mathrm{f}=-\frac{m(1+\beta)}{1+\lambda_2}\,. \tag{18}$$

3.2. Exact Solutions for the Thermal Boundary Layer Problem

The energy equation solution can be computed by adopting the procedure given in [29]. In order to solve this equation, a new variable $\varepsilon=\mathrm{Pr}\exp(-m\eta)/m^2$ is introduced such that $\theta(\eta)=\widehat{\theta}(\varepsilon)$. Substituting it into (8) yields the following differential equation:

$$\varepsilon\frac{\mathrm{d}^2\widehat{\theta}}{\mathrm{d}\varepsilon^2}+\left(1-\frac{\mathrm{Pr}}{m^2}+\varepsilon\right)\frac{\mathrm{d}\widehat{\theta}}{\mathrm{d}\varepsilon}=0 \tag{19}$$

with the boundary conditions

$$\widehat{\theta}'(\mathrm{Pr}/m^2)=\frac{\gamma m}{\mathrm{Pr}}[1-\widehat{\theta}(\mathrm{Pr}/m^2)]\quad \text{and}\quad \widehat{\theta}(0)=0. \tag{20}$$

The solution of (19) in terms of η is

$$\theta(\eta)=C_1+C_2\Gamma\left(\frac{\mathrm{Pr}}{m^2},\mathrm{Pr}\frac{\exp(-m\eta)}{m^2}\right), \tag{21}$$

where $\Gamma(B,z)$ is the incomplete Gamma function. Using the boundary conditions (10), (21) becomes

$$\theta(\eta)=\frac{\gamma\Gamma\left(\frac{\mathrm{Pr}}{m^2},0\right)-\gamma\Gamma\left(\frac{\mathrm{Pr}}{m^2},\frac{\mathrm{Pr}}{m^2}\exp(-m\eta)\right)}{m\exp\left(-\frac{\mathrm{Pr}}{m^2}\right)\left(\frac{\mathrm{Pr}}{m^2}\right)^{\frac{\mathrm{Pr}}{m^2}}+\gamma\Gamma\left(\frac{\mathrm{Pr}}{m^2},0\right)-\gamma\Gamma\left(\frac{\mathrm{Pr}}{m^2},\frac{\mathrm{Pr}}{m^2}\right)}\,. \tag{22}$$

Now, the heat transfer flux at the wall is

$$-\theta'(0)=\frac{\gamma m}{m+\exp\left(\frac{\mathrm{Pr}}{m^2}\right)\left(\frac{\mathrm{Pr}}{m^2}\right)^{-\frac{\mathrm{Pr}}{m^2}}\gamma\left[\Gamma\left(\frac{\mathrm{Pr}}{m^2},0\right)-\Gamma\left(\frac{\mathrm{Pr}}{m^2},\frac{\mathrm{Pr}}{m^2}\right)\right]}\,. \tag{23}$$

3.3. Exact Solution for the Concentration

Substituting (15) in (9) becomes

$$\phi''+\mathrm{Sc}\left(\frac{1-\mathrm{e}^{-m\eta}}{m}\right)\phi'-\mathrm{Sc}(\mathrm{e}^{-m\eta})\phi-L\mathrm{Sc}\phi=0 \tag{24}$$

with the boundary conditions given by

$$\phi(0)=1\,,\quad \phi(\infty)=0\,. \tag{25}$$

In order to solve (24), a new variable $\xi=\mathrm{Sc}\exp(-m\eta)/m^2$ is introduced such that $\phi(\eta)=\widehat{\phi}(\xi)$. Putting it into (24) and (25), we get following differential equation:

$$\xi\frac{\mathrm{d}^2\widehat{\phi}}{\mathrm{d}\xi^2}+\left[1-(m+1)\frac{\mathrm{Sc}}{m^2}+\xi\right]\frac{\mathrm{d}\widehat{\phi}}{\mathrm{d}\xi}-\frac{L}{m^2}\xi^{-1}\widehat{\phi}=0 \tag{26}$$

with the boundary conditions

$$\widehat{\phi}\left(\frac{\mathrm{Sc}}{m^2}\right)=1\,,\quad \widehat{\phi}(\infty)=0\,, \tag{27}$$

and by using boundary conditions (27), we get the exact solution [20] of (26) in the form

$$\widehat{\phi}(\xi)=\left(\frac{m^2}{\mathrm{Sc}}\xi\right)^{(\kappa_1+\kappa_2)}\cdot\frac{{}_1F_1[\kappa_1+\kappa_2-1;1+2\kappa_2;-\xi]}{{}_1F_1[\kappa_1+\kappa_2-1;1+2\kappa_2;-\frac{\mathrm{Sc}}{m^2}]}\,, \tag{28}$$

where ${}_1F_1$ are the confluent hypergeometric functions, and κ_1 and κ_2 are defined as

$$\kappa_1=\frac{\mathrm{Sc}}{2m^2}\,,\quad \kappa_2=\frac{\sqrt{4L\mathrm{Sc}\,m^2+\mathrm{Sc}^2}}{2m^2}\,. \tag{29}$$

The solution in terms of η is written as

$$\phi(\eta)=\exp[-(\kappa_1+\kappa_2)m\eta]\cdot\frac{{}_1F_1\left[\kappa_1+\kappa_2-1;1+2\kappa_2;-\mathrm{Sc}\frac{\exp(-m\eta)}{m^2}\right]}{{}_1F_1\left[\kappa_1+\kappa_2-1;1+2\kappa_2;-\frac{\mathrm{Sc}}{m^2}\right]}\,. \tag{30}$$

(30) can be used to calculate the dimensionless expression of the reduced Sherwood number as

$$-\phi'(0) = \frac{m(\kappa_1+\kappa_2) - \mathrm{Sc}(\kappa_1+\kappa_2-1)_1F_1\left[\kappa_1+\kappa_2; 2(1+\kappa_2); -\frac{\mathrm{Sc}}{m^2}\right]}{m(1+2\kappa_2)_1F_1\left[\kappa_1+\kappa_2-1; 1+2\kappa_2; -\frac{\mathrm{Sc}}{m^2}\right]}. \tag{31}$$

3.4. Results and Discussion

We have portrayed the graphical results to interpret the behaviour of embedding physical parameters on velocity, temperature, and concentration. Figures 1 and 2 are displayed to analyze the comparison of the exact solutions with the numerical solutions for θ and ϕ. An excellent agreement is found between the two solutions for different values of Deborah number β. Figure 3 elucidates the effects of the ratio λ_2 on the velocity and the boundary layer thickness. The velocity decreases and the profiles move closer to the boundary when λ_2 is increased. The velocity profiles shown in Figure 4 illustrate that as Deborah number β increases, the velocity and the thickness of the boundary layer increase and the curves become less steep. It is worth mentioning here that the Deborah number β is smaller for liquid like materials. Here the results for the Newtonian fluid case are recovered by choosing $\beta = 0$. The effect of Hartman number M on the velocity field f' is shown in Figure 5. Here the presence of a magnetic field creates a bulk known as Lorentz force which opposes the fluid velocity. As a consequence, the boundary layer thickness decreases with an increase in M. Figure 6 presents the influence of Prandtl number Pr on the temperature. It is clear that

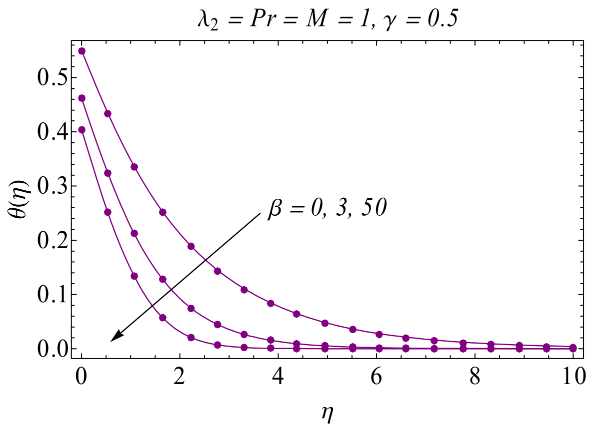

Fig. 1 (colour online). Influence of β on θ and ϕ. Solid lines: exact solutions; points: numerical solutions.

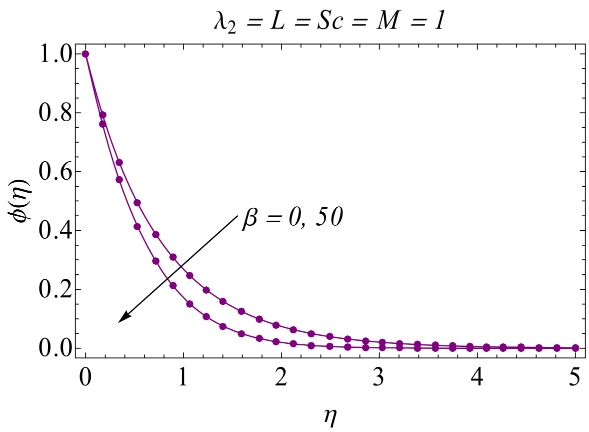

Fig. 2 (colour online). Influence of β on θ and ϕ. Solid lines: exact solutions; points: numerical solutions.

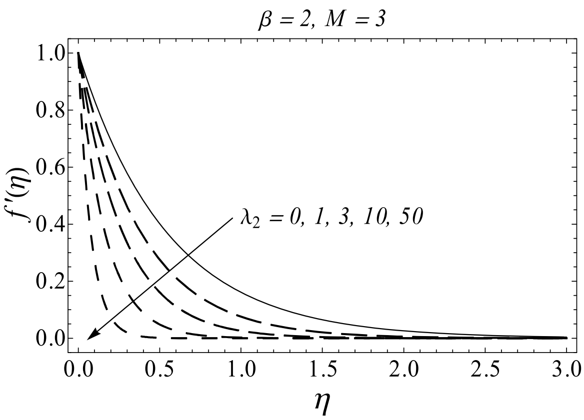

Fig. 3. Effect of λ_2 on f'.

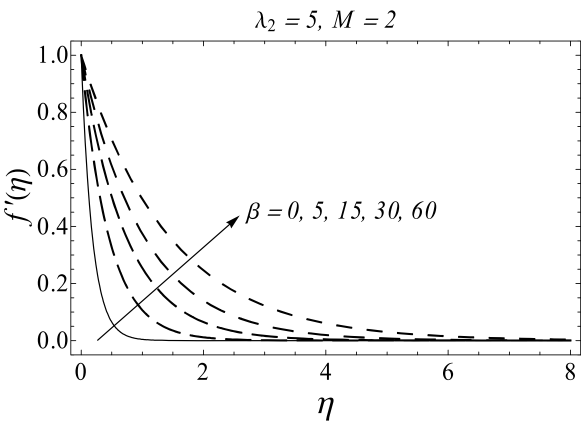

Fig. 4. Effect of β on f'.

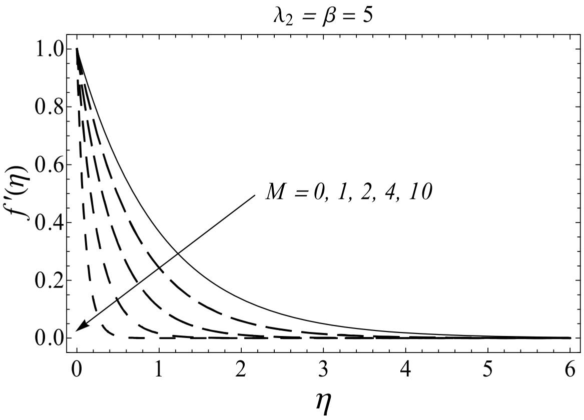

Fig. 5. Effect of M on f'.

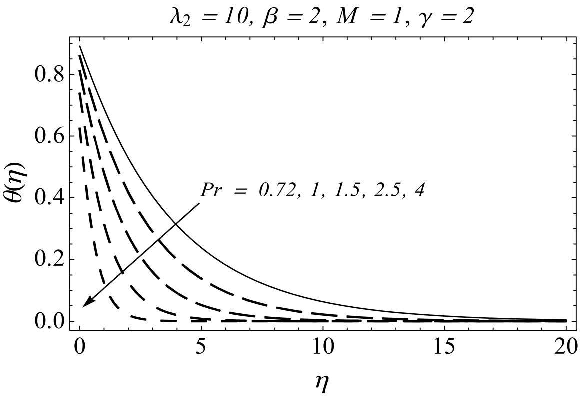

Fig. 6. Effect of Pr on θ.

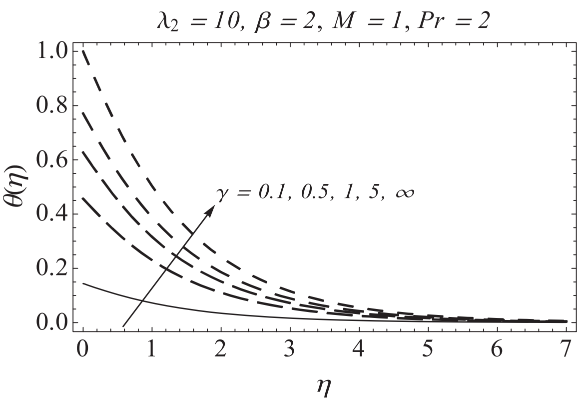

Fig. 7. Effect of γ on θ.

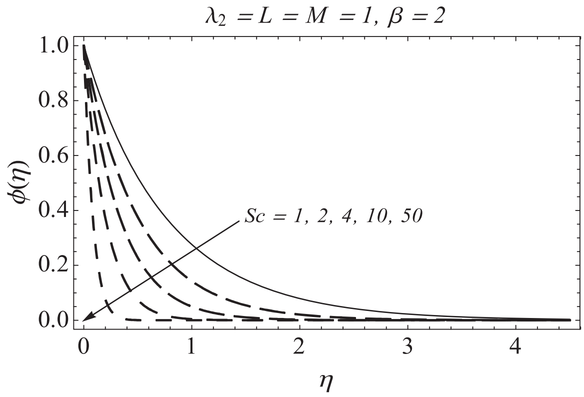

Fig. 8. Effect of Sc on ϕ.

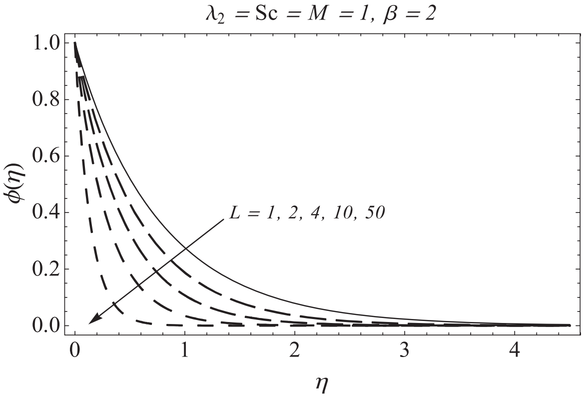

Fig. 9. Effect of L on ϕ.

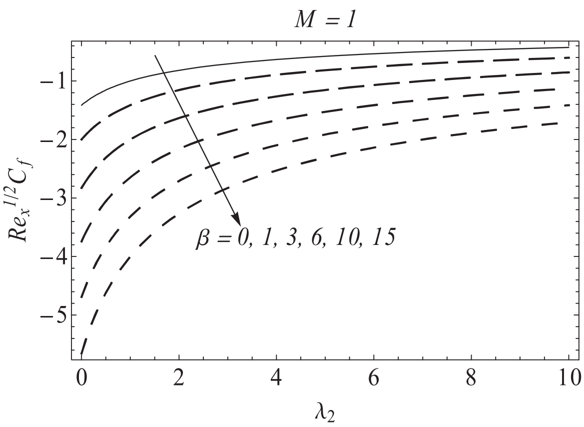

Fig. 10. Effect of β on $\mathrm{Re}_x^{1/2}C_\mathrm{f}$.

an increase in Pr corresponds to a decrease in thermal conductivity. Consequently, the thermal boundary layer thins, and the curves become increasingly steeper. Figure 7 characterizes the influence of Biot number γ on the dimensionless temperature. An increase in γ indicates higher surface temperatures which

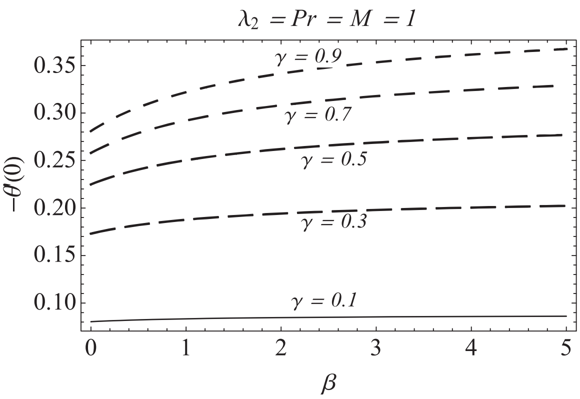

Fig. 11. Effect of γ on $-\theta'(0)$.

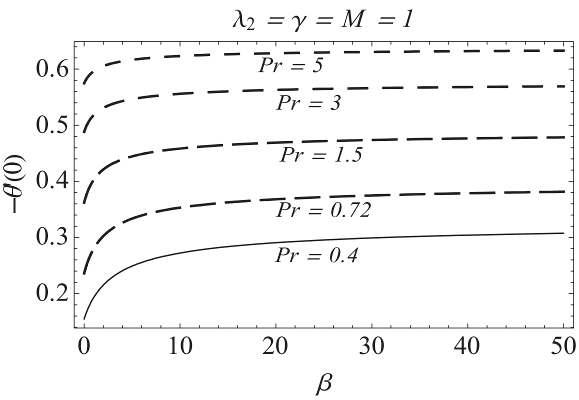

Fig. 12. Effect of Pr on $-\theta'(0)$.

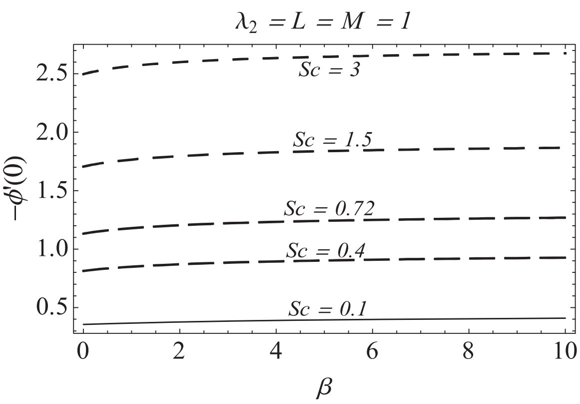

Fig. 13. Effect of Sc on $-\phi'(0)$.

lead to the thickness of thermal boundary layer. The influence of Schmidt number Sc on the concentration field is described in Figure 8. As Sc gradually increases, this corresponds to a weaker molecular diffusivity and thinner concentration boundary layer. Figure 9 shows the effects of the generative and destructive chemical reaction parameter L on the concentration. As the chemical reaction effect intensifies the concentration profiles shift towards the bounding surface indicating a decrease in the concentration boundary layer.

To analyze the effects of parameters on the skin friction coefficient, local Nusselt number, and local Sherwood number, we have displayed Figures 10 – 13. The behaviour of Deborah number β on the skin friction coefficient is seen in Figure 10. Here an increase in the viscoelastic effect enhances the stresses at the wall. From the industrial point of view, this outcome is not desireable since a larger drag force will be required to displace the fluid above the sheet when β is increased. This increase in the skin friction coefficient is significant only for small values of λ_2. Figure 11 shows that larger convection at the sheet results in a higher rate of heat transfer at the sheet which increases the magnitude of the local Nusselt number. It is already noticed form Figure 6 that the temperature profiles become more steep as Pr increases. Therefore the local Nusselt number being proportional to the initial slope increases with an increase in Pr (see Fig. 12). Figure 13 presents the influence of Schmidt number Sc on the local Sherwood number for a fixed value of the chemical reaction parameter. This is because the larger values of Sc shift the concentration profiles towards the boundary indicating the larger surface mass transfer.

4. Conclusions

In this study, the exact solutions for heat and mass transfer in the flow of a Jeffrey fluid over a linearly stretching sheet have been obtained. The exact solutions for temperature and concentration fields are in a very good agreement with the numerical solutions. Mass transfer with first-order chemical reaction is accounted. It is found that the velocity and the boundary layer thickness are increasing functions of the Deborah number β. An increase in the Biot number γ leads to a stronger convection at the sheet which results in an increment of temperature and the thermal boundary layer thickness. The concentration field ϕ is a decreasing function of the Schmidt number Sc and the chemical reaction parameter L. The analysis for constant wall temperature ($\theta(0) = 1$) which is not yet reported can be recovered by setting $\gamma \rightarrow \infty$. Further, the analytic solutions for the Newtonian fluid case can be obtained by choosing $\beta = 0$. To our knowledge, this study is a first attempt for exact solutions of two-dimensional flow of a Jeffrey fluid over a surface with convective boundary conditions and chemical reaction.

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[1] M. A. Al-Nimr, A. F. Khadrawi, and A. Othman, Chem. Eng. Sci. **60**, 7131 (2005).
[2] T. Hayat and M. Mustafa, Z. Naturforsch. **65a**, 711 (2010).
[3] T. Hayat, Z. Iqbal, M. Mustafa, and S. Obaidat, ASME J. Heat Transf. **134**, 011701 (2012).
[4] M. Mustafa, T. Hayat, and A. A. Hendi, ASME J. Appl. Mec. **79**, 024501 (2012).
[5] M. Kothandapani and S. Srinivas, Int. J. Nonlin. Mech. **43**, 915 (2008).
[6] T. Hayat, N. Ahmad, and N. Ali, Commun. Nonlin. Sci. Numer. Simul. **13**, 1581 (2008).
[7] M. Kothandapani and S. Srinivas, Int. J. Nonlin. Mech. **43**, 915 (2008).
[8] T. Hayat and N. Ali, Commun. Nonlin. Sci. Numer. Simul. **13**, 1343 (2088).
[9] S. Nadeem and N. S. Akbar, Z. Naturforsch. **64a**, 713 (2009).
[10] H. Blasius, Z. Angew. Math. Phys. **56**, 1 (1908).
[11] L. Howarth, P. R. Soc. Lond. A **164**, 547 (1938).
[12] B. C. Sakiadis, AIChE J. **7**, 26 (1961).
[13] L. J. Crane, Z. Angew. Math. Phys. **21**, 645 (1970).
[14] K. R. Rajagopal, T. Y. Na, and A. S. Gupta, Rheol. Acta **23**, 213 (1984).
[15] K. Sankara and L. T. Watson, Z. Angew. Math. Phys. **36**, 845 (1985).
[16] H. I. Andersson, K. H. Bech, and B. S. Dandapat, Int. J. Nonlin. Mech. **27**, 929 (1992).
[17] T. R. Mahapatra and A. S. Gupta, Int. J. Nonlin. Mech. **39**, 811 (2004).
[18] I. C. Liu, Int. Comm. Heat Mass Transf. **32**, 1075 (2005).
[19] R. Cortell, Chem. Eng. Process **46**, 721 (2007).
[20] H. A. M. El-Arabawy, J. Math. Stat. **5**, 159 (2009).
[21] T. R. Mahapatra, S. K. Nandy, and A. S. Gupta, Int. J. Nonlin. Mech. **44**, 124 (2009).
[22] M. M. Nandeppanavar, M. S. Abel, and J. Tawade, Commun. Nonlin. Sci. Numer. Simul. **15**, 1791 (2010).
[23] A. Ahmed and S. Asghar, Appl. Math. Lett. **24**, 1905 (2011).
[24] T. Hayat, M. Mustafa, and I. Pop, Commun. Nonlin. Sci. Numer. Simul. **15**, 1183 (2010).
[25] R. Cortell, Appl. Math. Comput. **206**, 832 (2010).
[26] A. Aziz, Commun. Nonlin. Sci. Numer. Simul. **14**, 1064 (2009).
[27] E. Magyari, Comment on "A similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition" by A. Aziz, Commun. Nonlin. Sci. Numer. Simul. **14**, 1064 (2009), Commun. Nonlin. Sci. Numer. Simul. **16**, 599 (2011).
[28] O. Makinde and A. Aziz, Int. J. Ther. Sci. **49**, 1813 (2010).
[29] S. Yao, T. Fang, and Y. Zhong, Commun. Nonlin. Sci. Numer. Simul. **16**, 752 (2011).