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Analysis and interpretation of Ibuji spring magnetic anomaly using the Mellin transform

Research Article

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Abstract: The Mellin transform is a mathematical tool which has been applied in many areas of Mathematics, Physics and Engineering. Its application in Geophysics is in the computation of solution of potential problems for the determination of the mass as well as the depth to the basement of some solid mineral deposits. In this study, the Mellin transform is used to determine the depth to the top (h) and the depth to the bottom (H) of the basement of a profile of an anomalous magnetic body. Ibuji, the study area is located in Ifedore Local Government area of Ondo state, Nigeria, underlain by Precambrian complex rocks and bounded by geographical co-ordinate of Easting $5°00'00''$ to $5°4'30''$ and Northing $7°24'00''$ to $7°27'36''$. The magnetic anomaly profile due to a two– dimensional body(vertical thin sheet)over magnetic spring of the study area was digitised and the values of magnetic amplitude (nT) with respect to its horizontal distance (say interval of 5 m) obtained from the digitized profile was then used in the computation of Mellin transform using Matlab programs. In order to determine the depths H and h, the amplitudes were considered at three arbitrary point ($s = \frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$) such that, ($0 < s < 1$), where s is a complex variable of real positive integer. The value obtained for H was 47.95 m, which compared favourably with the result obtained using other methods. Meanwhile, the value obtained for h has a convergence restriction, whereby, at lower values of s, there is divergence, while at higher values of s, (about 0.9), the result converges and h was obtained to be 32.56 m. The Ibuji magnetic anomaly was therefore analysed to have a depth to the bottom (H) of 47.95 m and depth to the top of 32.56 m using this mathematical tool.

Keywords: Mellin transform • Ibuji • Matlab • Ondo state • magnetic anomaly

1. Introduction

The application of integral transform like Fourier, Hilbert, Hankel and Mellin to geophysical data processing and interpretation has gained greater importance over the last two decades [1–5]. The increasing use of modern digital computer has facilitated the direct interpretation of geophysical data employing the Mellin transform.

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Mellin transform gives a large place to the theory of analytic function and relies essentially on Cauchy's theorem and the method of residue. It is also closely related to the Laplace transform and the Fourier transform and the theory of the gamma function and allied special functions [6].

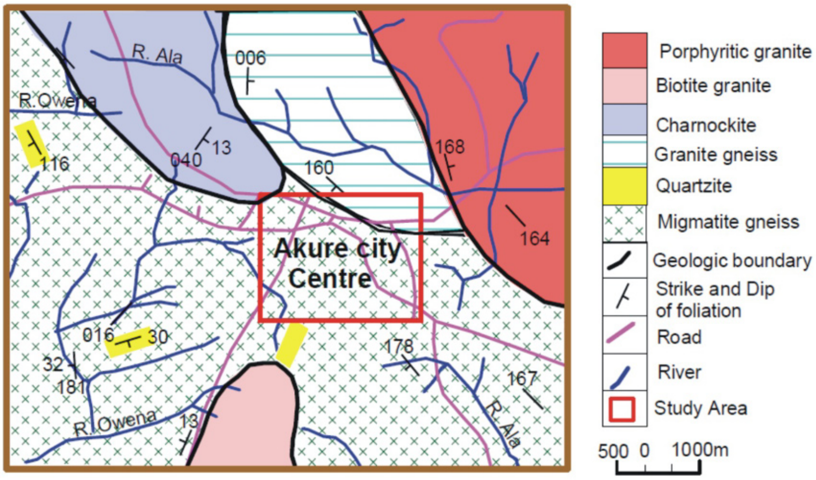

Figure 1. The structural map of Ondo–Akure area. (Adapted from [15]).

The first occurrence of the transformation is found in a memoir by Riemman in which he used it to study the famous Zeta function. However, it is the Finnish mathematician, R. Hjalmar Mellin (1854–1933) who was the first to give a systematic formulation of the transformation and its inverse. Hence the transform was named after the Finnish mathematician. By definition, the Mellin transform $F(s)$ and alternatively, by the more complete notation $MT\{f(x);s\}$, corresponding to a function $f(x)$ defined only for $x \geq 0$ is

$$F(s) = MT\{f(x);s\} = \int_0^\infty x^{s-1} f(x)dx \quad (1)$$

With certain restriction on $f(x)$, $F(s)$ considered as a function of a complex variable s is a function of exponential type, analytic in a strip parallel to the imaginary axis. The new variable s, which is taken to be complex, must be restricted to those values for which the integral converges. In general, we have convergence at $x = 0$ only if $Re\{s\}$ is larger than a certain value and at $x = \infty$ only if $Re\{s\}$ is smaller than a certain value. The reason for the restriction of $f(x)$ is that there are technical difficulties in defining Mellin transform directly for a function define over $(-\infty, +\infty)$. Mellin transform theory is given for functions defined only for positive values of the argument i.e. positive random variable. There is a reciprocal formula enabling one to go from the transform $F(s)$ to the function $f(x)$, the transformation is [7, 8]

$$f(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} x^{-s} F(s)ds \quad (2)$$

It is deducted that the Mellin transform is a generalised form of the gamma function

$$\Gamma(s) = \int_0^\infty x^{s-1} e^{-x} dx \quad (3)$$

Mellin transform is an improper integral of the third kind which obeys the convergence theorem at larger value of and also satisfy the divergence theorem at the smaller values of s.

In contrast to Fourier and Laplace transform that were introduced to solve physical problem, Mellin transform arose in a mathematical context. Besides its use in mathematics, Mellin transform has been applied in many different areas of physics and engineering [9]. This gives rise to its application in the analysis and the interpretation of magnetic anomaly that may be present around the magnetic spring in Ibuji, Ondo State Nigeria. Several attempt made by the Ondo State Government to provide portable water for the people in the community through groundwater development have not been successful due to the fact that the migmatite gneiss occur as a low lying outcrops across the community (as shown in Fig. 1), even around the spring. Many Authors have used integrated geophysical methods in the detection of groundwater in fractured media [10–15], it was shown that ground water can occur within rocky terrain, provided there is a discontinuity within the rock. In hard rock areas, the ground water is found in the weathered layer, joint, fracture and fault of the local area. Using Mellin transform, the depth to the top (h) and bottom (H) of the basement of the magnetic spring could be determined.

2. Geological and hydrogeological description

Ibuji, the study area, is located in Ifedore Local Government, Area of Ondo State. It is situated about 6 km North of Igbara-Oke along Igbara-Oke and Igbara-Odo road. It lies between the geographic co-ordinates of Easting 5°00′00″ to 5°4′30″ and Northing 7°24′00″ to 7°27′36″.

The study area is underlain by the Precambrian Basement Complex Rocks of South-western Nigeria [16]. The lithologic unit identified in the study area is the migmatite gneiss. The migmatite gneiss outcrop show evidence of structural deformation by series of approximately parallel East–West fractures, joints and lineaments defined by vegetation alignment.

Groundwater is found in the weathered layer, fractured and jointed column in a typical basement terrain. The weathered layer in the study area is generally thin due to the near-surface nature of the basement bedrock. The basement rocks are mainly exposed in the Northern part and in the area around the spring [17].

3. Methodology

The magnetic profile of a traverse (Fig. 6) obtained from the Proton Precision Magnetometer for Ibuji spring was digitised. The different value of magnetic amplitude with respect to its horizontal co-ordinate obtained from the digitized profile was then used in the computation of Mellin transform over a thin sheet (2-dimensional) body using MATLAB Programs to determine the depth to the top and the bottom of the rock basement of the spring.

4. Mathematical model

Here, the analysis of magnetic anomalies due to 2-dimensional body of a vertical thin sheet was used in the interpretation of the anomaly present around the spring using Mellin transform. The mathematical expression for potential field involving complicated algebra which was converted into simple function using Mellin transform as given by [20] is used

$$M(s) = \int_0^\infty x^{s-1} V(x) dx \tag{4}$$

where $V(x)$ is the potential field and s is the real variables which assume a positive real integer or fractional number in the Mellin's domain.

If the function of the potential field is asymmetric, then it is split into even and odd component and their corresponding Mellin transform are given as

$$VE(x) = \frac{V(x) + V(-x)}{2} \tag{5}$$

$$VO(x) = \frac{V(x) - V(-x)}{2} \tag{6}$$

where $VE(x)$ and $VO(x)$ are potential for even and odd respectively.

Furthermore, the amplitude $A(s)$ and phase component $P(s)$ are also given as

$$A(s) = ME(x)^2 + MO(x)^2 \tag{7}$$

$$P(s) = \tan^{-1}[MO(s)/ME(s)] \tag{8}$$

Also expression of continuous variable can be made applicable to the field data which is discrete by using the discrete Mellin transform given by [4],

$$ME(n.\Delta s) = \sum_{l=0}^{N} (l.\Delta x)^{n\Delta s-1} VE(l.\Delta x) \tag{9}$$

$$MO(n.\Delta s) = \sum_{l=0}^{N} (l.\Delta x)^{n\Delta s-1} VO(l.\Delta x) \tag{10}$$

where

$ME(n.\Delta s)$ = Discrete Mellin transform of even potential

$MO(n.\Delta s)$ = Discrete Mellin transform of odd potential

N = Total number of observe value

Δx = Station interval of the observed value

Δs = Interval of the discrete Mellin transforms

It should be noted that Interval of the discrete Mellin transform $(n.\Delta s)$ range between $(0 < n.\Delta s < 2)$ for even and $(-1 < n.\Delta s < 1)$ for odd potential.

4.1. Theoretical background

A vertical magnetic sheet of finite extent, extending infinitely in the direction parallel to the Y-axis, is considered. The section of the sheet in the X-Y plane is shown in Fig. 7. The vertical magnetic effect of such body is said to be related to the effect of depth to basement rock of Ibuji spring as given by [21].

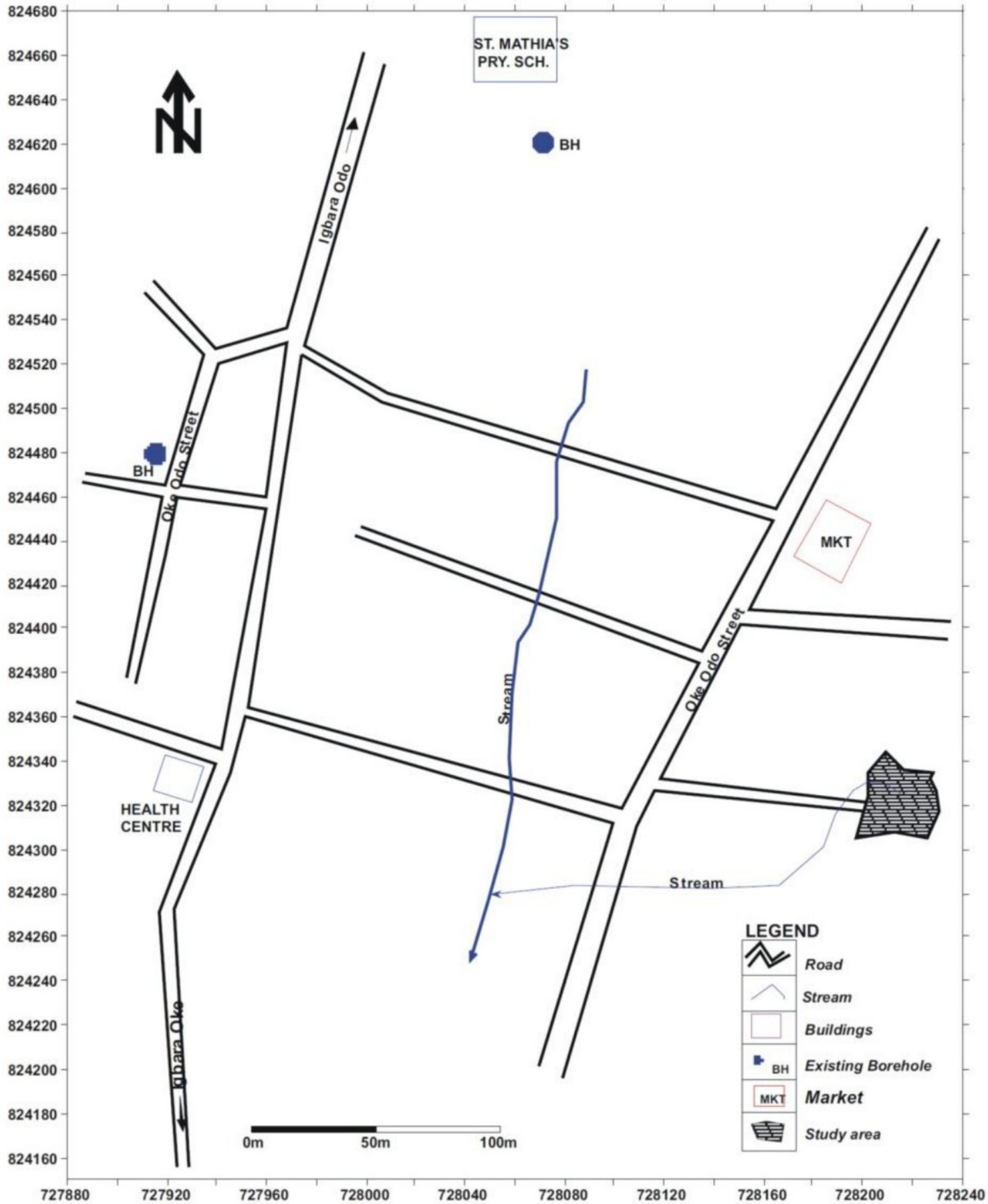

Figure 2. Basemap of Ibuji showing the study area.

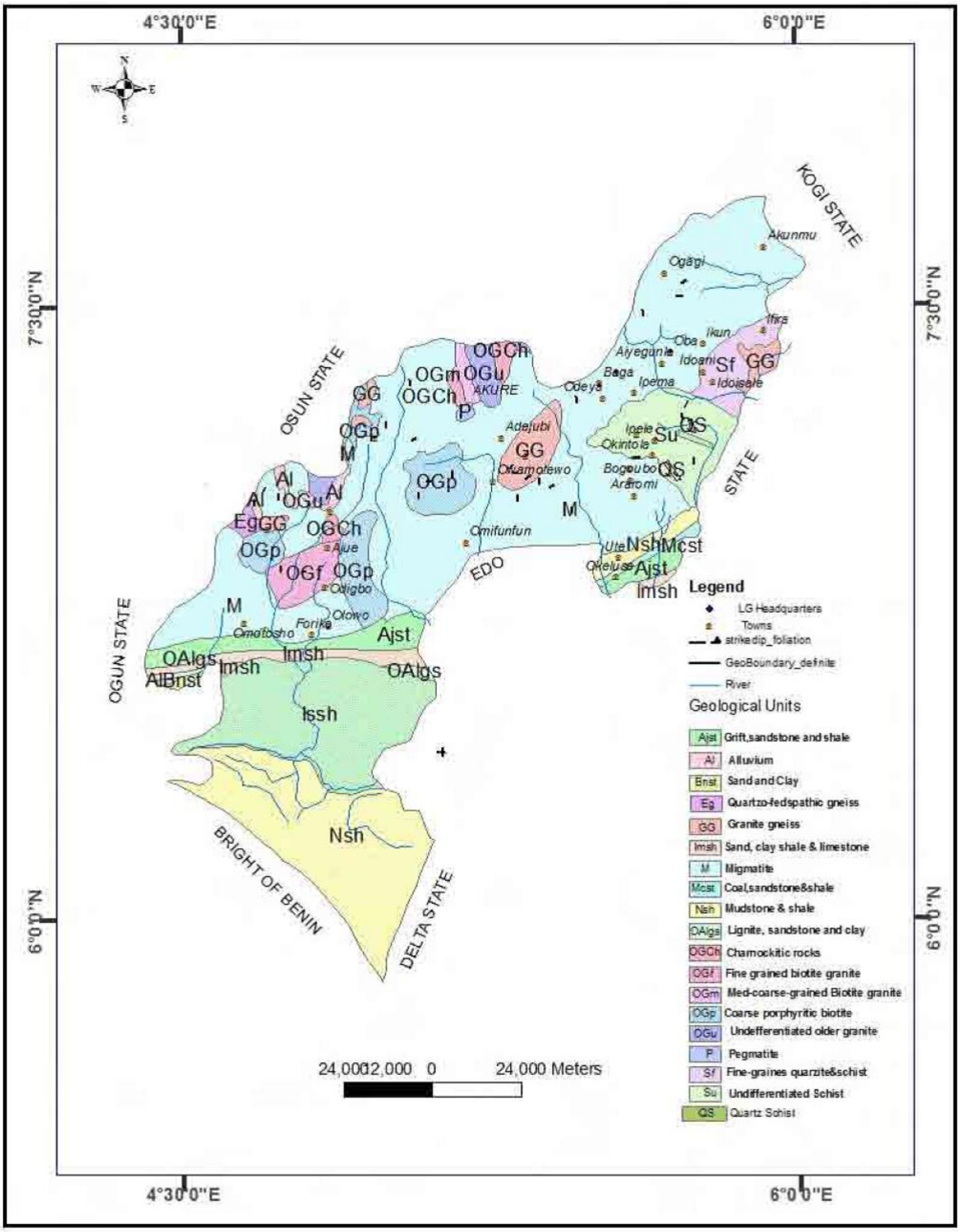

Figure 3. Geological Map of Ondo State showing the Study Area. (Adapted from [18]).

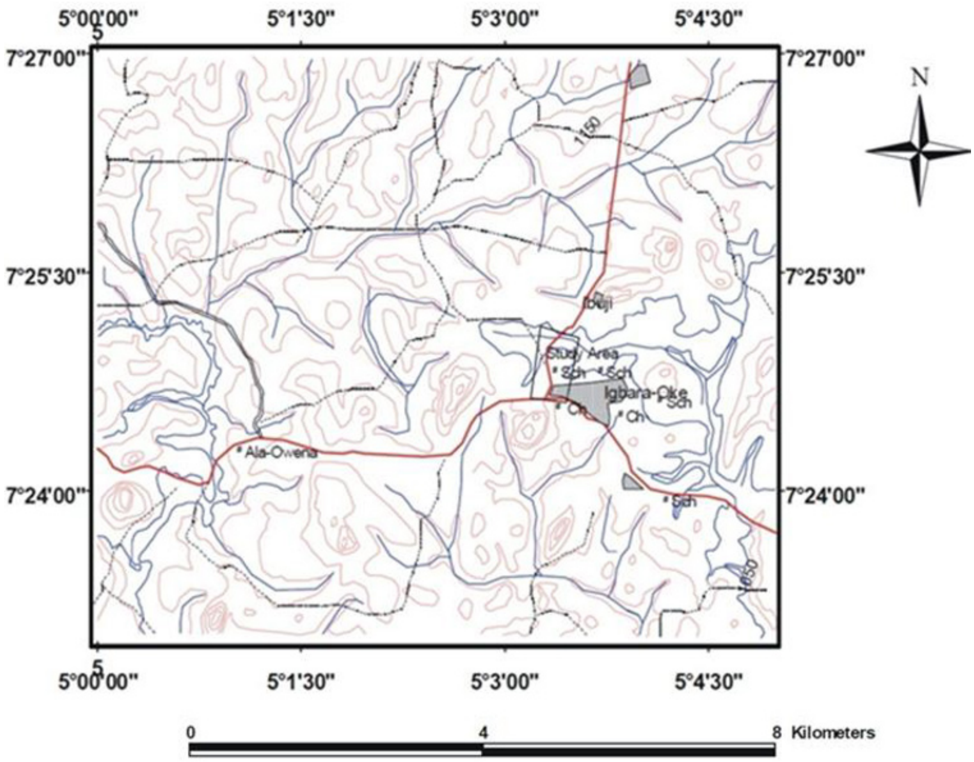

Figure 4. The Topographical map of the study area.

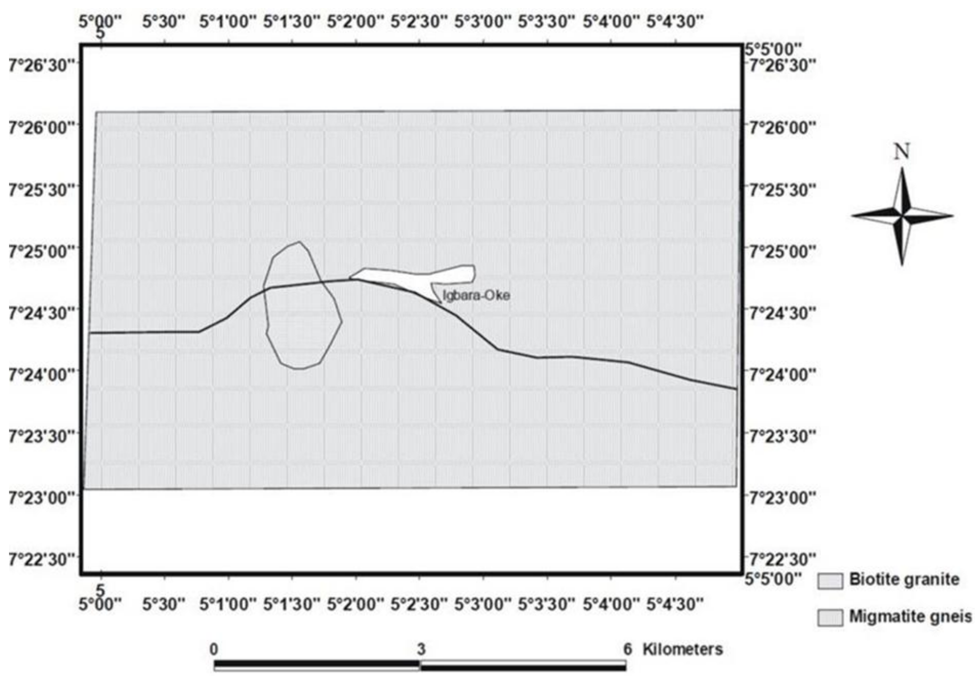

Figure 5. Geological map of the area. (After [19]).

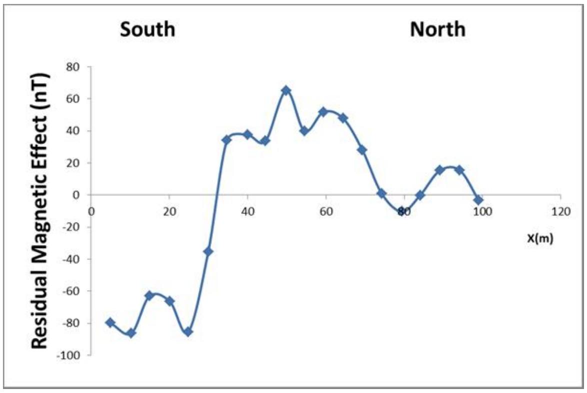

Figure 6. The magnetic profile of a traverse over the Ibuji Spring. (Adapted from [13]).

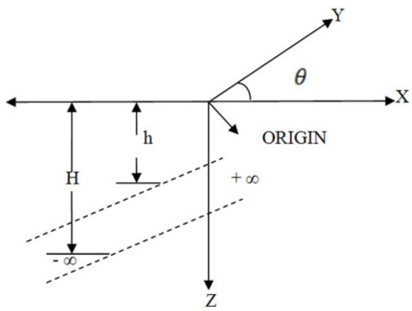

Figure 7. The cross section of the vertical magnetic sheet of the finite extent.

$$V(x) = k\left[\frac{x\sin\theta - H\cos\theta}{x^2 + H^2} - \frac{x\sin\theta - h\cos\theta}{x^2 + h^2}\right] \quad (11)$$

where h and H are the depth to the top and bottom of the sheet respectively. θ is the polarisation angle and k is the magnetic constant of the sheet.
Using equation (5) and (6), the even and odd components $V(x)$ are respectively given as follows:

$$VE(x) = k\cos\theta\left[\frac{h}{x^2 + h^2} - \frac{H}{x^2 + H^2}\right] \quad (12)$$

$$VO(x) = k\sin\theta\left[\frac{x}{x^2 + H^2} - \frac{x}{x^2 + h^2}\right] \quad (13)$$

Based on the theoretical background, the amplitudes are extracted as shown below:

$$A(s) = \left(\frac{k}{2}\right)(h^{s-1} - H^{s-1}[n^2(\cos\theta)^2 + m(\sin\theta)^2]^{\frac{1}{2}} \quad (14)$$

where

$$m = \Gamma\left[\frac{(1+s)}{2}\right]\Gamma\left(\frac{(1-s)}{2}\right) \quad (15)$$

$$n = [\Gamma(s/2)]\Gamma\left(\frac{(2-s)}{2}\right) \quad (16)$$

Γ is the gamma function.
For determining h and H, the amplitude of the model is considered at three arbitrary values of $'s'$ namely at $s = \frac{1}{4}$, $s = \frac{1}{2}$ and $s = \frac{3}{4}$ [22].

$$A\left(\frac{1}{4}\right) = C_1\left(h^{-\frac{3}{4}} - H^{-\frac{3}{4}}\right) \quad (17)$$

$$A\left(\frac{1}{2}\right) = C_2\left(h^{-\frac{1}{2}} - H^{-\frac{1}{2}}\right) \quad (18)$$

$$A\left(\frac{3}{4}\right) = C_3\left(h^{-\frac{1}{4}} - H^{-\frac{1}{4}}\right) \quad (19)$$

$$C_1 = \frac{k}{2}\left[\left(\Gamma\left(\frac{1}{8}\right)\Gamma\left(\frac{7}{8}\right)\cos\theta\right)^2 + \left(\Gamma\left(\frac{5}{8}\right)\Gamma\left(\frac{3}{8}\right)\sin\theta\right)^2\right] \quad (20)$$

$$C_2 = \frac{k}{2}\left[\left(\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)\cos\theta\right)^2 + \left(\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{1}{4}\right)\sin\theta\right)^2\right] \quad (21)$$

$$C_3 = \frac{k}{2}\left[\left(\Gamma\left(\frac{3}{8}\right)\Gamma\left(\frac{5}{8}\right)\cos\theta\right)^2 + \left(\Gamma\left(\frac{7}{8}\right)\Gamma\left(\frac{1}{8}\right)\sin\theta\right)^2\right] \quad (22)$$

From the above relation using simple algebra, we obtain:

$$h^{-\frac{1}{2}} + H^{-\frac{1}{2}} + h^{-\frac{1}{4}}H^{-\frac{1}{4}} = -\frac{A\left(\frac{1}{4}\right)C_2}{A\left(\frac{3}{4}\right)C_1} \quad (23)$$

and

$$h^{-\frac{1}{4}} + H^{-\frac{1}{4}} = \frac{A\left(\frac{1}{2}\right)C_3}{A\left(\frac{3}{4}\right)C_2} \quad (24)$$

Simplifying equation (23) and (24), we obtain:

$$H^{-\frac{1}{2}} - C_5 H^{-\frac{1}{4}} + (C_5 - C_4) = 0 \quad (25)$$

which is quadratic in $H^{-\frac{1}{4}}$, where

$$C_4 = -\frac{A\left(\frac{1}{4}\right)C_2}{A\left(\frac{3}{4}\right)C_1} \quad (26)$$

$$C_5 = \frac{A\left(\frac{1}{2}\right) C_3}{A\left(\frac{3}{4}\right) C_2} \tag{27}$$

where C_1, C_2, C_3, C_4 and C_5 are arbitrary constants. Solving equation (23), H is given by

$$H = \left[\left\{\frac{2}{C_5 \pm \sqrt{C_5^2 - 4(C_5 - C_4)}}\right\}^4\right] \tag{28}$$

Consequently,

$$h = \frac{1}{\left(C_5 - H^{-\frac{1}{4}}\right)} \tag{29}$$

From the equations above, the continuous Mellin transform can then be computed as:

$$ME(s) = \left(\frac{k}{2}\right) \sin\theta\Gamma\left(\frac{s}{2}\right)\Gamma\left(\frac{(2-s)}{2}\right)(h^{s-1} - H^{s-1}) \quad (0 < s < 2) \tag{30}$$

and

$$MO(s) = -\left(\frac{k}{2}\right) \sin\theta\Gamma\left(\frac{(1+s)}{2}\right)\Gamma\left(\frac{(1-s)}{2}\right)(h^{s-1} - H^{s-1}) \quad (-1 < s < 1) \tag{31}$$

where $ME(s)$ and $MO(s)$ are the continuous Mellin transform for the even and odd potentials respectively.

5. Results and discussion

The residual total field magnetic profile of the digitised traverse is shown in Fig. 6. The amplitude of the magnetic field varies between -100 nT and 80 nT. The digitised value of amplitude obtained from the profile is used in the computation of the potential $V(x)$ for both the discrete and the continuous model of Mellin transform to determine the depth to the top, h, and the depth to the basement, H, of the spring at the point of the arbitrary values of s i.e. $s = \frac{1}{4}$, $s = \frac{1}{2}$ and $s = \frac{3}{4}$ respectively.

According to [17] report of the study area, it was observed that the lithologic unit identified in the study area is said to be migmatite gneiss. The outcrop nature of the migmatite gneiss shows that there is an evidence of structural deformation along the top of the basement by a series of parallel East- West fractures, joints and lineaments defined by vegetation alignment. This observed structural deformation at this region brings about disparity in the value obtained from computation of the depth to the top of basement, when compared with those obtained by [17] as shown in Table 1.

According to [17] the profile shows that a vertical discontinuities was suspected along the traverse between amplitude 65-80 nT,which might have occurred due to an hard or impregnable rock such as outcrop which may bring about uncertainty in the value obtain in that region when compare with the theoretical findings. Comparing the magnetic survey result obtained with that of resistivity result over the same region, it was found that the region of disparity is a weathered zone (from the vertical electric sounding result), having overburden thickness > 2.5 m. This is closer to the calculated value of Mellin transform for h, as shown in Table 1. The weathered layer in the study area is said to be due to the near - surface nature of the basement rock and that the basement rock are mainly exposed in the northern part and in the area around the spring. All the aforementioned physical deformation may be the reason why there is disparity in the value of the top to the depth when compare with that of [17]. While the calculated depth to the basement of the spring rock, H, highly correlates with that of [17] as shown in the table.

An alternative explanation to the disparity could be obtained theoretically. From equation (29), we have h to be:

$$h = \frac{1}{\left(C_5 - H^{-\frac{1}{4}}\right)}$$

Using the convergence theorem, which states that convergence occurs for large value of s, while for the smaller values of s, there will be divergence in the value obtained. Then, from the mathematical model used, s is in the range: $0 < s < 1$, and applying the lager value of s say $\frac{9}{10}$, which is within the range, the value of h will be 32.56 m, assuming C_5 is the height of the outcrop and is negligible.

This result for h agrees favourably with that obtained by [17] as shown in Table 2.

6. Conclusion

In this study, Mellin transform is introduced for the interpretation of the magnetic anomaly over a vertical thin sheet extending to a finite depth; Using this approach, both the depth to the top and bottom of the basement of the spring was determined.

The present study indicates that the Mellin transform has a direct approach for the interpretation of the magnetic data and lead to the exposure of the effect of the geological feature such as the outcrop, to be present around the study area.

Mellin transform is said to be the third order kind of improper integral which obeys the convergence theorem at larger value of s and also satisfy the divergence theorem

Table 1. The comparism of value of computed depth over the depth of the basement.

Authors	Depth to bottom of basement (H) m	Depth to top of basement (h) m
Bayode and Akpoarebe (2011)	48.00	33.00
Present study	47.95	2.63

Table 2. The comparism of value of computed depth to depth of the basement at larger values of s.

Authors	Depth to bottom of basement (H) m	Depth to top of basement (h) m
Bayode and Akpoarebe (2011)	48.00	33.00
Present study	47.95	32.56

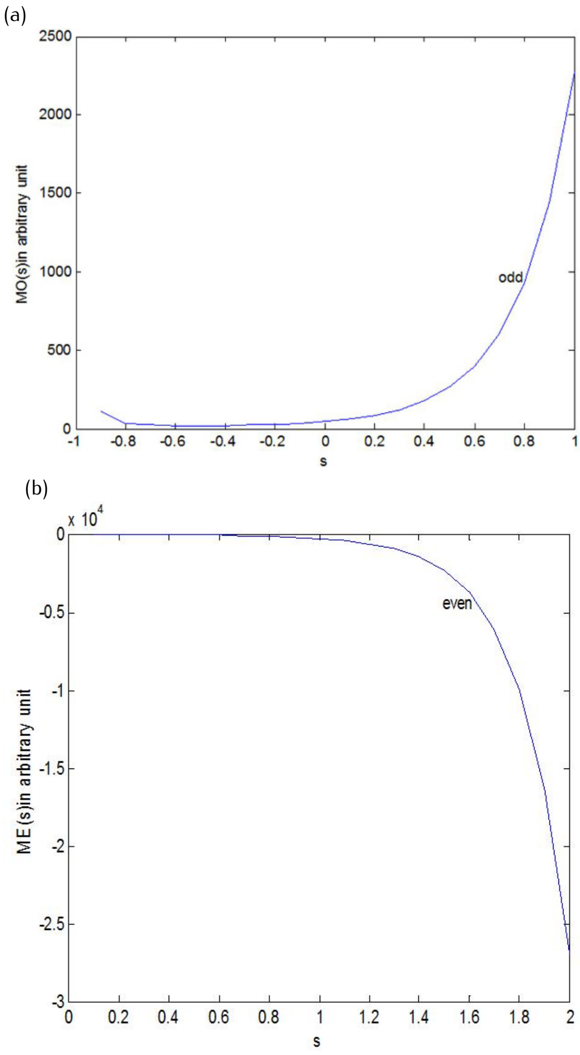

Figure 8. (a) The graph of the discrete Mellin transform for odd arbitrary values; (b) the graph of the discrete Mellin transform for even arbitrary value.

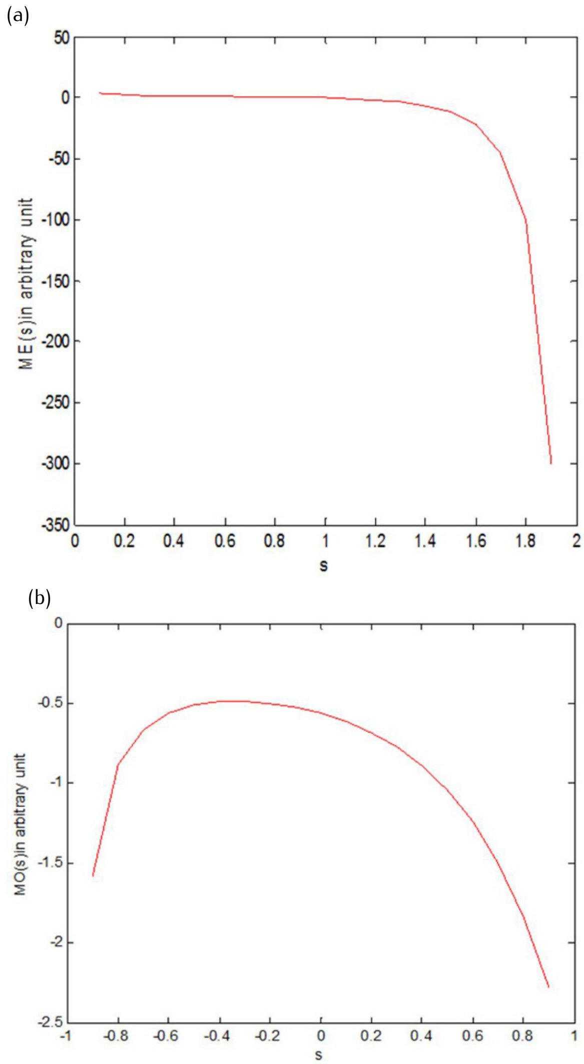

Figure 9. (a) The graph of the continuous Mellin transform for $0 < S < 2$; (b) the graph of the continuous Mellin transform for $-1 < S < 1$.

at the smaller values of s, using this approach the value for H and h were obtained as shown in Table 2.

The similarity of the curves of the transformed anomalies as shown in Fig. 8 and Fig. 9 and the gamma function curves is observed. This may be attributed to the fact that Mellin transform is a generalised form of a gamma function [23].

Finally, the study reveals that Mellin transform is an integral transform which can be used in the analysis and interpretation of potential problems.

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