PREFACE TO THE REVISED EDITION

The revisions have been extensive. Most of them made their way meanwhile into the Japanese edition, thanks to the patience of the Japanese translators and publishers. I am much relieved that these improvements are now to go out to readers of English.

The latest major change is the adoption in §23 of an axiom schema which provides in general that a class will exist if its members can be mapped on the ordinals less than some ordinal. In general my policy in the book has been to minimize existence assumptions and to postpone them; but this much compromise proved rewarding. While continuing to defer the assumption of infinite classes, it served to sweep away most of the existence premisses that had formerly cluttered the theorems of §§23-27 and also to simplify a number of proofs. One notable effect of the axiom schema is a proof that there is no last ordinal (new 24.9).

§§25-27 on transfinite recursion were already subject to another major change, prompted by Charles Parsons; see bibliography. He observed that some of my theorem schemata on this subject carried existence premisses that would rule out some of the clearly desirable interpretations of the schematic letters. He showed how to remedy the situation, and I have adopted his central idea.

Parsons also pointed out a second and lesser limitation in my treatment of transfinite recursion, and showed how to overcome it. But it was a limitation that operates only if there is a last ordinal; and this contingency is now excluded, as noted above. So I am not following Parsons on this point.

Another notable revision was instigated by Burton Dreben: the reorganization of §30 on infinite cardinals. Extensions and cor-

rections relating to the axiom of Fundierung and the axiom schema of replacement also have been prompted by Dreben and by Kenneth Brown. Findings by Brown and by Hao Wang have led me also to insert some further points on axioms in §13 and elsewhere. §37 on variants of the theory of types has been improved in the light mainly of findings by David Kaplan; a case of this is the elimination of $(\exists x)T_0x^2$ as axiom at one stage.

Minor improvements have been made throughout the book. Here and there a proof is shortened, a theorem strengthened, a theoretical or clerical error corrected, a space-saving lemma inserted, an obscurity clarified, a historical error corrected or omission supplied, a new event noted. In these connections I am indebted variously to Burton Dreben, John Denton, Jean van Heijenoort, Henry Hiz, Saul Kripke, David K. Lewis, Donald Martin, Akira Ohe, Charles Parsons, Dale R. Samson, Thomas Scanlon, Leslie Tharp, Joseph S. Ullian, and Natuhiko Yosida.

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W.V.Q.

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