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Quality Estimates in Geoid Computation by EGM08

Research article

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Abstract:

The high-degree Earth Gravitational Model EGM08 allows for geoid determination with a resolution of the order of 5′. Using this model for estimating the quasigeoid height, we estimate the global root mean square (rms) commission error to 5 and 11 cm, based on the assumptions that terrestrial gravity contributes to the model with an rms standard error of 5 mGal and correlation length 0.01° and 0.1°, respectively. The omission error is estimated to $-0.7\Delta\bar{g}\ [mm]$, where $\Delta\bar{g}$ is the regional mean gravity anomaly in units of mGal.

In case of geoid determination by EGM08, the topographic bias must also be considered. This is because the Earth's gravitational potential, in contrast to its spherical harmonic representation by EGM08, is not a harmonic function at the geoid inside the topography. If a correction is applied for the bias, the main uncertainty that remains is that from the uncertainty in the topographic density, which will still contribute to the overall geoid error.

Keywords:

EGM08 • Geoid • commission error • omission error

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1. Introduction

The Earth Gravitational Model EGM08 (Pavlis et al. 2008) provides a major step forward in geoid determination by a spherical harmonic model. As its resolution is as high as 5′, one may question how close it is to directly solve for "the 1-cm geoid". Xu and Rummel (1991) derived some basic formulas for the geoid height error propagated from the errors of an EGM, and Pavlis et al. (2005) used a similar technique in their preliminary study of EGM08, indicating that a global root mean square (rms) commission error of the geoid height of 20 cm was attainable. The omission error was not considered. The above studies did not consider correlations in the EGM data, and they can therefore hardly be used for regional studies. Also, the omission error was not considered. Independent evaluations of EGM08 vs. GPS/levelling derived geoid heights show, as one would expect, considerable regional variations in agreement (see Huang and Kotsakis 2009) due to lateral quality variations in both EGM08 and the GPS/levelling data. This study has the goal of investigating the regional variations in EGM08 from a theoretical point of view.

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2. The commission error

We will estimate the commission error a) directly by the error estimates of the EGM08 coefficients, b) by an integral formula and c) by a combination of these two methods.

2.1. Direct computation from potential coefficient error covariance matrix

Let us assume that the geoid height is estimated at the position P on the Mean Earth Sphere of radius R by the finite series of fully-normalized spherical harmonics $Y_{nm}(P)$, given by

$$\widetilde{N} = \frac{R}{\gamma}\sum_{n=2}^{L}\sum_{m=-n}^{n} A_{nm}Y_{nm}(P) = \frac{R}{\gamma}y^{T}a, \qquad (1)$$

where γ is normal gravity on the reference ellipsoid, A_{nm} are the unit-less harmonic coefficients of the disturbing potential with random errors of expectation zero and covariance matrix Q, and y and a are the vectors of the sets Y_{nm} and A_{nm}, respectively. Then the variance of $\widetilde{N}$ becomes

$$\sigma_{\widetilde{N}}^{2} = \left(\frac{R}{\gamma}\right)^{2} y^{T}Qy. \qquad (2)$$

As the Earth Gravitational Model 2008 (EGM08) with L =2160 includes about 4.7 million coefficients, it would be a huge task to compute the covariance matrix needed in Eq. (2), and consequently it is not available today. One may relax this task by considering only the mean of the geoid variance over the sphere. Due to the orthonormality of spherical harmonics when integrated over the sphere, the global average of the geoid variance becomes

$$\bar{\sigma}_{\widetilde{N}}^{2} = \left(\frac{R}{\gamma}\right)^{2}\sum_{n=2}^{L}\sum_{m=-n}^{n} Q_{nm}, \qquad (3)$$

where Q_{nm} are the diagonal elements of Q (i.e. the variances of A_{nm}). Using the standard values R = 6371 km, we obtain $\bar{\sigma}_{\widetilde{N}}$ =29.2 cm for EGM08 complete to L = 2160.

In a similar way we may estimate the global mean gravity anomaly variance by

$$\bar{\sigma}_{\Delta g}^{2} = \gamma^{2}\sum_{n=2}^{L}(n-1)^{2}\sum_{m=-n}^{n} Q_{nm}, \qquad (4)$$

yielding $\bar{\sigma}_{\Delta g}$ = 4.2 mGal and γ =981 Gal.

2.2. Solution by integral formula

The direct error propagation of the gravity anomaly error ε to the geoid error δN is given by Stokes' formula:

$$\delta N = \frac{R}{4\pi\gamma}\iint_{\sigma} S(\psi)\,\varepsilon\, d\sigma. \qquad (5)$$

For uncorrelated gravity anomalies with variance $\sigma_{\Delta g}^{2}$, the variance of the geoid height becomes (Pavlis and Saleh 2005):

$$\sigma_{\widetilde{N}}^{2} = \left(\frac{R}{4\pi\gamma}\right)^{2}\iint_{\sigma} S^{2}(\psi)\,\sigma_{\Delta g}^{2}\, d\sigma. \qquad (6)$$

Unfortunately, this assumption (with uncorrelated gravity anomalies) is not realistic. Mathematically it leads to an unlimited variance of $\widetilde{N}$, because by introducing the notation δ^{2} for the minimum value of $\sigma_{\Delta g}^{2}$ and considering the spectral form of Stokes' function (see Eq. 16 below) and the orthonormality of the spherical harmonics when integrated over the sphere one obtains

$$\iint_{\sigma} S^{2}(\psi)\, d\sigma = \sum_{n=2}^{\infty}\frac{2n+1}{(n-1)^{2}} = \infty, \qquad (7)$$

and therefore

$$\sigma_{\widetilde{N}}^{2} > \left(\frac{R}{4\pi\gamma}\right)^{2}\delta^{2}\iint_{\sigma} S^{2}(\psi)\, d\sigma = \infty. \qquad (8)$$

This problem can, of course, be avoided under the assumption that Stokes' function is band-limited, e.g. to the maximum degree of EGM08, as suggested by Pavlis and Saleh (2005). In this way Pavlis et al. (2005) estimated the global rms geoid height standard error of 20.1 cm. A corresponding estimate for the global rms error in the gravity anomaly determined from the error spectrum of the EGM was 7.0 mGal. However, even so, the neglected correlation of the data may significantly contribute to an erroneous estimate.

2.3. Combined solution

Let us now write Eq. (**??**) by the simplified notations as the sum

$$\widetilde{N} = \widetilde{N}_{1} + \widetilde{N}_{2} \qquad (9a)$$

where

$$\widetilde{N}_{1} = \sum_{n=2}^{M}\widetilde{N}_{n} \quad and \quad \widetilde{N}_{2} = \sum_{n=M+1}^{L}\widetilde{N}_{n} \qquad (9b)$$

i.e., $\widetilde{N}_{1}$ and $\widetilde{N}_{2}$ are the low- and high-degree components, respectively, of the geoid height estimator. Here M is the maximum degree to which the complete error spectrum of the EGM is estimated.

Then the error of $\widetilde{N}$ becomes

$$\widetilde{\varepsilon}_{\widetilde{N}} = \varepsilon_{1} + \varepsilon_{2}, \qquad (10)$$

where ε_{i} denotes the error of the component $\widetilde{N}_{i}$; i = 1, 2. Assuming that the errors are random with expectation zero, simple error propagation yields the variance of the geoid height estimator as

$$\sigma_{\widetilde{N}}^{2} = \sigma_{1}^{2} + \sigma_{2}^{2} + 2\sigma_{12}, \qquad (11)$$

where σ_{i}^{2} are the variances of components $\widetilde{N}_{i}$, and σ_{12} is their covariance.

In practice we can expect the covariance between the low- and high- degree components to be small/negligible, and from now on we omit it in the analysis. The variance component σ_{1}^{2} is directly obtained from Eq. (3), but now limited to degree M<L. σ_{2}^{2} needs further consideration to be estimated.

As $\widetilde{N}_{2}$ is band-limited, its error can be written as the Stokes integral

$$\varepsilon_{2} = \frac{R}{4\pi\gamma}\iint_{\sigma}\Delta S(\psi)\,\varepsilon_{\Delta g}\, d\sigma. \qquad (12a)$$

Here the kernel function is the band-limited Stokes function

$$\Delta S(\psi) = \sum_{n=M+1}^{L} \frac{2n+1}{n-1} P_n(\cos\psi) \tag{12b}$$

and $\varepsilon_{\Delta g}$ is the error of a gravity anomaly generated by the EGM components for degrees $M \le n \le L$, but otherwise arbitrary. (Note that Eq. 12 is blind to other harmonics.) Then it follows from Eq. (12a) that σ_2^2 can be written

$$\sigma_2^2 = \left(\frac{R}{4\pi\gamma}\right)^2 \iint_{\sigma} \iint_{\sigma'} \Delta S(\psi)\, \Delta S(\psi')\, C(Q, Q')\, d\sigma d\sigma', \tag{13}$$

where $C(Q, Q')$ is the covariance matrix for $\varepsilon_{\Delta g}$. By assuming that the covariance of the gravity anomaly errors can be omitted, one obtains also

$$\sigma_2^2 \approx \left(\frac{R}{4\pi\gamma}\right)^2 \iint_{\sigma} \{\Delta S(\psi)\}^2 \sigma_{\Delta g}^2 d\sigma. \tag{14}$$

Pavlis et al. (2004) used the technique of Pavlis and Saleh (2004), where σ_1^2 and σ_2^2 of our Eq. (11) were given by Eqs. (9a) and (14) for L = 2160 and the covariance among the data was omitted. In this way they estimated some regional geoid height commission errors of EGM08. The global rms of the commission error was 20.1 cm. In a similar approach they estimated the global rms commission error of the gravity anomaly computed by EGM08 to 7.0 mGal.

However, the high-degree components of EGM08 is heavily relying on regional gravity and satellite altimetry data, which should be regarded with coloured noise rather than white noise. Hence a better alternative should be to assume that the gravity anomaly error covariance function is homogeneous and isotropic (at least in a regional application), in which case it can be written

$$C(\psi) = \sum_{n=2}^{\infty} \sigma_n^2 P_n(\cos\psi), \tag{15}$$

where σ_n^2 are the gravity anomaly error degree variances. Inserting Eq. (15) into Eq. (13) and considering the spectral forms of Stokes' function,

$$S(\psi) = S(P, Q) = \sum_{n=2}^{\infty} \frac{1}{n-1} \sum_{n=-n}^{n} Y_{nm}(P)\, Y_{nm}(Q), \tag{16}$$

and Eq. (15),

$$C(\psi) = C(P, Q) = \sum_{n=2}^{\infty} \frac{\sigma_n^2}{2n+1} \sum_{m=-n}^{n} Y_{nm}(Q)\, Y_{nm}(Q') \tag{17}$$

as well as the orthogonality of spherical harmonics over the sphere, Eq. (13) reduces to

$$\sigma_2^2 = \left(\frac{R}{\gamma}\right)^2 \sum_{n=M+1}^{L} \frac{\sigma_n^2}{(n-1)^2}. \tag{18}$$

If the geoid height were only determined by gravity anomalies, the variance of the geoid height would be

$$\sigma_N^2 = \left(\frac{R}{\gamma}\right)^2 \sum_{n=2}^{\infty} \frac{\sigma_n^2}{(n-1)^2}. \tag{19}$$

In order to estimate the degree variances σ_n^2 we may consider the simple closed form covariance model for the (full spectrum) gravity anomaly (cf. Sjöberg 1984)

$$C(\psi) = c\left[\frac{1-k}{\sqrt{1-2k\cos\psi+k^2}} - 1 + k - (1-k)\, k\cos\psi\right], \tag{20}$$

where c and k are parameters to be fitted to some data. Expanding Eq. (20) as a series in Legendre polynomials and comparing the terms with those of Eq. (15), it follows that

$$\sigma_n^2 = ck^n (1-k). \tag{21}$$

We may fix the parameters c and k by assuming that the variance and correlation length (ψ_c) of the covariance function are known parameters. (The correlation length is the geocentric distance, whose covariance is 50% of the variance; e.g., Moritz 1980, p. 174.) By inserting $\psi = 0$ and $\psi = \psi_c$ in Eq. (20) one obtains the two new equations

$$\sigma_{\Delta g}^2 = ck^2 \tag{22a}$$

and

$$\frac{1}{2} = \frac{(1-k)/\sqrt{1-2k\cos\psi_c+k^2} - 1 + k - (1-k)k\cos\psi_c}{k^2} \tag{22b}$$

from which the parameters k and c can be determined as follows:

1) Reformulate Eq. (22b) as

$$f(k) = \frac{1-k}{\Omega} - (1-k)(1+kt) - \frac{k^2}{2} = 0, \tag{23}$$

where $\Omega = \sqrt{1-2kt+k^2}$ and $t = \cos\psi_c$,

which yields

$$f'(k) = -\frac{(1-t)(1+k)}{\Omega^3} + 1 - k - t - 2kt. \tag{24}$$

2) Determine k by Newton-Raphson's method, i.e. iterate

$$k_{i+1} = k_i - f(k_i)/f'(k_i);\ i = 0, 1, 2, \ldots \tag{25}$$

until convergence. A suitable start value for iteration could be $k_0 = 1$.

3) Determine c from Eq. (22a).

In Table 1 we summarize the results of the estimated $\sigma(N)$ for some scenarios with gravity anomaly standard errors 3 and 5 mGal and various correlation lengths. (15 iterations were sufficient for convergence of Eq. (25))

Table 1. Resulting parameters k, c and σ_N of Eqs. (14) and (12a) with $n_{max} = 2160$ for various standard errors ($\sigma_{\Delta g}$) and correlation lengths (ψ_c°) of gravity anomaly.

$\sigma_{\Delta g}$ mGal	ψ_c°	k	c	σ_N metre
3	0.001	0.999989	9.00	0.079
	0.01	0.999899	9.0	0.251
	0.1	0.998990	9.02	0.794
	0.5	0.994906	9.09	1.774
5	0.001	0.999990	25.00	0.132
	0.01	0.999899	25.00	0.419
	0.1	0.998990	25.05	1.324
	0.5	0.994906	25.26	2.957

We have to assume that the quality of the high-degree components of EGM08, represented by σ_2 of Eq. (18), varies from area to area over the Earth. If we have some information about the regional variance and correlation length of this data, here expressed as those parameters of the gravity anomaly error, the above method can be used to estimate the error degree variances σ_n^2. For example, by assuming a total gravity anomaly standard error of 5 mGal and correlation lengths 0.01° and 0.1° in determining the error degree variances in Eq. (18) from the above procedure, Eq. (18) yields with M =70 and L =2160 the high-degree commission errors (σ_2) as 3.8 and 11.0 cm, respectively. Hence, by taking the low-degree commission error as 3.8 cm as estimated above the total commission errors are estimated to

$$\sigma_N = \sqrt{3.5^2 + 3.8^2} = 5.1\ cm \quad \text{and} \quad \sigma_N = \sqrt{3.5^2 + 11.0^2} = 11.5\ cm \tag{26}$$

for the two stipulated correlation lengths of the data, respectively. At first glance it seems rather surprising that the contribution from the first terms under the square roots in Eq. (26) is only 3.5 cm. However, it is likely that this is just a manifestation of the high accuracy in the satellite-only part of EGM08. One may also assume that this contribution is rather constant over the surface of the Earth, while the second term under the square roots of Eq. (26) will change considerably from point to point with respect to the quality of the gravity anomaly data included in EGM08.

The commission error discussed above considers only the error propagation in the EGM. When applying it for geoid computations in continental areas it will also be deteriorated by a systematic error (topographic bias) stemming from the erroneous downard continuation of a harmonic function into the topographic masses (Sjöberg 2007) as discussed in Sect. 4.

3. The omission error

The omission/truncation error of EGM08 can be written

$$\delta N_O = R \sum_{n=n_{max}+1}^{\infty} \sum_{m=-n}^{n} A_{nm} Y_{nm}, \tag{27}$$

yielding the global mean square error

$$\tilde{m}_O^2 = \left(\frac{R}{\gamma}\right)^2 \sum_{n=2161}^{\infty} \frac{c_n}{(n-1)^2}, \tag{28}$$

where c_n are the gravity anomaly degree variances. By considering the gravity anomaly degree variances of Tscherning and Rapp (1974) one obtains the global rms truncation error of 23×10^{-4} mm.

A regional estimate of the truncation error is obtained as follows. First we rewrite the omission error as

$$\delta N_O = \tfrac{R}{4\pi\gamma} \iint_\sigma S^L(\psi)\, \Delta g\, d\sigma = \\ = \tfrac{R}{4\pi\gamma} \iint_\sigma S(\psi)\, \Delta g^L d\sigma = \tfrac{R}{4\pi\gamma} \iint_\sigma S^L(\psi)\, \Delta g^L d\sigma \tag{29a}$$

where

$$S^L(\psi) = S(\psi) - S_L(\psi) = S(\psi) - \sum_{n=2}^{L} \frac{2n+1}{n-1} P_n(\cos\psi) \tag{29b}$$

and

$$\Delta g^L = \Delta g - \sum_{n=2}^{L} \Delta g_n. \tag{29c}$$

Here Δg^L is a reduced gravity anomaly with only high-wavelength components. As $S^L(\psi)$ tapers off quickly with geocentric angle, Eq. (29a) can be approximated by

$$\delta N_O \approx \frac{R}{\gamma} \iint_{\sigma_0} S^L(\psi)\, \Delta g^L d\sigma, \tag{30}$$

and σ_0 is an arbitrary spherical cap. If the cap is selected such that its geocentric angel is equal to or larger than the resolution angle of EGM08 (i.e., about 5′) the missing integral of Eq. (30) can be regarded as negligible. Hence the truncation error can be approximated by

$$\delta N_O \approx \tfrac{R\Delta g^L}{2\gamma} [I_1(\psi_0) - I_2(\psi_0)]. \tag{31a}$$

$$I_1(\psi_0) = \int_0^{\psi_0} S(\psi) \sin\psi d\psi = \int_{t_0}^{1} S(t)\, dt = \\ \int_{t_0}^{1} \left[1 - 5t + y^{-1}(t) - 6y(t) - 3t \ln\left\{y(t) + y^2(t)\right\}\right] dt \tag{31b}$$

and

$$I_2(\psi_0) = \int_0^{\psi_0} S_{n_{\max}}(\psi)\sin\psi d\psi = \\ = \sum_{n=2}^{n_{\max}} \frac{P_{n-1}(\psi_0)-P_{n+1}(\psi_0)}{n-1} \tag{31c}$$

where $y(t) = \sqrt{(1-t)/2}$. For $\psi_0 = 5'$ numerical integration of Eq. (31b) yields $I_1(5') \approx 2.9 \times 10^{-3}$, and the summation in Eq. (31c) for $n_{\max} = 2160$ yields $I_2(5') = 3.1 \times 10^{-3}$. (Actually, the integral I_1 could be computed directly by the closed form expression of Lambert and Darling, 1936.)

Alternatively, one may directly compute Eq. (31a) as the spectral sum

$$\delta N_O \approx \frac{R\Delta\bar{g}^L}{2\gamma} \sum_{n=n_{\max}+1}^{\infty} \frac{P_{n-1}(t_0) - P_{n+1}(t_0)}{n-1}. \tag{32}$$

In any case, for $R = 6371$ km, $\gamma = 981$Gal , one obtains the numerical result

$$\delta N_O \approx -6.9x10^{-2}\Delta\bar{g}^L[\text{cm}] \tag{33}$$

for $\Delta\bar{g}^L$ in units of mGal.

This shows that for $\Delta\bar{g}$ set to 10 and 100 mGal the truncation error becomes -0.7 and -6.9 [cm], respectively. The rms gravity anomaly computed from EGM08 is 10.32 mGal, yielding an rms geoid height truncation error again of about --7 mm. Obviously the truncation error of the geoid height is negligible at the cm-level, unless the magnitude of $\Delta\bar{g}$ exceeds about 15 mGal in the near-zone of the computation point.

Another omission error for geoid height determination is related with the topographic bias. As we will see in the next section, in high mountain areas it becomes much more significant than the direct omission error in the EGM.

4. The analytical continuation error and the topographic bias

4.1. The analytical continuation error

In a strict sense it should not be expected that the representation of the gravity field in an external type spherical harmonic series be convergent inside the Brillouin sphere, i.e. the sphere enveloping all mass of the Earth. This implies also that a truncated series representation, e.g. created by EGM08, should suffer from this error when applied inside this sphere (e.g. Sjöberg 1977). However, although not yet proved, except for the limited numerical study by Jekeli 1981, as long as the series is applied on or outside the Earth's surface, the error is expected to be negligible. Hence, one should not expect this type of error to be significant in quasigeoid determination. One should also keep in mind that the quasigeoid agrees with the geoid over the oceans.

4.2. The topographic bias- commission error

The above effect changes drastically for the continental geoid, being normally located within the topographic masses. Here the analytical continuation error of the series is completely dominated by the topographic potential bias, originating with the fact that the harmonic representation of the series inside the Earth's masses, where the potential is not a harmonic function, is biased (Sjöberg 2007, Sjöberg 2009a and Sjöberg 2009b).

The bias of the geoid height can be written

$$\delta N_{bias}^{n_{\max}} = \frac{2\pi\mu}{\gamma} \sum_{n=0}^{n_{\max}} \sum_{m=-n}^{n} \left[\left(H^2\right)_{nm} + \frac{2}{3R}\left(H^3\right)_{nm} \right] Y_{nm}, \tag{34}$$

where $\mu = G \times \rho$ = gravitational constant times topographic density and $\left(H^v\right)_{nm}$; $v = 2, 3$ are the spherical harmonics of H^v, H being the topographic height (above the geoid). For $n_{\max}$= 2160 and using Pavlis et al. (2006) to create the spherical harmonics of H, the maximum value (in the Himalayas) of the bias is 5.153 m.

4.3. The topographic bias - omission error

The total topographic bias, given by the formula

$$\delta N_{bias} = \frac{2\pi\mu}{\gamma}\left(H^2 + \frac{2H^3}{3R}\right) \tag{35}$$

is significant at the 1-cm level already for the topographic height of about 300 m. For Mt. Everest it becomes 8.983 m.

In computing the geoid height omission error of EGM08 one should also consider the omission error in the topographic bias. This residual topographic bias is given by

$$\Delta\delta N_{bias}^{M} = \frac{2\pi\mu}{\gamma} \sum_{n=n_{\max}+1}^{\infty} \sum_{m=-n}^{n} [\left(H^2\right)_{nm} + \frac{2}{3R}\left(H^3\right)_{nm}] Y_{nm}, \tag{36}$$

or

$$\Delta\delta N_{bias}^{M} = \frac{2\pi\mu}{\gamma}\left(H^2 + \frac{2H^3}{3R}\right) - \delta N_{bias}^{n_{\max}}. \tag{37}$$

As the height of Mt. Everest is 8848 m, the maximum value of this residual bias becomes 8.983-5.153 = 3.830 m.

5. Concluding remarks

We have estimated the rms error of geoid (or quasi geoid) heights as determined by EGM08 to the order of 10 cm, which is completely dominated by the commission error. (The rms omission error is only 1 cm.) Both components vary locally, the commission error mainly with the quality of the gravity anomaly, the omission error with the magnitude of the gravity anomaly. In a region with an extreme gravity anomaly, e.g. of the order of 400 mGal, the omission error may increase to nearly 30 cm.

However, the error of the geoid, estimated in continental regions by EGM08, is affected also by the topographic bias, which ranges to 9 m for Mt. Everest. By applying the simple formula of Eq. (34), the commission error may be reduced but the remaining omission error is still of the order of 3.8 m in the highest mountains. This error may also be practically eliminated by subtracting the total topographic bias according to Eq. (35), and, theoretically, only the error due to the uncertainty in the topographic density remains in the topographic bias.

A related question is whether it is worthwhile to combine the EGM08 data with regional gravity data for improved geoid modelling towards the "1-cm geoid". As concluded above the EGM08 error is generally dominated by the commission error, which suggests that only the high-degree contribution (beyond degree 70) of EGM08 should possibly be down weighted in favour of improved gravity data to be used in Stokes' integration (with an integration cap size of at least 2.5 degrees). In areas with large and variable gravity anomalies, the inclusion of dense terrestrial gravity in a Stokes integration can also significantly reduce the omission error.

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