

Comparisons of geoid models over Alaska computed with different Stokes' kernel modifications

Research Article

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Abstract:

Various Stokes kernel modification methods have been developed over the years. The goal of this paper is to test the most commonly used Stokes kernel modifications numerically by using Alaska as a test area and EGM08 as a reference model. The tests show that some methods are more sensitive than others to the integration cap sizes. For instance, using the methods of Vaniček and Kleusberg or Featherstone et al. with kernel modification at degree 60, the geoid decreases by 30 cm (on average) when the cap size increases from 1° to 25°. The corresponding changes in the methods of Wong and Gore and Heck and Grüniger are only at the 1 cm level. At high modification degrees, above 360, the methods of Vaniček and Kleusberg and Featherstone et al become unstable because of numerical problems in the modification coefficients; similar conclusions have been reported by Featherstone (2003). In contrast, the methods of Wong and Gore, Heck and Grüniger and the least-squares spectral combination are stable at any modification degree, though they do not provide as good fit as the best case of the Molodenskii-type methods at the GPS/Leveling benchmarks. However, certain tests for choosing the cap size and modification degree have to be performed in advance to avoid abrupt mean geoid changes if the latter methods are applied.

Keywords:

Gravity • Geoid • Stokes's function • Stokes kernel modification

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1. Introduction

Since a local geoid is computed in a local area, the truncation error, that is the effect of the gravity data from the rest of the Earth, has to be taken into account (e.g., Molodenskii et al., 1962; Sjöberg, 1980, 1981, 1984, 2003a, 2003b; Jekeli 1981; Vaniček and Kleusberg, 1987; Vaniček and Featherstone 1998; Featherstone et al., 1998; Ellmann 2001, 2005 among others). This is usually done by using a global geopotential model, the reference model, which accounts for the contribution of the rest of the Earth but naturally also has a contribution inside the local computation area. The proper modification, or truncation, of Stokes's kernel is a critical step to optimally combine the long wavelength content of the global geopotential model with the medium and short

wavelength portion of the surface gravity data. The simplest kernel modification methods, such as that of Wong and Gore (1969), truncate the spherical harmonic representation of Stokes's function at a degree up to which the global model is more accurate than the gravity counterpart. More advanced methods have been developed, which take into consideration the errors of the surface gravity data and the reference model (Sjöberg 1980, 1981, 1991, 2003a, 2003b; Wenzel 1982; Wang, 1993). But these stochastic methods rely on accurate error models of the surface data and the reference model, which may not be easily obtained sometime; see Vaniček and Featherstone (1998), Featherstone (2003), and Ellmann (2005) for various discussions of and comparisons between the deterministic methods and the stochastic methods.

Unlike many previous studies that were done in relatively flat areas, this paper investigates the effects of different kernel modification methods on geoid computation in Alaska, which has complex geological rheology. Section 2 gives a brief review of all the available kernel modification methods. The details of the computation of the kernel modification are given in Section 3, followed by a brief discussion of characteristics of the modified kernels. Section 4 describes the gravity data, the elevation data, as well as the specific GPS/Leveling benchmarks (GPSBMs) that are used in the validation of different geoid models. The final geoid difference analysis is included in Section 5. Finally, some conclusions are given in Section 6.

2. Methods of kernel modification

In a remove-compute-restore scheme, the geoid is computed from the surface gravity data and a global reference model by:

$$N = \frac{R}{4\pi\gamma} \iint_{\sigma_0} [dg - (h - h_A) \frac{\partial(dg)}{\partial h}] S(\psi) d\sigma + \zeta_1(2, M) + \zeta_2(M + 1, M') + C \quad (1)$$

where R is the radius of the mean Earth, γ is the normal gravity, h_A is the height of the point level (see Moritz, 1980, p. 377), $S(\psi)$ is the Stokes function and ψ is the spherical distance between the computation and integration points (Heiskanen and Moritz, 1967, p.94), σ_0 is the integration area on the surface of a unit sphere; dg is the residual gravity anomaly computed from the surface gravity anomaly Δg^{obs} , the global reference gravity anomaly $\Delta g_1(2, M)$ up-to degree M , and the residual terrain effect on gravity $\Delta g_2(M + 1, M')$ from degree $M + 1$ to degree M' (e.g. $M' = 216,000$, if a 3 arc-seconds DEM is used), as shown in the following equation:

$$dg = \Delta g^{obs} - \Delta g_1(2, M) - \Delta g_2(M + 1, M') \quad (2)$$

If the EGM2008 (Pavlis et al., 2008) reference model is used to degree 2160, the residual gravity anomaly dg becomes very small and the downward continuation term in the integral of equation (1) can be safely neglected everywhere except in high mountains. The reference height anomaly $\zeta_1(2, M)$ is also computed from a global reference model on the Earth's surface, $\zeta_2(M + 1, M')$ is the contribution of the residual terrain to the height anomaly, and C is the correction to convert the height anomaly to the geoid height (Flury and Rummel, 2009):

$$C = \Delta g^{BO} \frac{H}{\bar{\gamma}} + \frac{H}{\bar{\gamma}} (V_{P_0}^{TOP} - V_P^{TOP}) \quad (3)$$

where Δg^{BO} is the refined Bouguer gravity anomaly, H is the orthometric height, $\bar{\gamma}$ is the mean normal gravity from the ellipsoid

to the telluroid along the ellipsoid normal, $V_{P_0}^{TOP}$ and V_P^{TOP} are the topographical potential at a geoid point (P_0) and the corresponding surface point (P), respectively.

To minimize the truncation error and optimally combine the surface gravity data with the global reference model, Stokes's kernel, $S(\psi)$, in the integral of equation (1) is replaced by a modified kernel, $\tilde{S}(\psi)$ in local geoid computations. Various kinds of kernel modification were developed over the years. In this paper, we consider and compare the performance of some of the most commonly used ones, listed in the following:

1. Wong and Gore (1969) method:

$$\tilde{S}_{WG}(p, \psi) = S(\psi) - \sum_{n=2}^p \frac{2n+1}{n-1} P_n(\cos\psi) \quad (4)$$

By completely removing the spectrum up to degree p , this modification eliminates the low degree contributions from the local surface data, replacing it by that of the reference model.

2. Heck and Grüniger (1987) method

$$\tilde{S}_{HG}(p, \psi) = \tilde{S}_{WG}(p, \psi) - \tilde{S}_{WG}(p, \psi_0) \quad (5)$$

where ψ_0 is the cap size. The extra correction term introduced by this method makes the error kernel function continuous through the boundaries, for a faster convergence.

3. Vaniček and Kleusberg (1987) method

$$\tilde{S}_{VK}(L, p, \psi) = \tilde{S}_{WG}(p, \psi) - \sum_{n=2}^L \frac{2n+1}{2} t_n(\cos\psi_0) P_n(\cos\psi) \quad (6)$$

where $t_n(\cos\psi_0)$ is the modification coefficient determined by minimizing the L_2 norm of the error kernel for the selected ψ_0 and L ($L \leq M$ and $L \leq p$). In most practical cases, L is set equal to p . This modification applies Molodensky's modification of the spherical Stokes's kernel to the spheroidal Stokes's kernel. Detailed derivations can be found in Vaniček and Kleusberg (1987), and Vaniček and Sjöberg (1991).

4. Featherstone et al. (1998) method

$$\tilde{S}_F(L, p, \psi) = \tilde{S}_{VK}(L, p, \psi) - \tilde{S}_{VK}(L, p, \psi_0) \quad (7)$$

Again the correction term is for a faster converging error kernel.

5. Method of the least squares spectral combination

One of other methods is the method of spectral combination (e.g., Wenzel, 1982, Wang 1993). Sjöberg's general kernel modification (2003a, b) is reduced to the same method, if the truncation error is ignored.

$$\tilde{S}_{SC} \begin{cases} S(\psi) - \sum_{n=2}^L \frac{2n+1}{2} S'_n P_n(\cos\psi) & \text{if } 0 < \psi \leq \psi_0 \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

where:

$$S'_n = \frac{2}{n-1} \frac{C_n}{C_n + d_n} \quad (9)$$

$$c_n = \sum_{m=0}^n (c_{nm}^2 + s_{nm}^2) \quad (10)$$

$$d_n = \sum_{m=0}^n (\delta c_{nm}^2 + \delta s_{nm}^2) \quad (11)$$

and c_{nm} , s_{nm} , δc_{nm} , and δs_{nm} are the spherical harmonic coefficients and their corresponding standard deviations from the global reference model.

Methods (1) through (4) are deterministic since they do not consider data or reference model errors while (5) is still considered to be stochastic (Ellmann 2005). All the methods are applied to compute corresponding geoid models for Alaska. The following section describes the data used in the computations.

3. Kernel function computation

To save time, the kernel functions should be prepared before the geoid computations start. Typically, they are evaluated at 0.1° resolution, and stored in a numerical table, called a “kernel table”. Then, a linear interpolation is employed to obtain the value of $\tilde{S}(\psi)$ at a given spherical distance, ψ . In the deterministic methods, the values of the modification degree, L or p , and cap size, ψ_0 , need to be selected prior to computing the kernel tables. We used $L=p=\{60, 360, \text{ and } 2160\}$ and $\psi_0=\{1^\circ, 2^\circ, 3^\circ, 5^\circ, 6^\circ, 25^\circ\}$. The combinations of these variables will give a clear picture of the behavior of the modified kernels without presenting too much redundant information.

It is relatively straightforward to prepare the tables for methods (1), (2), and (5) of Section 2. For methods (3) and (4), the modification coefficients $t_n(\cos\psi_0)$ have to be computed first. Featherstone (2003) has published software to compute these coefficients up-to degree 360. We extended this routine to compute up to degree 2160 in the case of high degree modifications. All the resulting kernel functions at modification degree 60 and their differences with the original Stokes kernel are shown in Figure 1. The kernels of Wong and Gore, Heck and Grüniger, and spectral combination behave in almost the same way: they all decrease sharply from spherical distance zero to 0.2 degrees, then slowly fall to zero at 1

degree. The characteristics of the kernels indicate that the largest contribution to the geoid comes from the area inside 0.2 degree radius. The contribution is marginal from the data outside of 1 degree spherical cap. The numerical tests in Section 5 verify this assertion. It can be seen in Figure 1 that the Vaniček and Kleusberg and Featherstone kernels are almost the same, except a constant shift, as expected. The direct effect of this similarity is that the remote zone may still have noticeable contribution. If geoid computations are done by using spherical caps, the computation results are directly dependent of the cap size. If the cap size is small, the truncation error will be significant. This property of the kernels is shown in Section 5.

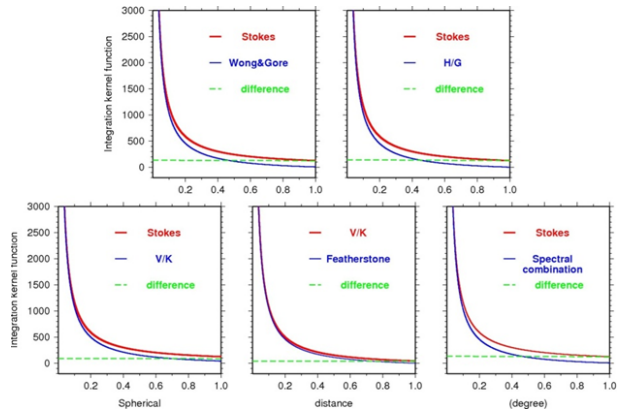


Figure 1. The kernel functions modified up-to degree 60 with 1° cap size (Other tables with different modification degree show similar behavior. To save space, they are not all shown here).

4. Data used

The computation area covers a geographic region from 49°N to 72°N in latitude and from 168°E to 237°E in longitude. There are about 532,000 surface gravity observations, archived by the National Geodetic Survey (NGS), the National Geospatial-Intelligence Agency (NGA), and Natural Resources Canada (NRCAN). The Arctic Gravity Project (ArcGP) airborne gravity data in the area (Forsberg and Kenyon, 2004) is also used. The altimetric gravity anomalies over ocean areas were extracted from the DNSC08GRA database (Andersen et al., 2010). The digital elevation model used corresponds with the Alaska DEM (Li et al., 2008) that is based on the 3" SRTM (Farr et al., 2007) below 64°N and the USGS National Elevation Data (NED) (Gesch et al., 2009) and the Canadian Digital Elevation Data (NRCAN 2007) at higher latitudes. The ASTER data from NASA's Land Processes Distributed Active Archive Center was also used separately, but did not show any advantages over our Alaska DEM (Li et al., 2010).

Table 1 shows that the above data sets refer to different coordinate systems. Thus, initially all data sets were converted into a common

Table 1. The amount of available data-points in the target area.

Data Base	NGS	NRCan	NGA ^a	DNSSC ^b
Horizontal Datum	NAD27	NAD83	WGS84	WGS84
Vertical Datum	NAVD29	CVGD28	EGM96Geoid	EGM96Geoid
Accuracy (mGal)	±3	±3	3.6±4.6	±3~±14
# of points	457,477	74,933	12,547	3,265,926

^aArcGP data from Forsberg and Kenyon (2004)^bAltimeter data from Andersen et al. (2010)

coordinate system - the ITRF2000 by using the NGS Geodetic Toolkit (Mulcare 2004a,b). Another related problem is that the gravity data come from different agencies which use different hardware and software. Systematic errors may exist among them, which obviously will distort the computed geoid models. Furthermore, to remove gravity data of suspicious quality, the K-nearest-neighbor collocation algorithm is employed, which removes about 6% of the total gravity points. The final cleaned gravity data are shown in Figure 2. It is worth mentioning that the ArcGP data (5'x5' mean) is only used in the Chukchi Peninsula area where no point data is available in the NGS archives.

A gravity anomaly grid with a 1'x1' spatial resolution is generated based on these cleaned data by least square collocation. Various methods are available to estimate the quality of interpolated gravity along data grid points (Li, 2010). For this investigation, a spline interpolation method was used to estimate the quality of the Alaskan grid from the available scattered points. Figure 3 shows the differences between the interpolated values and the original "true" gravity anomalies, resulting in better than 1 mGal standard deviation (STD) with almost zero mean bias. As such, we may conclude that the gravity grid has at least 1 mGal accuracy when compared to the cleaned observed point data.

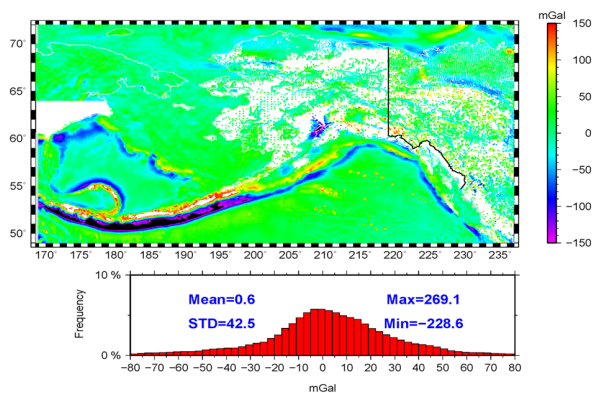


Figure 2. The edited gravity anomalies.

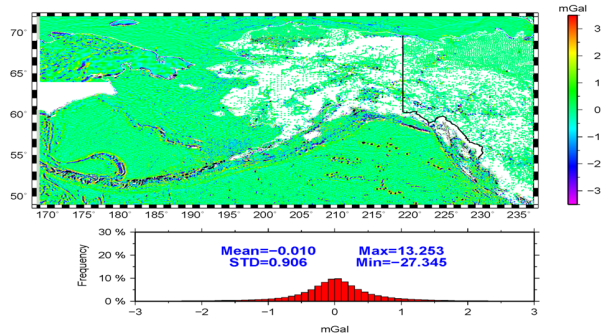


Figure 3. Error estimates of the gravity anomaly grid for Alaska Geoid computation.

5. Results and discussions

The cleaned observed gravity data discussed in Section 4, and the modified kernel tables of all the kernel modification methods described in Section 2 are inserted in the Stokes integral to compute the residual height anomalies, $\zeta^{residual}$. Then, at the same modification degree, all of these residual values are converted into geoid undulations by using the same additive terms in equation (1). Thus, the differences in the various geoid models are purely due to the effects of the differences in the modified kernels. To validate the performances of the kernel modification methods, their corresponding geoid undulations are compared with the geoid heights computed at the local GPS leveling benchmarks. The 89 GPSBs re-adjusted in 2007 by NGS in Alaska and the original 90 points on the adjacent the Canadian area (all shown in Figure 4) are used in the following analysis. Except a systematic bias between NAVD88 and EGM96 geoid, these benchmarks have a few cm precision, which are currently sufficient for evaluating the Alaska geoid changes, which have about 20 m amplitude changes from Canadian side to the U.S. side. The mean differences and the standard deviations of the geoid difference for different methods at different modification degrees, i.e., $L=60$, 360, and 2160, are shown in Table 2, Table 3, and Table 4, respectively.

From Table 2 ($L=60$), we see that all the methods show different standard deviations of the differences with the change of the cap sizes. The Wong and Gore (1969) method provides almost the same geoid models as the spectral combination method does, that is because at low degrees the ratio, $\frac{C_n}{C_n+d_n}$, in equation (9) is close to 1. The Heck and Grüniger (1987) method does not show much more improvement than the Wong and Gore (1969) method, especially in higher degrees (i.e., $L=360$, and $L=2160$; see Tables 3-4, respectively). The Featherstone et al. (1998) method is very similar to the Vaniček and Kleusberg (1987) method. The best fitted geoid model generated by the two methods is at cap size of 6° , with the biases change 3.3 cm. At cap size 25° , the methods of Vaniček and Kleusberg (1987) and Featherstone et al. (1998) become unstable; the bias changes almost 30 cm. The

Table 2. The statistics of the geoid differences at the local GPSBMs (Kernel modification up to degree 60. Parenthesis values are standard deviations. Outside values are mean.

Unit (m)	Wong& Gore(69)	LS Spectral Comb.	Heck& Grüninger(87)	Vaniček& Kleusberg(87)	Featherstone et al.(98)
Cap Size					
1°	1.752(0.278)	1.752(0.278)	1.752(0.278)	1.747(0.277)	1.752(0.278)
2°	1.758(0.282)	1.758(0.282)	1.747(0.276)	1.739(0.273)	1.742(0.274)
3°	1.769(0.287)	1.769(0.287)	1.756(0.281)	1.732(0.270)	1.733(0.271)
5°	1.766(0.287)	1.766(0.287)	1.783(0.293)	1.723(0.267)	1.723(0.267)
6°	1.762(0.288)	1.762(0.288)	1.774(0.291)	1.719(0.266)	1.719(0.266)
25°	1.762(0.289)	1.762(0.289)	1.763(0.289)	1.453(0.275)	1.454(0.275)

Table 3. The statistics of the geoid differences at the local GPSBMs (Kernel modification up to degree 360. Parenthesis values are standard deviations. Outside values are mean.

Unit (m)	Wong& Gore(69)	LS Spectral Comb.	Heck& Grüninger(87)	Vaniček& Kleusberg(87)	Featherstone et al.(98)
Cap Size					
1°	1.763(0.286)	1.763(0.286)	1.766(0.287)	1.757(0.280)	1.757(0.280)
2°	1.762(0.286)	1.762(0.286)	1.766(0.289)	1.753(0.279)	1.753(0.279)
3°	1.762(0.287)	1.762(0.287)	1.767(0.289)	1.710(0.271)	1.710(0.271)
5°	1.762(0.287)	1.762(0.287)	1.767(0.289)	1.720(0.265)	1.720(0.265)
6°	1.762(0.287)	1.762(0.287)	1.767(0.288)	2.619(0.967)	2.627(0.970)
25°	1.762(0.287)	1.762(0.287)	1.762(0.287)	1.343(0.361)	1.347(0.361)

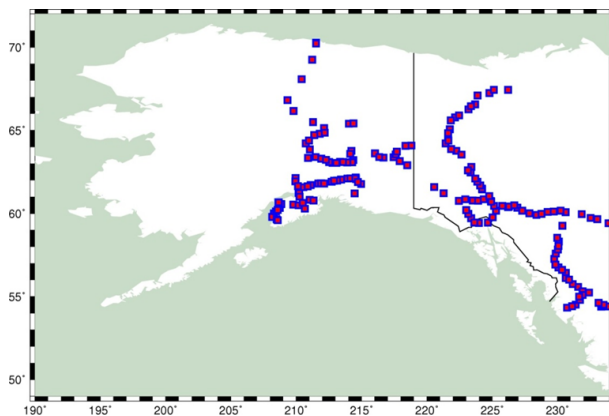


Figure 4. The available GPS leveling benchmarks in the target area.

corresponding change in the Wong and Gore (1969) method is only about 1 cm.

When modifying the kernel up to degree 360 (Table 3), the Wong and Gore (1969) method and the spectral combination method still generate very close results, and the geoid models are almost independent of the selected cap size. Nevertheless, at cap size of 5° the best results in the relative sense (minimum standard deviation) are still delivered by the methods of Vaniček and Kleusberg (1987) and Featherstone et al. (1998). However, the geoid model changes for these two methods due to the differences in cap sizes become

more significant than the modification up-to degree 60. As such, numerical tests have to be done at the GPSBMs for determination the optimal cap size when these two methods are desired. At the case of limited ground control data, the methods of Wong and Gore and the spectral combination should be applied to avoid large changes in the mean geoid models.

If we push the modification degree into the limit ($L=2160$), the methods of Wong and Gore (1969), spectral combination, and Heck and Grüninger (1987) still work normally, and the results do not have significant changes. The spectral combination method shows a marginal accuracy improvement than the methods of Wong and Gore (1969) and Heck and Grüninger (1987). However, the best fitted geoid models are still obtained by using the methods of Vaniček and Kleusberg (1987), and Featherstone et al. (1998) at cap size 5°. However, they have large disagreements with the rest of the methods at other integration cap sizes, because of the numerical instability of the modification coefficient, $t_n(\cos\psi_0)$. This problem does occur not only in small caps but also in large ones, when the modification degree (L) is high; see Featherstone 2003.

6. Conclusions

Based on the numerical results of the computation tests, we reached the following conclusions:

The kernel modification methods at low to medium modification degrees ($L \leq 360$) provide similar geoid estimators. The methods

Table 4. The statistics of the geoid differences at the local GPSBMs (Kernel modification up to degree 2160. Parenthesis values are standard deviations. Outside values are mean).

Unit (m) Cap Size	Wong& Gore(69)	LS Spectral Comb.	Heck& Grüniger(87)	Vaniček& Kleusberg(87)	Featherstone et al.(98)
1°	1.764(0.286)	1.763(0.285)	1.765(0.286)	1.608(0.469)	1.608(0.469)
2°	1.764(0.287)	1.763(0.285)	1.765(0.287)	0.730(1.556)	0.729(1.557)
3°	1.764(0.287)	1.763(0.285)	1.765(0.287)	1.523(0.388)	1.522(0.388)
5°	1.764(0.286)	1.763(0.285)	1.766(0.286)	1.711(0.266)	1.710(0.266)
6°	1.764(0.286)	1.763(0.285)	1.766(0.286)	2.806(1.706)	2.813(1.708)
25°	1.764(0.286)	1.763(0.285)	1.764(0.286)	1.160(0.610)	1.170(0.612)

of Vaniček and Kleusberg (1987), and Featherstone et al. (1998) are more versatile and fit the GPS/leveling data the best in the relative sense at various cap sizes. The drawback is the instability of the two methods. The differences in mean and standard deviation change from 1.720(0.265) m to 2.619(0.967) m just by increasing the computation cap from 5 to 6 degree by Vaniček and Kleusberg's method (Table 3). Featherstone et al.'s method shows similar differences. This unpleasant numerical feature comes from the numerical instability of the modification coefficients in these two Molodenskii-type kernel modification methods (Featherstone, 2003). This problem does not occur only in small caps but also in large ones, when the modification degree is high (Table 4). As a result, to avoid such problems, certain ground control data (GPSBMs) are needed in order to find the optimal cap sizes when these two methods are applied in regional geoid modeling. In the case when only limited amount of GPSBMs are available to evaluate the geoid models, the methods of Wong and Gore (1969), the spectral combination and Heck and Grüniger (1987) should be used, considering that their corresponding geoid estimators are almost independent of the modification degree and the computation cap size. There are no risks of inducing large errors by choosing different integration cap, though the fitting at GPS/Leveling data may not be the best one.

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