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SOME PROPERTIES OF HYPERIDEALS IN TERNARY SEMIHYPERGROUPS

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ABSTRACT. Ternary semihypergroups are algebraic structures with one ternary associative hyperoperation. In this paper we give some properties of left (right) and lateral hyperideals in ternary semihypergroups. We introduce the notion of left simple, lateral simple, left (0-)simple and lateral 0-simple ternary semihypergroups and characterize the minimality and maximality of left (right) and lateral hyperideals in ternary semihypergroups. The relationship between them is investigated in ternary semihypergroups extending and generalizing the analogues results for ternary semigroups.

1. Introduction and preliminaries

Algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory etc. Ternary algebraic operations were considered in the 19th century by several mathematicians such as Cayley [5] who introduced the notion of "cubic matrix" which in turn was generalized by Kapranov, et al. in 1990 [36]. Ternary structures and their generalization, the so-called n-ary structures, raise certain hopes in view of their possible applications in physics and other sciences. Some significant physical applications in Nambu mechanics, although still hypothetical, in the fractional quantum Hall effect, the non-standard statistics (the "anyons), supersymmetric theories, Yang-Baxter equation, etc. can be seen in [1, 10, 38, 39, 56, 57]). The notion of an n-ary group was introduced in 1928 by W. Dörnte [20] (under inspiration of Emmy Noether). The idea of investigations of n-ary algebras, i.e.,

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sets with one n-ary operation, seems to be going back to Kasners lecture [37] at the 53rd annual meeting of the American Association of the Advancement of Science in 1904. Sets with one n-ary operation having different properties were investigated by many authors. Such systems have many applications in different branches. For example, in the theory of automata [26] are used n-ary systems satisfying some associative law, some others n-ary systems are applied in the theory of quantum groups [49] and combinatorics [55]. Different applications of ternary structures in physics are described by R. Kerner in [38]. In physics there are used also such structures as n-ary Filippov algebras (see [52]) and n-Lie algebras (see [57]). Some n-ary structures induced by hypercubes have application in error-correcting and error-detecting coding theory, cryptology, as well as in the theory of (t, m, s)-nets (see for example [40]). Ternary semigroups are universal algebras with one associative operation. The theory of ternary algebraic system was introduced by D. H. Lehmer [41] in 1932. He investigated certain algebraic systems called triplexes which turn out to be commutative ternary groups. The notion of ternary semigroups was introduced by S. Banach (cf. [45]). He showed by an example that a ternary semigroup does not necessary reduce to an ordinary semigroup. In 1965, Sioson [54] studied ideal theory in ternary semigroups. He also introduced the notion of regular ternary semigroups and characterized them by using the notion of quasi-ideals. In [21, 22] Dudek et. al. studied the ideals in n-ary semigroups. In 1995, Dixit and Dewan [19] introduced and studied some properties of ideals and quasi-(bi-)ideals in ternary semigroups.

Hyperstructure theory was introduced in 1934, when F. Marty [46] defined hypergroups based on the notion of hyperoperation, began to analyze their properties and applied them to groups. In the following decades and nowadays, a number of different hyperstructures are widely studied from the theoretical point of view and for their applications to many subjects of pure and applied mathematics by many mathematicians. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. Several books have been written on hyperstructure theory, see [8,9,16,58]. A recent book on hyperstructures [9] points out on their applications in rough set theory, cryptography, codes, automata, probability, geometry, lattices, binary relations, graphs and hypergraphs. Another book [16] is devoted especially to the study of hyperring theory. Several kinds of hyperrings are introduced and analyzed. The volume ends with an outline of applications in chemistry and physics, analyzing several special kinds of hyperstructures: e-hyperstructures and transposition hypergroups. The theory of suitable modified hyperstructures can serve as a mathematical background in the field of quantum communication systems. Some principal notions about semihypergroups theory can be found in [3, 6, 13, 14, 17, 25, 42, 50]. Recently, Davvaz, Hila and et. al. [2, 29, 31, 48] introduced the notion of Γ-semihypergroup as a generalization of a semigroup, a generalization of a semihypergroup and a generalization of a Γ-semigroup. They presented many interesting examples and obtained a several characterizations of Γ-semihypergroups.

n-ary generalizations of algebraic structures is the most natural way for further development and deeper understanding of their fundamental properties. In [18], Davvaz and Vougiouklis introduced the concept of n-ary hypergroups as a generalization of hypergroups in the sense of Marty. Also, we can consider n-ary hypergroups as a nice generalization of n-ary groups. Leoreanu-Fotea and Davvaz in [43] introduced and studied the notion of a partial n-ary hypergroupoid, associated with a binary relation. Some important results, concerning Rosenberg partial hypergroupoids, induced by relations, are generalized to the case of n-ary hypergroupoids. Davvaz and et. al. in [14] considered a class of algebraic hypersystems which represent a generalization of semigroups, hypersemigroups and n-ary semigroups. Ternary semihypergroups are algebraic structures with one ternary associative hyperoperation. A ternary semihypergroup is a particular case of an n-ary semihypergroup (n-semihypergroup) for $n = 3$ (cf. [13, 14, 18, 43, 44]. Recently, Davvaz and Leoreanu [15] studied binary relations on ternary semihypergroups and studied some basic properties of compatible relations on them. In [33], we introduced and studied prime left, semiprime left, irreducible left hyperideals and hyperideal extensions in ternary semihypergroups and investigated some basic properties of them. The main purpose of this paper is to give some other properties of left (right) and lateral hyperideals in ternary semihypergroups. We introduce the notion of left simple, lateral simple, left (0-) simple and lateral 0-simple ternary semihypergroups and characterize the minimality and maximality of left (right) and lateral hyperideals in ternary semihypergroups. The relationship between them is investigated in ternary semihypergroups extending and generalizing the analogues results for ternary semigroups.

Recall first the basic terms and definitions from the ternary semihypergroups theory.

Definition 1.1. ([15]) A map $f\colon H \times H \times H \to \mathcal{P}^*(H)$ is called *ternary hyperoperation* on the set H, where H is a nonempty set and $\mathcal{P}^*(H) = \mathcal{P}(H)\backslash\{\emptyset\}$ denotes the set of all nonempty subsets of H.

Definition 1.2. ([15]) A *ternary hypergroupoid* is called the pair (H, f) where f is a ternary hyperoperation on the set H.

If A, B, C are nonempty subsets of H, then we define

$$f(A, B, C) = \bigcup_{a\in A, b\in B, c\in C} f(a, b, c).$$

Definition 1.3. ([15]) A ternary hypergroupoid (H, f) is called a *ternary semihypergroup* if for all $a_1, a_2, \dots, a_5 \in H$, we have

$$f(f(a_1, a_2, a_3), a_4, a_5) = f(a_1, f(a_2, a_3, a_4), a_5) = f(a_1, a_2, f(a_3, a_4, a_5)).$$

Since the set $\{x\}$ can be indentified with the element x, any ternary semigroup is a ternary semihypergroup. A ternary semigroup does not necessarily reduce to an ordinary semigroup. This has been shown by the following example.

Example 1.4. ([19]) Let $S = \{-i, 0, i\}$ be a ternary semigroup under the multiplication over complex numbers while S is not a binary semigroup under the multiplication over complex numbers.

Los [45] showed that an ternary semigroup however may be embedded in an ordinary semigroup in such a way that the operation in the ternary semigroup is an (ternary) extension of the (binary) operation of the containing semigroup.

Definition 1.5. ([15]) Let (H, f) be a ternary semihypergroup. Then H is called a *ternary hypergroup* if for all $a, b, c \in H$, there exist $x, y, z \in H$ such that:

$$c \in f(x, a, b) \cap f(a, y, b) \cap f(a, b, z).$$

Definition 1.6. ([15]) Let (H, f) be a ternary hypergroupoid. Then

(1) (H, f) is $(1,3)$-*commutative* if for all $a_1, a_2, a_3 \in H$,

$$f(a_1, a_2, a_3) = f(a_3, a_2, a_1);$$

(2) (H, f) is $(2,3)$-*commutative* if for all $a_1, a_2, a_3 \in H$,

$$f(a_1, a_2, a_3) = f(a_1, a_3, a_2);$$

(3) (H, f) is $(1,2)$-*commutative* if for all $a_1, a_2, a_3 \in H$,

$$f(a_1, a_2, a_3) = f(a_2, a_1, a_3);$$

(4) (H, f) is *commutative* if for all $a_1, a_2, a_3 \in H$ and for all $\sigma \in \mathbb{S}_3$,

$$f(a_1, a_2, a_3) = f(a_{\sigma(1)}, a_{\sigma(2)}, a_{\sigma(3)}).$$

Different examples of ternary semihypergroups can be found in [14, 15, 18].

Definition 1.7. Let (H, f) be a ternary semihypergroup and T a nonempty subset of H. Then T is called a ternary subsemihypergroup of H if and only if $f(T, T, T) \subseteq T$.

Definition 1.8. A ternary semihypergroup (H, f) is said to have a *zero element* if there exist an element $0 \in H$ such that for all $a, b \in H$, $f(0, a, b) = f(a, 0, b) = f(a, b, 0) = \{0\}$.

Definition 1.9. Let (H, f) be a ternary semihypergroup. An element $e \in H$ is called *left identity* element of H if for all $a \in H$, $f(e, a, a) = \{a\}$. An element $e \in H$ is called an *identity* element of H if for all $a \in H$, $f(a, a, e) = f(e, a, a) = f(a, e, a) = \{a\}$. It is clear that ([18]) $f(e, e, a) = f(e, a, e) = f(a, e, e) = \{a\}$.

Definition 1.10. A nonempty subset I of a ternary semihypergroup H is called a *left* (*right, lateral*) *hyperideal* of H if

$$f(H, H, I) \subseteq I(f(I, H, H) \subseteq I, f(H, I, H) \subseteq I).$$

A nonemtpy subset I of a ternary semihypergroup H is called a *hyperideal* of H if it is a left, right and lateral hyperideal of H. A nonemtpy subset I of a ternary semihypergroup H is called *two-sided hyperideal* of H if it is a left and right hyperideal of H. A lateral hyperideal I of a ternary semihyperideal H is called a *proper lateral hyperideal* of H if $I \neq H$.

Definition 1.11. A left hyperideal I of a ternary semihypergroup H is called *idempotent* if $f(I, I, I) = I$.

Example 1.12. Let $H = \{a, b, c, d, e, g\}$ and $f(x, y, z) = (x * y) * z$ for all $x, y, z \in H$, where $*$ is defined by the table:

$*$	a	b	c	d	e	g
a	a	$\{a, b\}$	c	$\{c, d\}$	e	$\{e, g\}$
b	b	b	d	d	g	g
c	c	$\{c, d\}$	c	$\{c, d\}$	c	$\{c, d\}$
d	d	d	d	d	d	d
e	e	$\{e, g\}$	c	$\{c, d\}$	e	$\{e, g\}$
g	g	g	d	d	g	g

Then (H, f) is a ternary semihypergroup. Clearly, $I_1 = \{c, d\}$, $I_2 = \{c, d, e, g\}$ and H are lateral hyperideals of H.

Example 1.13. Let $H = \{a, b, c, d, e, g\}$ and $f(x, y, z) = (x * y) * z$ for all $x, y, z \in H$, where $*$ is defined by the table:

$*$	a	b	c	d	e	g
a	$\{b, c\}$	$\{b, c\}$	$\{b, c\}$	$\{b, c\}$	$\{b, c\}$	$\{b, c\}$
b	$\{a, c\}$	$\{a, c\}$	$\{a, c\}$	$\{a, c\}$	$\{a, c\}$	$\{a, c\}$
c	$\{a, b\}$	$\{a, b\}$	$\{a, b\}$	$\{a, b\}$	$\{a, b\}$	$\{a, b\}$
d	$H - d$	$H - d$	$H - d$	$H - d$	$H - d$	$H - d$
e	$H - e$	$H - e$	$H - e$	$H - e$	$H - e$	$H - e$
g	$H - g$	$H - g$	$H - g$	$H - g$	$H - g$	$H - g$

Then (H, f) is a ternary semihypergroup. There is no proper lateral hyperideal of H.

Example 1.14. ([43]) Let $|H| \geq 4$ and $f\colon H \times H \times H \to \mathcal{P}^*(H)$, defined as follows:

$$f(x_0, x_0, x_0) = H - \{x_0, x_1\}$$
$$f(x, y, z) = H - \{x_0, x_2\}, \qquad \text{for all} \quad (x, y, z) \neq (x_0, x_0, x_0)$$

and $x_0 \neq x_1 \neq x_2 \neq x_0$. (H, f) is a ternary semihypergroup. It can be seen that $H - \{x_0\}$ and $H - \{x_0, x_2\}$ are proper lateral hyperideals of H.

2. Some elementary properties on left (0-)simple and lateral (0)-simple ternary semihypergroups

In this section we introduce and characterize the left simple, lateral simple, left (0-) simple and lateral (0-) simple ternary semihypergroups. Some properties of them are investigated in terms of left and lateral hyperideals.

Definition 2.1. Let (H, f) be a ternary semihypergroup without zero. H is called *lateral simple* if it has no proper lateral hyperideal.

Definition 2.2. Let (H, f) be a ternary semihypergroup without zero. H is called *left simple* if it has no proper left hyperideal.

The ternary semihypergroup of Example 1.13 is lateral simple.

It is clear that if H is a ternary semihypergroup with zero, then every lateral hyperideal of H contains a zero element.

Definition 2.3. Let (H, f) be a ternary semihypergroup with zero. H is called *lateral* 0*-simple* if it is has no nonzero proper lateral hyperideal and $f(H, H, H) \neq \{0\}$.

Definition 2.4. Let (H, f) be a ternary semihypergroup with zero. H is called *left* 0*-simple* if it is has no nonzero proper left hyperideal and $f(H, H, H) \neq \{0\}$.

It is clear that due to associative law in ternary semihypergroup H, for any elements $x_1, x_2, \ldots, x_{2n+1} \in H$ and positive integers m, n with $m \leq n$, one may write

$$\begin{aligned} f(x_1, x_2, \ldots, x_{2n+1}) &= f(x_1, \ldots, x_m, x_{m+1}, x_{m+2}, \ldots, x_{2n+1}) \\ &= f(x_1, \ldots, f(f(x_m, x_{m+1}, x_{m+2}), x_{m+3}, x_{m+4}), \ldots, x_{2n+1}). \end{aligned}$$

Let (H, f) be a ternary semihypergroup. It is clear that the intersection of all lateral hyperideals of a ternary subsemihypergroup T of H containing a nonempty subset A of T is the *lateral hyperideal of H generated by A*.

For every element $a \in H$, the left, right, lateral, two-sided and hyperideal generated by a are respectively given by

$$\begin{aligned}
\langle a\rangle_l &= \{a\} \cup f(H,H,a)\\
\langle a\rangle_r &= \{a\} \cup f(a,H,H)\\
\langle a\rangle_m &= \{a\} \cup f(H,a,H) \cup f(H,H,a,H,H)\\
\langle a\rangle_t &= \{a\} \cup f(H,H,a) \cup f(a,H,H) \cup f(H,H,a,H,H)\\
\langle a\rangle &= \{a\} \cup f(H,H,a) \cup f(a,H,H) \cup f(H,a,H) \cup f(H,H,a,H,H)
\end{aligned}$$

It can be easily verified the following lemmas.

Lemma 2.5. *Let (H,f) be a ternary semihypergroup. For any nonempty subset A of H, $f(H,H,A,H,H) \cup f(H,A,H) \cup A$ is the smallest lateral hyperideal of H containing A.*

Lemma 2.6. *Let (H,f) be a ternary semihypergroup. For any nonempty subset A of H, $f(H,H,A) \cup A$ is the smallest left hyperideal of H containing A.*

Lemma 2.7. *Let (H,f) be a ternary semihypergroup. For any nonempty subset A of H, $f(H,H,A,H,H) \cup f(H,A,H)$ is a lateral hyperideal of H.*

Lemma 2.8. *Let (H,f) be a ternary semihypergroup. For any nonempty subset A of H, $f(H,H,A)$ is a left hyperideal of H.*

Lemma 2.9. *Let (H,f) be a ternary semihypergroup without zero. Then the following statements are equivalent:*

(1) *H is lateral simple.*
(2) *for all $a \in H$, $f(H,H,a,H,H) \cup f(H,a,H) = H$*
(3) *for all $a \in H$, $\langle a\rangle_m = H$.*

Proof.

(1) $\Longrightarrow$ (2). Let H be lateral simple. By Lemma 2.7, we have

$$f(H,H,a,H,H) \cup f(H,a,H) = H \qquad \text{for all} \quad a \in H.$$

(2) $\Longrightarrow$ (3). By Lemma 2.5, we have

$$\langle a\rangle_m = f(H,H,a,H,H) \cup f(H,a,H) \cup \{a\} = H \cup \{a\} = H.$$

(3) $\Longrightarrow$ (1). Let I be a lateral hyperideal of H and $a \in I$. Then $H = \langle a\rangle_m \subseteq I \subseteq H$. So $I = H$. Hence H is lateral simple. □

Lemma 2.10. *Let (H, f) be a ternary semihypergroup without zero. Then the following statements are equivalent:*

(1) *H is left simple.*

(2) *for all $a \in H$, $f(H, H, a) = H$*

(3) *for all $a \in H$, $\langle a\rangle_l = H$.*

Proof.

(1) $\Longrightarrow$ (2). Let H be left simple. By Lemma 2.8, we have for all $a \in H$, $f(H, H, a) = H$.

(2) $\Longrightarrow$ (3). By Lemma 2.6, we have $\langle a\rangle_l = f(H, H, a) \cup \{a\} = H \cup \{a\} = H$.

(3) $\Longrightarrow$ (1). Let L be a left hyperideal of H and $a \in L$. Then $H = \langle a\rangle_l \subseteq L \subseteq H$. So $L = H$. Hence H is left simple. □

Lemma 2.11. *Let (H, f) be a ternary semihypergroup with zero. Then the following statements hold true:*

(1) *If H is lateral 0-simple, then for all $a \in H \setminus \{0\}$, $\langle a\rangle_m = H$.*

(2) *If for all $a \in H \setminus \{0\}$, $\langle a\rangle_m = H$, then either $f(H, H, H) = \{0\}$ or H is lateral 0-simple.*

Proof.

(1). Let H be lateral 0-simple. Then for all $a \in H \setminus \{0\}$, $\langle a\rangle_m$ is a nonzero lateral hyperideal of H. Hence for all $a \in H \setminus \{0\}$, $\langle a\rangle_m = H$.

(2). Let assume that for all $a \in H \setminus \{0\}$, $\langle a\rangle_m = H$ and $f(H, H, H) \neq \{0\}$. Let I be a nonzero lateral hyperideal of H and let $a \in I\setminus\{0\}$. Then $H = \langle a\rangle_m \subseteq I \subseteq H$. So $I = H$. Therefore H is lateral 0-simple. □

Lemma 2.12. *Let (H, f) be a ternary semihypergroup with zero. Then the following statements hold true:*

(1) *If H is left 0-simple, then for all $a \in H \setminus \{0\}$, $\langle a\rangle_l = H$.*

(2) *If for all $a \in H \setminus \{0\}$, $\langle a\rangle_l = H$, then either $f(H, H, H) = \{0\}$ or H is left 0-simple.*

Proof.

(1). Let H be left 0-simple. Then for all $a \in H \setminus \{0\}$, $\langle a\rangle_l$ is a nonzero left hyperideal of H. Hence for all $a \in H \setminus \{0\}$, $\langle a\rangle_l = H$.

(2). Let assume that for all $a \in H\setminus\{0\}$, $\langle a\rangle_l = H$ and $f(H, H, H) \neq \{0\}$. Let L be a nonzero left hyperideal of H and let $a \in L\setminus\{0\}$. Then $H = \langle a\rangle_l \subseteq L \subseteq H$. So $L = H$. Therefore H is left 0-simple. □

The following lemmas are trivially true.

Lemma 2.13. *Let (H, f) be a ternary semihypergroup and $\{I_\alpha, \alpha \in \Gamma\}$ be a family of lateral hyperideals of H. Then $\bigcup_{\alpha\in\Gamma} I_\alpha$ is a lateral hyperideal of H and $\bigcap_{\alpha\in\Gamma} I_\alpha$ is also a lateral hyperideal of H if $\bigcap_{\alpha\in\Gamma} I_\alpha \neq \emptyset$.*

Lemma 2.14. *Let (H, f) be a ternary semihypergroup and $\{L_\alpha, \alpha \in \Gamma\}$ be a family of left hyperideals of H. Then $\bigcup_{\alpha\in\Gamma} L_\alpha$ is a left hyperideal of H and $\bigcap_{\alpha\in\Gamma} L_\alpha$ is also a left hyperideal of H if $\bigcap_{\alpha\in\Gamma} L_\alpha \neq \emptyset$.*

Lemma 2.15. *Let (H, f) be a ternary semihypergroup, I be a lateral hyperideal of H and T a ternary subsemihypergroup of H. Then the following statements hold true:*

(1) *If T is lateral simple such that $T \cap I \neq$, then $T \subseteq I$.*
(2) *If T is lateral 0-simple such that $T \setminus \{0\} \cap I \neq \emptyset$, then $T \subseteq I$.*

P r o o f.

(1). Let assume T is lateral simple such that $T \cap I \neq \emptyset$. Then, let $a \in T \cap I$. By Lemma 2.7, since we have $(f(T, T, a, T, T) \cup f(T, a, T)) \cap T$ is a lateral hyperideal of T, it follows that $(f(T, T, a, T, T) \cup f(T, a, T)) \cap T = T$. Hence $T \subseteq f(T, T, a, T, T) \cup f(T, a, T) \subseteq f(H, H, I, H, H) \cup f(H, I, H) \subseteq f(H, I, H) \subseteq I$. So $T \subseteq I$.

(2). Let assume that T is lateral 0-simple such that $T \setminus \{0\} \cap I \neq \emptyset$. Then, let $a \in T \setminus \{0\} \cap I$. By Lemma 2.5 and Lemma 2.11(1), we have $T = \langle a \rangle_m = (f(T, T, a, T, T) \cup f(T, a, T) \cup \{a\}) \cap T \subseteq f(T, T, a, T, T) \cup f(T, a, T) \cup \{a\} \subseteq f(T, T, a, T, T) \cup f(T, a, T) \cup \{a\} = \langle a \rangle_m \subseteq I$. Therefore $T \subseteq I$. □

Lemma 2.16. *Let (H, f) be a ternary semihypergroup, L be a left hyperideal of H and T a ternary subsemihypergroup of H. Then the following statements hold true:*

(1) *If T is left simple such that $T \cap L \neq$, then $T \subseteq L$.*
(2) *If T is left 0-simple such that $T \setminus \{0\} \cap L \neq \emptyset$, then $T \subseteq L$.*

P r o o f. The proof is similar to the proof of Lemma 2.15 with the suitable modifications. □

Lemma 2.17. *Let (H, f) be a ternary semihypergroup. If A is a nonempty subset of a lateral hyperideal I of H such that $f(I, I, A, I, I) = f(I, A, I)$, then $f(I, A, I)$ is a lateral hyperideal of H.*

P r o o f. Let A be a nonempty subset of a lateral hyperideal I of H such that $f(I,I,A,I,I) = f(I,A,I)$. Then $f(H,I,H) \subseteq I$. Hence

$$f(H, f(I,A,I), H) = f(H, f(I,I,A,I,I), H) = f(f(H,I,I), A, f(I,I,H)) \\ \subseteq f(f(H,I,H), A, f(H,I,H)) \subseteq f(I,A,I).$$

Therefore $f(I,A,I)$ is a lateral hyperideal of H. □

3. Minimal lateral hyperideals of ternary semihypergroups

In this section, we give some properties of (0-) minimal left and lateral hyperideals of ternary semihypergroups and investigate the relationship between the (0-) minimal left and lateral hyperideals and the left (0-) simple and lateral (0-)simple ternary semihypergroups.

Definition 3.1. Let (H, f) be a ternary semihypergroup without zero. A lateral hyperideal I of H is called a *minimal lateral hyperideal* of H if there is no lateral hyperideal A of H such that $A \subset I$.

Equivalent definition. Let (H, f) be a ternary semihypergroup without zero. A lateral hyperideal I of H is called a *minimal lateral hyperideal* of H if for every lateral hyperideal A of H such that $A \subseteq I$, we have $A = I$.

The lateral hyperideal I_1 of the Example 1.12 is a minimal lateral hyperideal of H.

Definition 3.2. Let (H, f) be a ternary semihypergroup without zero. A left hyperideal L of H is called a *minimal left hyperideal* of H if there is no left hyperideal A of H such that $A \subset L$.

Equivalent definition. Let (H, f) be a ternary semihypergroup without zero. A left hyperideal L of H is called a *minimal left hyperideal* of H if for every left hyperideal A of H such that $A \subseteq L$, we have $A = L$.

Definition 3.3. Let (H, f) be a ternary semihypergroup with zero. A nonzero lateral hyperideal I of H is called a 0-*minimal lateral hyperideal* of H if there is no nonzero lateral hyperideal A of H such that $A \subset I$.

Equivalent definition. Let (H, f) be a ternary semihypergroup with zero. A nonzero lateral hyperideal I of H is called a 0-*minimal lateral hyperideal* of H if for every nonzero lateral hyperideal A of H such that $A \subseteq I$, we have $A = I$.

Equivalent definition. Let (H, f) be a ternary semihypergroup with zero. A nonzero lateral hyperideal I of H is called a 0-*minimal lateral hyperideal* of H if for every lateral hyperideal A of H such that $A \subseteq I$, we have $A = \{0\}$.

Definition 3.4. Let (H, f) be a ternary semihypergroup with zero. A nonzero left hyperideal L of H is called a 0-*minimal left hyperideal* of H if there is no nonzero left hyperideal A of H such that $A \subset L$.

Equivalent definition. Let (H, f) be a ternary semihypergroup with zero. A nonzero left hyperideal L of H is called a 0-*minimal left hyperideal* of H if for every nonzero left hyperideal A of H such that $A \subseteq L$, we have $A = L$.

Equivalent definition. Let (H, f) be a ternary semihypergroup with zero. A nonzero left hyperideal L of H is called a 0-*minimal left hyperideal* of H if for every left hyperideal A of H such that $A \subseteq L$, we have $A = \{0\}$.

Theorem 3.5. *Let (H, f) be a ternary semihypergroup without zero and I be a lateral hyperideal of H. Then the following statements hold true:*

1. *If I is a minimal lateral hyperideal without zero of H, then either there exists a lateral hyperideal A of I such that $f(I, I, A, I, I) \neq f(I, A, I)$ or I is lateral simple.*
2. *If I is lateral simple, then I is a minimal lateral hyperideal of H.*
3. *If I is a minimal lateral hyperideal with zero of H, then either there exists a nonzero lateral hyperideal A of I such that $f(I, I, A, I, I) \neq f(I, A, I)$ or I is lateral 0-simple.*

Proof.

(1). Let assume that I is a minimal lateral hyperideal without zero of H and $f(I, I, C, I, I) = f(I, C, I)$ for all lateral hyperideals C of I. Let A be a lateral hyperideal of I. Then we have $f(I, I, A, I, I) = f(I, A, I) \subseteq A \subseteq I$. By Lemma 2.17, we have $f(I, A, I)$ is a lateral hyperideal of H. Since I is a minimal lateral hyperideal of H, $f(I, A, I) = I$. Therefore, $A = I$. It follows that I is lateral simple.

(2). Let assume that I is lateral simple and A be a lateral hyperideal of H such that $A \subseteq I$. Then we have $A \cap I \neq \emptyset$. By Lemma 2.15(1), it follows that $I \subseteq A$. Hence $A = I$, so I is a minimal lateral hyperideal of H.

(3). The proof is similar to the proof of (1). □

Theorem 3.6. *Let (H, f) be a ternary semihypergroup without zero and L be a left hyperideal of H. Then the following statements hold true:*

1. *If L is a minimal left hyperideal without zero of H if and only if L is left simple.*
2. *If L is a minimal left hyperideal with zero of H, then L is left 0-simple.*

Proof. The proof is similar to the proof of Theorem 3.5. □

Theorem 3.7. *Let (H, f) be a ternary semihypergroup with zero and I be a nonzero lateral hyperideal of H. Then the following statements hold true.*

(1) *If I is a 0-minimal lateral hyperideal of H, then either there exists a nonzero lateral hyperideal A of I such that $f(I, I, A, I, I) \neq f(I, A, I) = \{0\}$ or I is lateral 0-simple.*

(2) *If I is lateral 0-simple, then I is a 0-minimal lateral hyperideal of H.*

Proof. The proof is similar to the proof of Theorem 3.5(1) and the Lemma 2.15(2). □

Theorem 3.8. *Let (H, f) be a ternary semihypergroup with zero and L be a nonzero left hyperideal of H. Then the following statements hold true.*

(1) *If L is a 0-minimal left hyperideal of H, then either there exists a nonzero left hyperideal A of I such that $f(L, L, A) = \{0\}$ or L is left 0-simple.*

(2) *If L is left 0-simple, then L is a 0-minimal left hyperideal of H.*

Proof. The proof is similar to the proof of above theorems. □

Theorem 3.9. *Let (H, f) be a ternary semihypergroup without zero having proper lateral hyperideals. Then every proper lateral hyperideal of H is minimal if and only if H contains exactly one proper lateral hyperideal or H contains exactly two proper lateral hyperideals I_1, I_2, such that $I_1 \cup I_2 = H$ and $I_1 \cap I_2 = \emptyset$.*

Proof.

$\Longrightarrow$: Suppose that every proper lateral hyperideal of H is minimal. Let I be a proper lateral hyperideal of H. Then I is a minimal lateral hyperideal of H. We have the following two cases:

Case 1. For all $a \in H \setminus I$, $H = \langle a \rangle_m$.

If P is also a proper lateral hyperideal of H and $P \neq I$, then since I is a minimal lateral hyperideal, we have $P \setminus I \neq \emptyset$. Thus there exists $a \in P \setminus I \subseteq H \setminus I$. Hence $H = \langle a \rangle_m \subseteq P \subseteq H$, so $P = H$. It is impossible. So we have $P = I$ and in this case I is the unique proper lateral hyperideal of H.

Case 2. There exists $a \in H \setminus I$ such that $H \neq \langle a \rangle_m$.

We have $\langle a \rangle_m \neq I$ and $\langle a \rangle_m$ is a minimal lateral hyperideal of H. Lemma 2.13 implies $\langle a \rangle_m \cup I$ is a lateral hyperideal of H. Since $I \subset \langle a \rangle_m \cup I$, by hypothesis we obtain $\langle a \rangle_m \cup I = H$. Since $\langle a \rangle_m \cap I \subset \langle a \rangle_m$ and $\langle a \rangle_m$ is a minimal lateral hyperideal of H, we get $\langle a \rangle_m \cap I = \emptyset$. Let P be an arbitrary proper lateral hyperideal of H. Then P is a minimal lateral hyperideal of H. We have $P = P \cap H = P \cap (\langle a \rangle_m \cup I) = (P \cap \langle a \rangle_m) \cup (P \cap I)$. If $P \cap I \neq \emptyset$, then since P and $\langle a \rangle_m$ are minimal lateral hyperideals of H, we have $P = \langle a \rangle_m$. In this case, H contains exactly two proper lateral hyperideals I and $\langle a \rangle_m$ such that $\langle a \rangle_m \cup I = H$ and $\langle a \rangle_m \cap I = \emptyset$.

$\Longleftarrow$: It is obvious. □

Theorem 3.10. *Let (H, f) be a ternary semihypergroup without zero having proper left hyperideals. Then every proper left hyperideal of H is minimal if and only if H contains exactly one proper left hyperideal or H contains exactly two proper left hyperideals L_1, L_2, such that $L_1 \cup L_2 = H$ and $L_1 \cap L_2 = \emptyset$.*

P r o o f. The proof is similar to the proof of the above theorem. □

Theorem 3.11. *Let (H, f) be a ternary semihypergroup with zero having nonzero proper lateral hyperideals. Then every nonzero proper lateral hyperideal of H is 0-minimal if and only if H contains exactly one nonzero proper lateral hyperideal or H contains exactly two nonzero proper lateral hyperideals I_1, I_2 such that $I_1 \cup I_2 = H$ and $I_1 \cap I_2 = \{0\}$.*

P r o o f. The proof is the same to the proof of Theorem 3.9. □

Theorem 3.12. *Let (H, f) be a ternary semihypergroup with zero having nonzero proper left hyperideals. Then every nonzero proper left hyperideal of H is 0-minimal if and only if H contains exactly one nonzero proper left hyperideal or H contains exactly two nonzero proper left hyperideals L_1, L_2 such that $L_1 \cup L_2 = H$ and $L_1 \cap L_2 = \{0\}$.*

P r o o f. The proof is the same to the proof of the above theorem. □

4. Maximal lateral hyperideals of ternary semihypergroups

In this section, we give some properties of maximal left and lateral hyperideals of ternary semihypergroups and investigate the relationship between the maximal left and lateral hyperideals and the union of all (nonzero) proper left and lateral hyperideals in ternary semihypergroups.

Definition 4.1. Let (H, f) be a ternary semihypergroup. A lateral hyperideal I of H is called a *maximal lateral hyperideal* of H if for every lateral hyperideal A of H such that $I \subset A$, we have $A = H$.

Equivalent definition. Let (H, f) be a ternary semihypergroup. A lateral hyperideal I of H is called a *maximal lateral hyperideal* of H if for every proper lateral hyperideal A of H such that $I \subseteq A$, we have $A = I$.

The lateral hyperideal I_2 of the Example 1.12 is a maximal lateral hyperideal of H.

Definition 4.2. Let (H, f) be a ternary semihypergroup. A left hyperideal L of H is called a *maximal left hyperideal* of H if for every left hyperideal A of H such that $I \subset A$, we have $A = H$.

Equivalent definition. Let (H, f) be a ternary semihypergroup. A left hyperideal I of H is called a *maximal left hyperideal* of H if for every proper left hyperideal A of H such that $A \subseteq I$, we have $A = I$.

Theorem 4.3. *Let (H, f) be a ternary semihypergroup without zero having proper lateral hyperideals. Then every proper lateral hyperideal of H is maximal if and only if H contains exactly one proper lateral hyperideal or H contains exactly two proper lateral hyperideals I_1, I_2 such that $I_1 \cup I_2 = H$ and $I_1 \cap I_2 = \emptyset$.*

Proof.

$\Longrightarrow$: Let assume that every proper lateral hyperideal of H is maximal. Let I be a proper hyperideal of H. Then I is maximal lateral hyperideal of H. We have the following two cases:

Case 1. For all $a \in H \setminus I$, $H = \langle a \rangle_m$.

If P is also a proper lateral hyperideal of H and $P \neq I$, then P is a maximal lateral hyperideal of H. It follows that $P \setminus I \neq \emptyset$, so there exists $a \in P \setminus I \subseteq H \setminus I$. Thus $H = \langle a \rangle_m \subseteq P \subseteq H$, so $P = H$. It is impossible. So we have $P = I$ and in this case I is the unique proper lateral hyperideal of H.

Case 2. There exists $a \in H \setminus I$ such that $H \neq \langle a \rangle_m$.

We have $\langle a \rangle_m \neq I$ and $\langle a \rangle_m$ is a maximal lateral hyperideal of H. Lemma 2.13 implies $\langle a \rangle_m \cup I$ is a lateral hyperideal of H. Since $I \subset \langle a \rangle_m \cup I$ and I is a maximal lateral hyperideal of H, we have $\langle a \rangle_m \cup I = H$. Since $\langle a \rangle_m \cap I \subset \langle a \rangle_m$ and by hypothesis, we obtain $\langle a \rangle_m \cap I = \emptyset$. Let P be an arbitrary proper lateral hyperideal of H. Then P is a maximal lateral hyperideal of H. We have $P = P \cap H = P \cap (\langle a \rangle_m \cup I) = (P \cap \langle a \rangle_m) \cup (P \cap I)$. If $P \cap I \neq \emptyset$, then since $P \cap \langle a \rangle_m$ and $\langle a \rangle_m$ are maximal lateral hyperideals of H, we have $P = I \, \langle a \rangle_m$. In this case, H contains exactly two proper lateral hyperideals I and $\langle a \rangle_m$ such that $\langle a \rangle_m \cup I = H$ and $\langle a \rangle_m \cap I = \emptyset$.

$\Longleftarrow$: It is obvious. □

Theorem 4.4. *Let (H, f) be a ternary semihypergroup without zero having proper left hyperideals. Then every proper left hyperideal of H is maximal if and only if H contains exactly one proper left hyperideal or H contains exactly two proper left hyperideals L_1, L_2 such that $L_1 \cup L_2 = H$ and $L_1 \cap L_2 = \emptyset$.*

Proof. The proof is similar to the proof of the above theorem. □

Theorem 4.5. *Let (H, f) be a ternary semihypergroup with zero having nonzero proper lateral hyperideals. Then every nonzero proper lateral hyperideal of H is maximal if and only if H contains exactly one nonzero proper lateral hyperideal or H contains exactly two nonzero proper lateral hyperideals I_1, I_2 such that $I_1 \cup I_2 = H$ and $I_1 \cap I_2 = \{0\}$.*

Proof. The proof is the same to the proof of Theorem 4.3. □

Theorem 4.6. *Let (H, f) be a ternary semihypergroup with zero having nonzero proper left hyperideals. Then every nonzero proper left hyperideal of H is maximal if and only if H contains exactly one nonzero proper left hyperideal or H contains exactly two nonzero proper left hyperideals L_1, L_2 such that $L_1 \cup L_2 = H$ and $L_1 \cap L_2 = \{0\}$.*

P r o o f. The proof is the same to the proof of the above theorem. □

Theorem 4.7. *Let (H, f) be a ternary semihypergroup. A proper lateral hyperideal I of H is maximal if and only if*

(1) *$H \setminus I = \{a\}$ and $f(H, a, H) \subseteq I$ for some $a \in H$ or*

(2) *$H \setminus I \subseteq f(H, H, a, H, H) \cup f(H, a, H)$ for all $a \in H \setminus I$.*

P r o o f.

$\Longrightarrow$: Let assume that I is a maximal lateral hyperideal of H. Then we have the two following cases:
Case 1. There exists $a \in H \setminus I$ such that $f(H, H, a, H, H) \cup f(H, a, H) \subseteq I$. We have $f(H, a, H) \subseteq I$. Lemma 2.5 implies $I \cup \{a\} = (I \cup f(H, H, a, H, H) \cup f(H, a, H)) \cup \{a\} = I \cup (f(H, H, a, H, H) \cup f(H, a, H) \cup \{a\}) = I \cup \langle a \rangle_m$. Thus since $I \cup \langle a \rangle_m$ is a lateral hyperideal of H, $I \cup \{a\}$ is a lateral hyperideal of H. Since I is a maximal lateral hyperideal of H and $I \subset I \cup \{a\}$, we have $I \cup \{a\} = H$. Hence $H \setminus I = \{a\}$.
Case 2. For all $a \in H \setminus I$, $f(H, H, a, H, H) \cup f(H, a, H) \nsubseteq I$. I $a \in H \setminus I$, then $f(H, H, a, H, H) \cup f(H, a, H) \nsubseteq I$ and $f(H, H, a, H, H) \cup f(H, a, H)$ is a lateral hyperideal of H by Lemma 2.7. Lemma 2.13 implies $I \cup f(H, H, a, H, H) \cup f(H, a, H)$ is a lateral hyperideal of H and $I \subset I \cup f(H, H, a, H, H) \cup f(H, a, H)$. Since I is a maximal lateral hyperideal of H, $I \cup f(H, H, a, H, H) \cup f(H, a, H) = H$. Hence for all $a \in H \setminus I$, we have that $H \setminus I \subseteq f(H, H, a, H, H) \cup f(H, a, H)$.

$\Longleftarrow$: Let P be a lateral hyperideal of H such that $I \subset P$. Then $P \setminus I \neq \emptyset$. If $H \setminus I = \{a\}$ and $f(H, a, H) \subseteq I$ for some $a \in H$, then $P \setminus I \subseteq H \setminus I = \{a\}$. Thus $P \setminus I = \{a\}$ and so $P = I \cup \{a\} = H$. Hence I is a maximal lateral hyperideal of H. If $H \setminus I \subseteq f(H, H, a, H, H) \cup f(H, a, H)$ for all $a \in H \setminus I$, then $H \setminus I \subseteq f(H, H, x, H, H) \cup f(H, x, H) \subseteq f(H, H, P, H, H) \cup f(H, P, H) \subseteq P$ for all $x \in P \setminus I$. Hence $H = (H \setminus I) \cup I \subseteq P \subseteq H$ and so $P = H$. Therefore I is a maximal lateral hyperideal of H. □

Let (H, f) be a ternary semihypergroup. Let $\mathcal{U}$ and $\mathcal{V}$ denote the union of all nonzero proper lateral hyperideals of H and the union of all nonzero proper left hyperideals of H respectively if H is a ternary semihypergroup with zero and let $\mathcal{U}$ and $\mathcal{V}$ denote the union of all proper lateral hyperideals of H and the union of all proper left hyperideals of H respectively if H is ternary semihypergroup without zero. It can be easily verified the following lemmas.

Lemma 4.8. *Let (H, f) be a ternary semihypergroup. Then $\mathcal{U} = H$ if and only if $\langle a\rangle_m \neq H$ for all $a \in H$.*

Lemma 4.9. *Let (H, f) be a ternary semihypergroup. Then $\mathcal{V} = H$ if and only if $\langle a\rangle_l \neq H$ for all $a \in H$.*

Theorem 4.10. *Let (H, f) be a ternary semihypergroup without zero. Then only one of the following statements is satisfied:*

(1) *H is lateral simple*

(2) *For all $a \in H$, $\langle a\rangle_m \neq H$.*

(3) *There exists $a \in H$ such that $\langle a\rangle_m = H$, $a \notin f(H, H, a, H, H) \cup f(H, a, H)$, $f(H, a, H) \subseteq \mathcal{U} = H \setminus \{a\}$ and $\mathcal{U}$ is the unique maximal lateral hyperideal of H.*

(4) *$H \setminus \mathcal{U} = \{x \in H : f(H, H, x, H, H) \cup f(H, x, H) = H\}$ and $\mathcal{U}$ is the unique maximal lateral hyperideal of H.*

P r o o f. Let assume that H is not lateral simple. Then there exists a proper lateral hyperideal of H. So $\mathcal{U}$ is a lateral hyperideal of H. We have the following two cases:

Case 1. $\mathcal{U} = H$.

Lemma 4.8 implies for all $a \in H$, $\langle a\rangle_m \neq H$. So the statement (2) is satisfied.

Case 2. $\mathcal{U} \neq H$.

We have $\mathcal{U}$ is a maximal lateral hyperideal of H. Assume that I is a maximal lateral hyperideal of H. Then since I is a proper lateral hyperideal of H, we have $I \subseteq \mathcal{U} \subset H$. Since I is a maximal lateral hyperideal of H, we have $I = \mathcal{U}$. Hence $\mathcal{U}$ is the unique maximal lateral hyperideal of H. Theorem 4.7 implies

(a) $H \setminus \mathcal{U} = \{a\}$ and $f(H, a, H) \subseteq \mathcal{U}$ for some $a \in H$ or

(b) For all $a \in H \setminus \mathcal{U}$, $H \setminus \mathcal{U} \subseteq f(H, H, a, H, H) \cup f(H, a, H)$.

Suppose that $H \setminus \mathcal{U} = \{a\}$ and $f(H, a, H) \subseteq \mathcal{U}$ for some $a \in H$. Then $f(H, a, H) \subseteq \mathcal{U} = H \setminus \{a\}$. Since $a \notin \mathcal{U}$, we have $\langle a\rangle_m = H$. If $a \in f(H, H, a, H, H) \cup f(H, a, H)$, then $\{a\} \subseteq f(H, H, a, H, H) \cup f(H, a, H)$. Lemma 2.5 implies $H = \langle a\rangle_m = f(H, H, a, H, H) \cup f(H, a, H) \cup \{a\} = f(H, H, a, H, H) \cup f(H, a, H) \subseteq f(H, \mathcal{U}, H) \cup \mathcal{U} = \mathcal{U} \subseteq H$. Thus we have $H = \mathcal{U}$. It is impossible. Hence $a \notin f(H, H, a, H, H) \cup f(H, a, H)$ and so the statement (3) is satisfied.

Let suppose now that for all $a \in H \setminus \mathcal{U}$, $H \setminus \mathcal{U} \subseteq f(H, H, a, H, H) \cup f(H, a, H)$. Let $x \in H \setminus \mathcal{U}$. Then $x \in f(h, H, x, H, H) \cup f(H, x, H)$. So $\{x\} \subseteq f(H, H, x, H, H) \cup f(H, x, H)$. Lemma 2.5 implies $\langle x\rangle_m = f(H, H, x, H, H) \cup f(H, x, H) \cup \{x\} = f(H, H, x, H, H) \cup f(H, x, H)$. Since $x \notin \mathcal{U}$, we have $\langle x\rangle_m = H$. Hence $H = \langle x\rangle_m = f(H, H, x, H, H) \cup f(H, x, H)$. Conversely, let $x \in H$ such that $f(H, H, x, H, H) \cup f(H, x, H) = H$. If $x \in \mathcal{U}$, then $\langle x\rangle_m \subseteq \mathcal{U} \subset H$. Lemma 2.5 implies $\langle x\rangle_m = f(H, H, x, H, H) \cup f(H, x, H) \cup \{x\} = H \cup \{x\} = H$. It is impossible. So we have $x \in H \setminus \mathcal{U}$. Hence we have that $H \setminus \mathcal{U} = \big\{x \in H : f(H, H, x, H, H) \cup f(H, x, H) = H\big\}$ and so, the statement (4) is satisfied. This completes the proof. □

Theorem 4.11. *Let (H, f) be a ternary semihypergroup without zero. Then only one of the following statements is satisfied:*

(1) *H is left simple*

(2) *For all $a \in H$, $\langle a\rangle_l \neq H$.*

(3) *There exists $a \in H$ such that $\langle a\rangle_l = H$, $a \notin f(H, H, a)$, $f(H, a, H) \subseteq \mathcal{V} = H \setminus \{a\}$ and $\mathcal{V}$ is the unique maximal left hyperideal of H.*

(4) *$H \setminus \mathcal{V} = \big\{x \in H : f(H, H, x) = H\big\}$ and $\mathcal{V}$ is the unique maximal left hyperideal of H.*

P r o o f. The proof is similar to the proof of the above theorem. □

Theorem 4.12. *Let (H, f) be a ternary semihypergroup with zero and $f(H, H, H) \neq \{0\}$. Then only one of the following statements is satisfied:*

(1) *H is lateral 0-simple.*

(2) *For all $a \in H$, $\langle a\rangle_m \neq H$.*

(3) *There exists $a \in H$ such that $\langle a\rangle_m = H$, $a \notin f(H, H, a, H, H) \cup f(H, a, H)$, $f(H, a, H) \subseteq \mathcal{U} = H \setminus \{a\}$ and $\mathcal{U}$ is the unique maximal lateral hyperideal of H.*

(4) *$H \setminus \mathcal{U} = \big\{x \in H : f(H, H, x, H, H) \cup f(H, x, H) = H\big\}$ and $\mathcal{U}$ is the unique maximal lateral hyperideal of H.*

P r o o f. The proof is the same to the proof of Theorem 4.10. □

Theorem 4.13. *Let (H, f) be a ternary semihypergroup with zero and $f(H, H, H) \neq \{0\}$. Then only one of the following statements is satisfied:*

(1) *H is left 0-simple.*

(2) *For all $a \in H$, $\langle a\rangle_l \neq H$.*

(3) *There exists $a \in H$ such that $\langle a\rangle_l = H$, $a \notin f(H, H, a)$, $f(H, a, H) \subseteq \mathcal{V} = H \setminus \{a\}$ and $\mathcal{V}$ is the unique maximal left hyperideal of H.*

(4) *$H \setminus \mathcal{V} = \big\{x \in H : f(H, H, x) = H\big\}$ and $\mathcal{V}$ is the unique maximal left hyperideal of H.*

P r o o f. The proof is the same to the proof of the above theorem. □

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