Cross Focusing of two Coaxial Gaussian Beams with Relativistic and Ponderomotive Nonlinearity

Prerana Sharma

Department of Physics, Ujjain Engineering College, Ujjain, M.P.465010, India

Reprint requests to P. S.; E-mail: preranaiitd@rediffmail.com

Z. Naturforsch. **67a,** 10 – 14 (2012) / DOI: 10.5560/ZNA.2011-0064
Received April 1, 2011 / revised July 8, 2011

This paper presents the cross focusing of two high power lasers by taking off-axial contributions of the laser beams in a collisionless plasma. Due to relativistic and ponderomotive nonlinearities the two laser beams affect the dynamics of each other and cross focusing takes place. The expressions for the laser beam intensities by using the eikonal method are derived. The contributions of the r^2 and r^4 terms are incorporated. By expanding the eikonal and the other relevant quantities up to the fourth power of r, the solution of the pump laser beam is obtained within the extended paraxial ray approximation. Filamentary structures of the laser beams are observed due to the relativistic and the ponderomotive nonlinearity. The focusing of the laser beams is shown to become fast in the extended paraxial region. Using the laser beam and the plasma parameters, appropriate for beat wave processes, the filaments of the laser beams are studied and the relevance of these results to beat wave processes is pointed out.

Key words: Filamentation; Ponderomotive Nonlinearity; Cross Focusing.

1. Introduction

There is considerable interest in the interaction of intense laser beams with plasmas because of its relevance to laser fusion [1] and charged particle acceleration [2, 3]. The generation of a large electric field in plasmas by high power lasers has been studied for several years in the context of particle acceleration, and new techniques have been investigated; one of these techniques is the beat wave acceleration [4, 5], in which two laser beams are propagating in the plasma simultaneously. As particle acceleration becomes a requirement of various modern physics experiments, alternative methods for acceleration by using laser beams has become important. Most of the studies related to laser plasma interactions are restricted to the case where the electron nonlinearity due to relativistic mass variation is accounted by cross focusing of two laser beams with paraxial approximation. But in many situations, the laser beam can create different types of nonlinearities at different time scales according to the inequalities (i) $\tau < \tau_{\mathrm{pe}}$ or (ii) $\tau_{\mathrm{pe}} < \tau < \tau_{\mathrm{pi}}$, hence one can have different time regimes. Here τ is the laser pulse duration, τ_{pi} the ion plasma period, and τ_{pe} the electron plasma period. In case (i), $\tau < \tau_{\mathrm{pe}}$, the relativistic nonlinearity is important. This nonlinearity is setup almost instantaneously. In case (ii), the relativistic and ponderomotive nonlinearities are operative [6 – 9]. In this case, electrons are expelled from the channel due to the electron ponderomotive force, while ions are much less expelled due to their inertia. The motivation of the present work is to study the nonlinear propagation of two laser beams, when relativistic and ponderomotive nonlinearities are effective, by considering an extended paraxial approximation. In this case the electrons are expelled from the high intensity region by the ponderomotive force. The nonlinearity in the dielectric constant of the plasma becomes effective through the electron mass variation due to the laser intensity and due to the change in electron density because of the ponderomotive force. Therefore, the laser beam propagation is expected to be drastically affected in comparison to the pure relativistic case. The contribution of the off-axial rays has been taken into account by incorporating the r^2 and r^4 terms in the present analysis. The solution of the pump laser beam has been obtained within the extended paraxial ray approximation by expanding the eikonal and the other relevant quantities up to the fourth power of r.

In Section 2, we derive the expression for the effective dielectric constant of the plasma in the presence of two laser beams when relativistic and ponderomotive nonlinearities are operative. The solution of self-focusing equations for the laser intensities is obtained by using the eikonal method [10] in Section 3, and a brief discussion of the results follows in Section 4.

2. Effective Dielectric Constant of the Plasma

Consider the propagation of two coaxial Gaussian laser beams of frequencies ω_1 and ω_2 along the z-direction. The initial intensity distributions of the beams are given by

$$\begin{aligned}\mathbf{E}_1\cdot\mathbf{E}_1^*\big|_{z=0} &= E_{10}^2\ \mathrm{e}^{-r^2/r_1^2},\\ \mathbf{E}_2\cdot\mathbf{E}_2^*\big|_{z=0} &= E_{20}^2\ \mathrm{e}^{-r^2/r_2^2},\end{aligned} \tag{1}$$

where r is the radial coordinate of the cylindrical coordinate system, r_1 and r_2 are the initial beam widths, and E_0 is the axial amplitude of the beam. The dielectric constant of the plasma is given by

$$\varepsilon_{01,02} = 1-\frac{\omega_{\mathrm{p}0}^2}{\omega_{1,2}^2}, \tag{2}$$

where $\omega_{\mathrm{p}0}$ is the plasma frequency given by $\omega_{\mathrm{p}0}^2 = 4\pi n_0 e^2/m_0$ (with e the charge of an electron, m_0 its rest mass, and n_0 the density of plasma electrons in the absence of the laser beam). The relativistic factor is given by

$$\gamma = \left(1+\frac{e^2}{c^2m_0^2\omega_1^2}E_1\cdot E_1^* + \frac{e^2}{c^2m_0^2\omega_2^2}E_2\cdot E_2^*\right)^{1/2}.$$

The above expression is valid if there is no change in the plasma density. The relativistic ponderomotive force is given by

$$F_{\mathrm{p}} = -m_0c^2\nabla(\gamma-1)\,. \tag{3}$$

Using the electron continuity equation and the current density equation up to the second order correction in the electron density equation (with the help of the ponderomotive force), the total density is given by

$$n = n_0+n_2 = n_0+\frac{c^2n_0}{\omega_{\mathrm{p}0}}\left(\nabla^2\gamma-\frac{(\nabla\gamma)^2}{\gamma}\right).$$

Now, the effective dielectric constant of the plasma at frequency ω_0 is given by

$$\varepsilon_{1,2} = \varepsilon_{01,2}+\phi_{1,2}(E_1\cdot E_1^*, E_2\cdot E_2^*), \tag{4}$$

where

$$\phi_{1,2}(E_1\cdot E_1^*, E_2\cdot E_2^*) = \frac{\omega_{\mathrm{p}0}^2}{\omega_{1,2}^2}\left(1-\frac{n}{n_0\gamma}\right).$$

Taylor-expanding the dielectric constant in (4) around $r=0$, one can write

$$\varepsilon_{1,2} = \varepsilon_{f1,2}+\gamma_{1,2}r^2,$$

where

$$\varepsilon_{f1,2} = \varepsilon_{01,02}+\frac{\omega_{\mathrm{p}0}^2}{\omega_{1,2}^2}\left[-\frac{1}{2}\left(\frac{\alpha_1}{f_1^2}+\frac{\alpha_2}{f_2^2}\right) + \frac{(\alpha_{00}-1)}{\gamma k_{\mathrm{p}}^4}\left(\frac{\alpha_1}{r_1^2f_1^4}+\frac{\alpha_2}{r_2^2f_2^4}\right)\right]$$

and

$$\begin{aligned}\gamma_{1,2} = -\frac{\omega_{\mathrm{p}0}^2}{\omega_{1,2}^2}\Bigg[&\frac{3(\alpha_{00}-1)}{\gamma^2k_{\mathrm{p}}^2}\left(\frac{\alpha_1}{r_1^4f_1^6}+\frac{\alpha_2}{r_2^4f_2^6}\right)\\ &+\frac{(4\alpha_{02}+\alpha_{00})}{2\gamma^3}\left(\frac{\alpha_1}{r_1^2f_1^4}+\frac{\alpha_2}{r_2^2f_2^4}\right)\\ &-\frac{3}{\gamma^4k_{\mathrm{p}}^2}\left(\frac{\alpha_1^2}{r_1^4f_1^8}-\frac{\alpha_2^2}{r_2^4f_2^8}\right)-\frac{6\alpha_1\alpha_2}{k_{\mathrm{p}}^2\gamma^4r_1^2r_2^2f_1^4f_2^4}\Bigg],\end{aligned}$$

α_{00} and α_{02} are the coefficients of r^2 and r^4, respectively, defined below in (9). $\alpha_{1,2} = \alpha_0 A_{01,2}^2$ is the square of the dimensionless vector potential, $E = -\mathrm{d}A/\mathrm{d}(ct)$, $\alpha_0 = e^2/m_0^2c^4$; f_1 and f_2 are the dimensionless beam width parameters at z as given by (11) in Section 3, and $k_{\mathrm{p}}^2 = -\omega_{\mathrm{p}0}^2/c^2$.

3. Cross Focusing of Laser Beams

The wave equation governing the electric vectors of two laser beams in a plasma can be written as

$$\frac{\partial^2E_{1,2}}{\partial z^2}+\frac{1}{r}\frac{\partial E_{1,2}}{\partial r}+\frac{\partial E_{1,2}}{\partial r^2}+\frac{\omega_{1,2}^2}{c^2}\varepsilon_{1,2}E_{1,2} = 0. \tag{5}$$

In (5), the $\nabla(\nabla\cdot E)$ term has been neglected, which is justified as long as

$$\left(\frac{\omega_{\mathrm{p}0}^2}{\omega_{1,2}^2}\right)\left(\frac{1}{\varepsilon_{1,2}}\right)\mathrm{Im}\varepsilon_{1,2}\le 1.$$

The variations of the electric fields are

$$E_{1,2} = A_{1,2}(x,y,z)\,\mathrm{e}^{-\mathrm{i}k_{1,2}z}.$$

The wave equation then becomes

$$-k_{1,2}^2 A_{1,2} - 2\mathrm{i}k_{1,2}A_{1,2} + \left(\frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial}{\partial r^2}\right)A_{1,2} + \frac{\omega_{1,2}^2}{c^2}\varepsilon_{1,2}A_{1,2} = 0. \tag{6}$$

$A_{1,2}$ are complex functions of the space variables: we assume furthermore the variation of $A_{1,2}$ as (cf. [8])

$$A_{1,2} = A_{01,2}(r,z)\,\mathrm{e}^{-\mathrm{i}k_{1,2}S_{1,2}(r,z)}, \tag{7}$$

where $A_{1,2}$ and $S_{1,2}$ are real functions of the space variables. Substituting the values of $A_{1,2}$ from (7) into (6) and separating real and imaginary parts of the resulting equation, the following set of equations is obtained:

The real part of (6) is

$$2\frac{\partial S_{1,2}}{\partial z} + \left(\frac{\partial S_{1,2}}{\partial z}\right)^2 = \frac{\omega_{1,2}^2\varepsilon_{1,2}}{c^2k_{1,2}^2} + \frac{1}{k_{1,2}^2A_{01,2}}\left(\frac{\partial^2 A_{01,2}}{\partial r^2} + \frac{1}{r}\frac{\partial A_{01,2}}{\partial r}\right), \tag{8}$$

where

$$A_{01,2}^2 = \left(1 + \frac{\alpha_{01,2}r^2}{r_{1,2}^2f_{1,2}^2} + \frac{\alpha_{21,2}r^4}{r_{1,2}^4f_{1,2}^4}\right) \cdot \left(\frac{E_{01,2}^2}{f_{1,2}^2}\right)\mathrm{e}^{\left(-\frac{r^2}{r_{1,2}^2f_{1,2}^2}\right)} \tag{9}$$

are the laser beam intensities, $f_{1,2}$ are the dimensionless beam width parameters for beam 1 and 2, respectively, and

$$S_{1,2} = \frac{r^2}{2f_{1,2}}\frac{\mathrm{d}f_{1,2}}{\mathrm{d}z} + \frac{r^4}{r_{1,2}^4}S_{21,2}. \tag{10}$$

By substituting (4), (9), and (10) into (8) and equating the coefficients of r^2 on both sides of the resulting equation, the governing equations for the beam width parameters $f_{1,2}$ are

$$\begin{aligned}\frac{\mathrm{d}^2 f_{1,2}}{\mathrm{d}\xi^2} &= \frac{1}{f_{1,2}^3}\left(-3\alpha_{00}^2 + 8\alpha_{02} + 1 - 2\alpha_{00}\right) \\ &- \frac{\omega_{\mathrm{p}0}^2}{\omega_0^2}\frac{k_0^2r_0^2f_{1,2}}{2\gamma^3}(\alpha_{00}-1)\left(\frac{\alpha_1^2}{f_1^3} + \frac{\alpha_2^2}{f_2^3}\right) \\ &- \frac{(4\alpha_{02}+\alpha_{00})f_{1,2}}{2\gamma^3}\left(\frac{\alpha_1^2}{f_1^6} + \frac{\alpha_2^2}{f_2^6}\right) \\ &+ \frac{(4\alpha_{02}+\alpha_{00})f_{1,2}}{\gamma^4}\left(\frac{\alpha_1^2}{f_1^8} + \frac{\alpha_2^2}{f_2^8}\right) - \frac{f_{1,2}}{\gamma^4}\left(\frac{6\alpha_1\alpha_2}{f_1^4f_2^4}\right).\end{aligned} \tag{11}$$

Similarly, by equating the coefficients of r^4 on both sides of the resulting equation, the following relation is obtained:

$$\frac{\partial S_{02}}{\partial z} = -\frac{1}{r_0^2k_0^2f_{1,2}^6}\left(2\alpha_{02} - \frac{3}{2}\alpha_{00}\alpha_{02} - \frac{3}{4}\alpha_{00}^2\right) + \frac{(-\alpha_{00}+2\alpha_{02})r_0^2}{k_0^2\gamma^3}\left(\frac{\alpha_1^2}{f_1^{10}} + \frac{\alpha_2^2}{f_2^{10}}\right). \tag{12}$$

Again, the imaginary part of (6) is given by

$$\frac{\partial A_{01,2}^2}{\partial z} + \frac{\partial S_{1,2}}{\partial r}\frac{\partial A_{01,2}^2}{\partial r} + A_{01,2}^2\left(\frac{\partial^2 S_{1,2}}{\partial r^2} + \frac{1}{r}\frac{\partial S_{1,2}}{\partial r}\right) = 0. \tag{13}$$

Inserting (9) and (10) into (13) and equating the coefficients of r^2 on both sides of the resulting equation, the equations for the coefficient $\alpha_{01,2}$ are obtained as

$$\frac{\partial \alpha_{01,2}}{\partial z} = -\frac{16S_{21,2}f_{1,2}^2}{r_{1,2}^2}. \tag{14}$$

In a similar way, equating the coefficients of r^4 gives the equation for the coefficients

$$\frac{\partial \alpha_{21,2}}{\partial z} = 8(1-3\alpha_{01,2})\frac{S_{21,2}f_{1,2}^2}{r_{1,2}^2}. \tag{15}$$

4. Discussion

Here, we have developed the theory of cross focusing of two laser beams by taking the off-axial parts of the laser beams into consideration, if relativistic and ponderomotive nonlinearities are operative. This is given by (11). The focusing/defocusing behaviour of the laser beams depends on the magnitudes of the nonlinear coupling term, i.e. on the nonlinear refraction of the laser beams and the diffraction term (second and first term on the right hand side of (11)). Equation (9) represents the intensity profile of the laser beams in the plasma in radial direction. The intensity profiles of both laser beams depend on the beam width parameters $f_{1,2}$ and the coefficients $\alpha_{01,2}$ and $\alpha_{21,2}$ of the r^2 and r^4 terms in the off-paraxial region. Equation (11) determines the focusing/defocusing of the laser beams along the distance of propagation in the plasma. In order to have a numerical appreciation of the cross focusing in this region and the effect of changing the parameters of the plasma and laser beams, we have performed numerical computations of (11), (12), (14), and (15). We have solved the coupled equations and obtained the numerical results with typical values for the plasma and laser beam parameters. The following set of parameters has been used in the numerical calculations: $r_1 = 15$ µm, $r_2 = 20$ µm, $\omega_1 = 1.776 \cdot 10^{14}$ rad/S, $\omega_2 = 1.716 \cdot 10^{14}$ rad/S, and $\omega_{p0} = 0.3\omega_1$. For the initial plane wave front of the laser beams the initial conditions used here are $f_{1,2} = 1$, $\mathrm{d}f_{1,2}/\mathrm{d}z = 0$, $\alpha_{01,2} = \alpha_{21,2} = 0$, and $S_{21,2} = 0$ at $z = 0$.

Before proceedings further, we discuss the effect of a change of the square of normalized beam radius R^2 on the dimensionless critical vector potential $a = A^2$ [11]. This is presented in Figure 1. For calculating the critical vector potential a, we put $\mathrm{d}^2 f_{1,2}/\mathrm{d}\xi^2 = 0$ and $f_{1,2} = 1$ in (11). If we consider the relativistic nonlinearity only, the square of the normalized beam radius has the form $R^2 \propto (1/a)$. But if we consider both relativistic and ponderomotive nonlinearities, the square of normalized beam radius has the form $R^2 \propto [2 + 6(a)^2 - 6a]/a$. From Figure 1, we got two different values for the critical vector potential corresponding to a given value of R^2 in the case of relativistic plus ponderomotive nonlinearity (for example: $R^2 = 10$, $a_{\mathrm{rel}} = 1.25$, $a_{\mathrm{rel+pon}} = 1.5$, 2.3). This also shows that if including the ponderomotive nonlinearity, we need more power for self-focusing.

For initial plane wave fronts of the beams the initial conditions for $f_{1,2}$ are $f_{1,2} = 1$ and $\mathrm{d}f_{1,2}/\mathrm{d}z = 0$ at $z = 0$. If two laser beams propagate simultaneously through the plasma, the density of the plasma will vary along the channel due to the ponderomotive force. But the relativistic and ponderomotive nonlinearities introduced in the plasma depend upon the total intensity of the two beams; therefore, the behaviour of f_1 is also governed by f_2 and vice versa. In other words, the self-focusing of one beam is affected by the presence of another beam; this is referred to as cross focusing. The properties of the cross focusing are influenced by the coupling term; this coupling term comes from the expansion of the $\gamma_{1,2}$ terms in the powers of r^2. We have solved (11) for various combinations of the initial fields of the lasers, which are greater than the critical values. In that case, f_1 and f_2 are obtained from the numerical solutions. Figure 2 illustrates the behaviour of the two beams when relativistic as well as ponderomotive nonlinearities are considered simultaneously. Both of the beams show oscillatory self-focusing when we use the potential for the first beam $a_1 = 1.5$ and for the second beam $a_2 = 0.3$. We have also studied the behaviour of one laser beam in the absence of the second beam. Figure 3 shows the self-focusing behaviour of a single laser beam (in absence of a second one) when both kinds of nonlinearities are operative. Figure 4 shows the effect of changing the potential of the second laser on the cross-focusing process (by keeping the potential of the first laser beam constant) when relativistic and ponderomotive nonlinearities are operative. It explicitly illustrates the effect

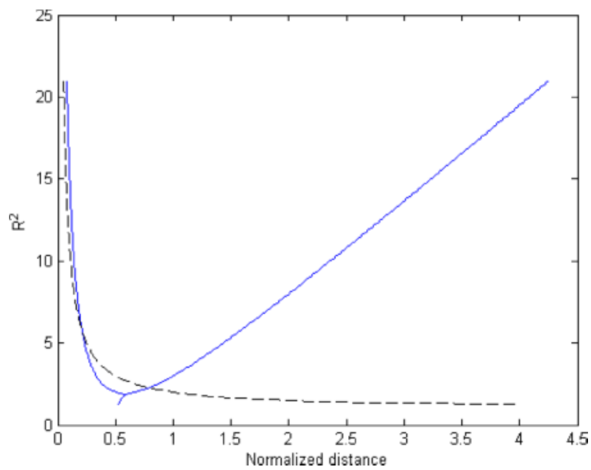

Fig. 1 (colour online). Variation of the square of the normalized beam radius R^2 with the normalized critical field a of the beam (solid line: relativistic case, dotted line: ponderomotive case).

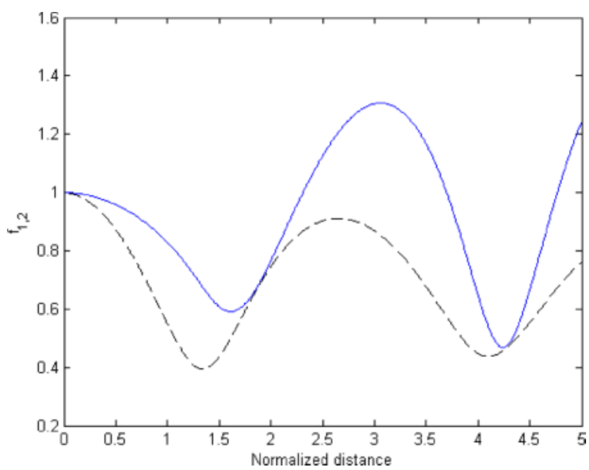

Fig. 2 (colour online). Variation of both beam width parameters f_1 and f_2 with the normalized distance ξ when only the relativistic nonlinearity has been considered for $a_1 = 1.5$ and $a_2 = 0.3$. Solid line: f_1; semi dotted line: represents f_2.

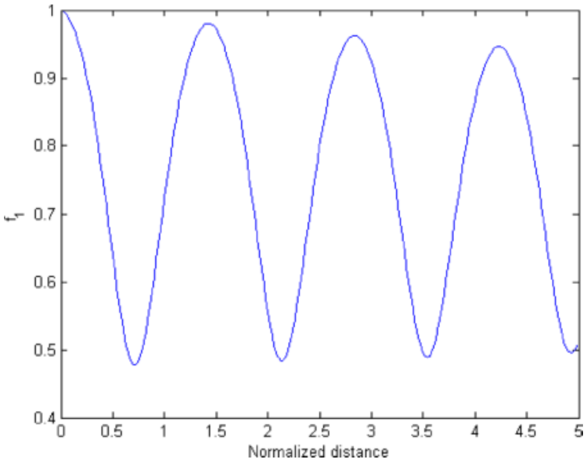

Fig. 3 (colour online). Variation of the beam width parameter f_1 with the normalized distance ξ for $a_1 = 1.5$, $a_2 = 0$ for a single laser beam (but both the relativistic and ponderomotive nonlinearities are operative).

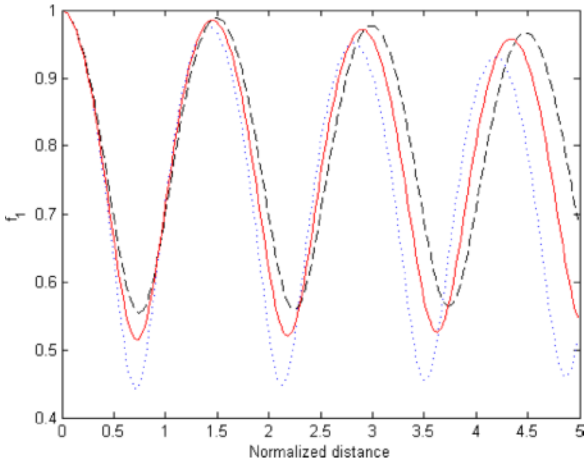

Fig. 4 (colour online). Variation of the beam width parameter f_1 with the normalized distance ξ by keeping $a_1 = 1.5$ constant when relativistic and ponderomotive nonlinearities are operative. Dashed line: f_1 at second beam potential $a_2 = 0.3$; solid line: f_1 at $a_2 = 0.5$; dotted line: f_1 at $a_2 = 0.7$.

of changing the potential of the second laser on the focusing/defocusing of the first laser beam in presence of both relativistic and ponderomotive nonlinearities. Note that the rate of focusing of the first laser beam becomes slower with the increase of the potential of the second laser beam. These two beams interact with each other and generate an electron plasma wave. This is due to the contribution of the second laser beam in (11), which governs the beam width profile f_1.

Acknowledgement

This work was partially supported by the Madhya Pradesh Council for Science and Technology Bhopal, India.

[1] W. L. Kruer, The Physics of Laser Plasma Interaction, Addison-Wesley Publishing Company, New York 1988.
[2] J. S. Wurtele, Phys. Today **47**, 33 (1994).
[3] A. Modena, Z. Najmudin, A. E. Dangor, C. E. Clayton, K. A. Marsh, C. Joshi, V. Malka, C. B. Darrow, C. Danson, D. Neely, and F. N. Walsh, Nature **377**, 606 (1995).
[4] E. Esarey, A. Ting, and P. Sprangle, Appl. Phys. Lett. **53**, 1266 (1988).
[5] I. Yousef, Phys. Rev. E **67**, 016501 (2003).
[6] A. M. Borisov, A. V. Borovskiy, O. B. Shiryaev, V. V. Korobkin, A. M. Prokhorov, J. C. Solem, T. S. Luk, K. Boyer, and C. K. Rhodes, Phys. Rev. A **45**, 5830 (1992).
[7] H. S. Brandi, C. Manus, and G. Mainfray, Phys. Rev. E **47**, 3780 (1993).
[8] H. S. Brandi, C. Manus, and G. Mainfray, Phys. Fluids B **5**, 3539 (1993).
[9] S. G.-Zheng, O. Edward, Y. C. Lee, and P. Guzdar, Phys. Fluids **30**, 526 (1987).
[10] S. A. Akhmanov, A. P. Sukhorukov, and R. V. Khokhlov, Sov. Phys. Usp. **10**, 609 (1968).
[11] M. S. Sodha, A. K. Ghatak, and V. K. Tripathi, Prog. Opt. **13**, 196 (1976).