PREFACE

THIS book provides an introduction to the differential geometry of curves and surfaces in three-dimensional Euclidean space. We first consider some basic concepts and facts of analytic geometry which will be useful for later investigations. The theory of space curves is presented in the second chapter. We then proceed to the foundations of the theory of surfaces. Problems closely related to the first and second fundamental forms are considered in the third and fourth chapter. Chapter V is devoted to geodesics. Several types of mappings of surfaces which are of theoretical or practical importance figure in Chapter VI, including some mappings of the sphere into the plane which are frequently used when constructing maps of the globe. In this connexion different types of special surfaces occur necessarily. This chapter is therefore related to Chapter VIII on special surfaces. The absolute differential calculus and the displacement of Levi-Cività, which is of interest especially in connexion with the theory of relativity, are investigated in Chapter VII. As is natural the results obtained in Chapters III and IV yield the foundations of the Chapters V–VIII.

In the theory of surfaces we make full use of the tensor calculus, which is developed as needed, cf. Sections 27–33. The student will quickly find that this calculus becomes a simple tool as soon as he is accustomed to the few basic concepts and rules, especially to the 'summation convention', cf. Section 27. He will perceive that the tensor method is helpful in achieving a simplification of the analytic formalism of many investigations. Hence tensors are important tools in modern differential geometry.

The presentation in this book may also be considered as a preparation for the Riemannian geometry of n dimensions.

As is well known, tensors are of increasing importance not only in mathematics, but also in the application of mathematics to physics and engineering. Since the problems treated in differential geometry by means of tensor calculus are relatively perspicuous, they enable us to understand not only the formalism but also the nature and essential background of this calculus. The student will thus gain by being able to apply his knowledge of tensors to fields other than that of differential geometry. In using tensor calculus one should never forget that the purpose of this calculus lies in its applications to certain problems; it is a tool only, albeit a very powerful one.

We should mention that many of the topics considered in this book can also be investigated by means of outer differential forms. Cf. E. Cartan, *Les Systèmes différentiels extérieurs et leurs applications géométriques* (Paris, 1945), W. Blaschke, *Einführung in die Differentialgeometrie* (Berlin, 1950).

In writing this book, the experiences which I gained during the period of personal co-operation with Professor H. Behnke (University of Münster in Westphalia), as well as his printed lecture notes on differential geometry, were of help to me. I have tried to present the whole subject-matter in the simplest possible form consistent with the needs of mathematical rigour, and to convey a clear idea of the geometric significance of the different concepts, methods, and results. For this reason also, numerous figures and examples are included in the text.

In order to lessen the reader's difficulties, especially for those who are approaching differential geometry for the first time, the discussion is relatively detailed. The selection of topics included in this book has been made with great care, consideration being given to the didactic point of view as well as the theoretical and practical importance of the different aspects of the subject.

Problems are included at the end of almost every section, and supplementary problems at the end of the book. These exercises should help the reader to become familiar with the material presented in the text and, what is more important, to get acquainted with the manner of reasoning in differential geometry.

Differential geometry has various relations to other fields of mathematics. Besides the calculus other branches, such as function theory, the calculus of variations, and the theory of differential equations, are also basically important in differential geometry. On the other hand, differential geometry is an essential part of the foundations of some applied sciences, for instance physics, geodesy, and geography. Differential geometry has therefore what we may call a 'general character'; I have tried to stress this point of view in connexion with several topics.

This book is a free translation of my *Differentialgeometrie* which appeared in the series *Mathematik und ihre Anwendungen in Physik und Technik* ('Mathematics and its applications to physics and technical science') (Series A, vol. 25) of the Akademische Verlagsgesellschaft, Geest und Portig, Leipzig, Germany. Some minor changes have been made in the course of translation.

Professor H. Behnke (University of Münster) and Professor H. Graf (Technical University of Darmstadt) have made valuable suggestions to

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E. K.

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Minor corrections have been made. To increase the value of the book as a university textbook, various supplementary problems have been included at the end.

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