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Utilization of a comparison of curvatures for land surface segmentation

Research Article

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Abstract: Utilization of a new geomorphometric variable for land surface segmentation - the angle of absolute curvatures - is a main goal of the paper. The angle of absolute curvatures is defined as the difference between the orientation of maximal curvature (field independent) and the orientation of the greater of the profile or the tangential curvature. Land-forms separated by three types of borders (A, B, C) can be delimited from the field of angles of absolute curvatures. Borders of A type are connected with a local extreme of slope. Borders of B and C type are connected with a change to the priority of either profile or tangential curvature, as shown in computation, respectively. Fields of altitude, slope, profile curvature, tangential curvature and rotor curvature are reflected by an algorithm. Distinct borders in the field of the angles of absolute curvatures are connected with a sudden change of value and with zero isolines in the previously mentioned fields. Spatially closed entities generated by this proposed algorithm are considered to be a variant of the elementary forms of the land surface. The quality of information generated by this algorithm depends on the size of the grid mesh of the input digital elevation model. The algorithm in its current state is suitable for locating the borders of some elementary forms in the first stage of geomorphology mapping.

Keywords:

1. Introduction

Geomorphometrical analysis (terrain analysis) has become current in geography (or more specifically, geomorphology). This was caused by marked progress in theoretical geomorphometry, which presents a background for geomorphometry. Theoretical geomorphometry has stood apart from the main stream of geomorphology and has emerged from empirical observations of land surface, mathematical analysis and computer science as a consequence of the quantitative revolution. This paper introduces a new geomorphometric variable, the angle of absolute curvatures, that results from comparing the orientation of the profile and tangential curvature on one hand, and the extreme (maximal and minimal) curvature of the land surface on the other hand. This variable has been primarily defined for testing a hypothesis that the angle of absolute curvatures can be used as a measure of the influence of endogenous and exogenous factors on land surface formation [1]. However, the first results showed a very interesting potential for using it for landform segmentation. In the paper it is therefore proposed as an

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algorithm for the conversion of a continuous land surface (in form of field) to a set of land-forms. Such a comparison and its interpretational potential have not been analyzed in geomorphometry until now. Forms derived by this algorithm are compared to elementary forms delimited by commonly used methods.

2. State of the art

Geomorphometry can be simply defined as a quantitative land-surface modeling, parametrization and analysis [2]. Quantification in geomorphology started around 1800. Hypothetically, the first article in this field of study was written by James Hutton [3]. Morphometric characteristics (topographic variables, terrain attributes, land surface parameters) are quantitative information about land surface, and are widely used in geomorphometry. A comprehensive system of land surface characteristics was suggested by Minár and Evans [4], and includes an older division proposed by Shary [5]. Morphometric characteristics were divided into two main groups: *field variables* and *object variables*. Field variables were subdivided into *variables specific to any field* (gravity, but also sun radiation or wind fields) and *field independent variables*. Characteristics from both mentioned categories were used as input fields into the proposed algorithm.

After the quantitative revolution (1960's), two basic streams of geomorphometry were defined by Evans [6], general and specific geomorphometry. The main object of study in *general geomorphometry* is the land surface as a continual field. On the other hand, the object of *specific geomorphometry* is the discrete land surface (the set of land surface forms). The segmentation of a land surface is a process of conversion (transformation) of a continual field of altitudes into a set of landforms.

A land surface can be divided into *elementary forms* (the most simple forms of a land surface), *composite forms* (complex forms of a land surface) and *land systems* (types of land surfaces) [7]. The segment of a land surface with homogenous geometrical attributes and with potentially uniform behaviour for geomorphological processes on a defined level of scale was named an elementary form. The border of an elementary form was defined by a discontinuity in genetic, geometric and dynamic homogeneity [8]. Segmentation can be considered as a use of two main principles for regionalization over a land surface – the principle of internal homogeneity (the maximal similarity of elements within one group) and of external separation (the maximal difference between groups) [9]. A wide range of segmentation processes has been developed. In accordance with Minár and Evans [7], these can be divided into *graph-based approaches* (by analysis of segment borders) and *classification approaches* (by analysis of internal attributes). In the proposed work, segmentation processes will be divided into several distinctive groups.

A set of procedures for segmentation processes that use abrupt changes in morphometrical characteristics (discontinuities) has been incorporated into empirical geomorphological mapping. The method of profiles has often been used for locating discontinuities, as mentioned above. One of the most advanced profile-based methods of geomorphological mapping was developed by Waters [10] and Savigear [11]. However, determining the locations of the side borders of segments was a crucial deficiency of this method. The definition of side borders, the building of a theoretical background, the algorithmization of procedures and the new use of a rising number of morphometrical characteristics were the main attributes of a subsequent group of mapping procedures developed without serious computer support [12–18].

A group of statistical approaches for the segmentation of a land surface has worked on principles similar to those for remote sensing. A set of (not only) morphometric characteristics is associated with each point of a land surface. The spatial database is input into cluster analysis [19]. The process has often been combined with a fuzzy approach [20, 21].

A separate group of approaches is represented by multiresolution segmentation e.g. those used by Dragut and Eisank [22]. The main principle is to merge spatial entities until the resultant entity (land-form) becomes internally homogenous. The segmentation of a land surface by isolines (a combination of isolines) of morphometric characteristics was the main principle of another group of analytical approaches. The aforementioned process has been used in several works [14, 23–30]. The approximation of the parts of a land surface (digital elevation model) by a fitted function was a further approach. This concept was used by Troeh [24] for an approximation of an alluvial fan. Later, it was mentioned by Parson [31]. The complete set of mathematical equations for elementary forms was derived by Minár [17].

All of the methods based on geometrical aspects of a field theory (mathematical analysis) can be included as analytical approaches – including the algorithm proposed here.

3. Materials and methods

A novel algorithm is proposed in this paper that uses relations between field invariant normal curvatures and normal curvatures specific to a gravity field for land surface segmentation. Three cases of relations between the afore-

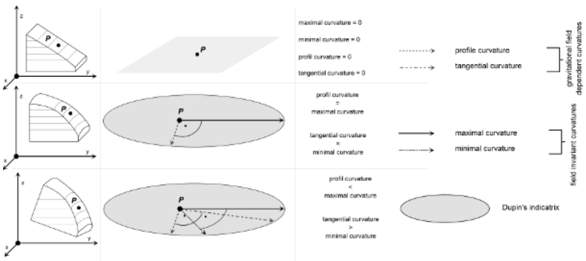

Figure 1. Illustration of the correlation between field-invariant curvatures and curvatures specific to a gravity field, where Dupin's indicatrix is defined as the conic section whose radius vector is defined by the magnitude and direction of a cut normal to the land surface.

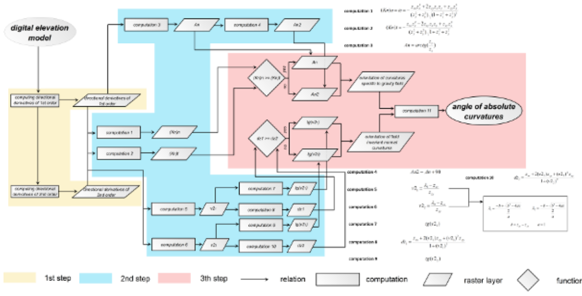

Figure 2. Workflow of the algorithm for a computation of the angle of absolute curvatures. $(K_n)_n$ - the profile curvature, $(K_n)_t$ - the tangential curvature, An, An_2 - the orientation of curvatures specific to a gravity field, $v2_1$, $v2_2$ - the partial results, $tg(v2_1)$,$tg(v2_2)$ - the orientation of field-invariant curvatures, dz_1, dz_2 - the total derivative of field-invariant curvatures. 1st step - the computation of spatial derivatives of the first and second order, 2nd step - the computation of the magnitude and direction of field-invariant curvatures and curvatures specific to a gravity field, 3rd step - the computation of the angle of absolute curvatures.

mentioned groups of curvatures can be described (see Figure 1):

- In the first class of land-forms, normal curvature is equal to zero in all directions.
- In the second, the directions of both curvatures are nearly identical and the differences of direction are equal (close) to zero. This case is rare.
- In the third, the directions of both curvatures are not identical and the difference of orientation is nonzero. This is the most common case, and the orientation (angle) of the difference is named the angle of absolute curvatures.

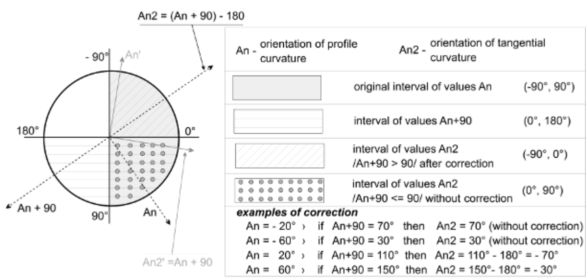

Figure 3. Explanation of the transformation of the orientation of the profile curvature (An) to the orientation of the tangential curvature (An_2).

These three cases are likewise represented by different shapes for Dupin's indicatrix [27]. While in the first case Dupin's indicatrix can not be defined, in the second case it is represented by a circle and in the third case by an ellipse. A computation of the angle of absolute curvatures is illustrated in Figure 2. A smoothed digital elevation model (DEM) in a raster layer format with grid mesh sized one or five meters was an input to the algorithm. The input DEM was smoothed three times by a focal statistic tool (mean of values in a 3 X 3 window) to avoid an accumulation of errors. The algorithm was built in *ArcGIS 10.0* with *Model builder*.

Directional derivatives of the first and second order were computed by the method of central differences in the first step of the algorithm. The computed fields of directional derivatives were inputs into the next computations of map algebra. The magnitudes of the field-invariant curvatures ($v2_1$, $v2_2$) and the magnitudes of curvatures specific to a gravity field ($(K_n)_n$, $(K_n)_t$) were computed in the second step. The orientation of the curvatures is identical with the orientation of a defined normal cut (for profile curvature is identical with the slope aspect An transformed to the interval <-90ï£¡, 90ï£¡>, and, by analogy, for the tangential curvature perpendicular to slope aspect (An_2) transformed to the interval <-90ï£¡, 90ï£¡>; see Figure 3).

The orientation of field-invariant curvatures ($tg(v2_1)$, $tg(v2_2)$) and the values of the total derivatives (dz_1, dz_2) were computed from the magnitudes of the field-invariant curvatures ($v2_1$, $v2_2$). The orientation of the maximal curvature was defined by the greatest value of the total derivation; by analogy, the orientation of minimal curvature was defined by the smallest value of the total derivation. The fields of orientations of the curvatures specific to a gravity field and the orientations of field-invariant curvatures were computed in the third step. The angle of

absolute curvatures was computed as an absolute value of the difference between the orientation of the greater of the absolute values of maximal and minimal field-invariant curvatures (absolute maximal curvature) and the orientation of the greater of the absolute values of the gravity-field-specific curvatures. Or, by analogy, it can be computed as the absolute value of the difference between the orientation of the smaller of the absolute values of maximal and minimal curvatures (absolute minimal curvature) and the orientation of the lower of the absolute values of gravity-field-specific curvatures.
Manual vectorization of structural lines (as well as "valley" and "ridge" lines) was carried out over the field of the angle of absolute curvatures. Significant borders between homogenous entities in the field of the angle of absolute curvatures were represented by these structural lines. The lines were in the next step transformed into spatially closed entities. A comparison of the profiles of the fields of altitude, slope, orientation, profile curvature, tangential curvature, rotor curvature (introduced by Shary [30]) and the angle of absolute curvatures was executed to analyze the types of structural lines present in the field of the angle of absolute curvatures.
Forms delimited by the angle of absolute curvatures were compared with:

- Forms defined on the basis of their curvatures by profile and tangential curvature.
- A Gaussian classification of landforms[28] adapted by Shary [30], where landform is defined by total (Gaussian) and mean curvatures:

$$k = \frac{z_{xx}z_{yy} - z_{xy}^2}{(1 + z_x^2 + z_y^2)^2} \quad (1)$$

$$2H = \frac{(1 + z_x^2)z_{yy} - 2z_x z_y z_{xy} + (1 + z_y^2)z_{xx}}{(1 + z_x^2 + z_y^2)^{\frac{2}{3}}} \quad (2)$$

where:
K - Total (Gaussian) curvature
H - Mean curvature
z_x - Partial derivatives of the first order by the X axis.
z_x - Partial derivatives of the first order by the Y axis.
z_{xx} - Partial derivatives of the second order by the X and the X axis.
z_{xy} - Partial derivatives of the second order by the X and the Y axis.
z_{yy} - Partial derivatives of the second order by the Y and the Y axis.

- Forms manually delimited by a geomorphologist (one of the authors of this study) from the field of altitudes.

The dependence of the quality of information in the field of the angle of absolute curvatures on the size of the grid mesh was tested by Moran's I index [32]. The tendency of a raster field to agglomeration is indicated by a high value of *Moran's I index* and the tendency to dispersion by low values. The significance of Moran's I index was quantified by a zs_i-*score* (the sum of the zs-score of each pixel) computed as:

$$zs_i = \frac{I - E[I]}{\sqrt{V[I]}} \quad (3)$$

$$E[I] = frac{-1}(n-1) \quad (4)$$

$$V[I] = E[I^2] - E[I]^2 \quad (5)$$

where:
I - Moran's I index

The *zs-score* is simply the standard deviation (associated with a standard distribution). A sudden decrease or increase in the change of the *zs-score* indicates that one cannot reject the hypothesis that the input field of the angle of absolute curvatures is spatially random.
The algorithm was tested in two areas of the Malá Fatra Mountains (Slovakia). The area of Vrátna - Grúň with 6 singularity points of altitude (3 peak points, 3 saddle points) was the first one. A DEM with a grid mesh size of 1 meter was used. The area of Vrátna - Žitné, with a slope of north-east aspect, was the second. A DEM with a grid mesh of 5 meters was used. Information, in the form of histograms, about both tested areas is shown in Figure 4 and Figure 5.

4. Results

Closed entities separated by borders of several types can be recognized in the field of the angle of absolute curvatures (see Figure 6).
Four types of borders (in the form of profiles) can be found in Figure 7. In borders of type A, a local minimum of the angle of absolute curvatures is connected to a local extreme of slope in a wide neighborhood of the border. Sometimes extreme values of one of the curvatures can be found in the wide neighborhood of border type A. Changes in the orientation of the absolute maximal curvature on the one hand and the profile curvature / the tangential curvature on the other are not connected to this type of

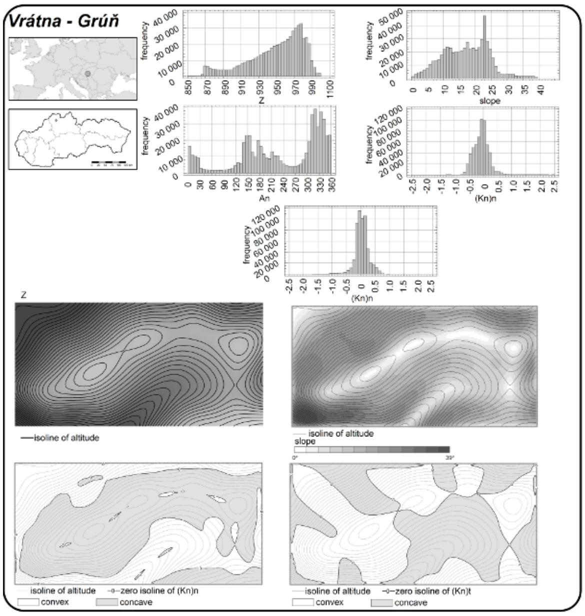

Figure 4. Basic information about the tested area of Vrátna - Grúň. Frequency - the number of pixels in the category, Z - the altitude, A_n - the slope aspect, $(K_n)_n$ - the profile curvature, $(K_n)_t$ - the tangential curvature.

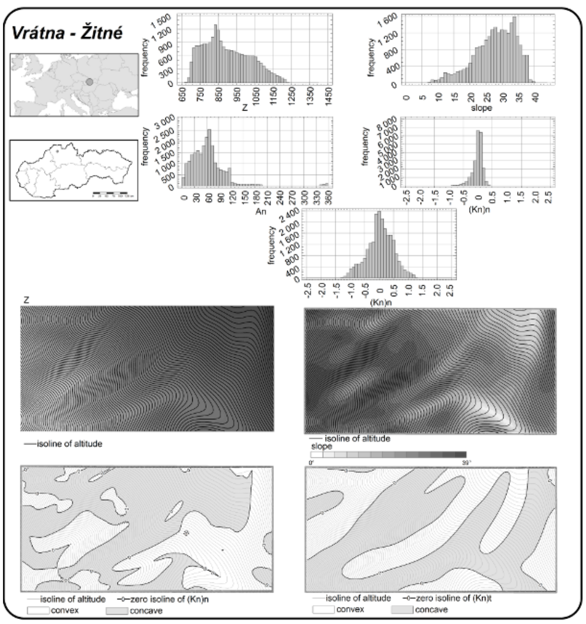

Figure 5. Basic information about the tested area of Vrátna - Žitné. Frequency - the number of pixels in the category, Z - the altitude, A_n - the slope aspect, $(K_n)_n$ - the profile curvature, $(K_n)_t$ - the tangential curvature.

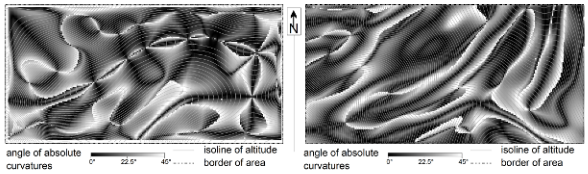

Figure 6. Field of the angle of absolute curvatures for the tested areas of Vrátna - Grúň (left) and Vrátna - Žitné (right).

border. A local maximum in the angle of absolute curvatures is the characteristic of border type B. Changes in the orientation of the absolute maximal curvature on one hand and the profile curvature / the tangential curvature on the other do occur in this type of border. A sudden change (increase or decrease) in the of angle of absolute curvatures is characteristic for border type C. This type of border, also named "breakpoint", is defined by a change in the input of the profile or tangential curvature connected to computational data for the angle of absolute curvatures. Coincidences among the magnitudes of the profile and tangential curvature and borders of types C and D are shown in Figure 7. Borders C and D are situated close to points where the priority of the profile curvature replaces the the priority of tangential curvature, and vice versa. Borders of type D are just a combination of types A, B and C and therefore will not be further considered as a special type. Coincidences between borders (types A, B and C) in the angle of absolute curvatures with local extremes of morphometric characteristics and coincidences between borders in the field of the angle of absolute curvatures and sudden changes of morphometric characteristics deserve further systematic study.

An analysis of the lengths of borders in the field of the angle of absolute curvatures showed that the majority of the borders are of type A (Figure 8). Borders of type D was not considered here as a particular type of border.

A comprehensive view of the coincidence between the angles of absolute curvatures and fields of other morphometric variables is offered in Figure 9. The spatial coincidence here is more visible than in the profiles in Figure 7. The classified raster of the slope and the unclassified raster of the profile curvature, the tangential curvature and the rotor curvature were used for this purpose. Coincidences of the zero values of morphometric variables and the borders in the angles of absolute curvatures are visible in Figure 9. For the case where the values of the curvatures change from maximal positive values to positive values close to zero, this change is not well visible. Coincidences between all of the changes to morphometric characteristics (zero isolines of morphometric characteristics, as well as sudden changes to morphometric characteristics) and borders in the field of the angle of absolute curvatures are

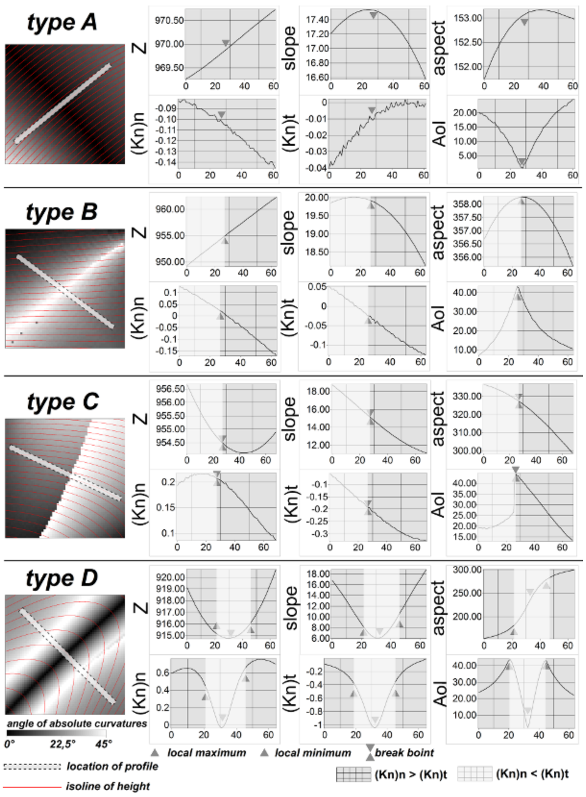

Figure 7. Three types of borders in field of angle of absolute curvatures (A, B, C). Type D was considered as a combination of preceding types of borders, where: Z - the altitude, $(K_n)_n$ - the profile curvature, $(K_n)_t$ - the tangential curvature, $AoAC$ - the angle of absolute curvatures.

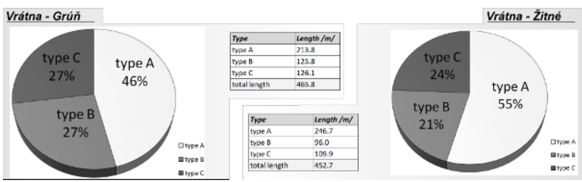

Figure 8. Lengths of borders of various types in the fields of the angle of absolute curvatures for the tested areas of Vrátna - Grúň (left) and Vrátna - Žitné (right).

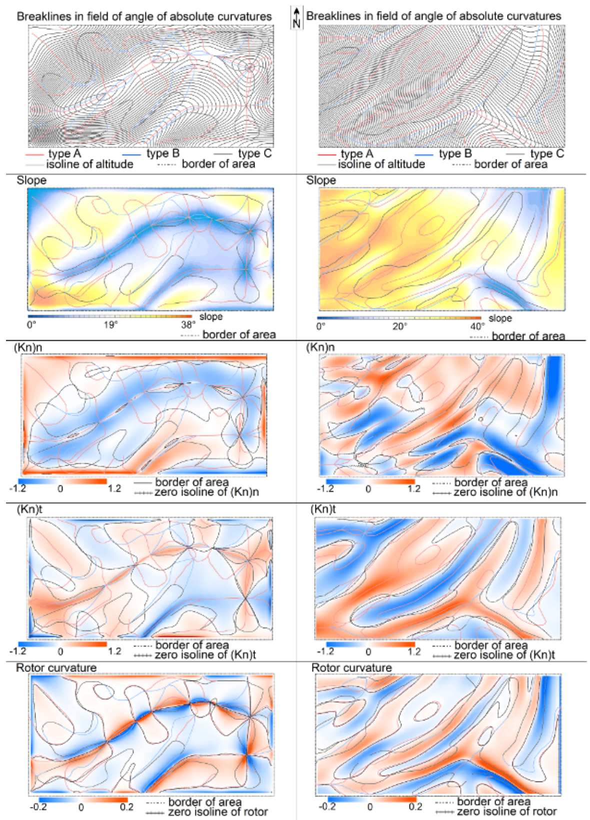

Figure 9. Coincidences between types of borders in field of the angle of absolute curvatures and morphometric characteristics for the tested areas of Vrátna - Grúň (left) and Vrátna - Žitné (right). $(K_n)_n$ - the profile curvature, $(K_n)_t$ - the tangential curvature.

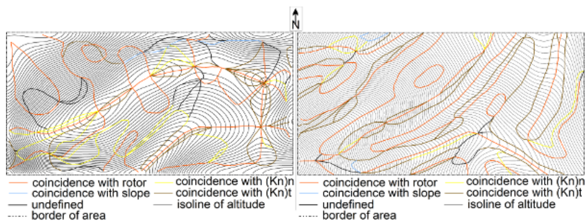

Figure 10. Coincidences between zero isolines of morphometric characteristics and sudden changes to morphometric characteristics on one hand, and borders in the field of the angle of absolute curvatures on the other, for the tested areas of Vrátna - Grúň (left) and Vrátna - Žitné (right). $(K_n)_n$ - the profile curvature, $(K_n)_t$ - the tangential curvature.

depicted in Figure 10.

An analysis of the lengths of the borders in the field of the angle of absolute curvatures and coincidences with all changes to morphometric characteristics is shown in Figure 11.

Borders in the field of the angle of absolute curvatures are dependent on the fields of the morphometric characteristics. Entities in the field of the angle of absolute curvatures can be defined as nonrandom forms of the land surface. A comparison among the forms of the land surface delimited by the angle of absolute curvatures and the forms of the land surface delimited by other analytical methods can be found in Figure 12. The forms of the land surface delimited through of the angle of absolute curvatures are not identical with the forms delimited by any of the other tested delimitation methods.

A combination of the borders delimited by the angle of

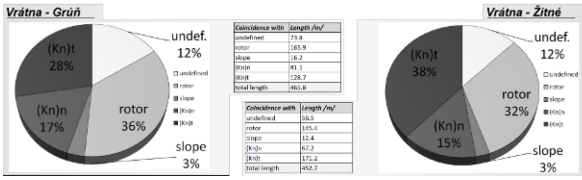

Figure 11. Lengths of the borders in the field of the angle of absolute curvatures and coincidences with all of the changes to morphometric characteristics for the tested areas of Vrátna - Grúň (left) and Vrátna – Žitné (right). $(K_n)_n$ - the profile curvature, $(K_n)_t$ - the tangential curvature.

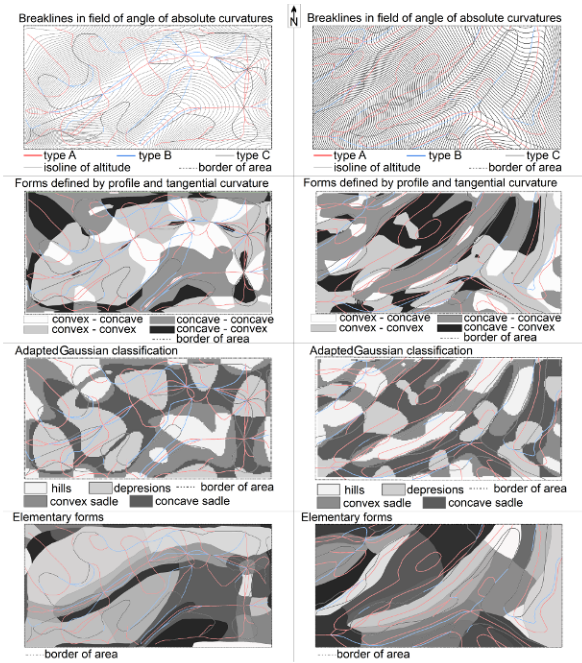

Figure 12. A comparison among the forms of land surface delimited by using the field of the angle of absolute curvatures and forms delimited by other selected analytical algorithms for the tested areas of Vrátna - Grúň (left) and Vrátna - Žitné (right). *Forms defined by profile and tangential curvature* - convex-convex, concave-convex, convex-concave, concave-concave; *Adapted Gaussian classification* - after Shary [30], *Elementary forms* - manually delimited elementary forms after Minár [8].

absolute curvatures and the method of manual delimitation of elementary forms of the land surface was used for the construction of a preliminary geomorphological map in Figure 13.

The quality of the information contained in the angle of absolute curvatures (represented by a raster layer) is at first sight influenced by the size of the grid mesh of the input DEM (Figure 14.

A quantitative representation of the quality of the information in the angle of absolute curvatures is represented by Moran's I index. The quality of the computation of Morans's I index is represented by a z-score (Figure 15).

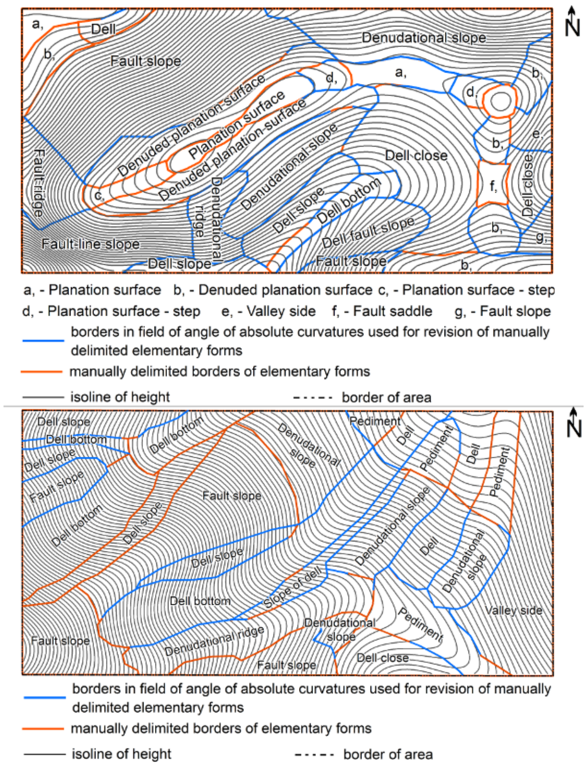

Figure 13. A preliminary geomorphological map constructed with borders delimited by the angle of absolute curvatures and the method of manual delimitation of elementary forms for the tested areas of Vrátna - Grúň (up) and Vrátna - Žitné (down).

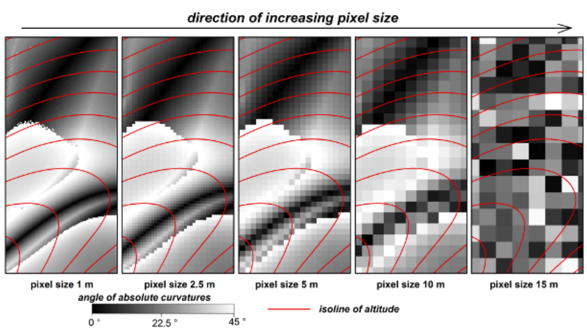

Figure 14. The dependence of the quality of the information in the field of the angle of absolute curvatures (represented by a raster layer) on the size of the grid mesh.

5. Discussion and conclusions

The proposed algorithm generates an angle of absolute curvatures that is not random and is characterized by spa-

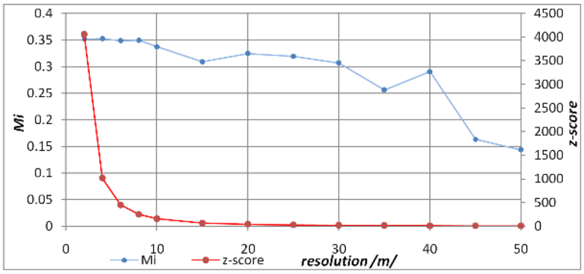

Figure 15. The dependence of Moran's I index and the z-score of that test on grid mesh size for the tested areas of Vrátna - Grúň (left) and Vrátna - Žitné (right)); where: *Mi* - Moran's I index, *resolution* - the computational size of the grid mesh of the angle of absolute curvatures.

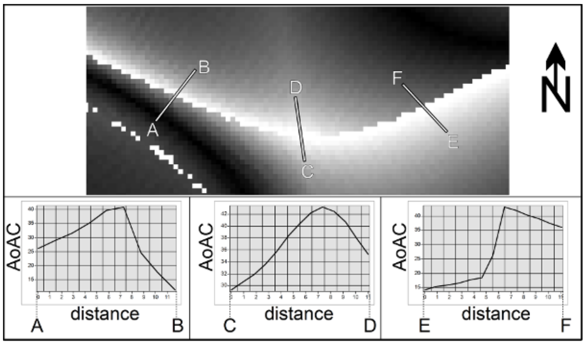

Figure 16. Transitions from left-sided border-type C (profile A - B) to right-sided border-type C (profile E - F) over transitional border-type B (profile C - D). *AoAC* - the angle of absolute curvatures.

tial autocorrelation. The angle of absolute curvatures is not comparable with horizontal excess curvature [5] (the difference between tangential and minimal curvatures) or with vertical excess curvature [5] (the difference between profile and minimal curvatures) due to their different methods of computation. The angle of absolute curvatures is defined in some cases by the difference in direction of the (absolute) maximal curvature and tangentional curvatrure, in other cases by the difference in direction of the (absolute) maximal curvature and the profile curvature. It is supposed that within the field of the angle of absolute curvatures there can be found nonrandom clusters of cells which may represent specific landforms. Clusters in the field of the angle of absolute curvatures are delimited by three types of borders. Borders of type A and C are considered as the most important for performing segmentation of the land surface. Borders of B type are often only transitions from left-sided C-type borders to right-sided C-type borders (see Figure16).

Borders of A type are connected with extreme values of slope in a wide neighborhood. Maximal value of slope can be interpreted as the scarp of a land surface. Borders of type B and C are connected with changes in the comparison of the orientation of absolute maximal curvature with profile curvature, and tangential curvature. Borders of type C are connected with the neighborhood of zero isolines of profile curvature, tangential curvature or rotor curvature. If borders of B type are grouped with C type, this common category represents 50% (more precisely, 46%) of the total length of the borders, as an analysis of length showed. Thus, interpretation of all groups of borders is necessary. Borders in the field of the angle of absolute curvatures coincide with sudden changes and zero isolines of other morphometric characteristics. Coincidences are not same for different fields of morphometric characteristics. Strong coincidences are observed between the borders in the field of the angle of absolute curvatures and the zero isolines of the rotor curvature. Coincidences between borders in the field of the angle of absolute curvatures and the zero isolines or sudden changes to other morphometric characteristics are noticeable, and future exploration is necessary. The land surface segments generated by the proposed algorithm are not identical with forms generated by other tested analytical segmentation processes. However, the forms of the land surface delimited from the angle of absolute curvatures are similar (though not identical) with manually delimited landforms. Differences rise from:

- The limitations of a person's eye that lead to inaccuracy in estimating structural lines' positions as well as the lesser ability of any geomorphologist to sense some complex fields of morphometric characteristics (e.g. profile curvature, rotor curvature).
- The selection of only genetically well-interpretable land surface segments and their borders by geomorphologists.

In contrast with statistical approaches, forms generated by the proposed algorithm are not fragmented into a huge set of forms. Forms generated by the algorithm are not identical (or similar) to forms generated by the overlay of isolines of other morphometric characteristics. Our algorithm reflects the character of a greater number of geomorphometric fields (slope, profile, tangential and rotor curvature) and not only their specific (zero) isolines but also sudden changes (discontinuities). The proposed algorithm is semiautomatic and the identification of the borders of elementary forms suitable for geomorphological mapping is its main purpose in its current state (see figure 13). Forms generated by the algorithm are land-forms of zero hierarchy level. Forms of several hierarchy levels are not recognizable in the field of the angle of absolute curvatures. However, by changing the attributes of the input DEM (e.g. smoothing, interpolation algorithm) we can obtain forms of higher hierarchy levels. The attributes of the input DEM will be crucial for practical use. The quality of information generated by the angle of absolute curvatures is dependent on the size of the grid mesh of the DEM used (see figure 16). The descent of the value of Moran's I index is continual, without a sudden change. A sudden decrease of the z-score of Moran's I index is visible for grid-mesh size 10 meters, which indicates the decreasing quality of the computation of the index (a non-normal distribution of the angle of absolute curvatures). Consequently, the continual decrease in Moran's I index and the sudden decrease of the corresponding z-score indicate a disintegration of field of the angle of absolute curvatures with increasing grid size. The results achieved are preliminary and additional tests are needed.

6. Acknowledgements

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