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L^p-SPACES ON LOCALLY COMPACT GROUPS

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ABSTRACT. In this paper, some relations between L^p-spaces on locally compact groups are found. Applying these results proves that for a locally compact group G, the convolution Banach algebras $L^p(G) \cap L^1(G)$ $(1 < p \leq \infty)$, and $A_p(G) \cap L^1(G)$ $(1 < p < \infty)$ are amenable if and only if G is discrete and amenable.

Introduction

In [4], Ghahramani and Lau studied a number of Segal algebras on locally compact groups. This paper is a continuation of the work of Ghahramani and Lau. The organization of this paper is as follows. In Section 1, it is shown that for a locally compact group G, $L^1(G) \cap L^p(G) \subseteq C(G)$, for some $1 \leq p \leq \infty$, if and only if G is discrete. Also it is proved that for $1 \leq p_1 \lneqq p_2 \leq \infty$, $L^1(G) \cap L^{p_1}(G) \subseteq L^{p_2}(G)$ if and only if G is discrete. In Section 2 for a locally compact group G, the Segal algebras $\mathcal{L}A_p(G)$ and $\mathcal{L}L^p(G)$, where $1 < p < \infty$, $\mathcal{L}UC(G)$, and the abstract Segal algebra $\mathcal{L}L^\infty(G)$ are introduced. Recall that the Segal algebra $\mathcal{L}A(G)$ was defined in [4]. It is shown that the convolution Banach algebra $\mathcal{L}L^\infty(G)$ is a Segal algebra on G if and only if G is discrete. Furthermore, it is proved that the convolution Banach algebras $\mathcal{L}A_p(G)$ and $\mathcal{L}L^p(G)$, where $1 < p < \infty$, $\mathcal{L}L^\infty(G)$, and $\mathcal{L}UC(G)$, are amenable if and only if G is discrete and amenable.

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1. Preliminaries

A linear subspace $S^1(G)$ of the convolution group algebra $L^1(G)$ of a locally compact group G is said to be a *Segal algebra* if it satisfies the following conditions:

(i) $S^1(G)$ is dense in $L^1(G)$.

(ii) $S^1(G)$ is a Banach space under some norm $\|\cdot\|_{S^1(G)}$ and $\|f\|_{S^1(G)} \geq \|f\|_1$ for all $f \in S^1(G)$.

(iii) $S^1(G)$ is left translation invariant and the map $x \mapsto \delta_x * f$ from G into $S^1(G)$ is continuous for all $f \in S^1(G)$.

(iv) $\|\delta_x * f\|_{S^1(G)} = \|f\|_{S^1(G)}$ for all $f \in S^1(G)$ and $x \in G$.

A Segal algebra $S^1(G)$ is *symmetric* if it is right translation invariant, for all $f \in S^1(G)$, $\|f * \delta_x\|_{S^1(G)} = \|f\|_{S^1(G)}$ $(x \in G)$, and the map $x \mapsto f * \delta_x$ from G into $S^1(G)$ is continuous.

Note that if $S^1(G)$ is a Segal algebra, then for each $f \in L^1(G)$ and $g \in S^1(G)$, $f * g \in S^1(G)$ and $\|f * g\|_{S^1(G)} \leq \|f\|_1 \|g\|_{S^1(G)}$ (see [7: Proposition 1, p. 19]), or equivalently $S^1(G)$ is an abstract Segal algebra with respect to $L^1(G)$.

Let G be a locally compact group. Let $C(G)$ ($C_b(G)$, respectively) be the space of all (bounded, respectively) continuous complex-valued functions on G, and we denote by $UC(G)$ the space of all *uniformly continuous* complex-valued functions on G i.e. all $f \in C_b(G)$ such that the mappings: $x \mapsto \delta_x * f$, $x \mapsto f * \delta_x$ from G into $C_b(G)$ is continuous whenever $C_b(G)$ has the sup norm topology. We recall that $C_{00}(G)$ is the space of compactly supported continuous functions on G.

For a locally compact group G and a function $f\colon G \to \mathbb{C}$, $\check{f}$ is defined by $\check{f}(x) = f(x^{-1})$. For $1 < p < \infty$, let $A_p(G)$ consist of all functions h in $C_0(G)$ that can be written in at least one way as $\sum\limits_{n=1}^{\infty} f_n * \check{g_n}$, where $f_n \in L^p(G)$, $g_n \in L^q(G)$, whenever $\frac{1}{p} + \frac{1}{q} = 1$, and $\sum\limits_{n=1}^{\infty} \|f_n\|_p \|g_n\|_q < \infty$. For $h \in A_p(G)$, define

$$\|h\|_{A_p(G)} = \inf\Big\{ \sum_{n=1}^{\infty} \|f_n\|_p \|g_n\|_q : \ h = \sum_{n=1}^{\infty} f_n * \check{g_n} \Big\}.$$

With this norm $A_p(G)$ is a Banach space. For more details see [6: 35.16]. This Banach space with the point-wise multiplication is a Banach algebra (see [2: Theorem 4.5.30]). The Banach algebra $A_2(G)$ is called the *Fourier algebra* of G and denoted by $A(G)$.

Let A be a Banach algebra, and X be a Banach A-bimodule. A bounded linear map $D\colon A \to X$ is called an X-*derivation*, if

$$D(ab) = D(a) \cdot b + a \cdot D(b) \qquad (a, b \in A).$$

For every $x \in X$, we define ad_x^A by $ad_x^A(a) = a \cdot x - x \cdot a$ $(a \in A)$. It is easily seen that ad_x^A is a derivation. Derivations of this form are called *inner derivations*.

A Banach algebra A is called *amenable* if every derivation from A into X^* is inner for all Banach A-bimodules X.

2. L^p-spaces on locally compact groups

In the proof of (c) $\implies$ (a) of [4: Proposition 2.3], it is shown that if for a locally compact group G, $L^1(G) \subseteq C_0(G)$, then G is discrete. The following lemma is a generalization of this fact by a quite different technique of proof.

Lemma 2.1. *Let G be a locally compact group with the Haar measure λ. If there exists a neighborhood U of the identity e of G, such that every function in*

$$L^\infty(G)1_U = \{f1_U :\ f \in L^\infty(G)\},$$

is equal λ-locally almost everywhere to a continuous function on G, then G is discrete.

P r o o f. Let U_0 be a symmetric open neighborhood of e with compact closure, such that $\overline{U_0} \subseteq U$. Then $G_0 = \bigcup\limits_{n=1}^{\infty} {U_0}^n$ is an open, closed, and σ-compact subgroup of G (see [5: Theorem 5.7]). Let λ_0 be the Haar measure of G_0. Since G_0 is an open and closed subset of G, so

$$\begin{aligned} L^\infty(G_0)1_{U_0} &= \{f|_{G_0} :\ f \in L^\infty(G)1_{U_0}\} \subseteq \{f|_{G_0} :\ f \in L^\infty(G)1_U\} \\ &\subseteq \{f|_{G_0} :\ f \in C(G)\} = C(G_0). \end{aligned}$$

It follows that there exists $f \in C(G_0)$ such that $f = 1_{U_0}$, λ_0-almost everywhere (note that for σ-compact groups, locally almost everywhere and almost everywhere are equivalent). Hence

$$\lambda_0\big(G_0 - f^{-1}\{0,1\}\big) = \lambda_0\big(G_0 - 1_{U_0}^{-1}\{0,1\}\big) = 0.$$

Now, since f is continuous, so the set $G_0 - f^{-1}\{0,1\}$ is open, and hence by [5: Remark 15.8(i)], is equal to the empty set. Therefore $G_0 = f^{-1}\{0,1\}$. It follows that $f^{-1}\{1\}$ is equal to the open set $f^{-1}(0,\infty)$, and so is an open set. Also we have

$$\lambda_0\left(f^{-1}\{1\} - \overline{U_0}\right) \le \lambda_0\big(f^{-1}\{1\} - U_0\big) \le \lambda_0\big(\{x \in G :\ f(x) \ne 1_{U_0}(x)\}\big) = 0.$$

But $f^{-1}\{1\} \setminus \overline{U_0}$ is open. It follows that $f^{-1}\{1\} \setminus \overline{U_0} = \emptyset$, and so $f^{-1}\{1\} \subseteq \overline{U_0}$. Similarly it can be proved that the Haar measure of the open set $U_0 \setminus f^{-1}\{1\}$ is zero, and so $U_0 \setminus f^{-1}\{1\} = \emptyset$. It follows that $U_0 \subseteq f^{-1}\{1\}$, and since $f^{-1}\{1\}$ is closed, so $\overline{U_0} \subseteq f^{-1}\{1\}$. Therefore $\overline{U_0} = f^{-1}\{1\}$. Since $f^{-1}\{1\}$ is open, so $\overline{U_0}$ is an open neighborhood of the identity e. But also $\overline{U_0}$ is compact. So by

[5: Theorem 7.5], $\overline{U_0}$ contains a subgroup H of G_0 that is compact and open. It follows that

$$\begin{aligned} L^\infty(H) &= \big\{f|_H :\ f \in L^\infty(G_0)\big\} = \big\{f|_H :\ f \in L^\infty(G_0)1_{U_0}\big\} \\ &\subseteq \big\{f|_H :\ f \in C(G_0)\big\} = C(H), \end{aligned}$$

and hence for the compact group H, $L^\infty(H) \subseteq C(H)$. On one hand by [6: Lemma 37.3], H is finite, and hence $\{e\}$ is an open subset of H. On the other hand H is an open subset of G_0, and G_0 is open in G. Hence $\{e\}$ is an open subset of G, and so G is discrete. □

Theorem 2.2. *Let G be a locally compact group and $1 \le p \le \infty$. Then*

$$L^1(G) \cap L^p(G) \subseteq C(G),$$

if and only if G is discrete.

P r o o f. Clearly if G is discrete, then $\ell^1(G) \cap \ell^p(G) \subseteq C(G)$. Conversely suppose $L^1(G) \cap L^p(G) \subseteq C(G)$. Let λ be a Haar measure for G, U a neighborhood of the identity with finite Haar measure. Thus if $f \in L^\infty(G)1_U$ and $p \neq \infty$, then $\|f\|_p \le \|f\|_\infty \lambda(U)^{\frac{1}{p}} < \infty$. It follows that for each $1 \le p \le \infty$, $L^\infty(G)1_U \subseteq L^1(G) \cap L^p(G)$. Therefore $L^\infty(G)1_U \subseteq C(G)$, and hence by Lemma 2.1, G is discrete. □

Note that in the above proposition, for $p = \infty$ one can not use the method of the proof of (c) $\Longrightarrow$ (a) of [4: Proposition 2.3] to get the result. But applying their method, we have the following result.

Theorem 2.3. *Let G be a locally compact group and $1 \le p_1 \lneqq p_2 \le \infty$. Then*

$$L^1(G) \cap L^{p_1}(G) \subseteq L^{p_2}(G),$$

if and only if G is discrete.

P r o o f. By a similar method as in the proof of [4: Lemma 2.1], we conclude that $L^1(G) \cap L^{p_1}(G)$ with the norm $\|f\|_{L^1(G)\cap L^{p_1}(G)} = \|f\|_1 + \|f\|_{p_1}$, $f \in L^1(G) \cap L^{p_1}(G)$, is a Banach space. Consider the linear map $\iota\colon L^1(G) \cap L^{p_1}(G) \to L^{p_2}(G)$; $f \mapsto f$. By applying the Closed Graph Theorem and using the same method as the proof of (c) $\Longrightarrow$ (a) of [4: Proposition 2.3], we see that ι is continuous. Suppose to the contrary, G is not discrete. Let λ be the Haar measure of G. Let U_0 be an open neighborhood of the identity e with compact closure, and $\{U\}$ be a system of open neighborhoods of e such that $U \subseteq U_0$, and $\lambda(U) \longrightarrow 0$ as

$U \downarrow \{e\}$. Let $g_U = \lambda(U)^{\frac{-1}{p_1}} 1_U$. Now since $p_1 \geq 1$, $1 - \frac{1}{p_1} \geq 0$, and hence we have

$$\begin{aligned}\|g_U\|_{L^1(G)\cap L^{p_1}(G)} &= \|g_U\|_1 + \|g_U\|_{p_1} \\ &= \lambda(U)^{\frac{-1}{p_1}} \|1_U\|_1 + \lambda(U)^{\frac{-1}{p_1}} \|1_U\|_{p_1} \\ &= \lambda(U)^{\frac{-1}{p_1}} \lambda(U) + \lambda(U)^{\frac{-1}{p_1}} \lambda(U)^{\frac{1}{p_1}} \\ &= \lambda(U)^{1-\frac{1}{p_1}} + 1 \leq \lambda(U_0)^{1-\frac{1}{p_1}} + 1.\end{aligned}$$

But since $p_1 \lneqq p_2$, so $\frac{1}{p_1} - \frac{1}{p_2} > 0$, and hence

$$\begin{aligned}\liminf_{U\downarrow\{e\}} \|g_U\|_{p_2} &= \liminf_{U\downarrow\{e\}} \lambda(U)^{\frac{-1}{p_1}} \|1_U\|_{p_2} = \liminf_{U\downarrow\{e\}} \lambda(U)^{\frac{-1}{p_1}} \lambda(U)^{\frac{1}{p_2}} \\ &= \left(\lim_{U\downarrow\{e\}} \frac{1}{\lambda(U)}\right)^{\frac{1}{p_1}-\frac{1}{p_2}} = \infty.\end{aligned}$$

This contradicts the continuity of ι. Hence G is discrete.

Conversely it is clear that if G is discrete, then $\ell^{p_1}(G) \subseteq \ell^{p_2}(G)$. □

Corollary 2.4. *Let G be a locally compact group and $1 \leq p_1 \lneqq p_2 \leq \infty$. Then $L^{p_1}(G) = L^{p_2}(G)$, if and only if G is finite.*

Proof. If $L^{p_1}(G) = L^{p_2}(G)$, then by Theorem 2.3, G is discrete. If G is an infinite discrete group, then there exists an infinite countable subset $\{x_n\}_{n\in\mathbb{N}}$ of distinct elements of G. It is easy to see that $\sum_{n\in\mathbb{N}} \left(\frac{1}{n}\right)^{\frac{1}{p_1}} \delta_{x_n} \in \ell^{p_2}(G) \setminus \ell^{p_1}(G)$, and so $\ell^{p_1}(G) \neq \ell^{p_2}(G)$. Clearly if G is finite, then $\ell^{p_1}(G) = \ell^{p_2}(G)$. □

3. Applications to certain Segal algebras on locally compact groups

We start this section with the following lemma.

Lemma 3.1. *Let $\mathfrak{A}$ be any of Banach spaces $(A_p(G), \|\cdot\|_{A_p(G)})$, $(L^p(G), \|\cdot\|_p)$ $(1 < p < \infty)$, and $(UC(G), \|\cdot\|_\infty)$. Let $\mathcal{L}\mathfrak{A} = L^1(G) \cap \mathfrak{A}$ with the norm $\|f\|_{\mathcal{L}\mathfrak{A}} = \|f\|_1 + \|f\|_{\mathfrak{A}}$, for $f \in \mathcal{L}\mathfrak{A}$. Then $\mathcal{L}\mathfrak{A}$ is a Segal algebra on G. Moreover if G is unimodular, then $\mathcal{L}\mathfrak{A}$ is a symmetric Segal algebra on G. Also if G is discrete, then $\mathcal{L}\mathfrak{A}$ is equal and Banach algebra isomorphic with $\ell^1(G)$.*

Proof. We first prove that $\mathcal{L}A_p(G)$ is a Segal algebra. By [6: 35.16(b)] and a similar method as the proof of [4: Lemma 2.1], we conclude that $\mathcal{L}A_p(G)$ is a left $L^1(G)$-module with convolution, and for $f \in L^1(G)$, $h \in A_p(G)$, and $x \in G$, $\|f * h\|_{\mathcal{L}A_p(G)} \leq \|f\|_1 \|h\|_{\mathcal{L}A_p(G)}$, and $\|\delta_x * h\|_{\mathcal{L}A_p(G)} = \|h\|_{\mathcal{L}A_p(G)}$. By [3: Proposition 2.42]) $L^1(G)$ has a bounded approximate identity in $C_{00}(G)$. This together with the fact that $C_{00}(G)$ is dense in $L^1(G)$, implies that $C_{00}(G) * C_{00}(G)$ is dense in $L^1(G)$. But $C_{00}(G) * C_{00}(G) \subseteq \mathcal{L}A_p(G)$, and so $\mathcal{L}A_p(G)$ is dense in $L^1(G)$. Let $x \in G$, $f \in A_p(G)$ and $\varepsilon > 0$. There exist $f_1, \dots, f_n \in L^p(G)$ and $g_1, \dots, g_n \in L^q(G)$ such that $\|f - \sum_{i=1}^{n} f_i * \check{g}_i\|_{A_p(G)} < \varepsilon$. Then

$$\|f - \delta_x * f\|_{A_p(G)} \leq 2\varepsilon + \sum_{i=1}^{n} \|f_i - \delta_x * f_i\|_p \|g_i\|_q,$$

and so

$$\limsup_{x \longrightarrow e} \|f - \delta_x * f\|_{A_p(G)} \leq 2\varepsilon + \sum_{i=1}^{n} \lim_{\alpha} \|f_i - \delta_x * f_i\|_p \|g_i\|_q = 2\varepsilon.$$

This together with the fact that $\varepsilon > 0$ is arbitrary, implies that

$$\lim_{x \longrightarrow e} \|f - \delta_x * f\|_{\mathcal{L}A_p(G)} = \lim_{x \longrightarrow e} \|f - \delta_x * f\|_{A_p(G)} + \lim_{x \longrightarrow e} \|f - \delta_x * f\|_1 = 0.$$

Therefore the mapping $G \to \mathcal{L}A_p(G)$; $f \mapsto \delta_x * f$ is continuous. So $\mathcal{L}A_p(G)$ is a Segal algebra on G. The other cases are proved similarly. Note that by [5: Theorem 20.4], for each $f \in L^p(G)$ $(1 < p < \infty)$, the mappings $G \to L^p(G)$, $x \mapsto \delta_x * f$, $f * \delta_x$ are continuous. Also by definition of $UC(G)$, for each $f \in UC(G)$ the mappings $G \to UC(G)$, $x \mapsto \delta_x * f$, $f * \delta_x$ are continuous. If G is unimodular, then for each $1 \leq p \leq \infty$, $L^p(G)\check{} = L^p(G)$, and for each $f \in L^p(G)$, $g \in L^1(G)$, $\|\check{f}\|_p = \|f\|_p$ and $\|f * g\|_p \leq \|f\|_p \|g\|_1$. So by a similar method as the first paragraph of the proof, we conclude that $\mathcal{L}\mathfrak{A}$ is an symmetric Segal algebra on G. If G is discrete, then $\delta_e \in \mathcal{L}\mathfrak{A}$. But $\mathcal{L}\mathfrak{A}$ is an ideal in $\ell^1(G)$, and so $\mathcal{L}\mathfrak{A} = \ell^1(G)$. Hence the mapping $\mathcal{L}\mathfrak{A} \to \ell^1(G)$; $f \mapsto f$ is continuous. Now, by the Inverse Mapping Theorem, $\mathcal{L}\mathfrak{A}$ is Banach algebra isomorphic with $\ell^1(G)$. □

Proposition 3.2. *Let G be a locally compact group. Then $\mathcal{L}L^\infty(G) = L^1(G) \cap L^\infty(G)$ with the norm $\|f\|_{\mathcal{L}L^\infty(G)} = \|f\|_1 + \|f\|_\infty$, for $f \in \mathcal{L}L^\infty(G)$ is a convolution Banach algebra. Moreover, $\mathcal{L}L^\infty(G)$ is a Segal algebra on G if and only if G is discrete.*

Proof. By a similar proof to that of [4: Lemma 2.1], one can show that $\mathcal{L}L^\infty(G)$ is a convolution Banach algebra. Suppose that $\mathcal{L}L^\infty(G)$ is a Segal algebra. Then by [7: Proposition 1(i), p. 34], $\mathcal{L}L^\infty(G)$ has a left approximate identity which is bounded in L^1-norm. Hence, by Cohen's Factorization Theorem, $\mathcal{L}L^\infty(G) = L^1(G) * \mathcal{L}L^\infty(G)$, and so by [5: Theorem 20.16]

$$\mathcal{L}L^\infty(G) = L^1(G) * \mathcal{L}L^\infty(G) \subseteq L^1(G) * L^\infty(G) \subseteq C(G).$$

Now by Theorem 2.2, G is discrete. □

Theorem 3.3. *Let G be a locally compact group, and $\mathfrak{A}$ be any of spaces $A_p(G)$ $(1 < p < \infty)$, $UC(G)$, and $L^p(G)$ $(1 < p \leq \infty)$. Then the convolution Banach algebra $\mathcal{L}\mathfrak{A}$ is amenable if and only if G is amenable and discrete.*

P r o o f. Suppose that $\mathcal{L}\mathfrak{A}$ is amenable. so it has a bounded approximate identity. By a result of Burnham ([1]) (see also [4: Lemma 1.1]), $\mathcal{L}\mathfrak{A} = L^1(G)$. Let $\mathfrak{A}$ be either $A_p(G)$ $(1 < p < \infty)$, or $UC(G)$. Then

$$L^1(G) = \mathcal{L}\mathfrak{A} = L^1(G) \cap \mathfrak{A} \subseteq \mathfrak{A} \subseteq C(G),$$

and so by Theorem 2.2, G is discrete. If $\mathcal{L}L^p(G)$ $(1 < p \leq \infty)$ is amenable, then

$$L^1(G) = \mathcal{L}L^p(G) = L^1(G) \cap L^p(G) \subseteq L^p(G),$$

and hence by Theorem 2.3, G is discrete. Now, by Lemma 3.1, $\mathcal{L}\mathfrak{A}$ is Banach algebra isomorphic with $\ell^1(G)$. It follows that $\ell^1(G)$ is amenable and hence G is amenable. Conversely if G is amenable and discrete, then by Lemma 3.1 $\mathcal{L}\mathfrak{A}$ is Banach algebra isomorphic with $\ell^1(G)$. Now, by Johnson's Theorem ([8: Theorem 2.1.8]) $\ell^1(G)$ is amenable. Hence $\mathcal{L}\mathfrak{A}$ is amenable. □

Lemma 3.4. *Let G be a compact group with the normalized Haar measure, and $\mathfrak{A}$ be any of the spaces $A_p(G)$ $(1 < p < \infty)$, $L^p(G)$ $(1 < p \lneqq \infty)$ and $C(G)$. Then $\mathfrak{A}$ is a symmetric Segal algebra on G which is Banach algebra isomorphic to $\mathcal{L}\mathfrak{A}$. Furthermore the convolution Banach algebra $L^\infty(G)$ is Banach algebra isomorphic to $\mathcal{L}L^\infty(G)$.*

P r o o f. Since G is compact, the mapping $\iota\colon \mathcal{L}\mathfrak{A} \to \mathfrak{A}$; $f \mapsto f$ is onto and continuous. Hence by the Inverse Mapping Theorem, $\mathcal{L}\mathfrak{A}$ is Banach algebra isomorphic with $\mathfrak{A}$. The result follows from Lemma 3.1, and Proposition 3.2. □

As a consequence of Lemma 3.4 and Theorem 3.3, we have the following result.

Proposition 3.5. *Let G be a compact group with the normalized Haar measure. Then the convolution Banach algebras $A_p(G)$ $(1 < p < \infty)$, $L^p(G)$ $(1 < p \leq \infty)$, and $C(G)$ are amenable if and only if G is finite.*

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