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General composition for high level Petri nets and its properties

Research Article

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Abstract: Composition of High Level Petri Nets in terms of joining relevant places and/or transitions is considered in the paper. In case of place composition type safe and combined types composition is contemplated. The process of composition is proposed and analysed in three separate cases with respect to the general approaches with minimal composition interface in each case but an analogous extension of the interface follows immediately from the approach introduced. Properties of the composition mechanism are analysed, namely preserving of boundedness, liveness and deadlock freedom. Conditions for preserving of the desired properties are introduced. Usability of the compositional mechanism is analysed in the process of de/compositional analysis.

Keywords: high level Petri nets • composition • composition properties

1. Introduction

Formal description techniques (FDT) are widely regarded as the only tool with ability to design, analyse and maintain complex discrete systems used in real word applications. Several FDT have been proposed in this field of academic research, the best known of which is probably Petri nets. These provide a very simple designing tool and they are appreciated especially for their simplicity and analytical properties.

Petri nets (PN) have been developed from the first proposal by C.A. Petri [12] and a wide family covering many aspects of real word systems and even including advantages of some other FDT (for instance stochastic/time extensions [7], object Petri nets [1], algebraic PN [2, 3] and so on) have been established. The most significant extension in general are High Level Petri Nets (HLPN) [9]. The extension was proposed for several types of Petri nets including time aspects. HLPN provide very high modelling power although their analysis is very difficult.

Since the first PN proposal one of the main reservations concerns their unability of de/composition which is actually not included in the original conception. This motivated a lot of research and several de/compositional approaches

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(e.g. [4, 5, 8, 14]) including separate classes of de/compositional PN [11] have been proposed for modelling and/or analysis of Petri nets.
In this paper we focus on composition of HLPN. Instead of defining a separate class of composable HLPN or defining composition by means of compositional operators similar to the ones used in process algebras which have been proposed early on [4, 14], we concentrate on the HLPN class defined in the international standard [9] and composition is carried out as joining relevant places and/or transitions forming the interface of composition. In the first section the HLPN definition is introduced. Subsequently the process of composition is considered in three separate cases - place composition, transition composition and place-transition composition. The cases are introduced with minimal composition interface but an analogous extension to more elements in the interface follows immediatly from the definitions established. In section 4 properties of the composition mechanism are analysed with respect to preserving some of the important Petri nets characteristics, namely boundedness, liveness and deadlock freedom. Since it turns out that in general these properties are not preserved, the conditions for preserving are stated in the relevant propositions and it is shown that a restricted version of the composition mechanism preserves all the desired characteristics. For the sake of simplicity propositions are proven by very trivial examples where it is possible.

2. HLPN definition

In order to investigate composition of HLPN we focus on the HLPN standard [9]. The authors of the standard claim that it covers the ideas forming basic HLPN classes, namely Pr/T nets [6], colored nets [10] and algebraic nets [15]. There are some preliminaries we leave out in this paper such as multiset, formal term definitions, binding of variables, transition enabling and so on. For more detailed information we refer to [9]. The standard includes two main definitions - HLPN and HLPN graphs. Since composition is more illustrative in the case of HLPN graphs, we consider them a base for our treatment and refer to this class as $HLPN(G)$. The following definitions of HLPN graph and marking are taken from [9], to which the reader may refer for more details regarding the theory of HLPN.

Definition 2.1.

HLPN graph. HLPN graph ($HLPNG$) is a structure

$$HLPNG = (NG,\ Sig,\ V,\ H,\ Type,\ AN,\ m_0),$$

where

$NG = (P,\ T,\ F)$ is a net graph with
P – set of places
T – set of transitions
$F \subseteq (P \times T) \cup (T \times P)$ – set of directed arcs referred to as flow relation
$Sig = (S,\ O)$ is a many-sorted Boolean signature with the set of sorts $S = \{Integer, Boolean, Natural...\}$ and operations $O = \{<, \leq, \neq, =, +, -, ...\}$
V – S-indexed set of variables, $V \cap O = \emptyset$
$H = (S_H,\ O_H)$ - many-sorted algebra for the signature Sig defining its meaning
$Type :\ P \to S_H$ – mapping assigning types (sorts) for places
$AN = (A,\ TC)$ – net annotation with
$A :\ F \to Term(O \cup V)$ - mapping assigning terms to arcs. The result of term evaluation is a multiset over the types of associated places, i.e. $\forall((p,\ t),\ (t',\ p) \in F)\ \forall\alpha : Val_\alpha(A(p,\ t))$, $Val_\alpha(A(t',\ p)) \in Bag(Type(p))$, where $Term(O \cup V)$ is a set of terms over variables and operations, α is an assigning of token values to variables, $Val_\alpha(term)$ is term evaluation and $Bag(B)$ is a set of multisets over B
$TC :\ T \to Term(O \cup V)_{Bool}$ is a mapping assigning boolean expressions to transitions
$m_0 :\ P \to \bigcup_{p \in P} Bag\,(Type\,(p))$ is initial marking

Definition 2.2.
HLPN marking. Marking of $HLPNG = (NG,\ Sig,\ V,\ H,\ Type,\ AN,\ m_0)$ is a mapping

$$m:\ P \rightarrow \bigcup_{p \in P} Bag\,(Type\,(p)),$$

such that $\forall p \in P:\ m(p) \in Bag\,(Type\,(p))$.

3. HLPN composition

HLPN composition in contrast to the low level one has to take into account the net notations, i.e. a set of arc expressions and transition conditions. Moreover, since HLPN from definition contain a number of types for particular places, composition must take into account the types of these places. Composition of HLPN viewed as bipartite graphs may be performed through a set of places P_c, transitions T_c or both. We call the places P_c and transitions T_c the interface of composition denoted as I_c. In terms of [8] we have P composition if $I_c = P_c$, T composition if $I_c = T_c$ and PT composition if $I_c = P_c \cup T_c$. Note that we consider composition a junction. Composition may be divided into the following two steps:

1. structural composition
2. composition of net annotation

In the following we consider particular composition approaches as inverse operations to decomposition [13] and define the resulting net as a compound of two subnets. It is clear that composition may be generalized for n subnets. We focus on the elementary cases (in case of P composition $I_c = \{p\}$, T composition $I_c = \{t\}$ and for PT we have $I_c = \{p,\ t\}$) but an analogous extension of I_c to more elements is possible provided that the composition definition is extended properly.

Definition 3.1.
Composition. Let $N_1,\ N_2 \in HLPN(G)$, $N_i = (NG_i,\ Sig_i,\ V_i,\ H_i,\ Type_i,\ AN_i,\ m_{0i})$, $i = \{1,\ 2\}$, and χ_C be a function defined on the $HLPN(G)$ domain such that

$$\chi_C:\ HLPN(G) \times I \times HLPN(G) \rightarrow HLPN(G),$$

where
$I : [P_1 \times P_2 \rightarrow P_C] \cup [T_1 \times T_2 \rightarrow T_C], (P_1 \cup P_2) \cap P_C = \emptyset, (T_1 \cup T_2) \cap T_C = \emptyset$, $[A \rightarrow B]$ is a set of functions defined from A to B. I is a set of the functions creating the interface of composition I_C,
$I_C = P_C \cup T_C$
$C \in \{P,\ T,\ PT\}$ is index determining the type of composition and its interface.

For χ_C we assume that the signatures $Sig_1 = (S_1,\ O_1)$ and $Sig_2 = (S_2,\ O_2)$ do not contain the same operator definitions using different number nor different types of arguments, i.e. the following hold

$$\forall op \in O_1 \cup O_2:\ op_{(\sigma_1,\ s_1)} \in O_1 \wedge op_{(\sigma_2,\ s_2)} \in O_2 \Longrightarrow \sigma_1 = \sigma_2 \wedge s_1 = s_2,$$

where op is the same operator defined in the signatures, σ_i is a string of the types of the operator's input arguments, s_i is the type of the operator result.

In order to extend the initial markings of subnets let us introduce an auxiliary function

$$ext_{P,P'}:\ \bigcup_{p \in P} Bag(Type(p)) \rightarrow \bigcup_{p' \in P'} Bag(Type(p')),$$

provided that $P \subseteq P'$. The function represents an extension of the set of multisets for places from P to the set of multisets for places from P' such that $bag(Type(p')) = bag(Type(p))$ iff $p \in P \cap P', bag(Type(p')) = \emptyset(Type(p'))$

otherwise. In this case $\emptyset(Type(p'))$ is an empty multiset over $Type(p')$.

We also use the overloaded function

$$ext_{A,B}: \ Bag(A) \rightarrow Bag(B),$$

provided that $A \subseteq B, (A \cup B) \cap (P) = \emptyset, P =$ set of places. The function represents an extension of the multiset over A to the multiset over B such that

$$\forall b \in bag\,(B): \ bag\,(b) = bag\,(b) \Leftrightarrow b \in A, \ bag(b) = 0 \Leftrightarrow b \notin A,$$

where $bag(x)$ stands for multiplicity of the x element in the relevant multiset and $bag(B) \in Bag(B)$.

3.1. P composition

P composition is carried out as an inverse operation to P decomposition, i.e. a junction of places. Let $p_1 \in P_1$ and $p_2 \in P_2$ and $Type_1(p_1) \subseteq Type_2(p_2) \vee Type_2(p_2) \subseteq Type_1(p_1)$. Then $N = \chi_P(N_1, \ \{(p_1, p_2) \rightarrow \{p\}\}, \ N_2)$ we call type safe composition. If $\exists (c_1 \in Type_1(p_1), \ c_2 \in Type_2(p_2)): \ c_1 \neq c_2$ then composition is of combined place types.

3.1.1. P type safe composition

Definition 3.2.

P composition. Let $N_1, N_2 \in HLPN(G), p_1 \in P_1, p_2 \in P_2, I = \{(p_1, p_2) \rightarrow \{p\}\}$, i.e. $I_P = \{p\}$. Then the resulting net $N = \chi_P(N_1, \ I, \ N_2)$ of composition of two subnets through places p_1, $p_2$is given as

$$N = (NG, \ Sig, \ V, \ H, \ Type, \ AN, \ m_0),$$

where

$NG = (P, \ T, \ F)$
$P \ = \ P_1 \cup P_2 \cup \{p\} - \{p_1, \ p_2\}$
$T \ = \ T_1 \cup T_2$
$F \ = \ F_1 \cup F_2 \cup \{(p, \ t'), \ (t', \ p) | t' \in T \wedge (p_i, \ t'), \ (t', \ p_i) \in F_i \Rightarrow (p, \ t'), \ (t', \ p) \in F\}$
$-\{(p_i, \ t'), \ (t', \ p_i) | \ t \in T\}, \ \ i \in \{1, \ 2\}$
$Sig = Sig_1 \cup Sig_2 = (S_1 \cup S_2, \ O_1 \cup O_2)$
$V = V_1 \cup V_2$
$H = H_1 \cup H_2 = (S_{H_1} \cup S_{H_2}, \ O_{H_1} \cup O_{H_2})$
$Type: \ P \rightarrow (S_{H_1} \cup S_{H_2}), \ \forall p' \in P_i \backslash \{p_1, \ p_2\}: Type(p') = Type_i(p'), Type(p) = Type_1(p_1),$
if $Type_2(p_2) \subseteq Type_1(p_1), Type(p) = Type_2(p_2)$, if $Type_1\,(p_1) \subseteq Type_2\,(p_2)$
$AN \ = \ (A, \ TC)\,,$
$A: \ F \rightarrow Term\,(O_1 \cup O_2 \cup V)\,, A(f) = A_i(f)$ for $f \in F_i \backslash \{(p_i, \ t'), \ (t', \ p_i) | t' \in T\},$
$A\,((p, \ t')) = A\,((p_i, \ t')) \ , \ A\,((t', \ p)) = A\,((t', \ p_i))$, for $(p_i, \ t')\,, \ (t', \ p_i) \in F_i, \ t' \in T$
$TC: \ T \rightarrow Term(O_1 \cup O_2 \cup V)_{Bool}, \forall t' \in T: TC(t') = TC_i(t'),$
$m_0 = ext_{P_1 - \{p_1\}, \ \ P}(m_{01}|_{p_1}) + ext_{P_2 - \{p_2\}, \ \ P}(m_{02}|_{p_2})$, where $m_{0i}|_{p_i}$ is a restriction of the marking m_{0i} to all places except p_i,
$m_0(p) = ext_{Type(p_1), Type(p_1) \cup Type(p_2)}(m_0(p_1)) + ext_{Type(p_2), Type(p_1) \cup Type(p_2)}(m_0(p_2)).$

The resulting initial marking after composition is a sum of initial markings of the subnets $N_1, \ N_2$ over P considering the new place added as well.

Notice that composition extends the state space of the resulting net to composition of state spaces of the subnets. By state space we mean a set of sets of multisets over the types of places, that is $\bigcup_{p \in P} Bag\,(Type\,(p))$. Composition of state spaces with respect to interface I, I_C is given as $\bigcup_{p \in P'} Bag(Type(p))$, where $P' = \{(P_1 \cup P_2 \cup \{p | p \in I_C\}) - \{p_1, p_2 | (p_1, p_2) \rightarrow p \in I, p_i \in P_i\}\}$. In our extended version, the state space is a set of all reachable markings of all nets over a set of places. Thus the reachability set of a net N is only a subset of state space as meant in this paper.

3.1.2. P combined types composition

It is possible to compose subnets through places of different types and the result is similar as in the case of P type safe composition with the following difference

$$Type: \ P \rightarrow (S_{H_1} \cup S_{H_2}), \ Type(p) = Type(p_1) \cup Type(p_2).$$

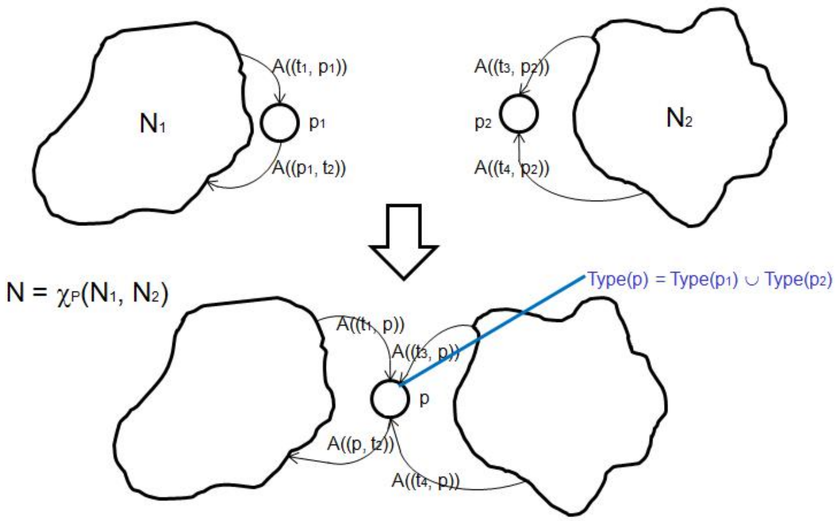

Figure 1. P composition principle.

Composition of different types extends the marking of p so that the type of p is union of types of p_1 and p_2. On the other hand P type safe composition does not extend the type of p.

3.2. T composition

T composition is carried out as an inverse operation to T decomposition, i.e. junction of transitions. Let $t_1 \in T_1,\ t_2 \in T_2$ and t be the resulting transition. The transition condition of t is the logical disjunction of the conditions of t_1 and t_2.

Definition 3.3.

T composition. Let $N_1, N_2 \in HLPN(G), t_1 \in T_1, t_2 \in T_2, I = \{(t_1, t_2) \to \{t\}\}$, i.e. $I_T = \{t\}$. Then the resulting net $N = \chi_T(N_1,\ I,\ N_2)$ of composition of two subnets through transitions $t_1,\ t_2$ is given as

$$N\ =\ (NG,\ Sig,\ V,\ H,\ Type,\ AN,\ m_0)\,,$$

where

$$\begin{aligned}
&NG = (P,\ T,\ F),\\
&\qquad P = P_1 \cup P_2\\
&\qquad T = T_1 \cup T_2 \cup \{t\} - \{t_1,\ t_2\}\\
&\qquad F =\ F_1 \cup F_1 \cup \{(p',\ t),\ (t,\ p')|p' \in P \wedge (p',\ t_i),\ (t_i,\ p') \cup F_i \Rightarrow\ (p',\ t),\ (t,\ p') \in F\}\\
&\qquad -\{(p',\ t_i),\ (t_i,\ p')|\ p' \in P\},\\
&Sig = Sig_1 \cup Sig_2 = (S_1 \cup S_2,\ O_1 \cup O_2)\\
&V = V_1 \cup V_2\\
&H = H_1 \cup H_2 = (S_{H_1} \cup S_{H_2},\ O_{H_1} \cup O_{H_2})\\
&Type:\ P \to (S_{H_1} \cup S_{H_2}),\ Type = Type_1 \cup Type_2, \forall p' \in P_i:\quad Type(p') = Type_i(p')\\
&AN\ =\ (A,\ TC)\,,\\
&\qquad A:\ F \to Term(O_1 \cup O_2 \cup V), A(f) = A_i(f) \text{ if } f \in F_i \backslash \{(p',\ t_i),\ (t_i,\ p')|p' \in P\},\\
&\qquad A\left((p',\ t)\right) = A\left((p',\ t_i)\right),\ \ A\left((t,\ p')\right) = A\left((t_i,\ p')\right), \text{for}\ \ (p',\ t_i)\,,\ (t_i,\ p') \in F_i,\ \ p' \in P,\\
&\qquad TC:\ T \to Term\left(O_1 \cup O_2 \cup V\right)_{Bool}, \forall t' \in\ T_i \backslash \{t_i\}:\ TC\left(t'\right) = TC_i\left(t'\right),\\
&\qquad TC(t) = TC_1(t_1) \vee TC_2(t_2)\\
&m_0 =\ ext_{P_1,P}(m_{01})\ +\ ext_{P_2,P}(m_{02}).
\end{aligned}$$

As in the case of P composition, the resulting initial marking is a sum of initial markings of the subnets $N_1,\ N_2$ over P. The transition condition of the new transition t is the logical disjunction of the original transition conditions of t_1 and t_2. The choice of logical disjunction of transition conditions is justified by the fact that logical conjunction would block

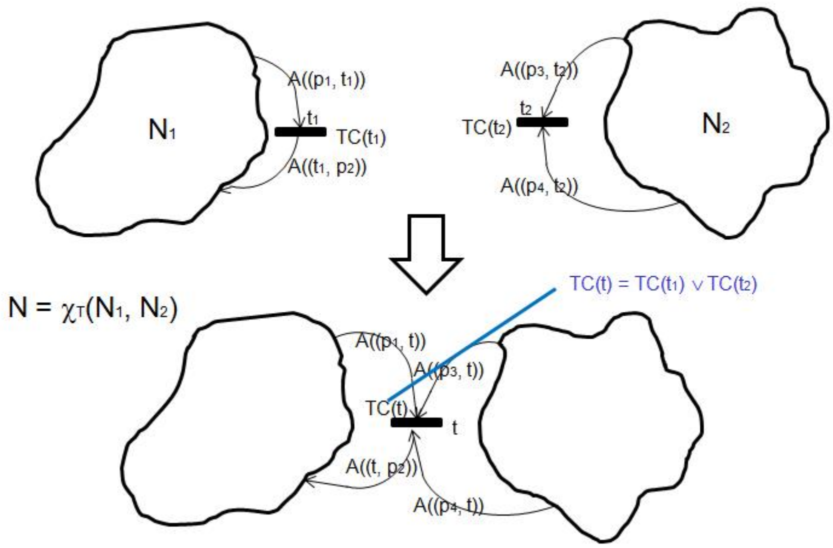

Figure 2. T composition principle.

the firing of the transition (if $TC(t_1) = \neg TC(t_2) \Rightarrow TC(t_1) \wedge TC(t_2) = false$). Disjunction is therefore a less strict condition and allows the execution in N in a more favorable way.
Notice that T composition extends the state space of the resulting net N to composition of the state spaces of the subnets. Furthermore it does not change the place types as well as P type safe composition.

3.3. PT composition

PT composition combines features of P and T composition so it is based on joining places and transitions at the same time as an inverse operation to PT decomposition. Consequently P and T composition are special cases of more general PT composition. In order to point out the principle let us consider the easiest case – composition through places $p_1 \in P_1,\ p_2 \in P_2$with the resulting place p and transitions $t_1 \in T_1,\ t_2 \in T_2$with the resulting transition t.

Definition 3.4.

PT composition. Let $N_1, N_2 \in HLPN(G), p_1 \in P_1, p_2 \in P_2, t_1 \in T_1, t_2 \in T_2, I = \{(p_1, p_2) \to \{p\}, (t_1, t_2) \to \{t\}\}$, i.e. $I_{PT} = \{p, t\}$. Then the resulting net $N = \chi_{PT}(N_1,\ I,\ N_2)$ is given as

$$N = (NG,\ Sig,\ V,\ H,\ Type,\ AN,\ m_0),$$

where

$$\begin{aligned}
&NG = (P,\ T,\ F),\\
&\quad P = P_1 \cup P_2 \cup \{p\} - \{p_1,\ p_2\}\\
&\quad T = T_1 \cup T_2 \cup \{t\} - \{t_1,\ t_2\}\\
&\quad F = F_1 \cup F_2 \cup \{(p',\ t),\ (t,\ p')| p' \in P \wedge (p',\ t_i),\ (t_i,\ p') \in F_i \Rightarrow (p',\ t),\ (t,\ p') \in F\}\\
&\quad \cup\{(p,\ t'),\ (t',\ p)|\ t' \in T\ \wedge (p_i,\ t'),\ (t',\ p_i) \in F_i \Rightarrow (p,\ t'),\ (t',\ p) \in F\}\\
&\quad \cup \{(p,\ t)\,,\ (t,\ p)|\,(p_i,\ t_i)\,,\ (t_i,\ p_i) \in F_i \Rightarrow (p,\ t),\ (t,\ p) \in F\}\\
&\quad -\{(p_i,\ t_i),\ (t_i,\ p_i),\ (p_i,\ t'),\ (t',\ p_i),\ (p',\ t_i),\ (t_i,\ p')| t' \in T_1 \cup T_2,\ p' \in P_1 \cup P_2\},\\
&Sig = Sig_1 \cup Sig_2 = (S_1 \cup S_2,\ O_1 \cup O_2)\\
&V = V_1 \cup V_2\\
&H = H_1 \cup H_2 = (S_{H_1} \cup S_{H_2},\ O_{H_1} \cup O_{H_2})\\
&Type:\ P \to (S_{H_1} \cup S_{H_2}),\ Type = Type_1 \cup Type_2,\\
&Type(p) = Type(p_1) \cup Type(p_2)\\
&AN = (A,\ TC)\,,
\end{aligned}$$

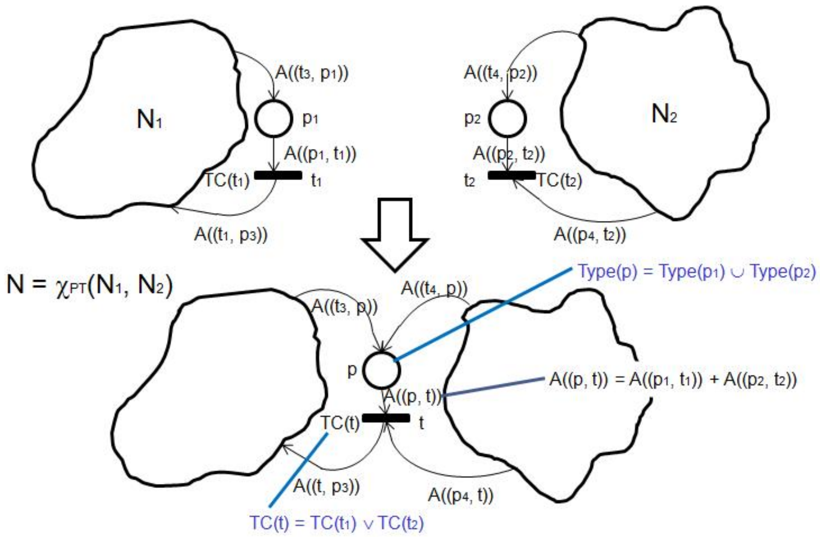

Figure 3. PT composition principle.

$A: F \rightarrow Term(O_1 \cup O_2 \cup V)$,
$A(f) = A_i(f)$ for $f \in F_i \backslash \{(p_i, t'), (t', p_i), (p', t_i), (t_i, p'), (p_i, t_i), (t_i, p) | t' \in T_1 \cup T_2, p' \in P_1 \cup P_2\}$,
$A((p', t)) = A((p', t_i)), A((t, p')) = A((t_i, p')), A((p, t')) = A((p_i, t')), A((t', p_i)) = A((t', p))$,
for $(p_i, t'), (t', p_i), (p', t_i), (t_i, p') \in F_i, t' \in T_1 \cup T_2, p' \in P_1 \cup P_2$
$A((p, t)) = A((p_1, t_1)) + A((p_2, t_2)), A((t, p)) = A((t_1, p_1)) + A((t_2, p_2))$, where $A(f) = \emptyset \Leftrightarrow f \notin F$,
$TC: T \rightarrow Term(O_1 \cup O_2 \cup V)_{Bool}, \forall t' \in T_i \backslash \{t_i\}: TC(t') = TC_i(t')$,
$TC(t) = TC_1(t_1) \vee TC_2(t_2)$
$m_0 = ext_{P_1 - \{p_1\}, P}(m_{01}|_{p_1}) + ext_{P_2 - \{p_2\}, P}(m_{02}|_{p_2})$, i.e. sum of initial markings of the subnets N_1, N_2 over P, where $m_{0i}|_{p_i}$ is a restriction of the marking m_{0i} to all places except p_i,
$m_0(p) = ext_{Type(p_1), Type(p_1) \cup Type(p_2)}(m_0(p_1)) + ext_{Type(p_2), Type(p_1) \cup Type(p_2)}(m_0(p_2))$.

As in the case of P composition, the resulting initial marking is a sum of initial markings of the subnets N_1, N_2 over P. Notice that PT composition is union of P and T composition as outlined above.

4. Properties of composition

In the following we are interested in properties of the composition mechanism, i.e. preserving some of the Petri net properties, namely boundedness, deadlock freedom and liveness.

4.1. Boundedness

Definition 4.1.
Boundedness. Let $N = (NG, Sig, V, H, Type, AN, m_0) \in HLPN(G)$.$N$ is bounded if $\forall p \in P\ \forall m \in R(N)\ \forall a \in Type(p)\ \exists k \in \mathbb{N} : k \geq bag(a)$ in $m(p)$.

In the definition $R(N)$ stands for the reachability set of net N, $\mathbb{N}$ is a set of natural numbers.

Proposition 4.1.
Let $N_i = (NG_i, Sig_i, V_i, H_i, Type_i, AN_i, m_{0i}) \in HLPN(G)$ be two bounded nets. Then $N = \chi_C(N_1, I, N_2)$ may not be neccessarily be bound, i.e. χ_C does not preserve boundedness for $C \in \{P, T, PT\}$.

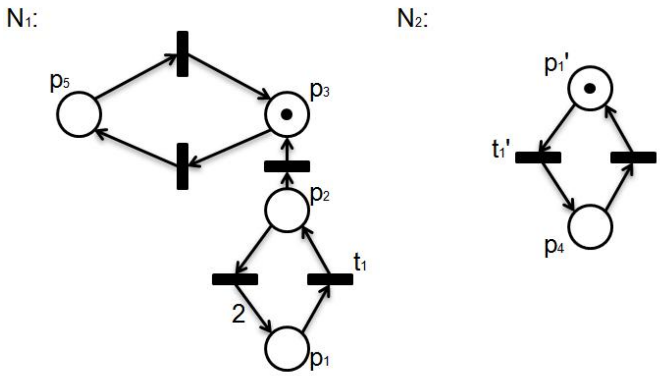

Figure 4. Source bounded nets for χ_P, χ_{PT}.

Proof. We are going to prove the proposition for each case separately.

1. χ_P

   For the sake of simplicity let us consider the low level Petri nets N_1, N_2 depicted in Fig. 4 where only relevant parts are labelled. Let us construct $N = \chi_P(N_1,\ I,\ N_2)$ by junction of places p_1 and p_1', i.e. $I = \{(p_1, p_1' \rightarrow \{p\})\}, I_P = \{p\}$. It is obvious that N is unbounded due to activation of the unbounded section. Similarly an equivalent high level net may be found.

2. χ_{PT}

   Since χ_P does not preserve boundedness, neither does χ_{PT}. If we construct $N = \chi_{PT}(N_1,\ I,\ N_2)$ from the source nets depicted in Fig. 4 by junction of places p_1 and p_1' and transitions t_1 and t_1', i.e. $I = \{(p_1, p_1' \rightarrow \{p\}), (t_1, t_1' \rightarrow \{t\})\}, I_{PT} = \{p, t\}$ the resulting net N is unbuounded. Again a high level net equivalent may be found.

3. χ_T

   Let us consider the two high level nets depicted in Fig. 5. For the sake of simplicity only relevant parts are labelled and places containing initial tokens are depicted with black dots. In this case black dots do not represent low level tokens. For these nets we have: $\forall p \in P_1 \cup P_2\ : Type(p) = \{2, 3\}, m_0(p_1) = \{0'2, 1'3\}, m_0(p_1') = m_0(p3) = \{1'2, 0'3\}, m_0(p_2) = m_0(p_4) = m_0(p_5) = \{0'2, 0'3\}$. Notation $x'y$ means that the number of y element in the multiset is x.

   Let us construct $N = \chi_T(N_1,\ I,\ N_2)$ by junction of transitions t_1 and t_1', i.e. $I = \{(t_1, t_1' \rightarrow \{t\})\}, I_T = \{t\}$. The resulting transition condition of the new transition is $TC(t) = x > 4 \vee x < 3$ and this enables the unbounded section. By $x{+}{+}x$ we mean that two tokens of type and value corresponding to the bounded variable x are added to p_1.

Remark 4.1.

Notice that in the proof we exploited the "dead" unbounded sections in separate subnets by their activating. Such a situation is not very likely in practise but it is sufficient to construct the proof. The situation may arise after decomposition of unbounded net.

Although χ_C does not preserve boundedness in general case under some conditions this may not hold.

Proposition 4.2.

Let $N_i = (NG_i,\ Sig_i,\ V_i,\ H_i,\ Type_i,\ AN_i,\ m_{0i}) \in HLPN(G), i \in \{1, 2\}$ be two bounded nets and $N = \chi_P(N_1,\ I,\ N_2)$ be the result of their composition. Then N is bounded if $\forall(p_1 \in P_1, p_2 \in P_2) : (p_1, p_2) \rightarrow p' \in I : Type(p_1) \cap Type(p_2) = \emptyset$.

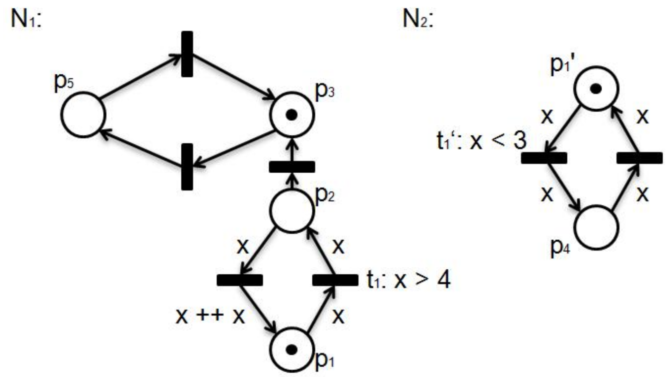

Figure 5. Source bounded nets for χ_T.

Proof. Since $\forall (p_1 \in P_1, p_2 \in P_2) : (p_1, p_2) \to p' \in I : Type(p_1) \cap Type(p_2) = \emptyset$, the reachability set of N (denoting $R(N)$) is given as

$$R(N) \subseteq \{m | m = ext_{P_1 - I_P - args(I_P),\ P}(m_1|_{args(I_P)}) + ext_{P_2 - I_P - args(I_P),\ P}(m_2|_{args(I_P)}), m_i \in R(N_i), \\ \forall p \in I_P : m(p) \in comb(p_1, p_2), (p_1, p_2) \in args(I_P), p_1 \in P_1, p_2 \in P_2\},$$

where

$$args(I_P) = \{(p_1, p_2) | (p_1, p_2) \to p' \in I, p' \in I_P\}$$
$$comb(p_1, p_2) = \{ext_{Type(p_1), Type(p_1) \cup Type(p_2)}(m_1(p_1)) + ext_{Type(p_2), Type(p_1) \cup Type(p_2)}(m_2(p_2)) | m_i \in R(N_i)\}.$$

Informally speaking $R(N)$ is "concatenation" of $R(N_1)$ and $R(N_2)$ such that markings in places from the composition interface are sum of markings of places which create the composition interface, i.e. if p_1 and p_2 are joined to p, marking in p is sum of markings in p_1 and p_2 at each step of net execution. This is a consequence of the fact that if the types of all the places to be junctioned are completely different the execution of the subnets N_1 and N_2 is not affected by each other since $\forall (p_1 \in P_1, p_2 \in P2) : (p_1, p_2) \in args(I_P)$ there is no arc from p_1 that can bind values from p_2 and conversely.

Having a look at $R(N)$ we can see that if N_1 and N_2 are bounded, N is bounded as well and $\forall m \in R(N) \forall p \in (P_1 \cup P_2) \cap P : m(p) \leq max(R(N(p))), \forall (p' \in I_P, (p_1, p_2) \in args(I_P) : (p_1, p_2) \to p' \in I) : m(p') \leq max(comb(p_1, p_2))$,

where

$R(N(p)) = \{m(p) | m \in R(N)\}$
$max(Bag(A))$ returns maximum of multisets from $Bag(A)$, i.e.
$max(Bag(A)) = bag(A) \Rightarrow \forall bag'(A) \in Bag(A) : bag'(A) \leq bag(A)$.

4.2. Deadlock freedom

Definition 4.2.

Deadlock-free net. Let $N = (NG, Sig, V, H, Type, AN, m_0) \in HLPN(G)$ and $R(N)$ be the reachability set of N. N is deadlock-free iff $\forall m \in R(N)\ \exists t \in T$ such that t is enabled in m.

Proposition 4.3.

Let $N_i = (NG_i, Sig_i, V_i, H_i, Type_i, AN_i, m_{0i}) \in HLPN(G), i \in \{1, 2\}$ be two deadlock-free nets. Then $N = \chi_C(N_1, I, N_2)$ may not be necessarily deadlock-free, i.e. χ_C does not preserve deadlock freedom for $C \in \{P, T, PT\}$.

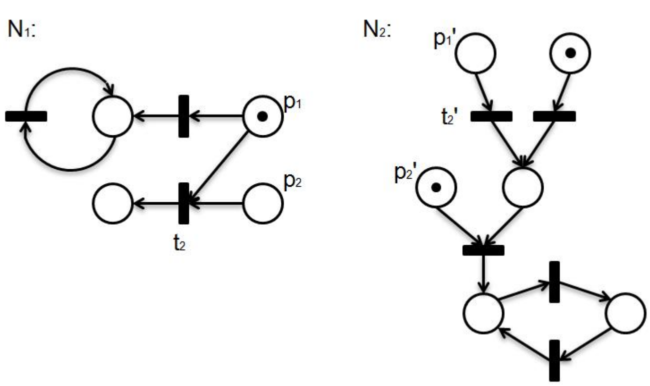

Figure 6. Source deadlock-free nets for χ_P, χ_{PT}.

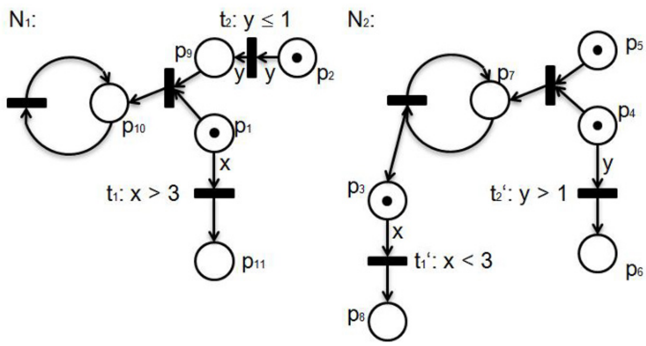

Figure 7. Source deadlock-free nets for χ_T.

Proof. Let us prove each case separately.

1. χ_P

   We are going to prove the proposition by an example. Let us consider the two low level nets depicted in Fig. 6 and construct $N = \chi_P(N_1,\ I,\ N_2)$ by junction of places p_1, p'_1 to pp_1 and p_2, p'_2 to pp_2, $I = \{(p_1, p'_1 \rightarrow \{pp_1\}), (p_2, p'_2 \rightarrow \{pp_2\})\}$, $I_P = \{pp_1, pp_2\}$. Obviously the resulting net N may reach a deadlock state, namely by firing t_2 first. Similarly a high level example may be easily found.

2. χ_{PT} As in the case of P composition let us construct $N = \chi_{PT}(N_1,\ I,\ N_2)$ from the same source nets depicted in Fig. 6 by junction of places p_1, p'_1 to pp_1 and p_2, p'_2 to pp_2 and transitions t_2, t'_2 to tt_2, $I = \{(p_1, p'_1) \rightarrow \{pp_1\}, (p_2, p'_2) \rightarrow \{pp_2\}, (t_2, t'_2) \rightarrow \{tt_2\}\}$, $I_{PT} = \{pp_1, pp_2, tt_2\}$. The resulting net N may reach a deadlock state, namely by firing tt_2 first. This obviously holds for high level nets as well since a high level equivalent may be found.

3. χ_T

   Let us consider composition of the two high level nets depicted in Fig. 7 so that the resulting net $N = \chi_T(N_1,\ I,\ N_2)$ is obtained by junction of the transitions t_1, t'_1 to tt_1 and t_2, t'_2 to tt_2, $I = \{(t_1, t'_1 \rightarrow \{tt_1\}), (t_2, t'_2 \rightarrow \{tt_2\})\}$, $I_T = \{tt_1, tt_2\}$. Note that in Fig. 7 only relevant parts are labelled and black dots do not stand for low level tokens but symbolize high level ones. The neccessary notation is as follows: $\forall p \in P_1 \cup P_2 : Type(p) = \{1, 2\}, m_0(p_1) = \{0'1, 1'2\}, m_0(p_2) = m_0(p_3) = m_0(p_4) = m_0(p_5) = \{1'1, 0'2\}, m_0(p_6) = ... = m_0(p_{11}) = \{0'1, 0'2\}, TC(tt_1) = x > 3 \vee x < 3, TC(tt_2) = y > 1 \vee y \leq 1$. Although N_1, N_2 separately can not deadlock, N on the other hand can, namely by firing tt_1 and tt_2 first since tt_1, tt_2 are enabled after composition.

As shown above χ_C in general does not preserve deadlock freedom. If we restrict the mechanism to place composition it can preserve deadlock freedom under some conditions which are described below. In the text below $p\bullet = \{t|(p,t) \in (P \times T)\}, \bullet p = \{t|(t,p) \in (T \times P)\}, t\bullet = \{p|(t,p) \in (T \times P)\}, \bullet t = \{p|(p,t) \in (P \times T)\}$.

Proposition 4.4.

Let $N_i = (NG_i,\ Sig_i,\ V_i,\ H_i,\ Type_i,\ AN_i,\ m_{0i}) \in HLPN(G), i \in \{1,2\}$ be two deadlock-free nets. Construct $N = \chi_P(N_1,\ I,\ N_2)$ such that $I = \{(p_1,p_2) \rightarrow \{p\}|p \in I_P, p_1\bullet = \emptyset\}$. Then $N = \chi_P(N_1,\ I,\ N_2)$ is deadlock-free.

Proof. Let $FS_1 = \{\sigma|\sigma = t_1, ..., t_n, t_i \in T \cap T_1\}$ be a set of all firing sequences of transitions in N_1. It is sufficient to show that after composition the set of firing sequences of subset N_1 does not change. Suppose contrary. Then $\exists p' \in \{\bullet t|t \in T \cap T_1\}\exists m \in R(N) : m(p') \notin \{m(p')|m \in R(N_1)\}$ and thus $\exists t \in T\backslash T_1 : p' \in t\bullet$. Since $T\backslash T_1 = T_2$ and $\forall t \in T_2 : t\bullet \subseteq P_2 \cup I_P$ such a t nor p' do not exist and therefore FS_1 does not change. In other words there is no token adding from subset N_2 through I_P to the subset N_1 and no place from N_1 which creates I_P adds tokens to N_1 (because $I = \{(p_1,p_2) \rightarrow \{p\}|p \in I_P, p_1\bullet = \emptyset\}$) and therefore all firing sequences in N for the subset N_1 are the same as in N_1 and from our assumption it follows that N_1 can not reach a deadlock state.

Remark 4.2.

According to the proposition if we compose nets and all the places of one of them which create the composition interface are final places containing no outgoing arcs, the resulting net is always deadlock-free.

Proposition 4.5.

Let $N_i = (NG_i,\ Sig_i,\ V_i,\ H_i,\ Type_i,\ AN_i,\ m_{0i}) \in HLPN(G), i \in \{1,2\}$ be two deadlock-free nets and $N = \chi_P(N_1,\ I,\ N_2)$ be the result of their composition. Then N is deadlock-free if $\forall(p_1 \in P_1, p_2 \in P_2) : (p_1,p_2) \rightarrow p' \in I : Type(p_1) \cap Type(p_2) = \emptyset$.

Proof. Since $\forall(p_1 \in P_1, p_2 \in P_2) : (p_1,p_2) \rightarrow p' \in I : Type(p_1) \cap Type(p_2) = \emptyset$, the reachability set of N is given as

$$R(N) \subseteq \{m|m = ext_{P_1 - I_P - args(I_P),\ P}(m_1|_{args(I_P)}) + ext_{P_2 - I_P - args(I_P),\ P}(m_2|_{args(I_P)}), m_i \in R(N_i),$$
$$\forall p \in I_P : m(p) \in comb(p_1,p_2), (p_1,p_2) \in args(I_P), p_1 \in P_1, p_2 \in P_2\},$$

where

$$args(I_P) = \{(p_1,p_2)|(p_1,p_2) \rightarrow p' \in I, p' \in I_P\}$$
$$comb(p_1,p_2) = \{ext_{Type(p_1),Type(p_1)\cup Type(p_2)}(m_1(p_1)) + ext_{Type(p_2),Type(p_1)\cup Type(p_2)}(m_2(p_2))|m_i \in R(N_i)\}.$$

As far as we consider $R(N)$ this is the same situation as in *proposition 4.2*, i.e. the subnets N_1 and N_2 behave like separate nets. It is straightforward that whenever $\exists t \in T_i$ that is enabled in N_i for some m, $i \in \{1,2\}$ then t is enabled in N. As N_1, N_2 are by assumption both deadlock-free thus $\forall m \in R(N_i)\exists t \in T_i : t$ is enabled in m, N is neccessarily deadlock-free as well.

4.3. Liveness

Definition 4.3.

Liveness. Transition t is live ($L(t)$) iff $\forall m \in R(N)\exists\sigma \in T^* : m(\sigma) \wedge \sigma(t) \geq 1$. Petri net is live iff $\forall t \in T : L(t)$.

In the definition T^* is Kleene's closure over the set of transitions T, $m(\sigma)$ is valid transition sequence from marking m and $\sigma(t)$ is number of occurrences of t in transition sequence σ.

As in the case of preserving boundedness and deadlock freedom, χ_C, not surprisingly, does not preserve liveness in general for $C \in \{T, PT\}$.

Proposition 4.6.

Let $N_i = (NG_i,\ Sig_i,\ V_i,\ H_i,\ Type_i,\ AN_i,\ m_{0i}) \in HLPN(G), i \in \{1,2\}$ be two live nets. Then $N = \chi_C(N_1,\ I,\ N_2)$ may not be necessarily live, i.e. χ_C does not preserve liveness for $C \in \{T, PT\}$.

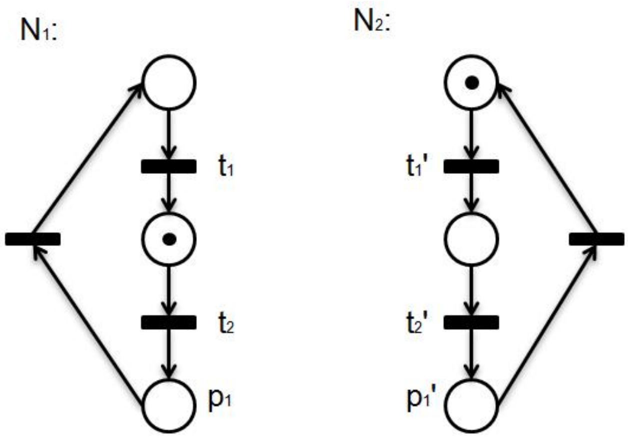

Figure 8. Source live nets for χ_T, χ_{PT}.

Proof. The proof is constructed for each case separately.

1. χ_T

   Let us prove the proposition using a small example. Consider the two live low level nets depicted in Fig. 8 and construct $N = \chi_T(N_1, I, N_2)$ with $I = \{(t_1, t'_1) \to \{tt_1\}, (t_2, t'_2) \to \{tt_2\}\}, I_T = \{tt_1, tt_2\}$. Obviously the resulting net N is not live since it is deadlocked and cannot fire tt_1 nor tt_2 from its initial marking. It is easy to find equivalent high level nets.

2. χ_{PT}

   As χ_T does not preserve liveness, neither does χ_{PT}. The proof is again trivial and it is sufficient to construct $N = \chi_T(N_1, I, N_2)$ with $I = \{(p_1, p'_1) \to \{pp_1\}, (t_1, t'_1) \to \{tt_1\}, (t_2, t'_2) \to \{tt_2\}\}, I_{PT} = \{pp_1, tt_1, tt_2\}$ from the source nets depicted in Fig. 8. By this construction we obtained a deadlocked net N since it cannot fire tt_1 nor tt_2 from its initial marking. A high level net example can be easily found.

Although χ_T, χ_{PT} do not preserve liveness in general, we can define a restricted version of the χ_P, which does.

Proposition 4.7.
Let $N_i = (NG_i,\ Sig_i,\ V_i,\ H_i,\ Type_i,\ AN_i,\ m_{0i}) \in HLPN(G), i \in \{1, 2\}$ be two live nets and $N = \chi_P(N_1,\ I,\ N_2)$ be the result of their composition. Then N is live if $\forall(p_1 \in P_1, p_2 \in P_2) : (p_1, p_2) \to p' \in I : Type(p_1) \cap Type(p_2) = \emptyset$.

Proof. As far as we consider $R(N)$ this is the same situation as in *proposition 4.2*, *proposition 4.5*, i.e. the subnets N_1 and N_2 behave like separate nets after composition without affecting each other during execution.
Thus we remind that since $\forall(p_1 \in P_1, p_2 \in P_2) : (p_1, p_2) \to p' \in I : Type(p_1) \cap Type(p_2) = \emptyset$, the reachability set of N is given as

$$R(N) \subseteq \{m | m = ext_{P_1 - I_P - args(I_P),\ P}(m_1|_{args(I_P)}) + ext_{P_2 - I_P - args(I_P),\ P}(m_2|_{args(I_P)}), m_i \in R(N_i),$$
$$\forall p \in I_P : m(p) \in comb(p_1, p_2), (p_1, p_2) \in args(I_P), p_1 \in P_1, p_2 \in P_2\},$$

where

$$args(I_P) = \{(p_1, p_2) | (p_1, p_2) \to p' \in I, p' \in I_P\}$$
$$comb(p_1, p_2) = \{ext_{Type(p_1), Type(p_1) \cup Type(p_2)}(m_1(p_1)) + ext_{Type(p_2), Type(p_1) \cup Type(p_2)}(m_2(p_2)) | m_i \in R(N_i)\}.$$

Due to the specific nature of composition it is obvious that whenever $\exists t \in T_i$ that is enabled in N_i for some m, $i \in \{1, 2\}$ then t is enabled in N. As N_1, N_2 are by assumption both live thus $\forall t \in T_i \forall m \in R(N_i) \exists \sigma \in T^* : m(\sigma) \wedge \sigma(t) \geq 1$, N is neccessarily live as well.

5. Composition in de/compositional analysis

So far we have defined a compositional mechanism and analysed its properties from general point of view. However, in practise composition is only a part of de/compositional analysis of nets performed after previous decomposition and analysis of the resulting subnets.
Let $\delta_C : HLPN(G) \times I \to HLPN(G)^2$ be a function decomposing a net through the interface $I \subseteq P \cup T$ duplicating elements $i \in I$ in each subnet. Index C has the same meaning as in the case of composition. We are not deeply interested in δ_C; it is sufficient to assume that such a function exists [13].
The compositional mechanism as introduced preserves any property of the analysed Petri net in de/compositional analysis under some conditions stated in the following proposition.

Proposition 5.1.

Let $N = (NG, Sig, V, H, Type, AN, m_0) \in HLPN(G), \exists \delta_C(N, I) = (N_1, N_2)$ such that no type of any place is affected and N, N_1, N_2 have a property π, i.e. δ_C preserves the property π. Let $I_C = \{(i, i) \to i | i \in I\}$. Then $N' = \chi_C(N_1, I_C, N_2)$ has the property π iff $\forall p \in I \cap P : m'_0(p) = m_0(p), \forall t \in I \cap T \forall \alpha : Val'_\alpha(TC'(t)) = Val_\alpha(TC(t)), \forall f \in \{(x, y)|x \in I \vee y \in I\} \cap F : Val'_\alpha(A'(f)) = Val_\alpha(A(f))$.

Proof. Trivial. From the definitions of χ_C for $C \in \{P, T, PT\}$ and construction of I_C in the proposition it directly follows that $NG' = NG, Sig' = Sig, V' = V, H' = H, Type' = Type$. Considering χ_C together with our requirement that $\forall p \in I \cap P : m'_0(p) = m_0(p)$ gives $m'_0 = m_0$. The definitions of χ_C with the requirements that $\forall t \in I \cap T \forall \alpha : Val'_\alpha(TC'(t)) = Val_\alpha(TC(t)), \forall f \in \{(x, y)|x \in I \vee y \in I\} : Val'_\alpha(A'(f)) = Val_\alpha(A(f))$ lead to equal operational semantics and the same reachability set, thus $R(N') = R(N)$. Since the static structure of N and N' is the same except the terms (arc expressions and transition conditions) possibly affected by decomposition and evaluation of such terms is the same in the both nets leading to the same reachability sets and considering the fact that each property depends on static structure and semantics of the nets we may conclude that if N has a property $\pi \implies N'$ has the same property.

Remark 5.1.

If we admit that in de/compositional analysis decomposing a net may lead to changes of the terms of the respective transitions and arcs between places/transitions from the de/compositional interface and those outside the interface, backward composition may be a bit different than that introduced in the definitions of χ_C. Namely the difference is given by the requirements in the proposition. It is not important to obtain the same terms after backward composition but their evaluation with the same bindings of variables must be the same in the original net and the resulting net after the composition. The usage of χ_C in de/compositional analysis must take into account the requirements therefore creating $TC'(t), m'_0(p)$ and $A'(f)$ for each $t \in I$ and $f = \{(x, y)|x \in I \vee y \in I\}$ may be different with respect to the requirements. In practise decomposition of a net leading to the necessity of modifying any terms is a very rare case. Thus the compositional approach χ_C may be used without any or with minimal changes. The proposition demonstrates its valid usage in de/compositional analysis.

6. Conclusion

The paper is divided into two main parts. In the first part composition of the chosen HLPN class defined according to the international standard is considered. The class covers three main HLPN classes - predicate/transition nets, colored nets and algebraic nets. The approach used comes out from general principles of composition used for low level PN in terms of joining relevant places and/or transitions. Other approaches are possible such as composition using special compositional operators but the aim of the paper is to study the extended high level version of composition for the chosen HLPN class without any extensions needed.
Since (HL)PN are bipartite graphs in their graphical form, composition in our approach is defined as place, transition or place-transition junction. The places/transitions form the interface of composition and we considered only the minimal interface in each case believing that the respective extension to more elements is clear enough from the sketched approach. In the case of place composition type safe and composition of places of different types are considered separately because of the natural feature of HLPN - types of places. Type safe composition combines places of the same type or may be used when the type of one place is a subtype of the second. Place and transition compositions are special cases of more general place-transition composition.
In the second main part of the paper we study the properties of the composition mechanism introduced, namely preserving boundedness, liveness and deadlock freedom. It turns out that the approach is very benevolent and in general it does not

preserve any of the desired PN characteristics. This fact is for the sake of simplicity proven using very simple examples instead of technical proofs. A restricted version of the compositional operator is defined in order to ensure the properties to be preserved. It is shown in the proofs of the corresponding propositions that boundedness, liveness and deadlock freedom are kept using this restricted operator. Subsequently usability of the composition mechanism in the process of de/compositional analysis is considered.

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