Gravity Modulation of Thermal Instability in a Viscoelastic Fluid Saturated Anisotropic Porous Medium

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The present paper deals with a thermal instability problem in a viscoelastic fluid saturating an anisotropic porous medium under gravity modulation. To find the gravity modulation effect, the gravity field is considered in two parts: a constant part and an externally imposed time-dependent periodic part. The time-dependent part of the gravity field, which can be realized by shaking the fluid, has been represented by a sinusoidal function. Using Hill's equation and the Floquet theory, the convective threshold has been obtained. It is found that gravity modulation can significantly affect the stability limits of the system. Further, we find that there is a competition between the synchronous and subharmonic modes of convection at the onset of instability. Effects of various parameters on the onset of instability have also been discussed.

Key words: Viscoelastic Fluid; Thermal Rayleigh Number; Gravity Modulation; Anisotropic Porous Medium; Floquet Theory.

1. Introduction

The stability problem of flow of viscoelastic fluids has fundamental importance in the technology of polymer products and viscosimetry. Lot of work is available on thermal instability in Newtonian and non-Newtonian fluids. However, in some situations, a viscoelastic model of a fluid serves to be more realistic than the Newtonian model. For example, the generalized Darcy equation of non-Newtonian fluids, such as heavy crude, is known to be more useful in the analysis of the mobility efficiency of oil recovery. Also, some oil sands contain waxy crudes at shallow depths of the reservoirs and these fluids are considered to be viscoelastic fluids [1]. One such fluid is Rivlin–Ericksen fluid for which, in the momentum equation, the usual viscous term is replaced by the resistance term $\left[-K\left(\tilde{\mu}+\mu'\frac{\partial}{\partial t}\right)\cdot\overrightarrow{V}\right]$; $\tilde{\mu}$ and μ' being the viscosity and viscoelascity of Rivlin–Ericksen fluid. Sharma and Kumar [2] examined the thermal instability of a layer of a Rivlin–Ericksen elastico-viscous fluid under the effect of uniform rotation and found that it has a stabilizing effect. Sharma et al. [3] investigated the Kelvin–Helmholtz instability of a Rivlin–Ericksen viscoelastic fluid in a porous medium.

Due to the promising applications in engineering and technology, the effect of complex body forces on the problem of convection in porous medium has gained considerable attention in recent years. One of the complex forces is the time-dependent gravitational field; it is of interest in space laboratory experiments, in the area of crystal growth etc. The gravity modulation in a convective stable configuration can significantly affect the stability limit of a system. It can stabilize or destabilize the system of a constant gravity field. It is found that the gravity modulation of convective flow of an ordinary fluid can substantially enhance or retard the heat transfer, thus drastically affecting the convection. Initial studies for flow and heat transfer in a pure fluid subjected to gravity modulation have been done by Gresho and Sani [4] and Gershuni et al. [5]. But the study of this phenomena in a porous medium is comparatively new, there are only a few articles available: Malashetty and Padmavathi [6], Alex and Patil [7, 8], Govender [9, 10], Bhadauria et al. [11], and Saravanan [12, 13]. Recently, Srivastava and Bhadauria [14] investigated the gravity modulation in mushy layer. A study related to the modulation in a viscoelastic fluid has been carried out by Yang [15].

The literature on gravity modulation of thermal instability in a viscoelastic fluid saturating a porous medium is scarce. To the best of authors' knowledge, no study is available in which the effect of gravity modulation is considered in a porous medium saturated by a viscoelastic fluid. Therefore, keeping in mind the importance of viscoelastic fluids, in this paper, we study the effect of gravity modulation on the convective threshold in horizontal anisotropic porous medium saturated by a Rivlin–Ericksen-type viscoelastic fluid. A sinusoidal function has been used to modulate the gravity field. The obtained results will be useful in the study of crystal growth under micro-gravity condition and in oil displace mechanism. It is believed that the current results will improve the available results and therefore improve the future experiments.

2. Governing Equations

Consider a viscoelastic fluid-saturated anisotropic porous medium, confined between two horizontal parallel surfaces at $z = 0$ and $z = d$, heated from below and cooled from above. The surfaces are infinitely extended in x- and y-directions, and are free. The porous medium is submitted to an oscillatory motion parallel to the gravitational field in the vertical direction as shown in Figure 1.

Since the porous medium is described by the Darcy model, therefore under the Boussinesq approximation

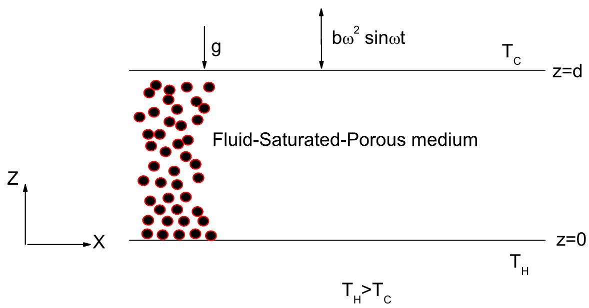

Fig. 1 (colour online). Differentially heated porous medium subjected to vibration.

the governing equations for the study of thermal instability in a Rivlin–Ericksen viscoelastic fluid saturating an anisotropic porous medium are

$$\frac{1}{\delta}\frac{\partial \vec{V}}{\partial t} = -\frac{1}{\rho_{\mathrm{R}}}\nabla p + \frac{\rho g}{\rho_{\mathrm{R}}} - \upsilon\left(1+\lambda\frac{\partial}{\partial t}\right)\vec{V}\cdot\overline{K}, \quad (1)$$

$$\gamma\frac{\partial T}{\partial t} + \vec{V}\cdot\nabla T = \nabla\cdot(D\cdot\nabla T), \quad (2)$$

$$\nabla\cdot\vec{V} = 0, \quad (3)$$

where $\gamma = \frac{(\rho c_p)_m}{(\rho c_p)_f}$ is the heat capacity ratio, $D = D_x(\hat{i}\hat{i} + \hat{j}\hat{j}) + D_z\hat{k}\hat{k}$ is the thermal diffusivity tensor, δ is the porosity, and $\nu = \frac{\tilde{\mu}}{\rho_{\mathrm{R}}}$ is the kinematic viscosity. Further, $\vec{V} = (u,v,w)$ is the velocity, p is the pressure, T is the temperature, $g = (0,0,-g)$ is acceleration due to gravity, while t is the time. $K = K_x^{-1}(\hat{i}\hat{i} + \hat{j}\hat{j}) + K_z^{-1}\hat{k}\hat{k}$ is the permeability tensor, $\tilde{\mu}$ is the fluid viscosity, and λ is the viscoelastic parameter. The relation between the reference density ρ_{R} and the reference temperature T_{R} is given by

$$\rho = \rho_{\mathrm{R}}[1 - \alpha(T - T_{\mathrm{R}})], \quad (4)$$

where α is the coefficient of thermal expansion. To consider the effect of gravity modulation, we write

$$g = g_0 + b\omega^2 \sin\omega t, \quad (5)$$

where g_0 is the mean gravity, b is the displacement amplitude, and ω is the vibration frequency. Since the porous medium is heated from below, the externally imposed wall temperatures can be defined as

$$\begin{aligned} T &= T_{\mathrm{R}} + \beta d, \quad & z &= 0, \\ &= T_{\mathrm{R}}, & z &= d, \end{aligned} \quad (6)$$

where β is the temperature gradient.

3. Linear Stability Analysis

The basic motionless state of system (1)–(4) can be written as

$$\begin{aligned} &V = (u,v,w) = (0,0,0), \quad T = T_{\mathrm{B}}(z), \\ &p = p_{\mathrm{B}}(z), \quad \rho = \rho_{\mathrm{B}}(z). \end{aligned} \quad (7)$$

The temperature $T_B(z)$, pressure p_B, and density ρ_B satisfy the following equations:

$$\frac{d^2T_B}{dz^2}=0, \tag{8}$$

$$\frac{dp_B}{dz}=-\rho_B g, \tag{9}$$

$$\rho_B=\rho_R[1-\alpha(T_B-T_R)]. \tag{10}$$

Equation (8) can be solved subject to the boundary conditions (6); we get

$$T_B=T_R+\beta d\left(1-\frac{z}{d}\right). \tag{11}$$

Let the basic state (7) be slightly perturbed according to

$$\begin{aligned}&\vec{V}=\vec{V}'=(u',v',w'),\;\; T=T_B(z)+\theta',\\ &p=p_B(z)+p',\;\; \rho=\rho_B(z)+\rho',\end{aligned} \tag{12}$$

where $\vec{V}'$, θ', p', ρ' represent the perturbed quantities. We substitute (12) into (1)–(3) and linearize the equations for the perturbation quantities $\vec{V}'$, θ', and p'. Now taking the curl of the momentum equation and then the vertical component of it, we get the linear equations for the perturbed variables, namely the vertical component of the velocity w and the temperature θ, as

$$\begin{aligned}\frac{1}{\delta}\frac{\partial}{\partial t}\nabla'^2 w'&=\alpha[g_0+b'\omega'^2\sin\omega t]\nabla'^2_H\theta'\\ &-\nu\left(1+\lambda'\frac{\partial}{\partial t}\right)\\ &\cdot\left(\frac{1}{K_z}\nabla'^2_H w'+\frac{1}{K_x}\frac{\partial^2 w'}{\partial z'^2}\right),\end{aligned} \tag{13}$$

$$\gamma\frac{\partial\theta'}{\partial t'}+w'\frac{dT_B'}{dz'}=D_x\nabla'^2_H\theta'+D_z\frac{\partial^2\theta'}{\partial z'^2}, \tag{14}$$

where ∇^2 is the Laplacian operator. We non-dimensionalize the above equations using the following scales:

$$\begin{aligned}&r'=dr^*,\;\; t'=\frac{d^2}{D_z}t^*,\;\; (T_B',\theta')=\Delta T(T_B^*,\theta^*),\\ &V'=\frac{D_z}{d}V^*,\;\; \Omega=\omega'\frac{d^2}{D_z},\end{aligned} \tag{15}$$

and obtain (where $(*)$ represent the non-dimensional quantity),

$$\begin{aligned}\chi\frac{\partial}{\partial t}\nabla^2 w+\left(1+\lambda\frac{\partial}{\partial t}\right)\left(\nabla^2_H+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)w\\ =R_a(1+\varepsilon\sin\Omega t)\nabla^2_H\theta,\end{aligned} \tag{16}$$

$$\left(\nu\frac{\partial}{\partial t}-\eta\nabla^2_H-\frac{\partial}{\partial z^2}\right)\theta=-w\frac{dT_B}{dz}, \tag{17}$$

where $\varepsilon=\kappa\mathrm{Fr}\Omega^2$ is the amplitude's ratio, and $\kappa=\frac{b}{d}$. The non-dimensional parameters in the above equations are: $R_a=\frac{\alpha g_0\Delta T K_z d}{\nu D_z}$ is the Darcy Rayleigh number, $\eta=\frac{D_x}{D_z}$ is the thermal anisotropy parameter, $\xi=\frac{K_x}{K_z}$ is the mechanical anisotropic parameter, $\Pr=\frac{\nu}{D_z}$ is the Prandtl number, and $D_a=\frac{K_z}{d^2}$ is the Darcy number. If D_a is very large, then the effect of porous media vanishes; however, if D_a is very small, then the porous media is densely packed. $\lambda'=\lambda\frac{D_z}{d}$ is a viscoelastic parameter, $\nabla^2_H=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$, and Ω is the non-dimensional vibration frequency. The non-dimensional group parameter $\chi=\frac{D_a}{\delta\Pr}$ is the reciprocal of the Vadasz number $Va=\frac{\delta\Pr}{D_a}$. As suggested by Vadasz [16], the value of Va in traditional applications of porous media is quite large and so χ is very small, therefore we neglect the time derivative term from (16). The asterisk '$*$' has been dropped in the above equations. The non-dimensional temperature gradient $\frac{dT_B}{dz}$ which appears in the above equation can be obtained from (11) as

$$\frac{dT_B}{dz}=-1. \tag{18}$$

Since free-free and perfect heat conducting boundaries are used, the boundary conditions at $z=0$ and 1 are

$$w=\frac{\partial^2 w}{\partial z^2}=\theta=0. \tag{19}$$

We seek the solutions of the unknown fields using normal mode technique as

$$\theta(x,y,z,t)=\theta(t)\exp[i(a_x x+a_y y)]\cdot\sin(\pi z), \tag{20}$$

where $a=(a_x^2+a_y^2)^{\frac{1}{2}}$ is the horizontal wave number. Eliminating the vertical component of the velocity by combining (16)–(17) after dropping the time derivative term in (16) and using (20), we get

$$\lambda\gamma\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}+[\gamma+\lambda(a^2\eta+\pi^2)]\frac{\mathrm{d}\theta}{\mathrm{d}t}+(a^2\eta+\pi^2)\theta = \frac{a^2\mathrm{R_a}}{\left(\frac{\pi^2}{\xi}+a^2\right)}(1+\varepsilon\sin(\Omega t))\theta. \tag{21}$$

The marginally stable solution for the unmodulated problem ($\varepsilon=0$) is given by

$$\mathrm{R_{a0}}=\frac{1}{a_0^2}\left[(\pi^2+\eta a_0^2)\left(\frac{\pi^2}{\xi}+a_0^2\right)\right]. \tag{22}$$

The minimum value (critical value) of $\mathrm{R_{a0}}$ and the corresponding value of the wave number $a=a_{0\mathrm{c}}$ are given by

$$\mathrm{R_{a0c}}=\pi^2\left(1+\sqrt{\frac{\eta}{\xi}}\right)^2, \tag{23}$$

$$a_{0\mathrm{c}}^2=\frac{\pi^2}{\sqrt{\xi\eta}}. \tag{24}$$

The results of (23) and (24) were first obtained by Epherre [17] in his study of convection in an anisotropic porous medium. It is very clear from (23) that an increase in the value of $\frac{\eta}{\xi}$ increases the value of the critical Rayleigh number $\mathrm{R_{a0c}}$, thus making the system more stable. However for isotropic porous medium, that is when $\eta=\xi=1$, we get

$$\mathrm{R_{a0c}}=4\pi^2 \quad \text{and} \quad a_{0\mathrm{c}}=\pi \tag{25}$$

which are same as obtained by Lapwood [18].

4. Solution

Now, to find the effect of gravity modulation, we consider $\varepsilon\neq 0$ and obtain the modified value of the critical Rayleigh number. For this we write (21) in the form

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}+2p\frac{\mathrm{d}\theta}{\mathrm{d}t}+h[\mathrm{R_{a0}}-\mathrm{R_a}(1+\varepsilon\sin\Omega t)]\theta=0, \tag{26}$$

where

$$2p=\frac{\gamma+\lambda(a^2\eta+\pi^2)}{\lambda\gamma}, \quad h=\frac{\lambda}{\gamma\lambda(\frac{1}{\xi}+q)} \quad \text{and} \quad q=\frac{a^2}{\pi^2}. \tag{27}$$

Now we put $2\tau=\Omega t-\frac{\pi}{2}$ and $\theta(t)=\mathrm{e}^{-pt}F(\tau)$ into (26) and get

$$\ddot{F}+[A-2B\cos(2\tau)]F=0, \tag{28}$$

where A and B are given by

$$A=\frac{4h}{\Omega^2}[R_N-\mathrm{R_a}], \quad B=\frac{2\mathrm{FrR_a}q}{\gamma\lambda(\frac{1}{\xi}+q)}, \tag{29}$$

$$R_N=\frac{-\mathrm{R_{a0}}[1-J(q)]^2}{4J(q)}, \tag{30}$$

$$J(q)=\frac{\gamma}{\lambda\pi^2(1+q\eta)}, \tag{31}$$

and $\mathrm{Fr}=\frac{\varepsilon}{\kappa\Omega^2}$ is the Froude number. Equation (28) is the well-known Mathieu equation. We use the Floquet theory to write the general solution of (28) in the form (Whittaker and Watson, [19])

$$F(\tau)=\mathrm{e}^{\mu\tau}P(\tau), \tag{32}$$

where μ is the Floquet exponent, which in general is a complex quantity, and $P(\tau)$ is a periodic function with period π or 2π. Then the solution of (26) is

$$\theta(t)=\mathrm{e}^{-pt}F(\tau)=\mathrm{e}^{\left(\frac{\mu\Omega}{2}-p\right)t}G(t), \tag{33}$$

where $G(t)$ is the periodic function with period π or 2π. From (33) we find the stability criterion as

$$\frac{\mu\Omega}{2}\le p. \tag{34}$$

The marginal stability condition can be given by

$$\frac{\mu\Omega}{2}=p. \tag{35}$$

We express the solutions of Mathieu's equation (28) in the form (Whittaker and Watson, [19])

$$F_\pi=\mathrm{e}^{\mu\tau}\sum_{-\infty}^{\infty}b_n\,\mathrm{e}^{2\mathrm{i}n\tau}, \tag{36}$$

$$F_{2\pi}=\mathrm{e}^{\mu\tau}\sum_{-\infty}^{\infty}b_n\,\mathrm{e}^{\mathrm{i}(2n+1)\tau}. \tag{37}$$

Expression (36) corresponds to the synchronous solutions, while (37) corresponds to the subharmonic ones. First, we consider the synchronous solution of (28). We put (36) in (28), which leads to the following linear equations:

$$\chi_n a_{n-1} + a_n + \chi_n a_{n+1} = 0, \quad (n = 0, \pm 1, \pm 2, \pm 3, \ldots), \tag{38}$$

where $\chi_n(\mu) = B/[(2n - \mathrm{i}\mu)^2 - A]$. For non-trivial solutions, the determinant of the matrix in (38) must vanish, thus we obtain Hill's determinantal equation

$$\Delta(\mathrm{i}\mu) = \begin{vmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \chi_{-2} & 1 & \chi_{-1} & 0 & 0 & 0 & 0 & \cdot \\ \cdot & 0 & \chi_{-1} & 1 & \chi_{-1} & 0 & 0 & 0 & \cdot \\ \cdot & 0 & 0 & \chi_0 & 1 & \chi_0 & 0 & 0 & \cdot \\ \cdot & 0 & 0 & 0 & \chi_1 & 1 & \chi_1 & 0 & \cdot \\ \cdot & 0 & 0 & 0 & 0 & \chi_2 & 1 & \chi_2 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix} = 0. \tag{39}$$

The above determinant equation (39) will be used to determine the value of μ for marginal stability. From (39) we write

$$\cosh(\mu\pi) = 1 - 2\Delta(0)\sin^2\left(\frac{\pi\sqrt{A}}{2}\right), \tag{40}$$

where $\Delta(0)$ can be obtained by setting $\mu = 0$ in the above determinant $\Delta(\mathrm{i}\mu)$. The value of the determinant $\Delta(0)$ has been obtained numerically using the Pivotal condensation method (Xavier, [20]):

$$\Delta(0) = \begin{vmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \chi_2 & 1 & \chi_2 & 0 & 0 & 0 & 0 & \cdot \\ \cdot & 0 & \chi_1 & 1 & \chi_1 & 0 & 0 & 0 & \cdot \\ \cdot & 0 & 0 & \chi_0 & 1 & \chi_0 & 0 & 0 & \cdot \\ \cdot & 0 & 0 & 0 & \chi_1 & 1 & \chi_1 & 0 & \cdot \\ \cdot & 0 & 0 & 0 & 0 & \chi_2 & 1 & \chi_2 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix} = 0. \tag{41}$$

Since the porous medium is heated from below, $\mathrm{R_a}$ is positive; so the constant A given by (29) is negative, and (40) becomes

$$\cosh(\mu\pi) = 1 + 2\Delta(0)\sinh^2\left(\frac{\pi\sqrt{-A}}{2}\right). \tag{42}$$

However, for subharmonic solutions, we obtain the characteristic equation as

$$\cosh(\mu\pi) = -1 - 2\Delta(0)\sinh^2\left(\frac{\pi\sqrt{-A}}{2}\right). \tag{43}$$

Equations (42) and (43) that relate Froude number Fr, Darcy–Rayleigh number $\mathrm{R_a}$, wave number a, non-dimensional frequency Ω, thermal anisotropy parameter η, mechanical anisotropic parameter ξ, viscoelastic parameter λ, and Floquet coefficient μ, have been solved numerically and marginal stability curves have been obtained. To obtain these stability curves, we fix the value of Ω first and then calculate the value of the critical Rayleigh number $\mathrm{R_{ac}}$ and the corresponding value of the wave number a_c at different values of the other parameters for the synchronous or subharmonic equations. Then we plot the curves for critical Rayleigh number and the corresponding wave number versus the modulation frequency Ω. Each one of these curves represent the minimum of the two modes of solutions (synchronous and subharmonic) in terms of the critical Rayleigh number.

5. Results and Discussion

For polymeric liquids the value of the viscoelastic parameter λ is in the range 10^{-1}–2, $D_z = 10^{-6}$ m^2/s, and $d = 0.1$ m. Also the value of the Froude number is of the order 10^{-5}. Since no experimental data is available, we consider $\lambda = 0.1$ and $\mathrm{Fr} = 10^{-4}$ to calculate the numerical results. Further, the values of ξ and η which are mechanical and thermal anisotropy parameters are considered to be 0.5 each.

In Figures 2–9, we have depicted the variation in $\mathrm{R_{ac}}$ and a_c with respect to Ω for the fixed values of the parameters $\lambda = 0.1$, $\eta = 0.5$, $\xi = 0.5$, and $\mathrm{Fr} = 10^{-4}$ with variation in one of these parameters. In all figures, we find qualitative similar results. There are two regions: the first one corresponds to the synchronous solution while the other one is related to the subharmonic solution. As we cross from one region to the other region, there is a crossover frequency $\Omega = \Omega^*$ which is a function of the parameters. When $\Omega < \Omega^*$, the values of $\mathrm{R_{ac}}$ corresponding to subharmonic solutions are bigger than such values for synchronous solution, while for $\Omega > \Omega^*$, the synchronous values are bigger than that for subharmonic solutions. Further, when Ω is close to zero, the values of $\mathrm{R_{ac}}$ and a_c correspond to the marginally stable solutions. As Ω increases in the synchronous region, the critical value of $\mathrm{R_{ac}}$ also increases. This shows that the effect of gravity modulation is to delay the onset of convection and as such making the system stable. On further increasing the value of Ω, the value of $\mathrm{R_{ac}}$ increases very fast thus making the system more and more stable. Further, this trend is reverse in the subharmonic region, where the value of $\mathrm{R_{ac}}$ decreases on increasing the value of Ω. Also the value of a_c decreases on increasing the values of Ω in the synchronous region, while it increases on increasing Ω in the subharmonic region. Lastly, when Ω becomes very large in the subharmonic region, the values of $\mathrm{R_{ac}}$ and a_c tend to some fixed values. This shows that beyond a certain value of Ω the gravity modulation does not effect the stability limit of the system.

In Figures 2 and 3, we consider respectively, the variation of $\mathrm{R_{ac}}$ and a_c with respect to Ω for three different values of the viscoelastic parameter $\lambda = 0.1, 0.2$, and 0.3. In Figure 2, we see from both synchronous and subharmonic regions that as the value of λ increases the value of $\mathrm{R_{ac}}$ decreases in the synchronous region thus advancing the convection, while it increases in subharmonic region thus delaying the onset of convection. Also the crossover frequency Ω^* shifts to the right as λ increases. Further, when Ω is very large in the subharmonic region, $\mathrm{R_{ac}}$ approaches to the values 480, 950, and 1400, respectively. From Figure 3 we find that the value of a_c decreases on increasing λ in the synchronous region, however, it does not have any effect in the subharmonic region.

In Figures 4 and 5, we have shown the variation of $\mathrm{R_{ac}}$ and a_c with respect to Ω for three different values of the thermal anisotropy parameter $\eta = 0.5, 1.0$, and 1.5. We can conclude from Figure 4 that the effect of increasing the value of η is to increase the value of $\mathrm{R_{ac}}$ in both regions thus suppressing the onset of convection. However, in Figure 5 the value of a_c decreases in both regions on increasing η.

In Figures 6 and 7, we depict the variation of $\mathrm{R_{ac}}$ and a_c with respect to Ω for three different values of the mechanical anisotropic parameter $\xi = 0.2, 0.5$, and

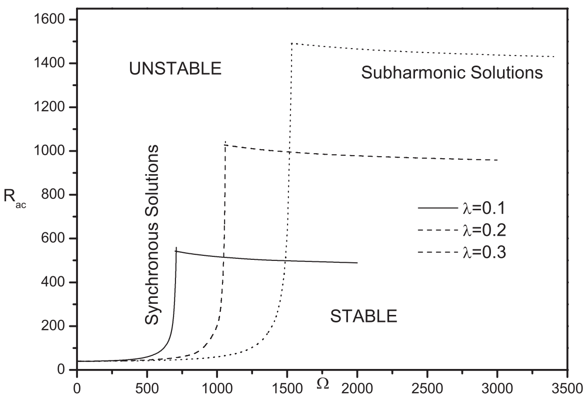

Fig. 2. Variation of $\mathrm{R_{ac}}$ with Ω. $\eta = 0.5$, $\xi = 0.5$, $\mathrm{Fr} = 10^{-4}$.

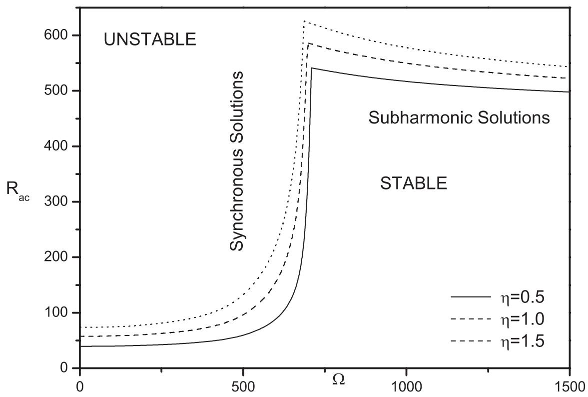

Fig. 4. Variation of $\mathrm{R_{ac}}$ with Ω. $\lambda = 0.1$, $\xi = 0.5$, $\mathrm{Fr} = 10^{-4}$.

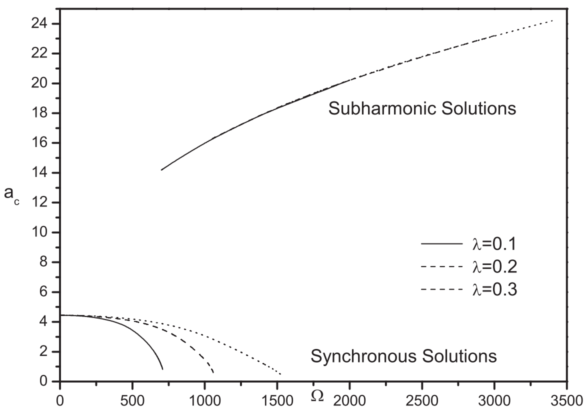

Fig. 3. Variation of a_c with Ω. $\eta = 0.5$, $\xi = 0.5$, $\mathrm{Fr} = 10^{-4}$.

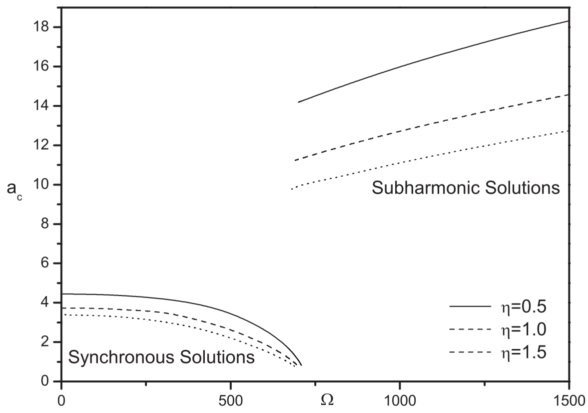

Fig. 5. Variation of a_c with Ω. $\lambda = 0.1$, $\mathrm{Fr} = 10^{-4}$, $\xi = 0.5$.

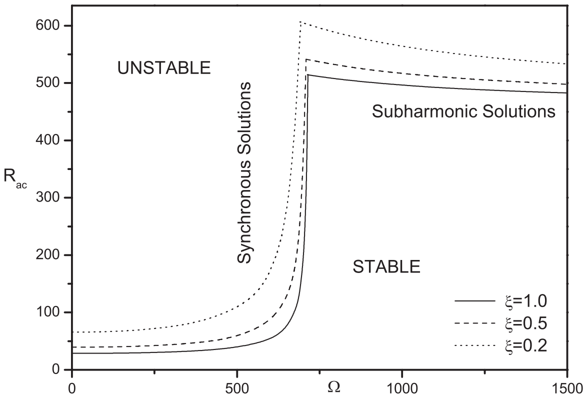

Fig. 6. Variation of R_{ac} with Ω. $\lambda = 0.1$, $\eta = 0.5$, $\mathrm{Fr} = 10^{-4}$.

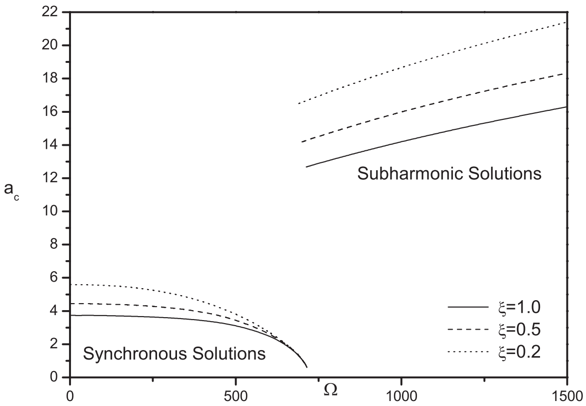

Fig. 7. Variation of R_{ac} with Ω. $\eta = 0.5$, $\lambda = 0.1$, $\mathrm{Fr} = 10^{-4}$.

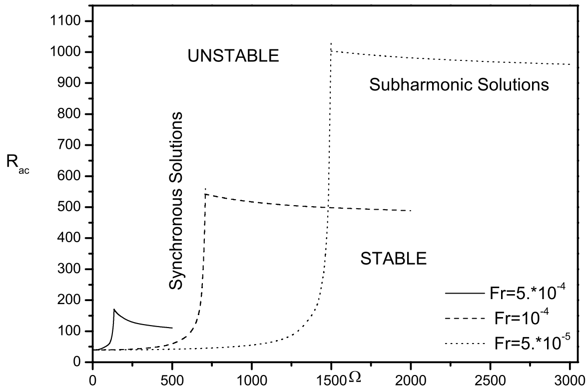

Fig. 8. Variation of R_{ac} with Ω. $\lambda = 0.1$, $\eta = 0.5$, $\xi = 0.5$.

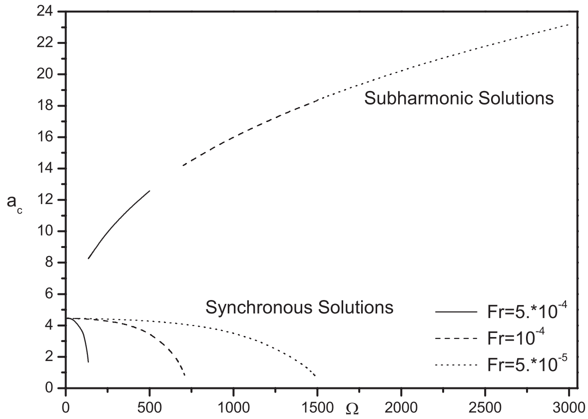

Fig. 9. Variation of a_{c} with Ω. $\lambda = 0.1$, $\eta = 0.5$, $\xi = 0.5$.

1.0. We observe from Figure 6 that the effect of increasing the value of ξ is to decreases the value of R_{ac} in the synchronous region as well as in the subharmonic region thus advancing the onset of convection and making the system less stabilized. In Figure 7, the value of a_{c} with respect to Ω decreases on increasing the value of ξ for both synchronous and subharmonic regions.

In Figures 8 and 9, we consider the variation of R_{ac} and a_{c} with respect to Ω for three different values of the Froude number $\mathrm{Fr} = 5 \times 10^{-5}, 10^{-4}$, and 5×10^{-4}. From Figure 8, we see that in the synchronous regions the value of R_{ac} increases thus stabilizing the system, while it decreases in the subharmonic region thus advancing the onset of convection on increasing the value of Fr. Furthermore, from Figure 9, we find that the value of a_{c} decreases on increasing Fr in the synchronous region, however, it remains the same in the subharmonic region.

In the last Figures 10–13, we have compared the results for viscoelastic and Newtonian fluids. It can be seen from Figures 10 and 12 that the value of R_{ac}

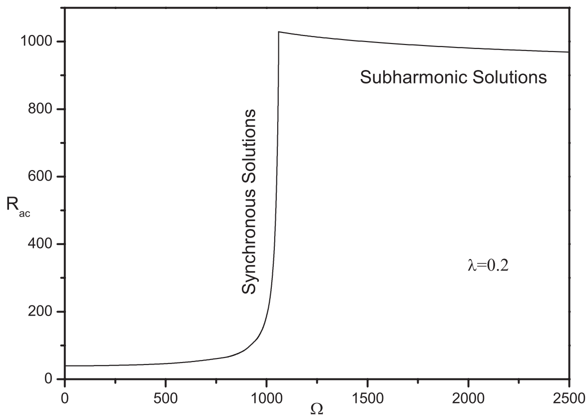

Fig. 10. Variation of R_{ac} with Ω. $\eta = 1.0$, $\xi = 1.0$, $\mathrm{Fr} = 10^{-4}$.

in the synchronous region is more for the Newtonian fluid than for the viscoelastic fluid, however it becomes more for the viscoelastic fluid in the subharmonic region. Thus the viscoelastic fluid has a stabilizing effect in the subharmonic region while the Newtonian fluid stabilizes the system in the synchronous region. The corresponding values of a_c have been plotted in Figures 11 and 13. From the figures it is found that the value of a_c is more in both regions, synchronous as well as subharmonic, for the viscoelastic fluid.

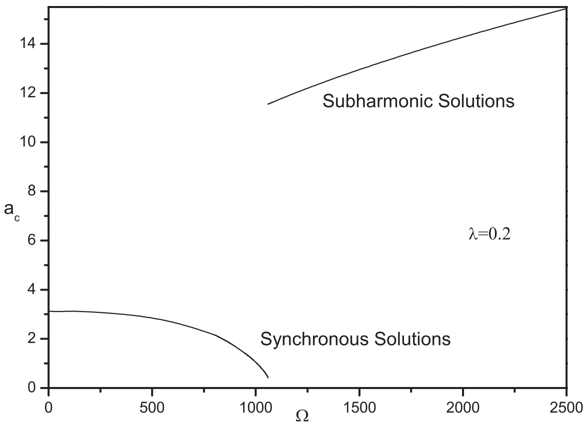

Fig. 11. Variation of a_c with Ω. $\eta = 1.0$, $\xi = 1.0$, $\mathrm{Fr} = 10^{-4}$.

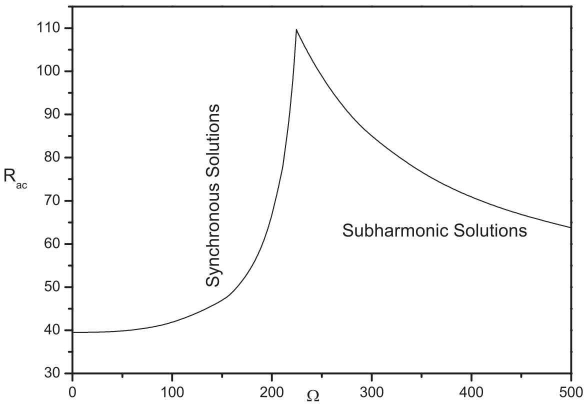

Fig. 12. Variation of R_{ac} with Ω. $\eta = 1.0$, $\xi = 1.0$, $\mathrm{Fr} = 10^{-4}$.

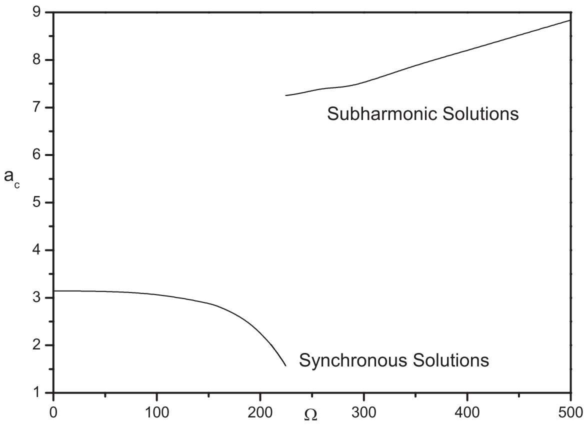

Fig. 13. Variation of a_c with Ω. $\eta = 1.0$, $\xi = 1.0$, $\mathrm{Fr} = 10^{-4}$.

6. Conclusions

In this paper, we have considered the thermal instability in a Rivlin–Ericksen-type viscoelastic fluid saturating a porous medium, confined between two free surfaces, and subjected to gravity modulation. The extended Darcy model has been used. The porous medium is heated from below and cooled from above. The following conclusions are drawn:

(i) The convective solution consists of two regions: one corresponding to the synchronous solution while the other corresponding to the subharmonic solution.
(ii) The value of the critical Rayleigh number increases while that of the critical wave number decreases in the synchronous region as Ω increases till the crossover frequency is reached.
(iii) In the subharmonic region the value of the critical Rayleigh number decreases while that of the critical wave number increases on increasing the value of the modulation frequency Ω. R_{ac} and a_c both approach to some fixed values at very large values of Ω.
(iv) The effect of increasing the viscoelastic parameter λ is to advance the onset of convection in the synchronous region while delaying it in the subharmonic region.
(v) On increasing the value of the thermal anisotropy parameter η, it is found that the value of R_{ac} increases in both the regions, thus suppressing the onset of convection.
(vi) An increment in the value of mechanical anisotropic parameter ξ decreases R_{ac} in both regions, thus advancing the onset of convection.
(vii) The effect of increasing the value of the Froude number Fr is to delay the onset of convection in the synchronous while advancing it in the subharmonic region.
(viii) A Newtonian fluid stabilizes the system in the synchronous region while a viscoelastic fluid in the subharmonic region.

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