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Yes, We Should Discount the Far-Distant Future at Its Lowest Possible Rate: A Resolution of the Weitzman–Gollier Puzzle

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Abstract

In this paper the author proves that the Expected Net Future Value (ENFV) criterion can lead a risk neutral social planner to reject projects that increase expected utility. By contrast, the Expected Net Present Value (ENPV) rule correctly identifies the economic value of the project. While the ENFV increases with uncertainty over future interest rates, the expected utility decreases because of the planner's desire to smooth consumption across time. This paper therefore shows that Weitzman (1998) is "right" and that, within his economy, the far-distant future should be discounted at its lowest possible rate.

JEL: D61, E43, G12, G31, Q51

Keywords: Discount rates; term structure; capital budgeting; interest rate uncertainty; environmental planning

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1 Introduction

In this paper I show that the Expected Net Future Value (ENFV) investment criterion is not necessarily consistent with expected utility maximisation. If it is chosen as the capital budgeting technique by risk neutral social planners then this can lead to them rejecting projects that increase expected utility. By contrast, the Expected Net Present Value (ENPV) criterion is always consistent with expected utility maximisation and therefore this should be the preferred technique. As a consequence, when future interest rates are persistent and unknown, the term structure of social discount rates should be downward sloping. This is consistent with recommendations contained within the UK Treasury Guidance on Appraisal and Evaluation in Central Government (the "Green Book"). The French government also requires that public institutions apply lower discount rates once the cash flow maturity becomes more than thirty years (Gollier 2009a). Furthermore, the findings in this paper support the analysis contained within the Stern Review (2007), where a baseline social discount rate of around 1.4% is used to evaluate the future costs of climate change even though many authors have noted that this is substantially below shorter-term market rates of return (for example, Nordhaus 2007, Weitzman 2007 and Dasgupta 2008).[1]

The theoretical justification for using a downward sloping term structure of social discount rates is given in Weitzman (1998, 2001) in an economy where policy makers are risk neutral but cannot perfectly forecast future interest rates. He shows that, through a Jensen's inequality effect, cost of capital uncertainty increases the ENPV of future cash flows. When interest rates are highly persistent, this leads to a sharply declining schedule of social discount rates. Calibration of the term structure for more realistic interest rate processes have been constructed by Newell and Pizer (2003, 2004), Guo et al. (2006), Groom et al. (2007) and Gollier et al. (2008). They demonstrate that this effect is of economic significance for far horizon projects and generates important policy implications for the evaluation of long term environmental and energy projects. Tackling climate change, for example, becomes a more urgent priority while using nuclear power as a means to reduce greenhouse gas emissions is relatively less attractive as an option given the future costs of decommissioning.

As a response to this, Gollier (2004) has extended a paradox that can be traced back to Pazner and Razin (1975). He proves that the recommendation that low discount rates should be applied at far horizons is highly sensitive to the chosen investment appraisal technique. If policy makers were to put money aside today to deal with environmental problems in the future, then again the effect of Jensen's inequality means that the expected future value of this saving increases with cost of capital uncertainty. As a consequence, this implies that policy makers, using an ENFV criterion, should give lower priority today to climate change abatement programmes. This apparent sensitivity of the optimal policy decision concerning long term initiatives to the method of capital appraisal is sometimes referred to as

[1]The relationship between the long-term discount rates recommended in the Green Book and those used in the Stern Review is discussed in the UK Treasury's supplementary guidance on intergenerational wealth transfers (HM Treasury 2008).

the Weitzman-Gollier puzzle.

In this paper, I resolve this paradox. Previous interesting proposed resolutions have been presented by Hepburn and Groom (2007), Gollier (2009a, 2009b) and Buchholz and Schumacher (2008). This paper makes a number of important extensions to this debate. In particular, I show that within the setting of Weitzman's (1998) paper, a fixed future cost becomes unambiguously more unattractive as interest rate uncertainty increases. As a consequence, it is the Weitzman ENPV that is the correct evaluation criterion and not the Gollier ENFV criterion. This contrasts with Gollier (2004, p.88): "Clearly, Weitzman and I cannot be both right. In fact, to tell the truth, I believe that we are both wrong", Hepburn and Groom (2007, p.107): "Our conclusion, perhaps surprisingly, is that Weitzman and Gollier are both right", Gollier (2009b, p.6): "This demonstrates that, as suggested by Hepburn and Groom (2007), both Weitzman (1998) and Gollier (2004) are right", Gollier (2009a, p.8): "In a sense, contrary to our conclusion in Gollier (2004), both Weitzman (1998) and Gollier (2004) are right..." and Buchholz and Schumacher (2008, p.4): "Much more is in favor of Gollier's approach because he puts the risk to the right place, i.e. to the future period".

This paper also makes methodological improvements on previous explanations for the puzzle. In contrast to Gollier (2009a, 2009b) and Buchholz and Schumacher (2008), the social planner remains risk neutral within the economy of this paper. This is consistent with the original paradox and shows that there is no requirement to call on risk aversion to resolve the problem. Further, in contrast to Hepburn and Groom (2007), there is no need to introduce arbitrary evaluation dates to reconcile the different approaches and show that policy makers are correct to use declining schedules of social discount rates.

The paper proceeds as follows. Section 2 briefly describes the paradox and the resolution proposed by Hepburn and Groom (2007). Section 3 develops the new proposed resolution, shows that the ENPV and ENFV criteria can be reconciled and demonstrates why the ENPV method is correct. Section 4 explains why the ENFV method cannot be used to evaluate projects when interest rates are stochastic. Section 5 concludes.

2 The puzzle

The Weitzman-Gollier puzzle arises in the following, highly stylised, economy. At time $-\delta$, the future short-term interest rate $\widetilde{r}$ is unknown but lies in the range $[r_{\min}, r_{\max}]$. The true interest rate, $\widetilde{r} = r$, will be revealed in the next instant, time 0, and then never change again.

For expected net present values (ENPV), a risk-neutral social planner contemplates spending p at time 0 to avoid a fixed cost, D_T, that will otherwise arise with certainty at time T. As all uncertainty is resolved at time 0, the planner values the proposal at this time using a discounted cash flow technique with the cost of capital equal to the risk-free rate; $NPV_0 = -p + D_T e^{-rT}$. At $-\delta$, through risk neutrality, the expected net present value is:

$$ENPV = -p + D_T E\left[e^{-\widetilde{r}T}\right] \tag{1}$$

The $T-$period discount rate, $r_d(T)$, is defined by:

$$e^{-r_d(T)T} = E\left[e^{-\widetilde{r}T}\right] \tag{2}$$

Under the expected net future value (ENFV) criterion, p has already been put aside to deal with a potential threat. The social planner is considering taking this money away from the preventative measure and investing it in a rolling portfolio of Treasury bills instead. The proceeds of this investment strategy will be used to deal with the threat, which results in a certain cost D_T, when it arises. The incremental cash flows from this change of strategy are zero at time 0 and $pe^{rT} - D_T$ at time T. At time $-\delta$, the expected net future value is given by:

$$ENFV = pE\left[e^{\widetilde{r}T}\right] - D_T \tag{3}$$

and the $T-$period compound rate, $r_c(T)$, is defined by:

$$e^{r_c(T)T} = E\left[e^{\widetilde{r}T}\right] \tag{4}$$

The paradox arises from differences between $r_d(T)$ and $r_c(T)$. As Gollier (2004) explains, (2) and (4) can be interpreted as exercises in exponential utility. $r_d(T)$ is the certainty equivalent of $\widetilde{r}$ when a pseudo-investor has a constant coefficient of absolute risk aversion T. As T gets larger, so the risk aversion of the pseudo-agent increases and $r_d(T)$ decreases. In the limit, as $T \to \infty$, the pseudo-agent becomes infinitely risk averse and $r_d(T) \to r_{\min}$. By contrast, $r_c(T)$ is the certainty equivalent of $\widetilde{r}$ when a pseudo-investor has a coefficient of absolute risk aversion $-T$. The pseudo-investor now becomes increasingly risk seeking with growing T and $r_c(T) \to r_{\max}$ in the limit.

Weitzman (1998, 2001) uses this argument in relation to $r_d(T)$ to contend that low discount rates should be applied to far-horizon costs, raising their perceived net present value. Newell and Pizer (2003, 2004), Guo et al. (2006), Groom et al. (2007) and Gollier et al. (2008) calibrate interest rate models to show that this effect can be of major economic significance. These recommendations currently influence both British and French governments' advice on social discounting.

Gollier (2004), by contrast, uses the argument in relation to $r_c(T)$ to contend that, if we start saving today to deal with threats in the future, then interest rate uncertainty increases our expected future wealth to deal with the problem when it arises. Equation (1) suggests that, with rising interest rate uncertainty, we should place more money today into preventing future environmental costs while (3) suggests that we should simultaneously take money away from similar existing projects and invest in financial assets instead. This is the puzzle.

This paradox had previously been recognised by Pazner and Razin (1975). It is also a restatement of a well-known result of Cox, Ingersoll and Ross (1981) that the local expectations hypothesis of the term structure of interest rates is inconsistent with the returns-to-maturity expectations hypothesis. If $B_{-\delta T}$ is the time $-\delta$ price of the default risk-free zero coupon bond, then Cox, Ingersoll and Ross (1981) show that:

$$\begin{array}{rclcrcl} & B_{-\delta T} & = & E[e^{-\widetilde{r}T}] & \text{i.i.w} & B_{-\delta T}^{-1} & = & E[e^{\widetilde{r}T}] \\ \Longrightarrow & 0 & = & E[e^{-\widetilde{r}T}] - B_{-\delta T} & \text{i.i.w} & 0 & = & B_{-\delta T}E[e^{\widetilde{r}T}] - 1 \end{array} \tag{5}$$

where "i.i.w" reads as "is inconsistent with". The right-hand sides of these two equations are respectively the ENPV and ENFV criteria (1) and(3) with $p = B_{-\delta T}$ and $D_T = 1$.

Hepburn and Groom (2007) propose a resolution. Their certainty equivalent discount rate, $r_{ca}(T, \tau)$ depends on both the horizon of the threat and an evaluation date, τ, and is defined by:

$$e^{-(T-\tau)r_{ca}(T,\tau)} = E[e^{-r(T-\tau)}] \tag{6}$$

This measure nests $r_d(T)$ when $\tau = 0$ and $r_c(T)$ when $\tau = 2T$. Now the coefficient of absolute risk aversion of the pseudo-investor is $T - \tau$. The appropriate cost of capital is decreasing in T, as with ENPV, but increasing in τ, as with ENFV. This analysis, though, provides no insights into the appropriate evaluation date and thus cannot objectively judge between the ENPV and ENFV criteria. I propose an alternate resolution to the paradox that overcomes these limitations.

3 Resolving the puzzle

In this section, I resolve the puzzle using two separate approaches. First, by using a utility of consumption argument, I show that the ENPV and ENFV criteria are consistent when agents are risk neutral. They agree, both qualitatively and quantitatively, that a fixed future cost becomes more unattractive as interest rate uncertainty increases. While the ENFV measure appears to show that cost of capital uncertainty increases the expected future wealth of an investor who saves today to spend in the future, here it is proved that such a strategy decreases expected future utility. This decline in expected utility with increased expected future wealth is not caused by risk aversion but instead by the desire of agents to smooth consumption intertemporally. I then demonstrate the consistency between the ENPV and ENFV approaches within the discounted cash flow setting of Ang and Liu (2004).

3.1 A utility-based approach

In this subsection, a utility based approach is taken to show that when agents are risk neutral (i) increased interest rate uncertainty unambiguously makes a fixed future cost more unattractive, (ii) that the ENFV and ENPV strategies have the same expected utility and (iii) that the ENPV investment criterion is consistent with expected utility maximisation while the ENFV criterion is not.

In this paper, I endogenise the interest rate uncertainty by calling on the pure exchange economy of Lucas (1978). The representative agent has an exogenous income stream of the single consumption good and this must be consumed immediately or it perishes. As a consequence, consumption in this model is exogenous but the risk-free rate adjusts to ensure that financial markets remain in equilibrium.

It is assumed that time zero consumption is fully known to the social planner at time $-\delta$, but that future consumption is not. Volatility is introduced into this economy by having a stochastic future income stream, which immediately translates to stochastic future consumption and interest rate uncertainty.

Assume that the representative agent has an intertemporal welfare function similar to that described by Gollier (2002):

$$\sum_{t=0}^{\infty} e^{-\rho t} u(m_t) \tag{7}$$

where $u(\cdot)$ is monotonic increasing and strictly concave and ρ is the constant time preference factor of utility. This function captures the agent's desire to smooth consumption across time. m_t is the certainty equivalent of consumption at time t and is defined by:

$$v(m_t) = E_0[v(\widetilde{c}_t)] \tag{8}$$

$\widetilde{c}_t$ is the agent's consumption at time t of the single consumption good, which is exogenously determined by aggregate output at this time. $v(\cdot)$ captures the agent's aversion to instantaneous risk and, when $v(\cdot) \equiv u(\cdot)$, the utility function becomes time separable. To capture risk neutrality, here $v''(\cdot) = 0$, so that $m_t = E_0[\widetilde{c}_t] = \overline{c}_t$. While Gollier (2009a, 2009b) and Buchholz and Schumacher (2008) also take a utility-based approach to address the paradox, their models are based within economies where the agent is risk averse. This is not consistent with the original puzzle, which is set in the context of a risk neutral social planner. Here the required curvature in the utility function is explicitly generated through $u(\cdot)$ rather than $v(\cdot)$, thus demonstrating that the puzzle arises from the desire to smooth consumption across time rather than across states.

Let B_{0T}, $R(T)$ denote respectively the time 0 price and yield of a default risk-free zero-coupon bond that matures at time T with a face value of \$1. By a standard Euler equation argument:

$$e^{-R(T)T} = B_{0T} = e^{-\rho T} u'(\overline{c}_T)/u'(c_0) \tag{9}$$

As interest rates are non-stochastic after time 0, the local expectations hypothesis can be invoked, so that $R(T) = r$ for all T:[2]

$$e^{-rT} = e^{-\rho T} u'(\overline{c}_T)/u'(c_0) \tag{10}$$

[2]In this setting interest rates are endogenous. That the interest rate will be constant after time zero is ensured by revealing to the agent that the consumption process will be generated in such a way that $u'(E_{t-1}(c_t))/u'(c_{t-1}) = u'(E_0(c_1))/u'(c_0)$ for all future t. At time $-\delta$, c_0 has already been revealed and so is non-stochastic. However, the agent is expecting important information to arrive in the next instant so that $E_0(c_1) \neq E_{-\delta}(c_1)$. It is this uncertainty about next period's expected consumption that drives the stochastic interest rate through the strict concavity of $u(\cdot)$ (not $v(\cdot)$). That the interest rate varies with $\overline{c}_T$ in this risk neutral pure-exchange economy contrasts with the situation in Cox, Ingersoll and Ross (1981), where the utility is time separable.

Gollier (2009b: equation 6) presents a similar result from a different economic framework. As interest rates are driven by the marginal utility of future expected consumption, high payouts from a strategy of investing in a rolling portfolio of Treasury bills are expected to occur at times when the consumption good gives low additional utility. By concentrating on wealth alone, the ENFV approach fails to reflect this.

Providing that the potential future threat is sufficiently small in relation to $\bar{c}_T$, the change in utility, Δu, from undertaking an ENFV strategy that results in an incremental future cash flow of $pe^{rT} - D_T$ is given by a first order Taylor's expansion:

$$\Delta u = e^{-\rho T}\left(pe^{rT} - D_T\right)u'(\bar{c}_T) \tag{11}$$

Substituting from (10):

$$\begin{aligned}\Delta u &= \left(pe^{rT} - D_T\right)e^{-rT}u'(c_0) \\ &= \left(p - D_T e^{-rT}\right)u'(c_0)\end{aligned} \tag{12}$$

The expected change in utility one instant earlier is:

$$\begin{aligned}E\left[\Delta u\right] &= \left(p - D_T E\left[e^{-\tilde{r}T}\right]\right)u'(c_0) \\ &= \left(p - D_T e^{-r_d(T)T}\right)u'(c_0)\end{aligned} \tag{13}$$

As $r_d(T)$ can be interpreted as the certainty equivalent of $\tilde{r}$ for an investor with coefficient of absolute risk aversion T, so $r_d(T)$ decreases as interest rate uncertainty increases, all else being equal. So, while the ENFV measure increases as interest rates become more uncertain, the associated expected change in utility decreases. This demonstrates the weakness of the ENFV approach, the limitations of which are discussed in more detail in the next section.

Turn next to the ENPV strategy of spending an amount p at time 0 to save D_T at time T. In this case, the change in utility is:

$$\begin{aligned}\Delta u &= -pu'(c_0) + D_T e^{-\rho T}u'(\bar{c}_T) \\ \implies E[\Delta u] &= (D_T e^{-r_d(T)T} - p)u'(c_0)\end{aligned} \tag{14}$$

and it follows immediately that rising interest rate uncertainty decreases the incentive to disinvest by exactly the same amount as it increases the incentive to invest in new similar projects. The equivalence between the ENPV and ENFV strategies when evaluated using expected utility is unsurprising. The ENFV approach apparently reveals value by investing in financial markets. However, in an efficient capital market, trading in financial assets always has a zero net effect on expected utility. Therefore the initiative is either attractive or not, irrespective of how it is funded. Gollier (2009b) presents a similar result for a risk averse planner by showing that the ENPV, ENFV and Ramsey discount rates are equivalent when the relationship between interest rates and marginal utility is explicitly modelled. Here, this result is extended to both a risk-neutral framework and to a setting where the project may have a non-zero effect on welfare. This allows for the main result of this paper:

Theorem 1. The ENPV criterion correctly identifies the attractiveness of social initiatives. The ENFV criterion, by contrast, can lead a risk-neutral social planner to reject projects that increase expected utility. ■

Proof. The first statement follows immediately from (14). A project increases expected utility if and only if $p < D_T e^{-r_d(T)T}$ as the ENPV criterion states. In addition, (14) shows that the current economic value of the initiative is $D_T e^{-r_d(T)T} - p$, which is exactly the same as the ENPV. For the second statement, consider a project where $D_T = p \exp\left[\left(r_c(T) + r_d(T)\right) T/2\right]$. We can express $r_c(T) = r_d(T) + \varepsilon/T$ for some $\varepsilon > 0$ when interest rates are stochastic, so $D_T = p \exp\left[r_c(T)T - \varepsilon/2\right]$ $= p \exp\left[r_d(T)T + \varepsilon/2\right]$. Then, from (14) the expected change in utility from this project is:

$$\begin{aligned} E[\Delta u] &= (pe^{r_d(T)T+\varepsilon/2}e^{-r_d(T)T} - p)u'(c_0) \\ &= p\left[e^{\varepsilon/2} - 1\right] u'(c_0) \end{aligned} \tag{15}$$

which is greater than zero. Therefore, the social planner should accept this initiative. However, from Gollier (2004), if she uses the ENFV criterion, a project is accepted if and only if $D_T - pe^{r_c(T)T} > 0$. In this case, according to the ENFV criterion:

$$\begin{aligned} D_T - pe^{r_c(T)T} &= pe^{r_c(T)T-\varepsilon/2} - pe^{r_c(T)T} \\ &= p\left[e^{-\varepsilon/2} - 1\right] e^{r_c(T)T} \end{aligned} \tag{16}$$

and this is negative, leading the social planner to reject a project that increases expected utility. ■

3.2 A discounted cash flow approach

We can also address the puzzle within the discounted cash flow setting of Ang and Liu (2004). Let P_t denote the present value at time t of any future cash flow, V_T, with terminal condition $P_T = V_T$. Define the variable λ_t by:

$$\exp(\lambda_t) = E_t\left[\frac{P_{t+1}}{P_t}\right] = E_t\left[\exp(\widetilde{r})\right] \tag{17}$$

Ang and Liu (2004: equation 17) show by repeated iteration of the single period discounted cash flow equation, that the relationship between V_T, λ_t and the appropriate discount rate to use for the NPV calculation, $R(T)$, is given by:

$$E_{-\delta}\left[V_T\right] \exp\left(-R(T)T\right) = E_{-\delta}\left[V_T \exp\left(-\sum_{t=0}^{T-1} \lambda_t\right)\right] \tag{18}$$

In the Weizman setting, $\lambda_t = r$ for all t as the interest rate is fully revealed next instant. Therefore,

$$\exp\left(-R(T)T\right) = \frac{1}{E_{-\delta}\left[V_T\right]} E_{-\delta}\left[V_T \exp\left(-rT\right)\right] \tag{19}$$

For the ENPV problem, $V_T = D_T$ which is non-stochastic, $R(T) = r_d(T)$ as expected. However, the same discount rate should not be used to discount the cash flow from the Gollier strategy of saving money now to spend later. In this case, $V_T = D_T - p\exp(rT)$, so:

$$\exp(-R(T)T) = \frac{D_T \exp(-r_d(T)T) - p}{[D_T - p\exp(r_c(T)T)]} \tag{20}$$

In this case, as the volatility increases, so $r_d(T)$ declines and $r_c(T)$ increases, pushing up the relevant cost of capital for this project. This is a central feature of the term structure of discount rates that is not often recognised in the literature but has been emphasised by Freeman (2009). Even for risk-neutral investors, the term structure of discount rates is determined by the stochastic characteristics of the cash flow. This is caused by the correlation between V_T and future single period costs of capital, λ_t. For the ENPV strategy, the correlation is zero because D_T is non-stochastic. However, the payoff to the ENFV strategy is highly correlated with the cost of capital by construction, meaning that it should be discounted at a higher rate than D_T.

The present value of the ENFV strategy is

$$[D_T - p\exp(r_c(T)T)]\,\frac{D_T \exp(-r_d(T)T) - p}{[D_T - p\exp(r_c(T)T)]} = D_T \exp(-r_d(T)T) - p \tag{21}$$

and, again, this is just minus the present value of the ENPV strategy confirming that the two methods are equivalent and that it is the ENPV method which is "right".

4 Why is the ENPV criterion better than ENFV?

To explain the weaknesses of the ENFV criterion, I turn to the economic framework of Jacquier et al. (2003, 2005), which is an extension of Blume (1974). Assume that there are two investors $i \in \{1, 2\}$ and that, rather that $\widetilde{r} \in [r_{\min}, r_{\max}]$, each investor has an estimate $\widetilde{r}_i \sim N(r, s_i^2)$. Both investors have unbiased forecasts of the true value but agent 1 is better informed than agent 2; $s_1^2 < s_2^2$. Both investors know that each will invest all their wealth in Treasury bills. Again it is assumed that the true value of r will be revealed in the next instant and then never change again.

For each \$1 that the agents invest, the actual future value at time T will be $\exp(rT)$. However, the ENFV of this investment as calculated by agent i is

$$E[\exp(\widetilde{r}_i T)] = \exp(rT + 0.5 s_i^2 T^2) \tag{22}$$

For both investors, this is an upward biased estimator of the true future value by a multiplicative factor $\exp(0.5 s_i^2 T^2)$. In particular, the ENFV of investor 2 is higher than the ENFV of investor 1 even though they both know that their future portfolio values will be identical. That ignorance increases the ENFV but does not affect the realised future investment value shows that there must be a weakness with this approach.

Jacquier et al. (2003, 2005), following Blume (1974) and Indro and Lee (1997), state that investors should lower their estimate $\widetilde{r}_i$ before estimating the ENFV. In particular, the compounding rate that could be used is:

$$r_c(T) = \widetilde{r}_i - 0.5s_i^2 T \tag{23}$$

Using this rate to calculate the ENFV gives an unbiased estimate of the true future payoff. However, we might set $r_c(T)$ using other criterion. Jacquier et al. (2005) also give an alternate formula for $r_c(T)$ that minimises the mean squared error of future estimated wealth. Within the context of the previous section, $r_c(T)$ could also be set to give an unbiased estimate of the expected future utility of the payoff. In any circumstance, the recommendation that the term structure of compounding rates should be declining with the horizon of the project is well established within the theory and practice of portfolio management.

While the realised future value of an investment portfolio is a function only of the true returns process and therefore does not depend on uncertainty at all, doubt does affect the true NPV. From the Ang and Liu (2004) model, it is clear that the appropriate discount rate depends on the current expectation of future single period costs of capital. These costs of capital are themselves related to the expected single period return at the time. Therefore, when there is uncertainty over the cost of capital, Itô's terms lead to an adjustment of the NPV. This asymmetric role of imperfect knowledge on compounding and discounting is another way of understanding this puzzle.

That the ENPV criterion has to be correct also follows from Cochrane (2001, p.27). He shows that, from the fundamental theorem of asset pricing, if the pricing kernel (stochastic discount factor) is denoted by $\pi_{t,t+j}$:

$$P_t = \sum_{j=1}^{\infty} \frac{E_t[d_{t+j}]}{R^f_{t,t+j}} + \sum_{j=1}^{\infty} cov_t(d_{t+j}, \pi_{t,t+j}) \tag{24}$$

where $R^f_{t,t+j}$=$E_t\left(\pi_{t,t+j}\right)^{-1}$ is the $j-$period risk-free rate, d_{t+j} is the dividend stream and P_t the current economic value of the asset. In the Weitzman (1998) setting, $d_{t+j} = D_T$ when $t + j = T$ and zero otherwise. As D_T is non-stochastic, it is clear that the ENPV approach is the one with the theoretical backing from the asset pricing literature.

5 Conclusion

In this paper I have shown that the ENPV criterion gives decisions that are consistent with expected utility maximisation. If social planners use the ENFV criterion, by contrast, then they may erroneously reject some viable initiatives. The ENFV criterion fails because, while trading in financial assets appears to increase expected future wealth, it does not capture changes in expected utility. Further, the apparent increase in wealth is in itself a mirage; the future value of a portfolio is only determined by the true underlying asset generation process rather than by investors knowledge about the process. This has led Jacquier et al. (2003, 2005) and other to argue that the term structure of compounding rates should be downward sloping.

By coming down so unambiguously on Weitzman's side in this debate, this paper differs from previous resolutions of the puzzle which either conclude that both measures are correct (Hepburn and Groom 2007; Gollier 2009a, 2009b) that both are wrong (Gollier 2004) or that Gollier's approach is to be preferred (Buchholz and Schumacher 2008). This paper also makes important methodological improvements on previous work in this area. In particular, it is shown that the puzzle can be resolved within a risk neutral framework without the need to introduce arbitrary evaluation dates. It is the social planner's desire to smooth consumption across time, rather than across states, that lies at the heart of this paradox.

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