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Combined approach for the unification of levelling networks in New Zealand
Research article

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Abstract:
The unification of levelling networks in New Zealand is done using a combined approach. It utilises the joint levelling network adjustment and the geopotential-value approach. The levelling and normal gravity data are used for a joint adjustment of the levelling networks at the South and North Islands of New Zealand while fixing the heights of tide gauges in Dunedin and Wellington. The results reveal a good quality of levelling data; the STD of residuals is 2 mm for the whole country. The comparison of the newly determined and original normal-orthometric heights confirms the presence of large local vertical datum offsets and systematic levelling errors. Since the geopotential-value approach is based on the Molodensky's theory, the newly adjusted normal-orthometric heights are converted to the normal heights. This conversion is based on applying the cumulative normal to normal-orthometric height correction computed from levelling and gravity anomaly data. In the absence of the observed gravity data the gravity anomalies along levelling lines are generated from EGM2008. The GPS-levelling data and EGM2008 are used to estimate the average offsets of the jointly adjusted levelling networks at the North and South Islands with respect to World Height System defined by the adopted geoidal geopotential value of $W_0 = 62636856 \pm 0.5\ m^2s^{-2}$; the estimated offsets are 10.6 cm and 27.5 cm.

Keywords:
Local vertical datum • Correction • GPS-levelling data • Gravity • Height • World Height System

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1. Introduction

The geodetic vertical reference system at the North, South and Stewart Islands of New Zealand was realised by 13 major local vertical datums (LVDs) based on precise levelling from 12 different tide gauges. The LVD Dunedin-Bluff 1960 was defined by fixing the heights of two levelling benchmarks from the LVDs Dunedin 1958 and Bluff 1955 instead of using the tide gauge as the origin. Moreover, additional LVDs were established for surveying purposes throughout the country based on precise levelling from tide gauges or connecting to existing levelling networks. For a more detailed overview of the levelling networks in New Zealand we refer readers to Gilliland (1987). The LVDs were defined in the system of the (approximate) normal-orthometric heights. The cumulative normal-orthometric correction to levelled height differences was defined based on the GRS67 normal gravity field parameters and computed approximately using a truncated form of the GRS67 normal-orthometric correction formula (Rapp, 1961). Since LVDs were referenced to the local mean sea level (MSL) determined based on the analysis of tide-gauge records, large discrepancies exist between individual LVDs.

The unification of LVDs can be done either by a joint adjustment of local levelling networks or by a determination of the gravimetric geoid/quasigeoid model and a subsequent combination of gravity and GPS-levelling data. Two methods were recently applied to unify LVDs in New Zealand based on the latter principle, namely the iterative gravimetric approach and the geopotential-value approach. The iterative gravimetric approach utilises an iterative

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determination of the regional gravimetric quasigeoid model and its comparison with the geometric quasigeoid model determined using GPS-levelling data for each LVD. The results of this method are provided in terms of the average LVD offsets relative to the regional quasigeoid model. Amos and Featherstone (2009) applied this method to estimate the LVD offsets relative to the NZGeoid05 quasigeoid model. The estimated LVD offsets relative to NZGeoid05 are between 26 cm (for the LVDs One Tree Point 1964, Nelson 1955, and Dunedin-Bluff 1960) and 59 cm (for the LVD Gisborne 1926). Claessens et al. (2011) used the same approach to estimate the LVD offsets relative to NZGeoid2009 which is the currently adopted official national quasigeoid model for New Zealand (see Amos, 2010). The estimated LVD offsets relative to NZGeoid2009 are between 6 cm (for the LVD One Tree Point 1964) and 49 cm (for the LVD Dunedin 1958). Tenzer et al. (2011) applied the geopotential-value approach to estimate the LVD offsets in New Zealand relative to World Height System (WHS). WHS is defined by the geoidal geopotential value of $W_0 = 62636856 \pm 0.5\ m^2 s^{-2}$ (Burša et al., 1997 and 2007) which is adopted by the International Astronomical Union (IAU). The geopotential differences were computed at the GPS-levelling points using the global geopotential model (GGM) coefficients and then averaged for each LVD. The estimated LVD offsets relative to WHS in New Zealand vary between 1 cm (for the LVD Wellington 1953) and 37 cm (for the LVD One Tree Point 1964). The geopotential-value approach was developed by Burke et al. (1996) and applied by Burša et al. (1999 and 2001) to estimate the average offsets of major LVDs in Europe, North America, and Australia. A similar method was used by Grafarend and Ardalan (1997) and Ardalan and Grafarend (1999) to calculate the LVD offsets in Baltic countries. It is worth mentioning that different values of W_0 were reported by Sanchez (2007) and Dayoub et al. (2011). Sanchez (2007) determined the value of W_0 using different MSL models and different GGMs showing that the choice of MSL and GGM is unimportant for estimating W_0 while the latitude domain of the altimetry-derived MSL models plays a major role. The value of W_0 estimated by Sanchez (2007) differs by 2.5 $m^2 s^{-2}$ from the value adopted by IAU. In a more recent study, Dayoub et al. (2011) reviewed previous studies using various methods and datasets. They confirmed the conclusions of Sanchez (2007) but reported and recommended a different value of $W_0 = 62636854.2 \pm 0.5\ m^2 s^{-2}$ and established that the dependency of W_0 on the latitude domain is merely due to the mean dynamic topography (MDT).

Amos and Featherstone (2009) argued that the practical implementation of the unified vertical datum in New Zealand through a joint levelling adjustment is problematic due to several reasons (e.g., vertical tectonic deformations, sea level variability, short term tide-gauge records, realisation of the levelling networks over several decades and their poor spatial coverage; see also Amos, 2010). Therefore, the official national vertical datum in New Zealand (NZVD2009) is realised based on the vertical reference surface defined by the NZGeoid2009 quasigeoid model. Despite the aforementioned deficiencies, the levelling and GPS data sets provide indispensable information required for the validation of gravimetric geoid/quasigeoid models and other geodetic applications.

In this study, the unification of LVDs in New Zealand is done based on a joint adjustment of local levelling networks at the North and South Islands and the subsequent application of the geopotential-value approach for estimating the average offsets of the jointly adjusted levelling networks relative to WHS using GPS-levelling data and the GGM coefficients. The methodology is briefly reviewed in Section 2. The input data are specified in Section 3. The numerical results are presented and discussed in Section 4. The summary and concluding remarks are given in Section 5.

2. Methodology

Since the normal gravity values at the surface points along levelling lines were calculated using the ellipsoid gravity formula for the parameters of the GRS67 reference ellipsoid, we firstly recomputed the cumulative normal-orthometric correction using the normal gravity field parameters of the GRS80 reference ellipsoid (Moritz, 1980). Our test results confirmed the finding of Filmer et al. (2010) that the differences between the values of the cumulative normal-orthometric correction computed for the GRS67 and GRS80 normal gravity field parameters are completely negligible.

The normal-orthometric-corrected loop closures are not independent on the levelling route taken (cf. Featherstone and Kuhn, 2006). However, the accurate computation of the cumulative normal to normal-orthometric height correction to levelled height differences is restricted (in the absence of observed gravity data along levelling lines) by the cumulative effect of the GGM commission and omission errors especially in mountainous regions of New Zealand with large spatial gravity and elevation gradients. Therefore, the observation equations in the joint adjustment of levelling networks were formed for the normal-orthometric-corrected loop closures, while disregarding the holonomity property (meaning, among other things, that the normal or orthometric corrected loop closures are equal zero independently on the leveling route; cf. Sansò and Vaníček, 2006).

The geopotential-value approach utilises Molodensky's concept of the normal heights according to which the normal gravity potential U evaluated on the telluroid equals the actual gravity potential W at the Earth's surface (cf. Molodensky et al., 1960). Hence, we write the following equality

$$U(H^N) = W(h) \tag{1}$$

where H^N denotes the normal height, and h is the geodetic (ellipsoidal) height. In practice, however, the condition in eqn. 1 does not hold due to the fact that the geopotential value at the tide gauge used as the reference for the normal heights is not the same as the geoidal geopotential value $W_{0,LVD}$. The geopotential difference between the values of and is then computed as (cf. Burša et al., 1999)

$$\delta W_{0,LVD} = W_0 - W_{0,LVD} = U(H)^{N} - W(h). \quad (2)$$

From eqn. 2, the LVD offset evaluated at the GPS-levelling point is defined as (*ibid.*)

$$\delta H_{0,LVD} = \frac{\delta W_{0,LVD}}{\bar{\gamma}} = \frac{U(H^{N}) - W(h)}{\bar{\gamma}}, \quad (3)$$

where $\bar{\gamma}$ is the integral mean of the normal gravity along the normal plumbline between the reference ellipsoid and telluroid. The gravity potential W in eqn. 3 is computed at the surface point using the GGM coefficients. The normal gravity potential in eqn. 3 is computed on the telluroid using, for instance, Somigliana's formula (Somigliana 1929; see also Heiskanen and Moritz 1967, eqn. 2–62).

When LVDs are defined in the system of the normal-orthometric heights, the cumulative normal to normal-orthometric height correction $\delta H^{N,N-O}$ is applied. The computation of this correction at the surface points along levelling lines is done using the following expression

$$\delta H^{N,N-O} = \frac{1}{\bar{\gamma}} \sum_i g_i \delta n_i - \frac{1}{\bar{\gamma}} \sum_i \left[\gamma_{0i} + \frac{\partial \gamma}{\partial h} H_i^{N-O} \right] \delta n_i = \frac{1}{\bar{\gamma}} \sum_i \Delta g_i \delta n_i, \quad (4)$$

where g_i are the observed gravity values, H_i^{N-O} are the normal-orthometric heights of the surface points, $\gamma_{0,i}$ are the normal gravity values computed at the reference ellipsoid surface, and $\partial\gamma/\partial h$ is the normal linear gravity gradient. As seen from eqn. 4, the normal to normal-orthometric height correction is calculated by a summation of the levelled height differences δn_i which are multiplied by the corresponding values of the gravity anomaly Δg_i. In the absence of the observed gravity data the gravity anomalies along levelling lines are generated from GGM. A similar method was used by Filmer et al. (2010) for the conversion of the normal-orthometric to normal heights in the Australian Height Datum. They used EGM2008 to reconstruct the observed gravity disturbances at the levelling benchmarks of the Australian National Levelling Network. Filmer et al. (2010) and Tenzer et al. (2011) computed the correction $\delta H_{N,N-O}$ as a function of the gravity disturbances δg_i instead of using the gravity anomalies Δg_i. Since the normal gravity data used for the definition of the normal-orthometric heights were calculated based on the levelled height differences, the definition of $\delta H_{N,N-O}$ in eqn. 4 as a function of the gravity anomalies Δg_i is more rigorous. However, our test results at the New Zealand's levelling networks revealed that the differences in the values of this correction computed using the gravity disturbances δg_i and the gravity anomalies Δg_i are below 0.1 mm.

When LVD is realised in the system of the orthometric heights, the geoid-to-quasigeoid correction is applied. Burša et al. (1999) applied the geoid-to-quasigeoid correction in estimating the average offset of the North American Vertical Datum 1988 (NAVD88) which is realised in the system of the Helmert's orthometric heights. The relation between the normal and (Helmert's) orthometric heights is defined as a function of the simple planar Bouguer gravity anomaly and the topographic height of the computation point (cf. Heiskanen and Moritz, 1967, Chapters 8–12 and 8–13). More rigorous definitions of the orthometric heights and the geoid-to-quasigeoid correction can be found in Tenzer et al. (2005) and Santos et al. (2006).

3. Input data

The precise levelling data used in this study comprise 10,150 benchmarks (5,967 levelling benchmarks at the North's Island and 4,183 levelling benchmarks at the South Island). The configuration of the New Zealand's levelling networks is shown in Fig. 1. The whole network consists of 14 LVDs (Auckland 1946, Gisborne 1926, Moturiki 1953, Napier 1962, One Tree Point 1964, Taranaki 1970, and Wellington 1953 at the North Island; Bluff 1955, Deep Cove 1960, Dunedin-Bluff 1960, Dunedin 1958, Lyttelton 1937, Nelson 1955, and Tarakohe 1982 at the South Island). As seen in Fig. 1, large parts of the South Island are not sufficiently covered by levelling profiles along the mountainous regions of the Southern Alps. Over most of the North Island the coverage of levelling networks is much better except for some irregularities along the mountainous regions of the central and lower North Island.

In total 1,452 co-located GPS-levelling points in New Zealand (consisting of 772 GPS-levelling benchmarks at the North Island and 680 GPS-levelling benchmarks at the South Island) were used to estimate the average offsets of the newly adjusted joint levelling networks at the South and North Islands with respect to WHS. The ellipsoidal heights are defined in the New Zealand Geodetic Datum 2000 (NZGD2000; GRS80 reference ellipsoid). The NZGD2000 is aligned to the International Terrestrial Reference Frame 1996 (ITRF1996) at the reference epoch of January 1st 2000 (Blick et al., 2005).

The GGM coefficients used in this study to generate the gravity field quantities were taken from the Earth Gravitational Model 2008 (EGM2008); see Pavlis et al. (2008).

4. Results

According to the numerical results based on the analysis of GPS-levelling data in New Zealand presented in Tenzer et al. (2011),

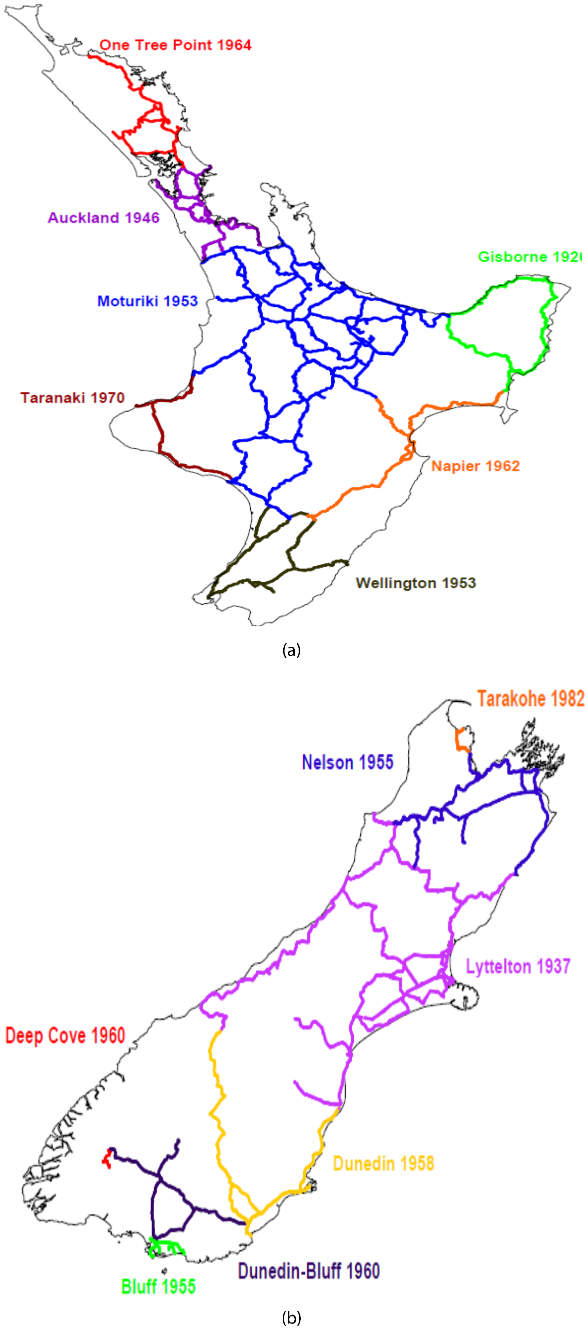

Figure 1. The levelling networks at the North and South Islands of New Zealand attributed to 14 LVDs (Auckland 1946, Gisborne 1926, Moturiki 1953, Napier 1962, One Tree Point 1964, Taranaki 1970, and Wellington 1953 at the North Island; Bluff 1955, Deep Cove 1960, Dunedin-Bluff 1960, Dunedin 1958, Lyttelton 1937, Nelson 1955, and Tarakohe 1982 at the South Island).

Table 1. Statistics of the least-squares residuals between the measured and adjusted normal-orthometric-corrected height differences between the levelling benchmarks at the North and South Islands of New Zealand.

LVD	**Min** [cm]	**Max** [cm]	**STD** [cm]
Auckland 1946	-0.8	0.8	0.1
Gisborne 1926	-1.0	0.9	0.2
Moturiki 1953	-0.8	0.01	0.2
Napier 1962	-0.7	0.6	0.1
One Tree Point 1964	-2.5	2.6	0.3
Taranaki 1970	-0.6	0.7	0.1
Wellington 1953	-1.9	1.9	0.2
North Island	**-2.5**	**2.6**	**0.2**

LVD	**Min** [cm]	**Max** [cm]	**STD** [cm]
Bluff 1955	-0.5	0.5	0.1
Deep Cove 1960	-0.2	1.4	0.1
Dunedin-Bluff 1960	-0.2	0.03	0.1
Dunedin 1958	-1.0	1.0	0.2
Lyttelton 1937	-1.0	1.1	0.2
Nelson 1955	-1.3	1.3	0.2
Tarakohe 1982	-0.3	0.3	0.1
South Island	**-1.3**	**1.4**	**0.2**

the LVDs Wellington 1953 and Dunedin 1958 have the smallest average offsets relative to WHS. The estimated average offsets of these two LVDs are 1±2 cm (for the LVD Wellington 1953) and 7±18 cm (for the LVD Dunedin 1958). Therefore, the tide gauges in Wellington and Dunedin were chosen as the origins for a definition of the new normal-orthometric heights within the North and South Islands. The large uncertainty in the estimated offset of the LVD Dunedin 1958 is mainly due to the commission and omission errors of EGM2008 (cf. Tenzer et al., 2011). It is worth mentioning that the choice of different tide gauges as the origins is optional without affecting the final results of the combined approach. The minimally constrained least-squares adjustment of the leveling networks at the South and North Island (fixed at the respective tide gauges) was realised using the Survey Network Adjustment Program (SNAP) developed at Land Information New Zealand (LINZ). No a priori error model was applied. The least-squares residuals between the measured and adjusted normal-orthometric-corrected height differences between the levelling benchmarks are shown in Fig. 2 and the histograms of residuals are given in Fig. 3. The statistics of the residuals at the levelling benchmarks of individual LVDs at the North and South Islands are summarised in Table 1.

The results of the joint levelling adjustment revealed a good quality of levelling data by means of the residuals between the measured and adjusted normal-orthometric-corrected height differences between the levelling benchmarks. The standard deviation (STD) of the least-squares residuals is 2 mm for the whole country. At the South Island's levelling benchmarks the residuals range within ±1.3 cm, and they are between -2.5 and 2.6 cm at the North

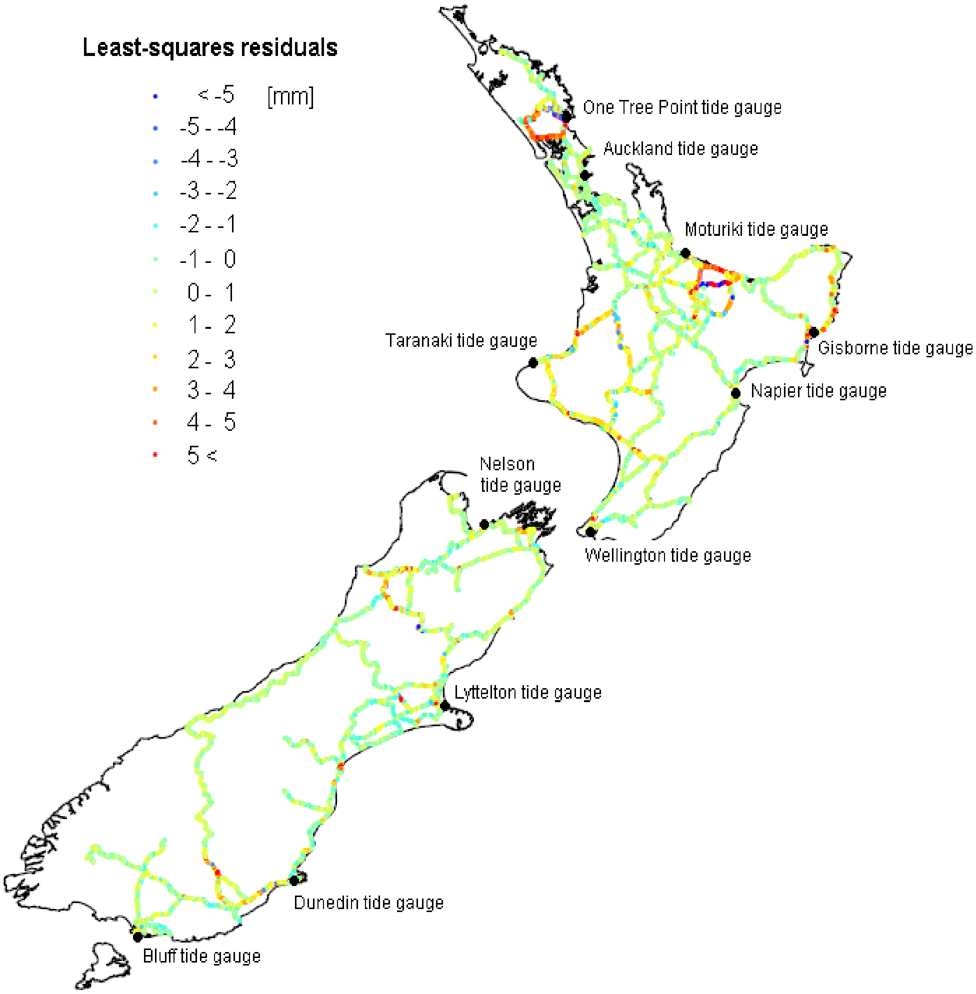

Figure 2. Results of the joint adjustment of the local levelling networks at the North and South Islands: The least-squares residuals between the measured and adjusted normal-orthometric-corrected height differences between the levelling benchmarks. The units are in millimetres.

Island's levelling benchmarks. The smallest residuals are found at the levelling networks of the LVDs Tarakohe 1982 (within $\pm$0.3 cm) and Dunedin-Bluff 1960 (between -0.2 and 0.03 cm) at the South Island. The largest residuals are found at the levelling networks of the LVDs One Tree Point 1964 (between -2.5 and 2.6 cm) and Wellington 1953 (within $\pm$1.9 cm) at the North Island. As seen in Table 1, the levelling networks of the LVDs Moturiki 1953 (at the North Island) and Deep Cove 1960 (at the South Island) have a systematic trend (mostly either positive or negative values of the least-squares residuals). A possible reason is due to the location of these LVDs in mountainous regions with large horizontal elevation gradients. The statistics of the least-squares residuals provide information only on the internal precision of levelling networks. Additional mainly systematic errors in levelling data and the deficiencies of LVDs in New Zealand are discussed in detail by Amos (2010).

The new normal-orthometric heights at the levelling benchmarks were computed from the heights of the fixed tide-gauge reference benchmarks and the adjusted normal-orthometric-corrected height differences. The differences between the newly determined and original normal-orthometric heights of the levelling benchmarks in New Zealand are shown in Fig. 4 and their statistics are given in Table 2. These differences are between -26.5 and 23.4 cm at the North Island's levelling benchmarks and between -21.6 and 6.5 cm at the South Island's levelling benchmarks. The individual comparison of the differences between the newly determined and original normal-orthometric heights at 14 LVDs indicates that these discrepancies are mainly due to the existing LVD offsets and systematic errors in levelling data. The averaged values of the relative offsets between the LVDs within the North Island taken with respect to the LVD Wellington 1953 are: 2.3 cm for the LVD Auckland 1946, 0.1 cm for the LVD Gisborne 1926, and -3.0 cm for

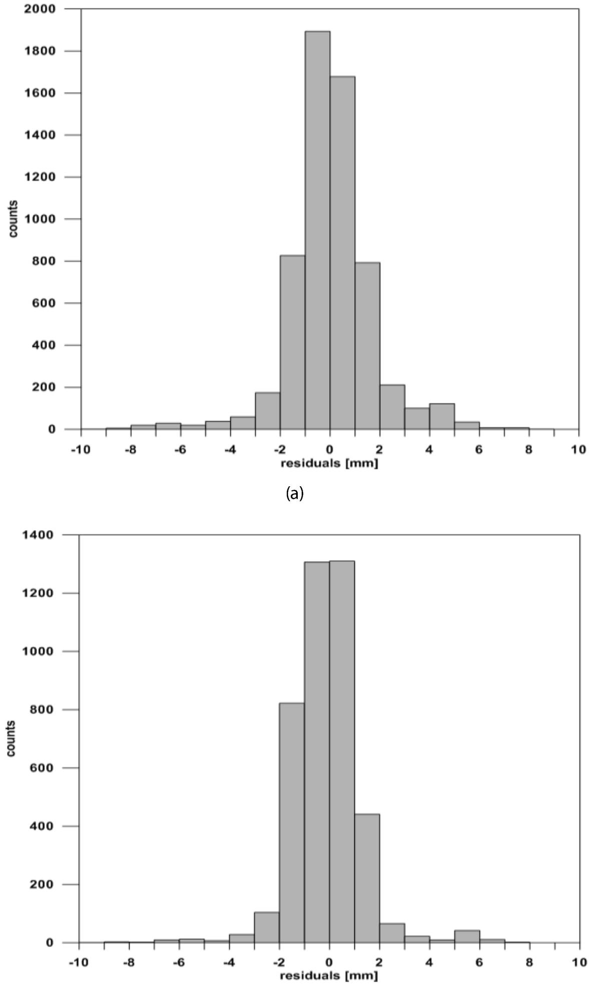

Figure 3. Histograms of the least-squares residuals between the measured and adjusted normal-orthometric-corrected height differences at: (a) the North Island's levelling benchmarks, and (b) the South Island's levelling benchmarks.

the LVD One Tree Point 1964. Similarly, the averaged values of the relative offsets between the LVDs within the South Island taken with respect to the LVD Dunedin 1958 are: 6.3 cm for the LVDs Bluff 1955 and Deep Cove 1960, 6.0 cm for the LVD Dunedin-Bluff 1960, and -11.6 cm for the LVD Tarakohe 1982. These relative LVD offsets only partially agree with the global principal pattern of the increasing MDT due to a typical south-north horizontal temperature gradient. The reasons are more likely to be due to the spatial variations in MSL that can be attributed to coastal configuration, the geometry of the ocean bottom relief and the seawater circulation around the coast of New Zealand which is dominated by the East Auckland Current, East Cape Current, Westland Current and D'Urville Current. Moreover, large discrepancies are expected due to the errors of the estimated MSL (caused by using short term

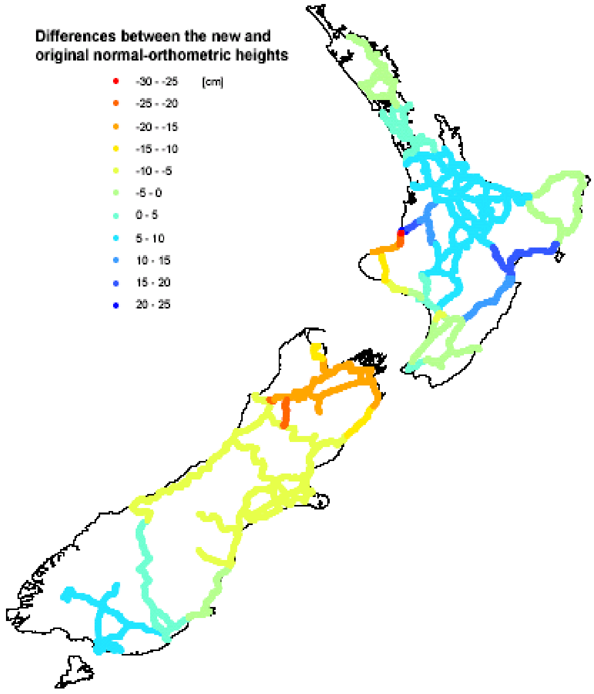

Figure 4. Differences between the original and newly determined normal-orthometric heights of the levelling benchmarks at the North and South Islands of New Zealand. The units are in centimetres.

tide-gauge records), levelling network realisation over different time epochs and the effects of vertical motions (cf. Amos, 2010). These systematic errors are not yet fully understood and need to be examined.

The computation of the cumulative normal to normal-orthometric height correction was done using the levelling and gravity anomaly data according to eqn. 4. The gravity anomaly values at the surface points along levelling lines were calculated using the EGM2008 coefficients complete to degree 2160 of spherical harmonics in the tide-free system. The computed values of the EGM2008 gravity anomalies at the levelling benchmarks vary from -157.1 to 115.4 mGal (the gravity anomalies vary from -84.2 to 115.4 mGal at the South Island's levelling benchmarks, and between -157.1 and 102.5 mGal at the North Island's levelling benchmarks). The values of the normal to normal-orthometric height correction computed at the levelling benchmarks in New Zealand are shown in Fig. 5, and their statistics are given in Table 3. At the North Island's levelling benchmarks this correction varies from -4.9 to 10.7 cm, while it varies between -2.6 and 5.7 cm at the South Island's levelling benchmarks. The mostly positive values of this correction have their maxima at the levelling lines crossing mountainous regions of the central North Island (LVD Moturiki 1953) and the upper South Island (LVD Lyttelton 1937). The largest negative values of this correction are at the central levelling segment of the LVD Napier 1962. Moreover, large negative values of this correction are also

Table 2. Statistics of the differences between the original and newly determined normal-orthometric heights of the levelling benchmarks at the North and South Islands of New Zealand.

LVD	Min [cm]	Max [cm]	Mean [cm]	RMS [cm]
Auckland 1946	0.5	0.6	0.5	0.1
Gisborne 1926	-2.2	-1.4	-1.7	0.2
Moturiki 1953	1.5	23.4	7.6	2.5
Napier 1962	1.7	12.4	15.0	1.0
One Tree Point 1964	-4.8	-4.7	-4.8	0.1
Taranaki 1970	-26.5	-0.1	-12.9	6.1
Wellington 1953	-5.9	1.0	-1.8	1.7
North Island	**-26.5**	**23.4**	**3.2**	**6.9**

LVD	Min [cm]	Max [cm]	Mean [cm]	RMS [cm]
Bluff 1955	6.4	6.5	6.4	0.1
Deep Cove 1960	6.4	6.5	6.4	0.1
Dunedin-Bluff 1960	6.0	6.1	6.1	0.1
Dunedin 1958	-0.4	0.8	0.1	0.3
Lyttelton 1937	-10.7	-5.8	-7.4	0.6
Nelson 1955	-21.6	-13.3	-18.0	1.7
Tarakohe 1982	-11.5	-11.4	-11.5	0.1
South Island	**-21.6**	**6.5**	**-6.5**	**7.4**

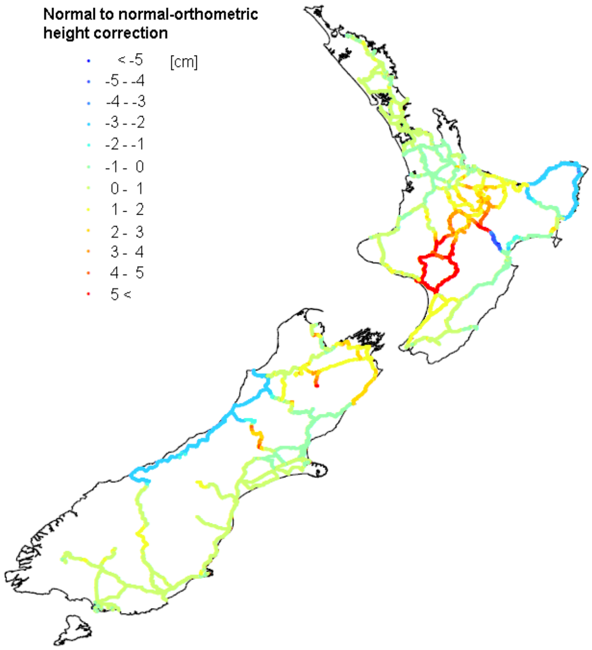

Figure 5. The normal to normal-orthometric height correction computed at the levelling benchmarks in New Zealand.

Table 3. Statistics of the normal to normal-orthometric height correction computed at the levelling benchmarks at the North and South Islands of New Zealand.

LVD	Min [cm]	Max [cm]	Mean [cm]	RMS [cm]
Auckland 1946	0.0	1.4	0.3	0.3
Gisborne 1926	-3.3	2.6	-1.6	1.4
Moturiki 1953	-0.5	10.7	2.8	3.2
Napier 1962	-4.9	2.9	-0.9	1.6
One Tree Point 1964	-0.2	2.3	0.3	0.4
Taranaki 1970	-0.1	1.5	0.6	0.4
Wellington 1953	-0.5	2.2	0.7	0.6
North Island	**-4.9**	**10.7**	**0.3**	**0.7**

LVD	Min [cm]	Max [cm]	Mean [cm]	RMS [cm]
Bluff 1955	-0.1	0.2	0.0	0.1
Deep Cove 1960	-0.1	0.1	0.0	0.01
Dunedin-Bluff 1960	-0.1	1.6	0.6	0.3
Dunedin 1958	-0.1	2.2	0.0	0.3
Lyttelton 1937	-2.6	5.7	-0.5	1.2
Nelson 1955	-0.1	5.6	1.4	0.8
Tarakohe 1982	0.0	5.1	1.0	1.5
South Island	**-2.6**	**5.7**	**0.4**	**0.4**

found at levelling lines along the west coast of the South Island (LVD Lyttelton 1937) and the east coast of the North Island (LVD Gisborne 1926). Both, the maxima and minima of this correction are situated at the levelling segments with the largest horizontal gravity and terrain elevation gradients.

The EGM2008 coefficients complete to degree 2160 of spherical harmonics (in the tide-free system) were used to compute the gravity potential values at the surface points. The parameters of the level ellipsoid in the tide-free system were adopted for the evaluation of the normal gravity potential on the telluroid. For a definition of tidal systems we refer readers, for instance, to Vatrt (1999). The geopotential differences at the GPS-levelling points were computed according to eqn. 2 and then averaged individually for the North and South Islands. The estimated average offsets are 10.6 cm and 27.5 cm for the joint levelling networks at the North and South Islands, respectively. The corresponding RMS of differences between the GPS-levelling geometric height anomalies and the EGM2008 gravimetric height anomalies are 1.6 cm and 0.5 cm. The inaccuracy of the estimated average offsets is mainly due to the commission and omission errors of EGM2008 (cf. Tenzer et al., 2011). The additional errors up to several centimetres are expected due to inaccuracies within the GPS and levelling measurements. Moreover, large errors are expected due to vertical deformations, sea level variations, short term tide-gauge records used for a definition of MSL, and other systematic factors already discussed in the previous paragraphs.

5. Summary and concluding remarks

We have used the levelling and normal gravity data for the joint adjustment of the local levelling networks at the North and South Islands of New Zealand fixing the heights of the tide gauges in Dunedin and Wellington. The geopotential-value approach was then applied for the estimation of the average offsets of the jointly adjusted levelling networks relative to WHS using the GPS-levelling data and EGM2008. Since the geopotential-value approach is based on the Molodensky's theory, the newly adjusted normal-orthometric heights were first converted to the normal heights based on applying the cumulative normal to normal-orthometric height correction. In the absence of observed gravity data along levelling lines, the gravity anomaly values were generated from EGM2008.

The results of the joint levelling adjustment revealed that the STD of least-squares residuals of the normal-orthometric-corrected height differences is 2 mm in New Zealand. The residuals between the levelling benchmarks are within $\pm$1.3 cm at the South Island and between -2.5 and 2.6 cm at the North Island.

The comparison of the newly determined normal-orthometric heights (defined with respect to the tide gauges in Dunedin and Wellington) with the original ones (defined individually in 14 LVDs) confirmed the presence of large offsets between individual LVDs as well as systematic levelling errors.

The computed values of the cumulative normal to normal-orthometric height correction at the levelling benchmarks in New Zealand vary from -4.9 to 10.7 cm. The uncertainties in the computed values of this correction are mainly due to the commutative contributions of the EGM2008 commission and omission errors.

The estimated average offsets of GPS-levelling points at the North and South Islands are 10.6 cm and 27.5 cm, respectively. The inaccuracy in the estimated values of these offsets is mainly due to the EGM2008 commission and omission errors, existing systematic distortions of the levelling networks, errors in the GPS solutions and the geocentric reference frame realisation, tectonic and other vertical movements, sea level variations, short term tide-gauge records used for the estimation of MSL, levelling network realisation over different time epochs.

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