Contents

PREFACE ix

1
AFFINE PLANES 3
1.1 The axioms 3
1.2 Examples 7
1.3 A co-ordinate plane 8
1.4 Finite planes 10
Exercises 13

2
COLLINEATIONS 14
2.1 Bijections 14
2.2 Collineations 15
2.3 Fixed elements 19
2.4 Homotheties 21
2.5 Translations 23
2.6 Dilatations 26
2.7 Axial affinities 28
Exercises 30

3
TRANSLATION PLANES 32
3.1 Linear transitivity 32
3.2 The configurations of translation planes 32
3.3 The prime kernel of a translation plane 44
Exercises 49

4
DESARGUESIAN PLANES 52
4.1 The dilatation groups $D(O)$ 52
4.2 The shear theorem 57

4.3 The linear transitivity of the groups $A(a)$ 61
Exercises 64

5
PAPPUS PLANES 66

Appendix 69
Exercises 71

6
CO-ORDINATES IN DESARGUESIAN PLANES 72

6.1 Co-ordinate planes 72
6.2 Co-ordinates in desarguesian planes 74
6.3 The fundamental theorem of affine geometry 80
Exercises 84

7
THE PROJECTIVE CLOSURE OF AN AFFINE PLANE 85

7.1 Motivation 85
7.2 The axioms of a projective plane 86
7.3 The duality principle 88
7.4 Collineations 90
7.5 The canonical duality 95
7.6 Affine restrictions of collineations of projective planes 96
Exercises 99

8
DESARGUESIAN PROJECTIVE PLANES 101

8.1 Projective and affine desarguesian planes 101
8.2 The projective theorem of Desargues 102
8.3 Projective Pappus planes 104
Exercises 105

APPENDIX 107

A.1 Groups 107
A.2 Skew fields 108
A.3 Right vector spaces 108

REFERENCES 111

INDEX 113