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Polarization, Growth and Social Policy in the Case of Israel, 1997–2008

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Abstract In this paper we apply two statistical models to the measurement of polarization to Israeli income data over the past decade in order to empirically detect income classes as sub-populations of incomes concentrated around an optimal number of poles. The statistical models compared are a multi-resolution analysis (MRA) and a log-normal approach (LNA). We find the MRA to be superior to the LNA, by providing a more efficient allocation of households into each of the classes, reducing the overlap between the classes around the cut-values for each class. We then study polarization by use of the MRA in a multinomial logit-analysis by including ethnic-cultural, individual, family and other characteristics. We use a multiplicative normalized polarization measure developed by Palacios and Garcia (2010) which consists of presenting the interaction of three components, consistent with the axioms spelled out by Esteban and Ray (1994): alienation and identification, the number of income classes and the size distribution of the groups. The strong cultural heterogeneity of Israeli society, the sharp shifts in social policy during the observation period and the generally high quality of yearly Israeli income data render this dataset particularly useful for analyzing polarization. We find polarization to be significantly affected by cultural classes, by social policy and by standard demographic and individual characteristics. A comparison of our results with those of Esteban and Ray and Zhang and Kanbur reveals some similarity with our normalized version of Zangh and Kanbur (2001).

JEL H54, I21, I3, J1, O15, O53

Keywords Polarization; poverty; multi-resolution analysis; income distribution

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1. INTRODUCTION

Among 19th century economists the question concerning the partition of the income distribution into income classes was a natural one to ask. In the last two decades the interest in such questions has reappeared, following the contributions of Foster and Wolfson (1992, 2009), Wolfson (1994) and Esteban and Ray (1994). Societies are believed to consist typically of three classes – the poor, the middle class and the rich. However, empirically one may find that the classes vary in number and sizes. In the recent literature the determination of the number of classes itself has been an issue of interest, especially due to the importance of the middle class in securing social stability. The possibility of the vanishing middle class[1] becomes thus not only a theoretical problem but also an interesting empirical question.

We focus on detecting empirically the number and sizes of income classes as sub-populations of incomes concentrated around an optimal number of poles by allocating micro data of the Israeli income survey for the years 1997 to 2008 to each of the detected groups. We proceed by calculating a three-pronged polarization index as suggested by Palacios and Garcia (2008, 2010). The index is based on the axiomatic approach of identification and alienation within and between the estimated income groups, their sizes and the number of groups, as laid out in Esteban and Ray (1994).

We then identify the variables and characteristics affecting the allocation of households among the classes in a multinomial logit analysis. The Israeli economy is particularly interesting for the study of polarization, due to the cultural heterogeneity of its population, its exposure to various macroeconomic and other shocks, as well as due to its dynamic economic development. The economy experienced sharp economic

[1] See a discussion of this phenomenon in Esteban and Ray (1994), Duncan, Smeeding and Rodgers (1993), Horrigan and Haugen (1988), Kosters and Ross (1988), and Atkinson and Brandolini, 2011.

fluctuations, from rapid growth to a severe recession, followed by a quick turnaround and a growth period of four and a half years, which was interrupted by the global crisis of 2008/9. During the 1990s there was a large influx of temporary migrant workers, allowed to work only in low-skilled occupations, thus putting downward pressure on wages for low-skilled workers with the effect of crowding them out of the labor market and causing many of them to become recipients of social benefits. During the recession of 2002/3, when the cyclical developments would normally cause an increase in the number of social benefit receivers, two consecutive governments carried out a harsh social policy reform, cutting deeply into the availability of social benefits (especially the size of child benefits and eligibility and size of benefits of unemployment and income support, including a temporary freeze on the indexation of social benefits). This development was accompanied by a small scale pilot project of pro-active labor market policy.[2] The largely export-led growth period thereafter was mainly concentrated in medium and hi-tech industries, thus benefiting mainly the high skilled labor force. As is well captured in the official poverty reports of the National Insurance institute these forces had a detrimental effect on poverty incidence and particularly on poverty severity.[3] The effect of these developments on polarization and social stability is part of the present analysis. We show that the values of the new measure proposed as well as those of Zhang and Kanbur (2001) are consistent with the micro-economic multinomial logit analysis of income class association.

Distinctly from polarization measures, such as Foster and Wolfson (1992, 2009), Silber et al. (2007), which use the Gini coefficient, the polarization indicator suggested here avoids social weighting by concentrating on positive rather than normative aspects of

[2] According to an OECD report on the Israeli labor market and social policy, (2010), Israeli budgets on active labor market policies (ALP) was only about 0.1% of GDP compared to an average corresponding figure for OECD countries for 2006 of 0.6%.
[3] See official poverty reports at www.btl.gov.il.

polarization, such as the variance. Consequently, the alienation-identification component of the PG polarization index, rather than reflecting a welfare measure, should be understood as a mirror of class society, giving an equal relative weight to each class, notwithstanding the ranking of the income of its members. This approach views polarization as a neutral phenomenon, differentiating it from the concept of social weighting which is an important feature of social welfare functions such as the poverty indices of Sen, Foster, Greer and Thorbecke or the Gini-inequality index. To stress this conceptual difference, imagine a society in which poverty has been eradicated. The issue of polarization will still be relevant, focusing for example on the extremely rich and the resulting concentration of political power and threat to democracy.[4] In this case the social weighting such as the squared income gap as in the FGT measures or the rank in the Gini index would give a lower weight to the richest in the society despite the considerable potential harm of social stability implied in their influence on policy makers. While being a crucial ingredient in poverty and inequality measures the Pigou-Dalton transfer axiom does not constitute a necessity in the context of polarization.

After the introduction, the methodology for measuring polarization is presented in the second section. Empirical results are presented in the third section. After a description of the data and of relevant stylized facts about the Israeli economy we compare the various approaches to polarization by use of Israeli data on net equivalised income. In the third section we analyze the allocation of households to each estimated class, as produced by the algorithms, for each of the years 1998, 2004 and 2008 by a multinomial logit analysis. The explanatory variables are personal and demographic characteristics, as well as variables reflecting socioeconomic policies of the period 2002-2004. We chose the years that best reflected the three periods: (1) before the restrictive policy of

[4] See for example Rubinstein, 2009, p. 186-189. In that section there is also a reference to a newspaper article on the problem of economic abundance by the same author in 2003.

cuts in social benefits, (2) immediately after the policy and (3) after three consecutive years of economic growth. Conclusions are drawn in the last section.

2. STATISTICAL APPROACH

The aim of using a specific statistical model in a polarization exercise is to allocate each household in the sample to its appropriate income class, such that the emerging classes will be more homogeneous in the households' net incomes than in the overall distribution. The most frequently used statistical approach is to estimate a mixture of gamma, normal and log-normal distributions, referred to here as the traditional approach. Such an approach may be found in the work of Paap and van Dijk 1998, Pittau and Zelli, 2006, Flaichare and Nuñez, 2007, Chotikapanich and Griffiths 2008, Pittau et al. 2010**,** among others. Maximum homogeneity is achieved by having a maximum of unique allocations of households into income groups. Unfortunately, in such exercises statistical models typically provide overlapping results, in which one household has a positive probability to be allocated to more than one group. As is well accepted in the literature (see for instance Esteban and Ray, 1994 and Zhang and Kanbur, 2001), one of the most important characteristics of polarization is the alienation-identification property. Homogeneity-heterogeneity is the statistical interpretation of this property. One of our purposes in this paper is therefore to keep overlapping results to a minimum by choosing a statistical model that enables us to reduce overlapping results to a minimum, in order to provide subpopulations that are as homogeneous as possible and less disputable.

In this section we compare two estimations of unknown probability density functions of a given population. The first is a mixture of a log-normal distribution and the second is a mixture of densities based on multi-resolution analysis.

The empirical application of the two mixtures is carried out using Israeli income data for the year 2005[5].

The estimated parameters and coefficients of the mixtures of log-normal distributions and of the MRA are given in figure 1 and tables 1.a and 1.b.

Figure 1. Components of the LNA (left panel) and MRA mixtures (right panel)

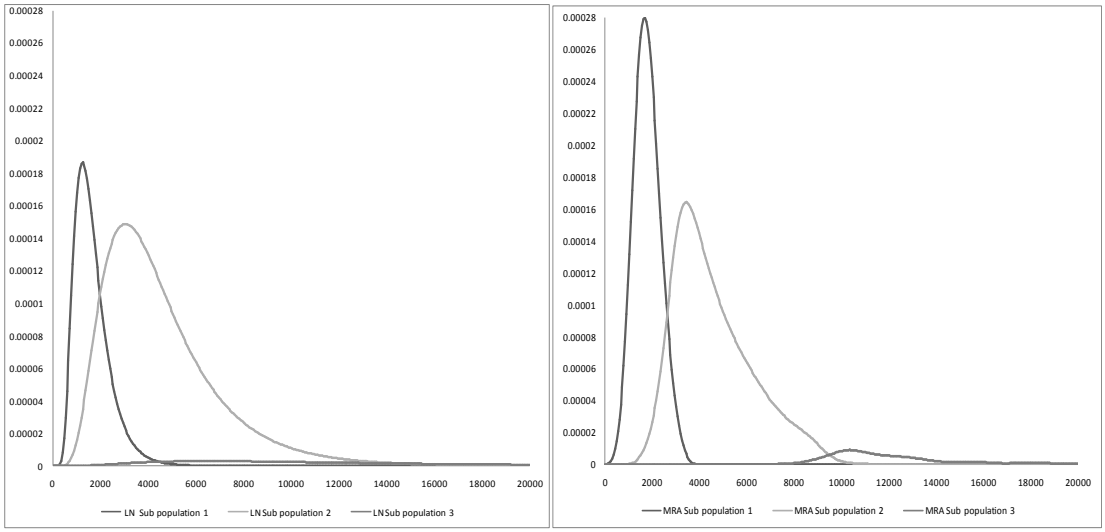


Table 1.a: Parameters of the mixture of log-normal pdf

	Expected Values	Expected Standard Deviation	Parameters of the Log Normal Mixture		
			$\hat{\mu}$	$\hat{\sigma}$	$\hat{p}$
Component 1	1658.11253	757.130889	7.31874368	0.43518162	0.27936849
Component 2	4579.03612	2546.53659	8.29450515	0.51911203	0.67728094
Component 3	5203.50434	17689.0709	7.29197697	1.59066692	0.04335057

[5] We use this year for the analysis because this is the first year in which the full effect of the harsh social policy carried out in the years 2002 to 2004 is fully reflected in the data, thus providing sufficient variance in the microeconomic information on homogeneity-heterogeneity and making it a good test case.

Table 1.b: Estimation of the mixture of MRA pdf

	Expected Values	Expected Standard Deviation	$\hat{p}$
Component 1	1728.6713	2398.93129	0.41010639
Component 2	4599.11435	2553.50559	0.55083056
Component 3	13414.2513	10597.3815	0.03906305

Figure 1 shows that in this sample the MRA mixture produces less overlap for each of the subgroups than the log-normal mixture. This has an important economic interpretation, since as mentioned earlier one of the major purposes of a polarization exercise is to allocate each household to a unique income-subgroup. However, in the zones of overlap such uniqueness is impossible.[6] In such cases it is only possible to assign to these households probabilities of belonging to each of the groups. This is a result to be expected, since we show in sections 2.1 and 2.2 that the components of the MRA mixture are found by a process which optimizes homogeneity whereas the components of the log-normal mixture are a result of maximizing the likelihood function, without including any consideration about the homogeneity of the components.

As is easily seen from figure 1 the overlap of the log-normal mixture is particularly high in the third group of high incomes. As a matter of fact it is easy to see that in the LNA there is quite a large overlap between the poor and the rich, a result that is strongly counterintuitive and the overlap of the rich and the middle class in the LNA model is almost complete. In contrast there is no overlap at all between the poor and the rich in the MRA and the overlap between the rich and the middle class is confined to a

[6]In fact this is the case for all households when the model is a mixture of normal, log-normal or gamma pdfs. Although based on an intuitive reasoning, in this paper we have truncated the tails of these distributions for the analysis of overlap and the creation of table 2.

relatively limited range of incomes. The considerable overlap between the poor and the middle class at the lower end of the middle class distribution may well reflect the phenomenon of blurred identification and alienation at the high end of the poor class and the lower tail of the middle class. These results have an important economic implication: at the overlapping tails people find it hard to identify with one or the other group and as for the top-income group (the "tycoons"), certain members of this group often have direct access to economic policy decision making, especially concerning those decisions that directly affect rich people's economic welfare. Since the number of households tends to decline strongly, the richer the households become, their group will typically be very small and hard to identify uniquely in a polarization exercise, while at the same time their economic importance increases. Therefore the homogeneity optimization, characteristic of the MRA procedure, has an inherent advantage, since the relative efficiency of identification by the MRA seems to increase with the reduction in the size of groups.

Focusing on this problem of overlapping by giving the share out of total observations in the various groups we can see from table 2 that the share of households allocated uniquely to the lowest class is about twice as big in MRA compared to the same group in LNA. The households uniquely allocated to the middle and high classes is smaller in the MRA. In LNA 73.7% of the households are uniquely allocated to anyone class whereas in the MRA the share is 92.6%. Accordingly the share of overlapping allocations is 26.3% in the LNA and only 7.4% in the MRA, thus resulting in an overlap that is 3.6 times higher in LNA than in MRA.

Table 2: Comparison of overlapping results in the log-normal and MRA mixtures

Unique and overlapping areas of the components of the mixtures			
Log Normal Mixture		MRA Mixtures	
G1	0.1726614	G1'	0.36767
G2	0.5562972	G2'	0.51708
G3	0.0085047	G3'	0.04147
G1G2	0.100947	**G1'G2'**	0.06775
G1G3	0.0637854	**G1'G3'**	0
G2G3	0.0340189	**G2'G3'**	0.00603
G1G2G3	0.0637854	G1'G2'G3'	0
Total unique allocations	0.7374634		0.92622
Total overlapping allocations	0.2625366		0.07378
total overlaps of LNA/MRA	3.6		

Given the widely accepted view that homogeneity or alienation-identification is a crucial feature of polarization, we may conclude that the MRA approach is shown in this example to be superior to the traditional approach. Following this empirical evidence we prefer to use a family of density functions based on multi-resolution analysis (henceforth MRA) as suggested in Palacios and Garcia, 2009.

2.1 *The model*

A mixture of Multi-resolution Analysis probability density functions, at the level of resolution *j*, is defined as follows

$$f_j(x) = \sum_{k \in S \subset Z}^{m} a_{jk} \lambda_{jk}(x) \qquad (1)$$

where S is a finite subset of integer numbers, $a_k \geq 0 \quad \forall k \in S; \sum a_k = 1$ and

where $\lambda_{j,k}(x)$ is a pdf with compact support[7], $\left[\frac{k-2}{2^j}, \frac{k+2}{2^j}\right]$ that results from making the variable $y = 2^j x - k$ in the Cubic Box Spline $\theta(y)$ given by

$$\theta(x) = \begin{cases} p_1(2+x) & if \ -2 \le x \le -1 \\ p_2(2+x) & if \ -1 \le x \le 0 \\ p_2(2+x) & if \quad 0 \le x \le 1 \\ p_2(2+x) & if \quad 1 \le x \le 2 \end{cases}$$

where

$$p_1(x) = \frac{x^3}{6} \text{ and } p_2(x) = \frac{-x^3}{2} + 2x^2 - 2x + \frac{2}{3}.$$

The mean and the variance of a MRA pdf are given by

$$E_{f_j}[X] = \frac{\mu}{2^j} \text{ and } V_{f_j}[X] = \frac{\sigma^2 + 1/3}{2^{2j}}$$

Where $\mu = \sum_{k \in Z} a_k k$ and $\sigma^2 = \sum_{k \in Z} a_k (k - \mu)^2$.

The coefficients of the mixture of MRA pdf given by (1) are estimated by the maximum likelihood procedure for a given value of *j j* using the EM algorithm (Hartley, 1958; Dempster et al., 1977; McLachlan and Krishman, 1997) and therefore they are consistent, asymptotically unbiased and asymptotically efficient. After estimating the population density for a given value of *j* we validate the model using the test of Kolmogorov-Smirnov (KS). If the fitted model is rejected by the test of KS, this is due

[7]Note that $x_k = \left\{\frac{k}{2^j}\right\}_{k \in Z}$ is a regular grid of points over $\Re$ equally spaced at distance $\frac{1}{2^j}$.

to the insufficient flexibility of the model or because the level of resolution used is too low. Of all the resolution levels that provide valid estimations for population density, we use the lowest[8] as explained in Palacios-Gonzalez and García-Fernández, 2012.

We refer to any mixture of MRA pdfs as a new MRA pdf. Furthermore it should be clear that any MRA pdf can be broken down in mixtures of MRA pdfs. The decomposition task can be made by multiple forms which allow us to obtain, from the infinite possible decompositions, an optimal decomposition according to the homogeneity of the groups around some selected modes. This is the principle on which is based the algorithm used to obtain the mixture whose components (or subpopulations) are more homogeneous.[9]

As in any other mixture of pdf, once the MRA pdf model is generated, we can calculate conditional probabilities that a household with a certain level of income comes from a component of the mixture. These probabilities allow us to classify each household into a specific income group. In particular, we cluster data by assigning each household to the level of income to which it has the highest conditional probability of belonging.[10] In the empirical section of the paper we use the classification into income groups provided by the posterior probabilities to estimate a Multinomial Logit model. In this way we can study the position of the households in the income distribution according to their socioeconomic characteristics.

2.2 Measurement of polarization

The notion of polarization was introduced by Wolfson (1994) and Esteban and Ray (1994) independently to explain distributional changes that are not explained by the

[8] Note that the test of KS is used to validate the model and not for detecting sub-populations.
[9] See the algorithm 1 in Palacios-Gonzalez and García-Fernández, 2012 for a detailed explanation of this process.
[10] See McLachlan and Peel, 2000.

standard measures of inequality. Following Esteban and Ray (1994) "polarization is viewed as the sum of antagonisms between individuals that belong to different groups. Antagonism is the joint result of inter-group alienation, combined with the sense of identification with the own group". According to the previous concept of polarization, they pointed out the following basic features that the polarization of a distribution of individual attributes must present:

1. There must be a high degree of homogeneity within each group.
2. There must be a high degree of heterogeneity across groups.
3. There must be a small number of significantly sized groups. Groups of insignificant size (e.g. isolated individuals) carry little weight.[11]

Since the mid-nineties, several measures of polarization have been defined attending to different approaches [see among others, Esteban, Gradín and Ray (1999), Tsui and Wang (2000), D'Ambrosio (2001), Zhang and Kanbur (2001), Duclos et al (2004), and Silber et al (2007)]. The measure of polarization used in this paper is developed considering the three contributing polarization factors, suggested by Esteban and Ray (1994): the alienation and the identification felt by individuals, the number of significantly sized groups and the distribution of the size of the groups. The calculation of identification and alienation is somewhat modified (as explained in Palacios and Garcia, 2008, 2010). To evaluate the effect that the listed factors have on polarization, three indices, $\mathrm{I_{ia}}$, I_m and I_g, are defined. Since the values of the polarization components I_{ia}, I_m and I_g are defined over the interval $[0,1]$, their product provides a normalized and non-dimensional index of polarization, that is

[11] Of course one may argue that the higher up in the income distribution one is positioned, the higher the importance of ever small groups become. In an extreme case, a single extremely rich person may exert more power on government decisions and distort democratic decisions than a larger group of people.

$$PG = I_{ia} I_g I_m \in [0,1] \qquad (2)$$

where

$$I_{ia} = \frac{V_B}{V} = 1 - \frac{V_W}{V}, \quad I_g(k) = \begin{cases} 0 & k = 1 \\ \frac{2}{k} & k = 2,3,\ldots \end{cases} \qquad I_m = \begin{cases} \frac{1-2d}{(1+d)} \text{ for } k = 2 \\ \frac{3-2d}{3(1+d)} \text{ for } k = 3,4,\ldots \end{cases}.$$

V_B, V_W and V are the between groups variance, the intra-group variance and the total variance respectively. The variance intra group is given by

$$V_W = \frac{1}{n}\sum_{i=1}^{k} n_i \sigma_i^2$$

that is the average of the within group variances weighted by the group sizes.

The expression

$$V_B = \frac{1}{n}\sum_{i=1}^{k} n_i (\mu_i - \mu)^2$$

is the between groups variance, that is the variance of the means of the groups.

The index k indicates the number of groups and d is the Euclidean distance between the distribution of the size of the groups and the distribution of maximum polarization, which is given by $d = \sum_{j=1}^{k} \left(p_j - p^H\right)^2$, where $p^H = \left(\frac{1}{2}, \quad \frac{1}{2}\right)$ for $k = 2$ and $p^H = \left(\frac{1}{2}, \quad 0, \quad \ldots, 0, \quad \frac{1}{2}\right)$ for $k \geq 3$.

The index I_{ia} complies with the first and second basic features of Esteban and Ray. We assume that identification increases with the similarity of the income within the group. An individual feels a sense of identification with the group to which he belongs when his income is closer to the average income of the group. In keeping with the second feature, we presume that alienation is positively linked to the distance among the mean

incomes of the groups. Attending to the previous arguments we consider, that a global measure of identification should be inversely proportional to the intra-group variance (V_W) and that a global measure of alienation felt by individuals belonging to the same group with respect to individuals belonging to the other groups should be proportional to the variance between groups (V_B). The ratio of the inter-groups variance to the intra-group variance quantifies the contribution of identification-alienation to polarization. This ratio has been normalized using the decomposition property of the variance obtaining I_{ia}. The index I_g is related to the third feature and is decreasing with the number of groups, in such a way that the higher the number of groups, the smaller is the contribution of this index to polarization. I_m captures the effect of the clustering of population around the extremes of the income distribution, or equivalently the influence of a diminishing middle class on polarization. Movements of individuals from the middle to the bottom and the top of the income distribution will thus involve a diminution of the middle class and an increase in I_m and hence in polarization.

The measure described above assumes that the population is bunched into income groups. In this paper, the number of groups and their sizes are obtained using the estimated coefficients of the MRA model and the algorithm described in section 2.1. For the data used, the estimated number of groups is equal to three (excepting the year 1997 in which there are four groups). For this reason, we compare the proposed measure with the measures of Esteban and Ray (1994) and Zhang and Kanbur (2001) which can be computed for any number of poles and are also obtained following an alienation and identification framework.

The measure of Esteban, and Ray (1994, henceforth ER) is given by the expression

$$ER = \sum_{i=1}^{n}\sum_{j=1}^{n} p_i^{1+\alpha} p_j \left| y_i - y_j \right| \quad 1 \le \alpha \le 1.6$$

In which $\left| y_i - y_j \right|$ represents the alienation (distance) felt by individuals of incomes y_i and y_j. The share of population is given by p_i, and p_i^{α} represents the sense of group identification of each of the p_i members of group i within their own group. The sense of identification increases with the number of people in the group which have the same income level. The parameter α falls into the interval $[1, 1.6]$ to be consistent with the set of axioms proposed by Esteban and Ray (1994).

Zhang and Kanbur (2001, henceforth ZK) provided an alternative approach to polarization based on the idea that polarization is generated by two tendencies: for k exogenously given groups, as income differences within the group decrease, that is as the groups are more homogeneous internally, differences across groups are, magnified and polarization is higher. In a similar way, for given within group differences, the further apart are the means of the groups the higher is polarization. These authors quantified these tendencies by the ratio of the between groups inequality to the within group inequality, that is

$$ZK = \frac{between - group\ inequality}{within - group\ inequality}.$$

For the Theil index the above expression can be written as follows

$$ZK = \frac{T_B}{T_W} = \frac{\sum_{j=1}^{K} \frac{n_j}{N}\frac{\mu_j}{\mu} ln\left(\frac{\mu_j}{\mu}\right)}{\sum_{j=1}^{K} \frac{n_j}{N}\frac{\mu_j}{\mu} T_j}$$

where

$$T_j = \frac{1}{n_j}\sum_{j=1}^{K}\frac{y_j}{\mu_j} ln\left(\frac{y_j}{\mu_j}\right)$$

k is the number of groups; N is the total population; n_j is the population of the j^{th} group; μ is the total sample mean; μ_j is the mean of the j^{th} group and y_j is the j^{th} income.

Our polarization measure has the following advantages with respect to those provided by of ER and ZK. In contrast to ER and ZK, the PG is a normalized measure, taking values between 0 and 1. It can thus be interpreted as a percentage portraying the degree of polarization. The expressions of Zhang and Kanbur (2001) and Esteban and Ray[12] (1994) are not normalized and consequently the results cannot be interpreted in terms of percentages. Indeed the results of both measures are difficult to interpret since there is no established standard of measurement. For example it can be shown that the Zhang and Kanbur polarization measure increases systematically with the number of groups. The introduction of the I_g index in the PG measure compensates the effect that the increasing of the number of groups has on the intra-group variance and hence on polarization, thus correcting this drawback of the ZK measure.

Furthermore it is easy to see, that the Zhang and Kanbur measure tends to infinity when the within-group inequality tends to zero. However, this drawback of the index can be corrected by normalizing their measure, using the decomposition property of the Theil index[13], as follows

[12]Although Esteban and Ray (1994) made an attempt of normalizing their measure, using log income and replacing the population weights by the population frequencies, it is easy to show that this measure can take values higher than one.

[13] The index of Theil can be broken down in a similar way as the variance. That is, the overall inequality is equal to the inter-groups inequality plus the intra-group inequality. This property is also verified by the Gini index if the groups do not overlap.

$$ZKN = 1 - \frac{T_W}{T}$$

where $T = T_W + T_B$.

Observe that such a normalized Zhang and Kanbur measure resembles the alienation-identification index (I_{ia}) in PG. The main modification introduced by I_{ia}, concerns the way in which we compute identification and alienation. According to the concept of polarization, if there is a high degree of homogeneity within each group and a high degree of heterogeneity across groups, society is polarized. In other words polarization focuses on dispersion and for this reason we prefer the use of the intra-group and the inter-groups variance to that of the intra-group and inter-group inequality to quantify the contribution of identification and alienation to polarization.[14] Indeed, from a statistical point of view, the intra-group variance and the inter-groups variance are the most appropriate approaches to evaluate the homogeneity within a group and the heterogeneity across groups respectively, when the representative magnitude of each group is the mean of the variable of interest, in our case the mean income (see among others Fisher, 1958). Moreover the concept of polarization, on the contrary to the inequality indices, is not linked directly to welfare. For this reason we think that positive measures, such as the variance, are more appropriate for the computation of alienation and identification and consequently for polarization.

3. EMPIRICAL RESULTS

Israel's society is highly heterogeneous both culturally and also with respect to the standards of living of the various population groups. Heterogeneity is driven mainly by

[14]As mentioned above, the negative effects of polarization may occur both at the bottom and the top of the distribution. Therefore the higher ranking of lower incomes, as for example in the Gini measure, may diminish the indicator's measurement of the damage caused by the concentration of excessive economic power at top incomes.

cultural differences based on nationality and religiosity: four fifth of the population being Jewish and one fifth Arab and within the Jewish population there is a significant cultural divide concerning religiosity between orthodox (henceforth Haredi) Jews, who account for about 10% of Israeli Jewish population, and the rest. Heterogeneity is emphasized by Haredi preference to let the men concentrate on theological studies, rather than earning a living, leaving this task to the wives. This tendency is underlined by the de facto exemption of the young Haredi from army service. Marriage at an early age, large family size and low labor market participation are typical in the Haredi society and create large (equivalised) income differences in favor of the non-orthodox Jewish majority. Important cultural differences as well as differences in opportunities for the Arabs create a further possible source for polarization between Jews and Arabs. However, in contrast to the Haredi society the Arab society is in a process of rapid reduction in family size, thus reducing heterogeneity over time.

A further source of polarization stems from government policy and the economic environment. The Israeli economy being small and open has been subject to significant shocks during the observation period. These shocks may affect various population groups differently, for example, depending on their involvement in the labor market. During the second half of the 1990's the Israeli economy had become increasingly open, not only due to its high and rising share of imports and - largely hi-tech oriented - exports, but also due to the increasingly liberal regime of flows of international capital and of migrant workers.[15] Economic vulnerability and polarization have been enhanced by the Israeli-Arab conflict which brought about repeated outbursts of violence, thus exposing the Israeli economy to politico-economic shocks. Such a shock occurred from the last quarter of 2001 to early 2003. Another cause of sharp changes in the income

[15] See Gottlieb and Blejer (2001).

distribution was the harsh mix of macroeconomic and socio-economic policies implemented during the years 2002 to 2004 and a previously started de facto liberal policy towards the influx of migrant workers[16], coupled with a policy of lax compliance and enforcement of labor laws among their employers.[17] This policy caused a significant influx of migrant workers[18] since 1993, affecting negatively the employment prospects and salaries of low skilled Israeli workers and thus possibly exacerbating polarization. A fiscal policy, led by a tax reform (from 2006 onward) which reduced income tax rates mainly for the well-to-do, coupled with severe cuts in social benefits (2002 to 2004) - particularly in child benefits, income support of families whose head of household was in working age, and in the eligibility criteria for unemployment – further emphasized the tendency of economic hardship for the low-skilled. The main goal of these cuts in welfare budgets which occurred mainly between 2002 and 2004, was aimed at raising labor market participation of income support receivers and at reducing the budget deficit through a reduction in social expenditure, which in the past was characterized by a higher degree of solidarity.[19] The worldwide economic crisis of 2008/9 was not significantly felt in the Israeli economy until the last two months of the year of 2008, such that it is hardly reflected in our observation period.[20] The above mentioned intense economic history of Israel thus presents a unique opportunity for studying polarization during the period of 1997 to 2008.

[16] The migrant workers which started to flow into Israel from 1993 onward in reaction to the gradual closure of the borders for Palestinian workers, due to cycles of political violence, are not to be confounded with Jewish immigrant workers who have been entering Israel for many decades and particularly since 1990.
[17] The government has undertaken several attempts over recent years to regulate migrant workers' influx but until now without much success (see various Bank of Israel Annual Reports and Gottlieb, 2002).
[18] Migrant workers, whose sole aim is to come to work in Israel are not to be confused with new immigrants, who immigrate to Israel by the law of return.
[19] See National Insurance Institute, Annual Surveys, 2004 to 2008.
[20] See Annual Survey, 2008, National Insurance Institute, p. 15-18.

3.1 Description of the survey

The data is from the annual income surveys for the years 1997 to 2008, carried out by the Israeli Central Bureau of Statistics (CBS).[21] The number of households surveyed each year varies between 12,815 and 15,000. The cash income data used in the analysis throughout the observation period are in constant 2006 prices. The mean net equivalised income varied between 2,577 NIS and 4,222 NIS per month, implying a real growth rate of that income by about 2.3% p.a.[22]

Table 3: Basic data[23]

Total population	Number of households in sample	Mean net equivalized income in sample	Standard deviation in sample	Average number of school years	Average family size	Average number of earners in household
1997	12,815	3,263	2,554	12.3	3.41	1.41
1998	13,266	3,324	2,512	12.4	3.36	1.37
1999	13,273	3,406	2,876	12.6	3.35	1.24
2000	13,424	3,523	2,697	12.4	3.33	1.25
2001	13,608	3,683	3,016	12.6	3.30	1.19
2002	13,955	3,519	2,647	12.7	3.31	1.18
2003	14,112	3,505	2,618	12.7	3.31	1.18
2004	14,337	3,634	2,788	12.8	3.30	1.20
2005	14,239	3,755	3,088	12.9	3.28	1.21
2006	14,282	3,989	3,452	13.0	3.28	1.24
2007	13,879	4,112	3,265	13.1	3.26	1.26
2008	13,854	4,139	3,285	13.2	3.26	1.27

[21] The CBS began to top-code the highest incomes since 2006. At first we analysed the non-top-coded data in the present framework, but eventually concluded that the top-coding had no significant effect on the results derived from income surveys.

[22] We use the official Israeli equivalence scale which is based on the traditional food-share scale of the Engel type: The values of the scale are 1.25, 2, 2.65, 3.2, 3.75, 4.25, 4.75, 5.2 for one, two, persons etc. respectively until it reaches 6 for 10 persons, continuing with an addition of 0.4 for each additional person. Israeli families are in general much larger than those found in Western countries. This is mainly due to Jewish and Muslim religiosity. Therefore the scale is not truncated for particularly large families. The numbers for the lower sized families are quite similar to those of the OECD scale, that prevailed before the OECD switched to using the square root of family size.

[22] In order to be able to analyse polarization over time we had to exclude the Jerusalem-Arabs from our data set, since they had not been surveyed in the years 2000 and 2001.This was necessary to ascertain a consistent, though incomplete measurement of polarization for Israel. Their population has been growing rapidly from somewhat more than 10% to nearly 20% of Israel's Arab population. They mostly belong to the poorest class of the income distribution. Their omission may thus slightly bias the overall results for polarization.

[23] The data are in real New Israeli Shekel (NIS in 2006 prices). Due to problems of collecting data on Jerusalem Arabs in the years 2000 and 2001 we excluded them from the sample throughout the observation period in order to analyse a consistent data set. Negative and zero incomes were excluded from the analysis. The data in table 3 are calculated from non-weighted household survey data.

3.2 Analysis and results

To model the equivalised net income distribution the MRA pdf given by (1) is used. The coefficients of the MRA model given by expression (1) are estimated by the maximum likelihood procedure using the EM algorithm (Hartley, 1958; Dempster et al., 1977; McLachlan and Krishman, 1997). Different approximations to the theoretical distribution, are performed by increasing the resolution level *m*. Attending to the parsimony principle, the model with minimum *m* which is non-rejected by the test of Kolmogorov-Smirnov fits well to the pdf and will be used to apply the measure of polarization.

After estimating the MRA pdf, the number of groups and their location are obtained by applying the algorithm 1 described in Palacios-Gonzalez and García-Fernández (2012).

The results presented in Appendix figures A.1 to A.12 reveal that according to the algorithm the number of significant income groups shrank during the observation period – from 4 groups in the first year (1997) to 3 groups in the following years.[24]

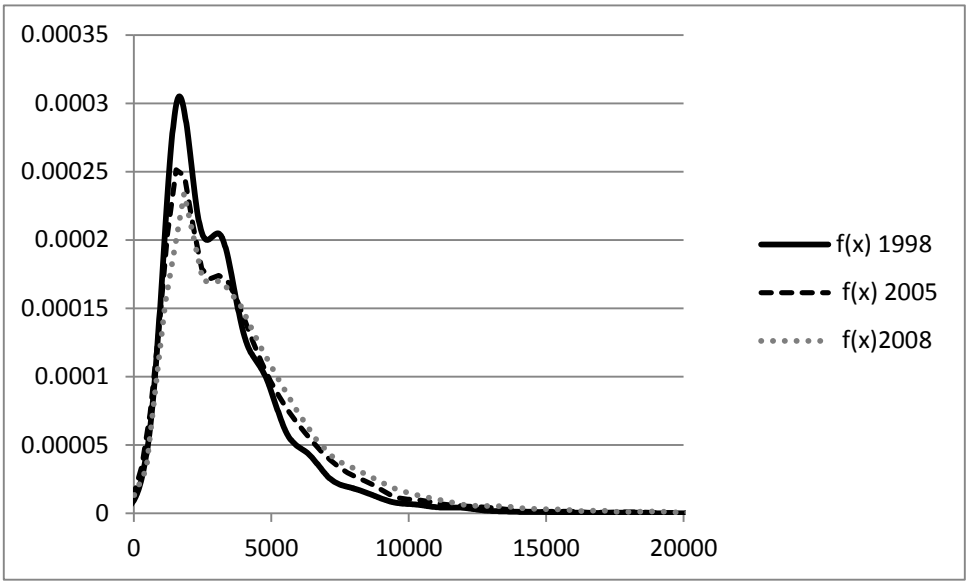
Figure 2 displays the overall probability density function of net incomes and reveals that over the three years compared[25] – 1998, 2005, 2008 – the shape of the overall distribution underwent important changes: while in 1998 there were two distinctive modes to the distribution, over time the second mode became more flattened. This flattening process was accompanied by an increase in dispersion as can be observed by an outward shift of the right hand side of the distribution, suggesting a movement within the middle class to its upper part. This reminds of a similar development for UK data, as

[24] Possibly the sample of 1997 was of lesser quality, compared to the following years, since this was the first year for which the income survey combined information from two sources – from the labour force survey and the expenditure survey – thus implying that the results from 1998 onwards are more qualitative. When we accidentally used original data (including Arabs from East Jerusalem) the algorithms produced only two classes, implying a vanishing middle class in 2008.

[25] The years 1998, 2005 and 2008 reflect respectively the first year of qualitatively improved data, the first year after the harsh social policy and the first year after a prolonged period of growth in GDP.

reported in Jenkins (1995), reflecting the 'shrinking middle class' phenomenon of the income distribution during the 1980s.

Figure 2: Changes in the overall probability density function of the equivalised net income distribution over time: 1998 – 2005 - 2008



The estimated MRA pdfs in appendix figures A.1-A.12, given for the overall population and for each group from 1997 to 2008 reflect two major forces that were at work: (1) a harsh socio-economic policy carried out from 2002 to 2004, with an emphasis on 2003-2004; (2) a sharp fluctuation in the per capita growth environment of GDP over the observation period of 1.7%, a negative growth rate (-1.9%) during the recession of 2001 to 2003 and renewed positive growth of 3.2% per capita over the years 2005-2008.[26] In the context of polarization the average per capita growth rate differed for each income group. When splitting down the changes into net household income by group, income actually declined during the recession for the lower and middle classes while it increased for the upper class. In the period of enhanced growth of 2005 to 2008, though

[26] We neglect 2004 in the calculation of sub-period p.c. growth because it reflects a year of transition.

while all three classes benefitted, the increase in the lower class was almost nil and in the middle class significantly smaller than in the upper class.[27]

The polarization indicator as defined in equation (2) is presented in figure 3 and in Appendix table A.3. Over the period 1998 to 2006 this indicator fluctuated around a negative trend. This trend was sharply reversed in the years 2007 and 2008.

Figure 3: The Polarization Index, its trend and the confidence intervals (95%)

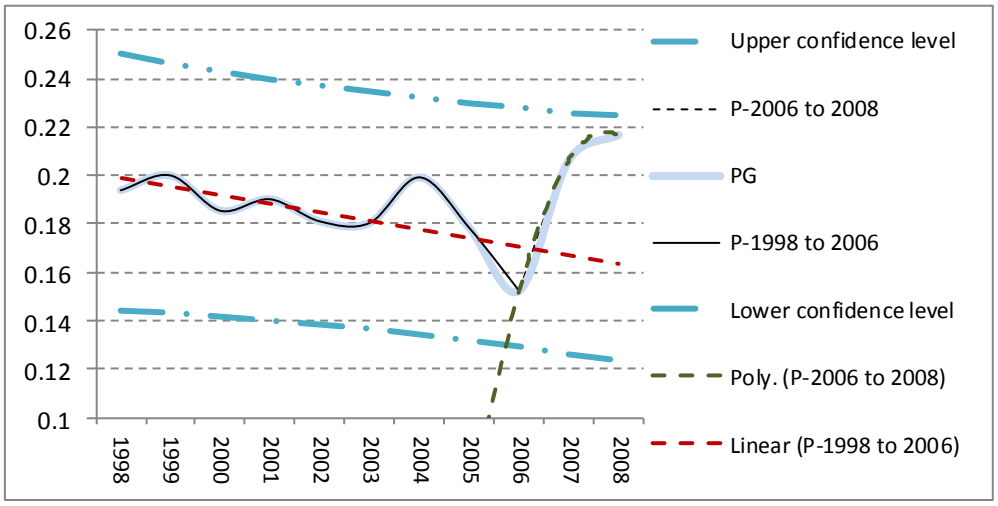


Table A.2 shows that during the tri-polar period (1998 to 2008) the size of the middle class was not stable and in 2007 and 2008 there was a tendency towards a 'shrinking middle class'. As shown below, the harsh (permanent) change in social policy which occurred in 2003 and 2004, following the recession of 2001/3, raised the probability for economically disadvantaged households to remain in (or fall into) the lower class. This policy included sharp cuts in various social security benefits, especially in child benefits, income support to heads of households in working age and unemployment benefits as well as a tightening of their eligibility criteria. This is shown in the Multinomial Logit equations below.

[27] These results can be calculated from table A.1. The p. c. GDP calculations are based on data from the Central Bureau of Statistics.

The polarization index given by expression (2) is presented in Figure 3 and in appendix tables A.3 and A.4. The figure of the components of the polarization index is given in Appendix figure A.13.

I_{ia}: At the heart of the polarization measure is the measure of identification and alienation. After some fluctuation it increased during 2001 to 2004, a period of harsh socio-economic policy (2002 -2004), which coincided with a severe recession (late 2001 – late 2003). This cut in social expenditure, during an economic downturn not only worsened the economic situation of the low and middle class but probably deepened the downturn by neutralizing the expected built-in-stabilizer. This effect was somewhat dampened during the period of enhanced growth[28], only to deteriorate again towards the end of the period.

I_g: As explained above this indicator reflects the number of groups, raising polarization, when the number of groups is falling. As mentioned above, this happened from 1997 to 1998, and possibly from 2007 to 2008. The I_g factor compensates for the "squeezing effect" on the intra-group variance of the subgroup probability density functions, when introducing an additional class. We give great importance to this factor even if the empirical observation of a reduction in the number of classes from 1997 to 1998 and again from 2007 to 2008 (when Arabs living in East Jerusalem are included – see also footnote 24) is not a robust econometric result at this stage.[29]

I_m: While the size of the middle class increased (except during the period of harsh socio-econoic policy) during the years 1998 to 2006, this tendency was reversed towards the end of the period. The present analysis suggests that when the

[28] An econometric test of this hypothesis exceeds the scope of the present paper and will be taken up in future research.
[29] The possibility of a polarization model to accommodate endogenously for a change in the number of classes can be useful with the availability of qualitative data for longer observation periods.

identification/alienation factor and the size effect (reflected in I_g and I_m) complement each other, then the chances of a shrinking middle class are enhanced.

We limit the comparison of our polarization measure with other measures to the tri-polar case (1998-2008). Given that the important measure of Foster and Wolfson is bi-polar and thus not strictly comparable, it has been excluded from this comparison. The resemblance between our identification-alienation component (I_{ia}) and the normalized expression of the measure of ZK (henceforth ZKN) is apparent in table A.4 and appendix table A.6, and obviously stems from the fact that both measures are based on the relationship between income homogeneity within the group and income heterogeneity between groups. Their focus on dispersion is nonetheless different since I_{ia} uses the variance to compute dispersion whereas ZK use concentration indices. The ER measure is different in nature, a fact reflected in table A.4.

3.2.1 A Multinomial Logit Analysis

An interesting question to analyze is the effect on various variables on the probability of belonging to a specific income class. For that purpose we use a Multinomial Logit analysis of households' group membership, explaining it by three types of variables: (1) demographic variables, such as belonging to a specific cultural group, (2) characteristics of the household or its head, such as the head's age group, his number of school years, the number of children and the share of employed adults in the household; (3) a variable of socio-economic policy, measuring the degree of dependence on social benefits as the ratio of benefits out of the family's total income. In order to test the robustness of the coefficients over the economic cycle we repeated the same regression for three years – 1998, 2004, 2008. The results are reported in tables 4 and 5. The odds-ratios reported in

table 5 correspond with ex-ante expectations. They show that the regression coefficients remain stable over time, which makes them robust.[30]

Table 4. Multinomial Logit Model: Dependent Variable – Group Membership

Model: Multinomial Logit		Coefficients		
Variable	**# eq.**	**1998**	**2004**	**2008**
Intercept	**1**	0.8933	-0.4412	0.8182
	2	1.9113	1.1304	1.9604
Arab	**1**	2.3549	2.3109	3.6107
	2	1.0061	0.6759	1.7077
New immigrant from 2000	**1**	3.0219	2.612	2.0556
	2	2.1366	1.7481	1.3329
8 years of education	**1**	4.3554	3.4644	3.1754
	2	2.8824	2.2772	2.0288
9-12 years of education	**1**	2.0734	1.9883	2.0961
	2	1.2545	1.2258	1.3943
age to 30	**1**	3.1276	3.1649	3.1423
	2	1.4797	1.4793	1.2043
age 31 to 45	**1**	2.4628	2.6222	2.2627
	2	0.9573	1.1409	0.5699
age 46 to pension age	**1**	0.8754	0.8776	0.8658
	2	0.1253	0.2616	-0.0496
Haredi	**1**	2.7582	3.2896	3.3314
	2	1.031	1.1475	1.12
family size*	**1**	1.3192	1.6184	1.4067
	2	0.9898	1.1099	0.9494
employment*	**1**	1.5454	2.3952	2.0063
	2	0.3094	0.7831	0.456
social benefit dependence*	**1**	-4.012	-3.5035	-4.3111
	2	-1.3053	-0.9324	-1.515

* The value of 'family size' is 1 for families larger than 4 and for the variable of dependence on social benefits the value is 1 if the share of social benefits exceeds half the income. The value of the employment variable is 1 if the employment of adults is below half the potential.

The regressions show for example that being a young, relatively low educated Haredi (Jewish Ultra-orthodox) male is associated with a high odds ratio of being associated with the low income group. The chances of a Haredi as compared to those of a non-

[30] The year 1998 was chosen as a representative year for the period preceding the harsh social-policy reform. In 2004 the social-policy reform's effects dominated and in 2008 the dominating effect on incomes was rapid economic growth (2005-3rd quarter of 2008).

orthodox Jew to be in the lowest income group rather than in the middle class are 9 times higher for the former.

Table 5. Multinomial Logit Model: Odds ratios

	X1	X2	X3	X4	X5	X6
Intercept	1	1	1	1	1	1
Arab	1	0	0	0	1	1
New Immigrant since 2000	0	0	0	0	0	0
8 years of schooling	1	1	1	1	0	0
9 to 12 years of schooling	0	0	0	0	1	1
age to 30	1	1	1	1	0	0
age 31-45	0	0	0	0	1	1
age 46 to Pension	0	0	0	0	0	0
haredi	0	0	1	1	0	0
family size*	1	1	1	1	1	1
employment*	1	1	1	0	1	1
social benefit dependence*	1	1	1	1	1	0
odds ratio	X1/X2		X3/X2	X4/X2		X6/X5
Conditioned on the event that Y equals either 1 or 3, then the odds for observing 1 rather than 3 are exp(beta(1)'x	37.0		28.0	3.8		74.5
Conditioned on the event that Y equals either 2 or 3, then the odds for observing 2 rather than 3 are exp(beta(2)'x	5.5		3.1	1.9		4.5
Conditioned on the event that Y equals either 1 or 2, then the odds ratio for observing 1 rather than 2 are exp((beta(1)-beta(2))'x	6.7		9.1	0.2		16.4

* The value of 'family size' is 1 for families larger than 4 and for the variable of dependence on social benefits the value is 1 if the share of social benefits exceeds half the income. The value of the employment variable is 1 if the employment of adults is below half the potential.

His chances of being in the top class are almost zero. This result is consistent with the results known from the poverty analysis for Israel, according to which their poverty incidence is very high.[31] This effect is mitigated with an increased employment effort.[32] Being Arab yields a similar result. As expected, risk is also negatively associated with

[31] See official poverty reports at www.btl.gov.il

[32] Distinctly from other poor groups, the Haredis' low labor force participation as well as the high number of children reflect to some extent a self-conscious choice.

age and exposure to welfare funds.[33] On the other hand, labor force participation and small family size (as a ratio) increase the chances of belonging to a higher income group. The estimated coefficients are remarkably stable during the years 1998, 2004 and 2008, especially for the first regression (indicated by "1" next to the variable name). The intuitively sensible results arising from this analysis support the consistency of the model's suggested allocation to income groups.

4. CONCLUSIONS

We propose an alternative measure of polarization to the main existing ones. On the one hand it fits easily into the framework suggested by the existing measures, since it explicitly includes an Identification-Alienation index as proposed by the axiomatic approach (Esteban and Ray, 1994). We prefer the use of a purely statistical measure, built on the variance, rather than on measures satisfying axioms in the realm of welfare measures, such as for example the transfer axiom. Our preference is based on the view that polarization should be treated as a positive rather than normative measure, keeping it non-weighted (or non-ranked as distinct for example from the Gini measure, which constitutes part of some of the polarization measures suggested in the literature). In a political economic context this is related to the question whether it is important that the measure should capture small classes of the super-rich as it is to capture changes in the poorest class. Raising the weight of the poor in the index, such as by use of the ranking characteristic in the Gini-index might obscure the importance of the super-rich for polarization. For example democratic decision making by the super-rich may be jeopardized by the existence of super-rich families, which may influence politicians'

[33]We found the social benefits policy to affect the Haredi population more strongly than the Arab population, possibly due to their higher dependence on social benefits due to the Haredi men's low labour force participation.

decisions through their financial support. This implies that the presence of the super-rich may well exacerbate polarization, an effect that may get lost when the normative aspect of the measure is stressed (by attaching an ever lower weight, the richer the family).

Furthermore the present index, being bounded between 0 and 1, makes it a scale-free and thus suitable index for comparisons over time and space.

The present index is able to capture changes in the number of significant groups and their size distribution thus making it possible to empirically test the phenomenon of the "shrinking or vanishing middle class".

The Israeli economy is a useful study ground for polarization, given the high heterogeneity of its society. The sharp economic fluctuations during the observation period – from rapid growth to a severe recession in 2002/3 – and back to renewed rapid growth during four and a half years thereafter, as well as the harsh and mostly permanent shift in social policy make the Israeli data a particularly interesting case of studying polarization. The analysis suggests that economic growth increased the chances of 'mobility' from a lower class to a higher one. Casual observation suggests that this effect is very weak in the case of the lower class. Our indicator incorporates the 'shrinking middle class effect' by two indicators – the identification-alienation measure and the indicator measuring clustering around extreme values of the income distribution. Their combined effect makes the observation of this important effect more gradual and thus more easily observable. This effect coincides with the flattening of the overall income distribution, observed in the Israeli context. This increase in dispersion is also manifest in the outward shift of the distribution at the high end of incomes.

The identification-alienation index, which is at the heart of many polarization measures, showed an increase from 2001 to 2004, coinciding with the harsh socio-economic policy. It dropped with the acceleration of economic growth (2005 to 2007). The Multinomial Logit analysis reveals that polarization analysis can be enriched by explaining income class membership by use of various characteristics, such as ethnic, cultural and other demographic and individual characteristics. Belonging to the Haredi (Jewish Ultra-orthodox) community sharply raises their probability of belonging to the low income group, as expected also from the poverty analysis for Israel. Being Arab yields a similar though less pronounced result. The Arabs' income performance has been improving, especially since their average family size has been decreasing lately. As expected, risk is also negatively associated with age and exposure to welfare funds. On the other hand, labor force participation and small family size (as a ratio) increase the chances of belonging to a higher income group. The results support the quality of the model's predictions of group membership.

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APPENDIX

Figure A.1. Global pdf and group pdfs - 1997

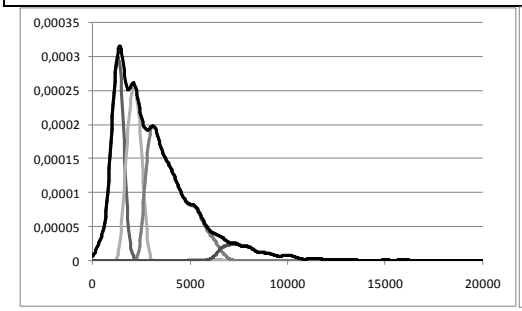

Figure A.2. Global pdf and group pdfs - 1998

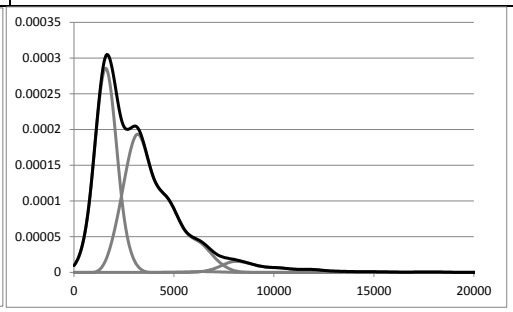

Figure A.3. Global pdf and group pdfs – 1999

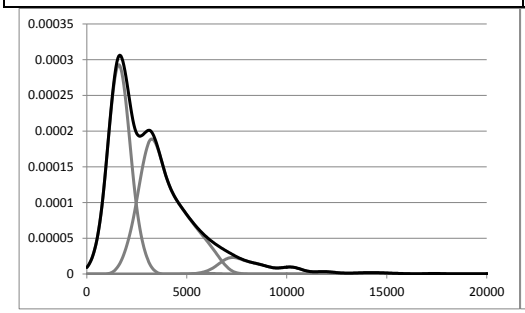

Figure A.4. Global pdf and group pdfs - 2000

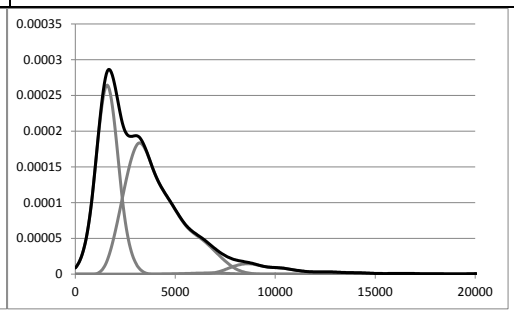

Figure A.5. Global pdf and group pdfs – 2001

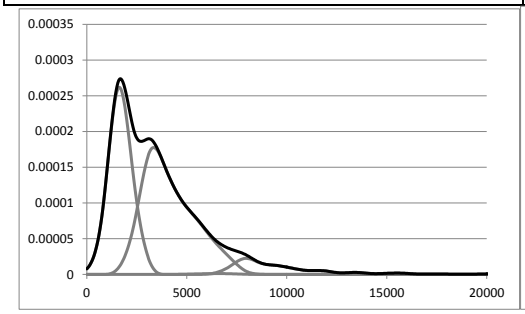

Figure A.6. Global pdf and group pdfs - 2002

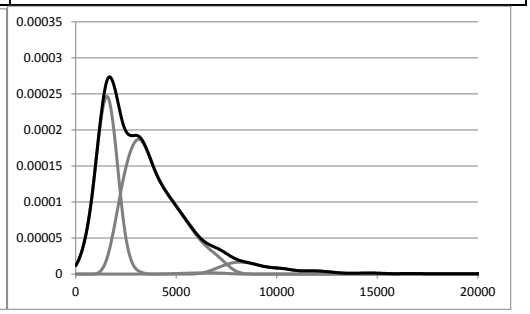

Figure A.7. Global pdf and group pdfs – 2003

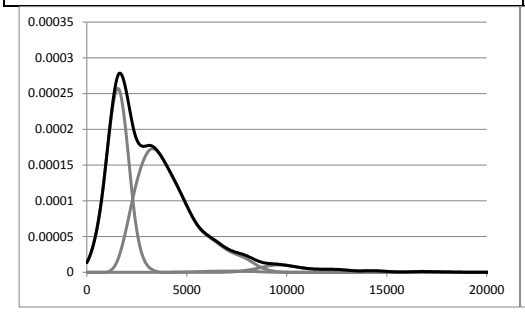

Figure A.8. Global pdf and group pdfs - 2004

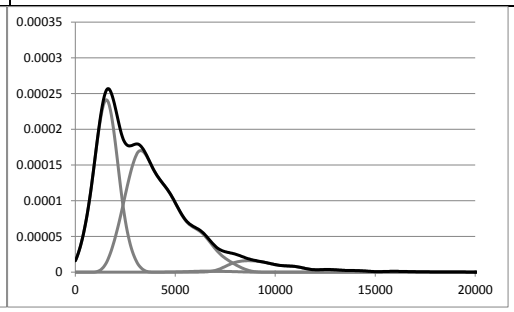

Figure A.9. Global pdf and group pdfs - 2005

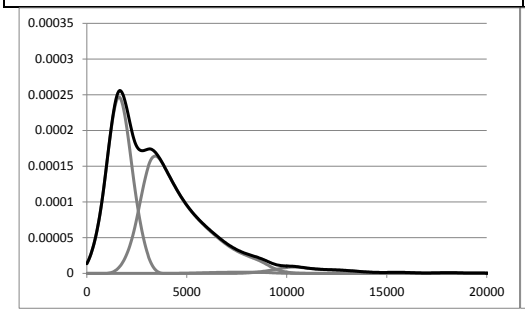

Figure A.10. Global pdf and group pdfs - 2006

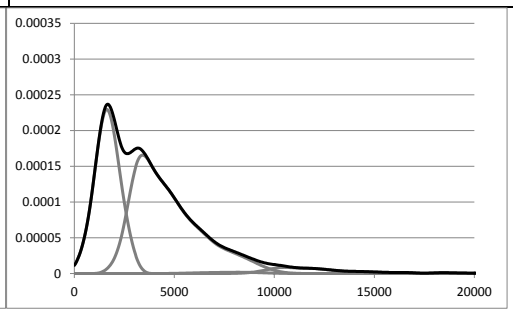

Figure A.11. Global pdf and group pdfs - 2007

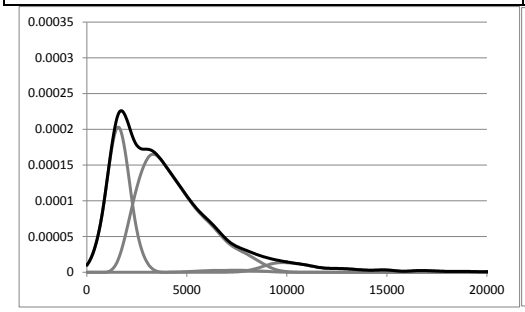

Figure A.12. Global pdf and group pdfs – 2008 (three groups)

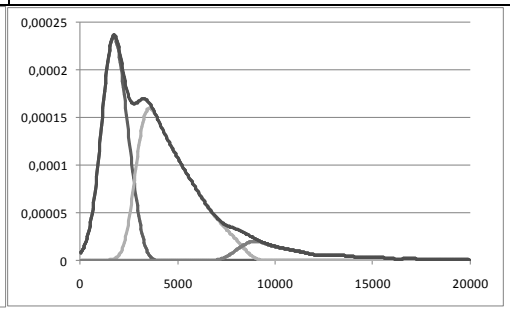

Figure A.13. The components of the polarization index

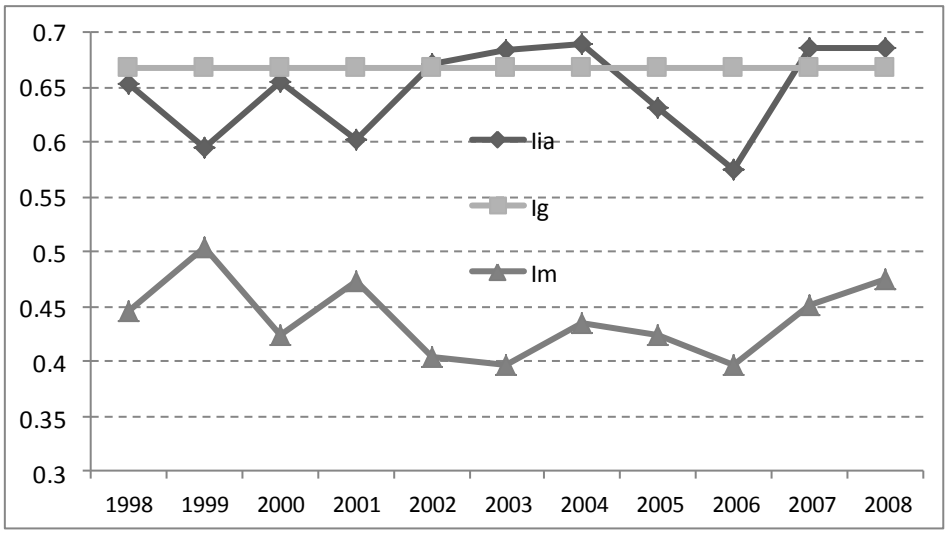

Table A.1: Net equivalised mean income by income groups, 2006 prices

	Lower class	Middle Class	Top class	Overall average	Relative income: Upper versus lower class	Relative income: Middle versus lower class
1997	1,181	3,084	9,266	3,263	7.8	2.6
1998	1,535	4,007	10,300	3,324	6.7	2.6
1999	1,571	3,981	9,925	3,406	6.3	2.5
2000	1,540	4,149	10,940	3,523	7.1	2.7
2001	1,635	4,243	10,565	3,683	6.5	2.6
2002	1,455	4,021	10,536	3,519	7.2	2.8
2003	1,463	4,245	11,850	3,505	8.1	2.9
2004	1,482	4,256	10,850	3,634	7.3	2.9
2005	1,590	4,686	13,496	3,755	8.5	2.9
2006	1,605	4,785	14,170	3,989	8.8	3.0
2007	1,650	4,560	11,744	4,112	7.1	2.8
2008	1,728	4,725	11,853	4,139	6.9	2.7
Average of yearly ratios of the means					7.4	2.8

Table A.2: The weights of the classes in the tri-polar period

	lower class	middle class	upper class	sum of weights
1998	0.410	0.538	0.052	1.000
1999	0.431	0.491	0.078	1.000
2000	0.383	0.562	0.055	1.000
2001	0.401	0.522	0.077	1.000
2002	0.353	0.585	0.062	1.000
2003	0.376	0.584	0.040	1.000
2004	0.378	0.557	0.065	1.000
2005	0.409	0.553	0.038	1.000
2006	0.374	0.584	0.042	1.000
2007	0.366	0.549	0.086	1.000
2008	0.393	0.525	0.083	1.000

Table A.3: The Polarization measure and its components

	Iia	Ig	Im	P
1998	0.652	0.667	0.446	0.194
1999	0.596	0.667	0.504	0.2
2000	0.655	0.667	0.425	0.185
2001	0.603	0.667	0.473	0.19
2002	0.671	0.667	0.405	0.181
2003	0.683	0.667	0.396	0.18
2004	0.688	0.667	0.434	0.199
2005	0.63	0.667	0.424	0.178
2006	0.575	0.667	0.397	0.152
2007	0.686	0.667	0.451	0.206
2008	0.685	0.667	0.474	0.217

Table A.4 The Polarization measure and other tri-polar measures

	P	Interval of Conf. 95%		ZK	Interval of Conf. 95%	
1998	0.194	0.144	0.250	3.107	2.331	4.104
1999	0.200	0.143	0.246	3.199	2.297	4.027
2000	0.185	0.142	0.243	2.983	2.260	3.954
2001	0.190	0.140	0.240	3.149	2.218	3.885
2002	0.181	0.139	0.237	2.834	2.173	3.820
2003	0.180	0.137	0.234	2.789	2.124	3.758
2004	0.199	0.134	0.232	3.203	2.071	3.701
2005	0.178	0.132	0.230	2.773	2.014	3.648
2006	0.152	0.129	0.228	2.604	1.952	3.598
2007	0.206	0.126	0.226	3.427	1.887	3.553
2008	0.217	0.123	0.225	3.163	1.817	3.512

	ZKN	Interval of Conf. 95%		ER	Interval of Conf. 95%	
1998	0.757	0.703	0.830	0.213	0.198	0.235
1999	0.762	0.700	0.824	0.218	0.198	0.234
2000	0.749	0.697	0.818	0.212	0.198	0.234
2001	0.759	0.694	0.813	0.214	0.199	0.233
2002	0.739	0.690	0.808	0.208	0.199	0.233
2003	0.736	0.686	0.803	0.218	0.199	0.233
2004	0.762	0.682	0.798	0.219	0.199	0.233
2005	0.735	0.677	0.794	0.231	0.199	0.233
2006	0.723	0.672	0.790	0.224	0.199	0.233
2007	0.774	0.667	0.786	0.217	0.198	0.233
2008	0.760	0.662	0.783	0.214	0.198	0.233

Table A.5. An Index (1998=1) of the measures in table A.3 during the Tri-polar period

	P	ZK	ZKN	ER
1998	1	1	1	1
1999	1.032	1.029	1.007	1.024
2000	0.956	0.960	0.990	0.996
2001	0.981	1.013	1.003	1.004
2002	0.934	0.912	0.977	0.977
2003	0.931	0.898	0.973	1.021
2004	1.028	1.031	1.007	1.029
2005	0.920	0.893	0.972	1.083
2006	0.786	0.838	0.955	1.049
2007	1.064	1.103	1.023	1.017
2008	1.118	1.018	1.004	1.006