

Least Squares Spectral Analysis for Detection of Systematic Behaviour of Digital Level Compensator

Research Article

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Abstract:

Levelling is the most precise technique for height difference measurements in geomatics engineering. Various systematic errors affect precise levelling observations and reduce the precision of the observed height differences. This study investigates digital levels residual compensator error and observational method for its elimination. For this purpose the levelling data, which was collected with Zeiss DiNi 12 digital levels, was analysed. There are different statistical and spectral methods that can reveal the presence of systematic errors in levelling results. In this study, the Least Squares Spectral Analysis (LSSA) method is used. The analysis confirmed that using alternating pointing method (BFFB, FBBF) instead of usual observation routine (BFFB) will eliminate the Zeiss DiNi 12 digital levels residual compensator error from section height differences and discrepancies. In this way, it does not matter using different instruments in the forward and backward section runs and the discrepancies can be used to investigate other systematic errors.

Keywords:

Precise levelling • digital levels • residual compensator error • least squares spectral analysis

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1. Introduction

During the last two decades, the geodetic instruments have become more automatic and electronic; finely constructed and externally well operating systems. The software has replaced more and more observer's task. Also the levelling experienced the similar development. The discovery of the digital levelling in the beginning of the 90's really conducted the leveling into the new era (Takalo and Rouhiainen 2004). A digital levelling system consists of: two bar code staffs, the optical components of the level, the compensator and the electro-optical linear array. Systematic errors of each component affect the digital levelling observations.

Hence, the precision of the measured height differences is decreased (Rouhiainen and Takalo 2008). The scale of the code is a function of temperature and a constant, both of which are determined by the staff calibration. To check the behaviour of the whole system, a "system calibration" procedure is used, where the staff readings are taken from different sectors on the bar code staff and compared with the true values obtained by a laser interferometer (Takalo and Rouhiainen 2004). The simultaneous calibration of the digital levelling system Zeiss DiNi12 and the bar code staff (system calibration) showed that even large graduation errors of the staff have only a small effect on the staff readings (Takalo et al. 2001).

Digital levels can be regarded as a fusion of a digital camera and an automatic level (Figure 1). The optical components of digital level are the same as those of an automatic level. It has a telescope with upright image and a compensator to stabilise the line-of-sight. Additionally, a position sensor coupled with the focus lens may supply rough distance information. A tilt sensor observes

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the compensator position and a beam-splitter guides part of the light to the Charge Coupled Device (CCD) sensor. The CCD array converts the bar code staff optical image to a digital image (data) such as a digital camera (Ingensand 2001).

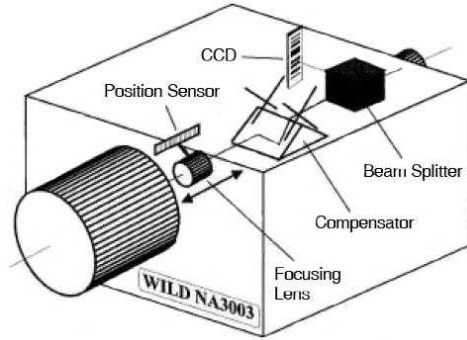


Figure 1. Basic optical design of a digital level.

2. Residual Compensator Error

The collimation axis or line of sight is the line that connects the centre of the cross hairs to the focal point of objective lens. The spot bubble is not very sensitive and is not the sole means of levelling the level. Older levels will have tubular bubbles attached to the side of the telescope, and a tilting crew is used to level this bubble, which then provides a horizontal line of sight in the direction of the collimation axis. Automatic levels use an automatic compensator, which allows the user to level the instrument with the spot bubble only. Any small departures are compensated by the compensator (Figure 2).

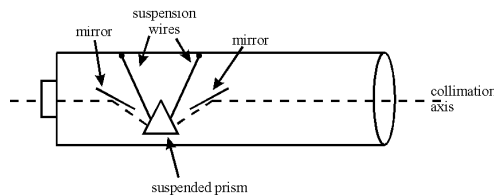


Figure 2. Schematic illustration of one type of compensator.

In this device the image of the object is deflected by a fixed mirror to pass through a prism, after which it is deflected by another mirror to the eyepiece. The prism is suspended by wires and its orientation changes as the telescope tube is tilted. The geometry of the device is designed so that any tilt of the telescope tube is compensated by a tilt of the prism and the collimation axis remains horizontal. The compensator has a limited range (a few minutes of an arc) and the level must be levelled reasonably well using the spot bubble before the compensator will work correctly (Merry

1988).

The problem to be always considered in precise levels is the compensator error. The pendulum movements in precise levels are almost linear in the range in which the pendulum is to correct the instrument's inclination. Manufacturers take every effort to make the pendulum set to the horizontal as precisely as possible. Despite adjustment to a pendulum and despite meticulous care in designing and assembling the pendulum and the vertical axis system, a tiny angular error may occur between foresight and backsight. Provided that lines of levels are run with consistent procedures, this angle constitutes a systematic error. The advantages of the digital level (fast measurement, no subjective errors) help identify this error better than ever before. In the past, these errors were part of the random error and could not be ascertained, while now they can be found out even if they are very small (Menzel 1998).

Precise Levelling is done in closed paths that are called loops, in forward and backward directions. The joint component of two adjacent loops is called levelling line. The levelling lines consist of some sections. The section starts and ends on bench mark and consists of some setups. In precise levelling, a levelling section is always measured forward, i.e., in A-direction, and backward, in B-direction. Therefore, dH is measured twice, in opposite directions A and B. Assuming that the observations are unbiased, the following equations are valid:

$$dH_A = -dH_B, \quad dH_A + dH_B = 0, \quad dH = (dH_A - dH_B)/2 \quad (1)$$

where dH_A is the section height difference in forward levelling, dH_B is the section height difference in backward levelling and dH is the section's mean height difference.

The horizontal level as given by the compensator of the Zeiss DiNi12 can be changed when turning the instrument from the back staff to the fore staff. Thus, the observed height difference is biased. While measuring the height difference, $BS_0 - FS_0 = 0$, in A-direction the back staff reading BS_A is assumed to be correct, but the foresight reading FS_A includes the error v_A (Figure 3). The length of the sighting distances are taken as being equal. In B-direction (Figure 4), the corresponding error is v_B . Thus, the forward and backward height difference will be obtained according to Eqs. (2) and (3). Assuming that the tilt effect in both directions is equal ($|v_A| = |v_B| = v$) then according to Eq. (4) the difference between forward and backward measurement includes the error ($-2v$). If the number of setups in forward and backward levelling section is equal and the tilt effect is assumed equal in the forward and backward section levelling, the section mean height difference will be free of tilt effect (Eq. (5)) (Takalo et al. 2002).

$$dH_A = BS_A - FS_A = BS_0 - (FS_0 + v_A) = -v_A \quad (2)$$

$$dH_B = BS_B - FS_B = BS_0 - (FS_0 + v_B) = -v_B \quad (3)$$

$$dH_A + dH_B = -(v_A + v_B) = -2v \quad (4)$$

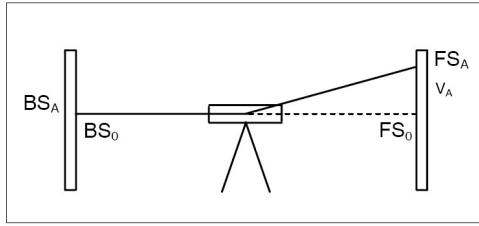


Figure 3. Levelling in A-direction.

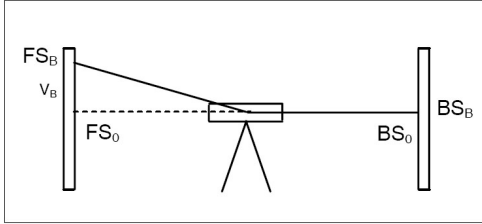


Figure 4. Levelling in B-direction.

$$dH = (dH_A - dH_B)/2 = [(-v_A) - (-v_B)]/2 = (-v + v)/2 = 0 \quad (5)$$

Since different levels may be used in the forward and backward runs of sections, this assumption ($|v_A| = |v_B| = v$) may not be true and Eq. (5) will not be equal to zero. Therefore, the residual compensator error (residual tilt effect) ($v_B - v_A$) affects section mean height difference.

3. Least Squares Spectral Analysis

A set of observations or results obtained from a physical process, arranged in a specific manner, is called a data series. A data series is called time series if the data is ordered chronologically. A single time history representing a random phenomenon is called a sample function (or a sample record when observed over a finite time interval). The collection of all possible sample functions (also called ensemble) which the random phenomenon might have produced is called a random process or a stochastic process. If the statistical properties (i.e., the statistical moments describing the mean, variance . . .) of the data series defined over the ensembles are independent of the value of the argument(s), the random process is stationary. Otherwise, the stochastic process is non-stationary (Bendat et al. 1971), (Blais 1988) and (Craymer 1988).

There are different statistical and spectral methods to detect the presence of systematic errors in levelling results. In this paper, the Least Squares Spectral Analysis (LSSA) method is used, the efficiency of which was confirmed by many researchers (Vosoghi 1994). The least squares spectral analysis is the only method which is able to adequately deal with non-equidistant and non-stationary data series of geodetic levelling. The problem is as follow:

1. Vector of observation time or argument: $\mathbf{t} = \{t_i\}$; $i = 1, 2, \dots, n$
2. Vector of observed values or functional values: $\mathbf{f}^T = [f(t_1) f(t_2) \dots f(t_n)]$
3. Vector of frequencies for which spectral values are desired: $\omega = \{\omega_j\}$, $j = 1, 2, \dots, m$

The $S(\omega_j)$ vector is sought where the $S(\omega_j)$ are the spectral values of the ω_j frequency. The least squares spectral analysis is an application of least squares approximation. Least squares approximation is a case of best interval approximation. The least squares spectral analysis is explained as follows (Vanicek and Wells 1972): in Hilbert space with L_2 norm with independent vector $\phi = (\phi_1, \phi_2, \dots, \phi_n)$ the problem is finding the polynomial $\mathbf{p}$ in subspace M that is spanned by the base vectors $\{\phi_1, \phi_2, \dots, \phi_m\}$ as the best approximation of the vector $\mathbf{f}$ in Hilbert space:

$$d(\mathbf{f}, \mathbf{p}) = \|\mathbf{f} - \mathbf{p}\| \rightarrow \min \quad (6)$$

If $\mathbf{p} = \sum_{i=1}^m \hat{c}_i \phi_i$, the unknown vector $\hat{\mathbf{c}}$ and the residual vector $\hat{\mathbf{v}}$ are:

$$\hat{\mathbf{C}} = [\Phi^T \Phi]^{-1} \Phi^T \mathbf{f} \quad (7)$$

$$\mathbf{v} = \mathbf{f} - \mathbf{p}$$

In the least squares spectral analysis the independent vector ϕ is made of base functions: $\phi = (\cos \omega_j t, \sin \omega_j t)$

$$\mathbf{p} = \hat{c}_1 \cos \omega_j t + \hat{c}_2 \sin \omega_j t \quad (8)$$

where ω_j is known and the vector $\mathbf{C}^T = [\hat{c}_1, \hat{c}_2]$ is evaluated by Eq. (7) for each ω_j .

$$\Phi = \begin{bmatrix} \cos \omega_j t_1 & \sin \omega_j t_1 \\ \cos \omega_j t_2 & \sin \omega_j t_2 \\ \vdots & \vdots \\ \cos \omega_j t_n & \sin \omega_j t_n \end{bmatrix} \quad (9)$$

If the functional values (f) are discrete, the Φ matrix in Eq. (7) will be a Vandermond matrix (Eq. (9)). According to the projection theorem, the $\mathbf{p}$ vector is an orthogonal projection of the vector $\mathbf{f}$ in subspace M and the vector $\hat{\mathbf{v}} \in H$ has a minimum norm between all of the vectors $\mathbf{v}$ which satisfies the equation $\mathbf{f} = \mathbf{p} + \mathbf{v}$ (Wells et al. 1985). $S(\omega_j)$ shows the harmonic base functions $\sin(\omega_j t)$ and $\cos(\omega_j t)$ ability to approximate the data series $f(t)$. The aim of LSSA is to minimise the norm of the residuals vector ($\|\hat{\mathbf{v}}\|$). When $\mathbf{p}$ is the best approximation of $\mathbf{f}$ this norm ($\|\hat{\mathbf{v}}\|$) will be minimum. Therefore:

$$S(\omega_j) = \mathbf{f}^T \mathbf{C}_f^{-1} \mathbf{f} - \hat{\mathbf{v}}^T \mathbf{C}_f^{-1} \hat{\mathbf{v}} \quad (10)$$

where C_f^{-1} is the observation (f) weight matrix. If we rewrite the Eq. (10) as follows, the vertical scale of all frequencies spectra will be the same.

$$S(\omega_j) = 1 - \{(\hat{v}^T C_f^{-1} \hat{v}) / (f^T C_f^{-1} f)\} \quad (11)$$

In this case the spectral values $S(\omega_j)$ will be limited to the range $[0, 1]$ and form a normalized least squares spectrum. The systematic effects have an accumulative behaviour and appear in the data series as a trend with a long period and low frequency. Therefore the significant low frequency peak in the spectrums means that there are the systematic errors in the data series. The null hypothesis $H_0 : S(\omega_j) = 0$ is used to test the significance of this decision function:

$$S(\omega_j) \begin{cases} \leq (1 + (\alpha^{\frac{-2}{v}} - 1)^{-1})^{-1} & : \text{accept } H_0 \\ > (1 + (\alpha^{\frac{-2}{v}} - 1)^{-1})^{-1} & : \text{reject } H_0 \end{cases} \quad (12)$$

where v is the degree of freedom ($v = n - 2$), n is the data length and α is the significance level (usually 5%).

The circuit misclosures and the discrepancies between forward and backward runs of the section height differences can be used as functional values to construct the data series in precise levelling. In levelling networks, the section discrepancies are invariably more numerous than the circuit misclosures. Investigations have shown that traces of different types of systematic errors can be found in the discrepancies whereas some types of systematic errors tend to cancel in the circuit misclosures. In this paper, the section discrepancies are used as functional values of the data series. Many parameters such as observed mean height difference of section (H), section average slope and total number of turning points (TP) (staff setups) in forward and backward runs can be used as data series argument.

4. Levelling Data Analysis

This study investigates digital levels residual compensator error and observational method for its elimination. For this purpose, the two levelling test data sets, which were done with Zeiss DiNi 12 digital levels, were analysed. In the first data set, the observation method for the height difference at each setup is routine (BFFB, BFFB, BFFB, BFFB ...). The underscored B indicates that the telescope always points to the backsight when the spot bubble is adjusted at each station. The height difference at each setup will be the average of two height differences ($B_1 - F_1$) and ($B_2 - F_2$). Where (B_1) is the first reading backwards, (F_1) is the first reading forwards, (F_2) is the second reading forwards and (B_2) is the second reading backwards. In precise levelling, the discrepancy between these two heights difference should not exceed 0.25 mm. Otherwise, the readings at the setup should be repeated (Figure 5). The data of many lines were analysed. Here, the result of one of them will be discussed as example. The results of the other ones

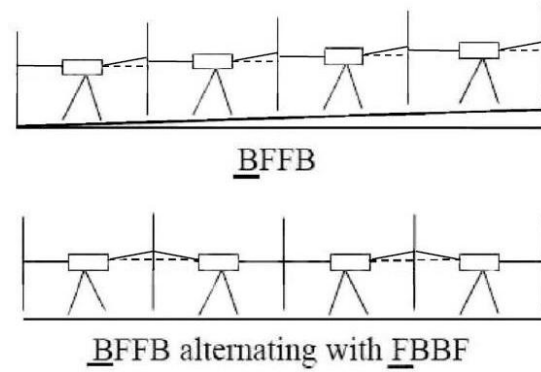


Figure 5. Measurement methods and the effect of the obliquity of horizon.

are the same. Figure 6 shows the accumulated discrepancies of this line. A linear trend (of about 0.84 mm/km) is considered significant. These figures show that the discrepancies in this levelling line accumulate and exhibit a systematic behaviour. Figure 7 shows the normalised least squares spectrum of the section discrepancies of this line with argument H . The low frequency significant peak in the spectrums indicates systematic behaviour in the functional values (discrepancies) of the data.

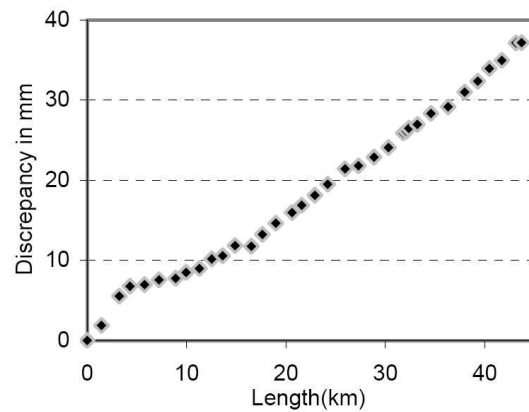


Figure 6. Accumulated discrepancy of a first data set sample levelling line.

The second data set is performed by the alternating pointing method (BFFB, FBBF) for each two sequential setups of section (Menzel 1998). The (BFFB) method in odd setups, and the (FBBF) method in even setups, respectively. The underscored B or F indicates that the spot bubble of the instrument is adjusted when the telescope is pointing at the staff indicated by the underscore. The number of setups is even and often sight lengths of two sequential setups are equal (Figure 5). Many lines of the second data set were analysed. Here, the result of one of them will be discussed as example. Figures 8 and 9 show the accu-

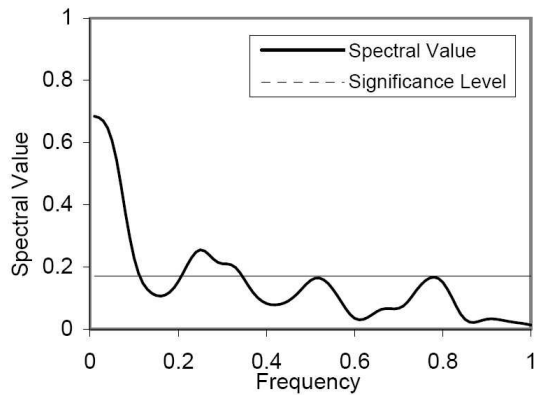


Figure 7. Normalised least squares spectrum of a first data set sample levelling line discrepancies with Argument H.

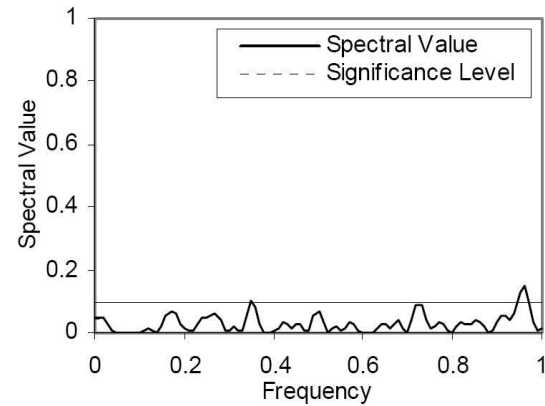


Figure 9. Normalised least squares spectrum of a second data set sample levelling line discrepancies with argument H.

ulated discrepancies behavior and the normalised least squares spectrum of the discrepancies of the levelling line, respectively. The accumulated discrepancies have a random behaviour. A very small apparent linear trend (of about 0.08 mm/km) is considered insignificant and not considered further. There is no significant low frequency peak in the normalised least squares spectrum of the line discrepancies with H as argument. This demonstrates the random behaviour of the data series.

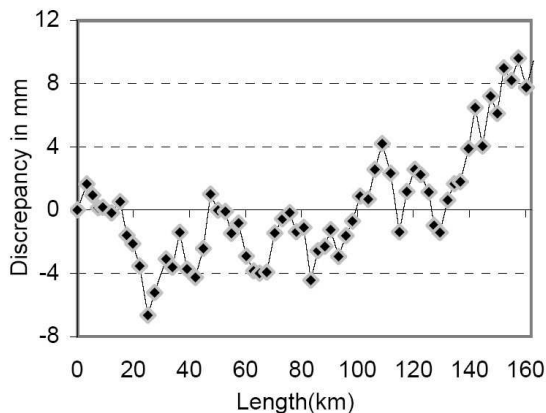


Figure 8. Accumulated discrepancy of a second data set sample levelling line.

In first order levelling, the maximum acceptable misclosure of a levelling loop with L km length is $3\text{mm}\sqrt{L}$. For a levelling line it will be the maximum acceptable discrepancy (the summation of the line height differences in forward and backward runs) of the levelling line. Table 1 includes the maximum acceptable errors and line discrepancies of the First and Second levelling lines. The results confirm the previous result.

Table 1. Maximum acceptable error and discrepancy of the test levelling lines.

	first	second
Maximum acceptable error (mm)	19.83	38.28
Discrepancy of line (mm)	37.18	9.45
One way length of line (km)	43.7	162.8

5. Conclusions

In the observation routine (BFFB), when using different levels in the forward and backward section runs, the residual compensator error affects the section mean height differences. If the same instrument is used in the forward and backward runs, this error is zero in section mean height difference, but it affects the sections discrepancy and any investigations about other systematic errors will be impossible using spectral analysis. By alternating pointing method (BFFB, FBBF) for each two sequential setups of section, residual compensator error will eliminate from two sequential setup height differences and eliminated from the sections mean height differences and discrepancies. Discrepancies can be used to investigate other systematic errors. By alternating pointing method (BFFB, FBBF), it does not matter to use different instruments in the forward and backward section runs.

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