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ON COMPLETION IN THE CATEGORY SSNσFRM

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ABSTRACT. We introduce the category **SSNσFrm** of super strong nearness σ-frames and show the existence of a completion for a super strong nearness σ-frame unique up to isomorphism by the similar construction presented in [WALTERS-WAYLAND, J. L.: *Completeness and Nearly Fine Uniform Frames.* PhD Thesis, Univ. Catholique de Louvain, 1996] and [WALTERS-WAYLAND, J. L.: *A Shirota Theorem for frames*, Appl. Categ. Structures **7** (1999), 271–277]. Completion is also shown to be a coreflection in **SSNσFrm**. We also engage with the notion of total boundedness for nearness σ-frames and provide a characterization of the Samuel compactification of a nearness σ-frame alternative to the description in [NAIDOO, I.: *Samuel compactification and uniform coreflection of nearness σ-frames*, Czechoslovak Math. J. **56(131)** (2006), 1229–1241].

1. Introduction

In [16, 17] the category of uniform σ-frames and uniform σ-frame homomorphism **UσFrm** was introduced. Further, Walters-Wayland in [18, 19] provided an external construction of the completion of a uniform σ-frame via the corresponding construction of the completion of a uniform frame (*e.g.* [3, 6]). For a uniform σ-frame (L, μ) one passes over to the uniform frame $(\mathcal{H}L, \mathcal{H}\mu)$ of the σ-ideals of L with uniformity generated by $\{\downarrow A : A \in \mu\}$. The uniform completion $((C(\mathcal{H}L), C(\mathcal{H}\mu)), \gamma_{\mathcal{H}L})$ of the uniform frame $(\mathcal{H}L, \mathcal{H}\mu)$, the description of which is given below, is then considered. The uniform cozero part of this completion, $(\mathrm{Coz}_u(C(\mathcal{H}L), C(\mathcal{H}\mu)), \mathrm{Coz}_u\, \gamma_{\mathcal{H}L})$, is then realized as the completion of the uniform σ-frame (L, μ) unique up to isomorphism.

Banaschewski and Pultr in [6] describe the completion of a nearness frame in the following way. For a frame L, the frame $\mathfrak{D}L$ of all non-empty down-sets

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in L is considered in [6] with the frame homomorphism $\bigvee : \mathfrak{D}L \longrightarrow L$, given by join $U \rightsquigarrow \bigvee U$, and right Galois adjoint $\downarrow : L \longrightarrow \mathfrak{D}L$ taking each $x \in L$ to $\downarrow x$. For a nearness frame (L, μ), $x \in L$ and $A \in \mu$, $k(x) = \{y \in L : y \triangleleft x\}$ and $x \wedge A = \{x \wedge a : a \in A\}$. The system CL of all $U \in \mathfrak{D}L$ such that $x \in U$ whenever $k(x) \subseteq U$, and $x \in U$ whenever $x \wedge A \subseteq U$ for some $A \in \mu$, is a frame with intersection for meet and $\gamma_L : CL \longrightarrow L$ given by $\bigvee$ is a dense homomorphism with right Galois adjoint $\downarrow$. Then $\{\downarrow A : A \in \mu\}$ generates a nearness $C\mu$ on CL and $(CL, C\mu)$ is complete. Finally $((CL, C\mu), \gamma_L)$ is the completion of the nearness frame (L, μ) unique up to isomorphism. Further, if (L, μ) is a uniform frame, then $(CL, C\mu)$ is also its uniform completion.

The structure of a nearness on a σ-frame generalized the notion of uniformity in [10, 12] and the category $\mathbf{N}\sigma\mathbf{Frm}$ of nearness σ-frames and uniform σ-frame homomorphisms was introduced therein. The aim within this paper is to continue with the study on the subcategory $\mathbf{SN}\sigma\mathbf{Frm}$ of strong nearness σ-frames depicted in [13] where the notion of *complete* was investigated. Although the construction of the completion of an arbitrary nearness σ-frame remains elusive, we restrict our attention to those strong nearness σ-frames L for which the underlying frame of the completion of the nearness frame $\mathcal{H}L$ is Lindelöf. We will call such nearness σ-frames *super strong*. It is within this restrictive category (which contains uniform σ-frames) that an external construction of the completion is realized.

When a nearness is generated by its finite members the resulting structured frame is totally bounded or precompact. We carry forth this notion to σ-frames which realizes an alternate description of the totally bounded coreflection of the nearness frame of all σ-ideals. We also show that for σ-frames, nearness and uniformity coalesce under strongness and total boundedness. An internal description of the Samuel compactification $\mathfrak{NR}_\sigma L$ of a nearness σ-frame is provided in [10]. A recent discussion in [14] provides for an alternative description of the compact regular coreflection of a nearness σ-frame. We use this illustration to bring together the completion, total boundedness, (super) strongness and the Samuel compactification in the realm of σ-frames.

We provide the necessary background on σ-frames and frames and their required structures in the next section. Since any uniform σ-frame is a strong nearness σ-frame, the machinery in [18], with certain appropriate modifications in proof based on the theory governing nearness structures, will be shown to inadvertently provide for the construction of the completion in the larger category $\mathbf{SSN}\sigma\mathbf{Frm}$ of super strong nearness σ-frames. We continue in Section 3 by also showing that completion is a coreflection for super strong nearness σ-frames using the analogous result for nearness frames in [3].

The last part of the paper deals with totally bounded and uniformly normal nearness σ-frames. The Samuel compactification in [10] is revisited in Section 4 which culminates in amalgamating the concepts discussed in this paper.

2. Preliminaries

We recall some basic notions and facts about *σ-frames*. A *σ-frame* L is a lattice admitting countable suprema satisfying the join distributivity condition for any $x \in L$ and any countable $Y \subseteq L$, $x \wedge \bigvee Y = \bigvee_{y \in Y} (x \wedge y)$. The unit of L is denoted by 1 and the zero by 0. σ-frames are the objects of the category σ**Frm** whose morphisms are the lattice homomorphisms which preserve 1, 0, finite meets and countable joins.

L will always be a σ-frame (unless stated otherwise) and we will use the following terminologies and notions. We will denote countable (finite) subsets of L by using $\subseteq_c$ ($\subseteq_f$). For $x, y \in L$, we say that y is *rather below* x and write $y \prec x$ if there is $s \in L$ such that $y \wedge s = 0$ and $s \vee x = 1$. L is called a *regular* σ-frame if for each $x \in L$ there is $Y \subseteq_c \{y \in L : \ y \prec x\}$ such that $x = \bigvee Y$. **Reg**σ**Frm** is the corresponding category of regular σ-frames and σ-frame homomorphisms.

A *cover* on L is any member of the collection $\text{cov}(L) = \{A \subseteq_c L : \ \bigvee A = 1\}$. For $A, B \in \text{cov}(L)$, $A \wedge B = \{a \wedge B : \ a \in A \text{ and } b \in B\}$. We write $A \leq B$ if for each $a \in A$ there is $b \in B$ such that $a \leq b$ and we consequently say that A *refines* B. We also denote $A^* = AA = \{Ax : \ x \in A\}$. We then say that A *star-refines* B and write $A \leq^* B$ in case $A^* \leq B$. If $x \in L$ and $A \in \text{cov}(L)$, then $Ax = \bigvee \{a \in A : \ a \wedge x \neq 0\} \in L$ is the *star* of x relative to the cover A. If $\mu \subseteq \text{cov}(L)$ and $x, y \in L$ then $x \lhd_\mu y$ (or for brevity $x \lhd y$) will mean that $Ax \leq y$ for some cover $A \in \mu$. If $A, B \in \mu$, then $A \lhd_\mu B$ will mean that for each $a \in A$ there is $b \in B$ such that $a \lhd_\mu b$. A *nearness* on L is a filter of covers μ (relative to the refinement relation $\leq$) that is necessarily *admissible*, *i.e.* for each $x \in L$, $x = \bigvee Y$ where $Y \subseteq_c \{y \in L : \ y \lhd_\mu x\}$. The members of a nearness μ are called *uniform* covers. The pair (L, μ) is then called a *nearness σ-frame*. These are the objects in the category **N**σ**Frm** whose morphisms are the *uniform σ-frame homomorphisms*, the morphisms on the underlying σ-frames that preserve uniform covers. A nearness μ is *strong* if for each $B \in \mu$ there is $A \in \mu$ such that $A \lhd_\mu B$. **SN**σ**Frm** is the resulting subcategory of the strong nearness σ-frames and uniform σ-frame homomorphisms. A *uniformity* on L is a nearness $\mu \subseteq \text{cov}(L)$ such that each uniform cover has a uniform star refinement. **U**σ**Frm** is the resulting category of uniform σ-frames and uniform homomorphisms treated expansively in [16] and [17].

We will also consider the following categories and subcategories from which the above generalizations originated. **Frm** of *frames* and *frame* homomorphisms are treated in [8] and [15]. Frames are σ-frames that accept *arbitrary* suprema in which $\wedge$ distributes over all $\bigvee$. Their homomorphisms also preserve all joins. *Regular* frames are those frames in which each element is a join of elements rather below it and the resulting category is denoted by **RegFrm**. In a frame L, $\mathrm{cov}(L)$ includes any subset whose join is the unit. *Lindelöf* (*Compact*) frames are those frames in which each cover has a countable (finite) subcover. For any σ-frame L, an ideal I is a σ*-ideal* if $X \subseteq_c I$ implies that $\bigvee X \in I$. The collection of all σ-ideals of $L \in \sigma\mathbf{Frm}$ is denoted by $\mathcal{H}L$. $\mathcal{H}L$ is a Lindelöf frame and $\mathcal{H}L \in \mathbf{RegFrm}$ whenever $L \in \mathbf{Reg}\sigma\mathbf{Frm}$. Given any σ-frame homomorphism $h\colon L \longrightarrow M$, $\mathcal{H}h\colon \mathcal{H}L \longrightarrow \mathcal{H}M$ defines a frame homomorphism where $\mathcal{H}h(I) = \langle h(I)\rangle$ is the σ-ideal generated by $h(I)$. $\mathcal{H}$ is functorial and is left adjoint to the functor $\mathrm{Coz}\colon \mathbf{Frm} \longrightarrow \mathbf{Reg}\sigma\mathbf{Frm}$ with unit $\downarrow\colon L \longrightarrow \mathrm{Coz}(\mathcal{H}L)$ an isomorphism taking any $x \in L$ to the principal ideal generated by x, ${\downarrow}x = \{y \in L : \ y \le x\}$. The counit, $\bigvee\colon \mathcal{H}(\mathrm{Coz}\, L) \longrightarrow L$ is the join map, which is an isomorphism provided that L is regular and Lindelöf. For any (regular) frame L, $\mathrm{Coz}\, L$ is the (regular) σ-frame consisting of all *cozero* elements of L and $c \in \mathrm{Coz}\, L$ provided that $c = h((0,1])$ for some frame homomorphism $h\colon \mathfrak{O}[0,1] \longrightarrow L$ with $\mathfrak{O}[0,1]$ the frame of all open sets of the unit interval $[0,1]$. Important details on $\mathrm{Coz}\, L$ may be found in [4]. In [9] it is shown that $\mathcal{H}$ and Coz induce an equivalence between the categories **LRegFrm** (regular Lindelöf frames) and $\mathbf{Reg}\sigma\mathbf{Frm}$.

NFrm, the category of nearness frames and uniform homomorphisms, is studied in [2,6]. In [11], the category **TNFrm** of totally bounded nearness frames is introduced. The totally bounded coreflection of a nearness frame (L,μ) is shown to be the nearness frame (L,μ_*) where μ_* is the nearness generated by the finite members of μ. The coreflection map is the identity $\mathrm{id}_L\colon (L,\mu_*) \longrightarrow (L,\mu)$. **SNFrm** consists of the *strong* nearness frames *viz.* those nearness frames (L,μ) in which the cover $\check{A} = \{x \in L : \ x \lhd_\mu a \ \text{ for some } \ a \in A\}$ is uniform for each $A \in \mu$. A uniform homomorphism $h\colon (L,\mu) \longrightarrow (M,\nu)$ between nearness frames is a *surjection* if h is onto such that $\big\{h_*(A) : \ A \in \nu\big\} = h_*[\nu]$ generates the nearness μ on L. The *right adjoint* of the frame homomorphism $h\colon L \longrightarrow M$ is the meet-preserving map $h_*\colon M \longrightarrow L$ where $h(x) \le y$ if and only if $x \le h_*(y)$, given explicitly by $h_*(x) = \bigvee\big\{a \in L : \ h(a) \le x\big\}$. We always have that $\mathrm{id}_L \le h_*h$ whilst $hh_* = \mathrm{id}_M$ whenever h is onto.

The following lemma pertaining to the right adjoint, strongness and surjections will be used in Section 3, the first of which is a consequence of the observation in [2: pp. 2], the second is [2: Corollary 2] and the last is [13: Lemma 2.1]. We recall that a frame (or σ-frame) homomorphism $h\colon L \longrightarrow M$ is *dense* if $x = 0$ whenever $h(x) = 0$ (equivalently for frames, $h_*(0) = 0$), and h is *codense* if $x = 1$ whenever $h(x) = 1$. In **RegFrm** and **RegσFrm** dense homomorphisms are monic whilst codense ones are injective.

Lemma 2.1.

1. *Let* $(L, \mu) \in$ **SNFrm**. *If* $h\colon (L, \mu) \longrightarrow M$ *is a dense and onto frame homomorphism, then* $h_* h(\check{A}) \leq A$ *for each* $A \in \mu$.
2. *For any dense surjection* $h\colon (L, \mu) \longrightarrow (M, \nu)$*, if* (M, ν) *is strong then so is* (L, μ).
3. *If* $h\colon (L, \mu) \longrightarrow (M, \nu)$ *is a dense surjection in* **SNFrm**, *then* $h_*[\nu]$ *generates* μ *if and only if* $h[\mu] = \big\{h(U) : U \in \mu\big\}$ *generates* ν.

A nearness frame is *separable* if its nearness is generated by the countable uniform covers. The category **SepSLNFrm** consists of those nearness frames in which the nearness is separable and strong and where the underlying frame is Lindelöf. The adjunction described above between **LRegFrm** and **RegσFrm** extends to the structured nearness setting as described in [12] which we will require in the sequel.

The functors Coz and $\mathcal{H}$ induce an equivalence between the categories **SepSLNFrm** and **SNσFrm**. Given any $(L, \mu) \in$ **SepSLNFrm**,

$$\operatorname{Coz} \mu = \{A \in \mu : A \subseteq_c \operatorname{Coz} L\}$$

defines a strong nearness on $\operatorname{Coz} L$. For any $(L, \mu) \in$ **SNσFrm**,

$$\mathcal{H}\mu = \{\mathcal{A} \in \operatorname{cov} \mathcal{H}L : \downarrow A \leq \mathcal{A} \text{ for some } A \in \mu\}$$

with $\downarrow A = \{\downarrow a : a \in A\}$ defines a separable strong nearness on the Lindelöf frame $\mathcal{H}L$. The adjunctions are given by the join map $\varepsilon_L\colon (\mathcal{H} \operatorname{Coz} L, \mathcal{H} \operatorname{Coz} \mu) \longrightarrow (L, \mu)$ and $\eta_L\colon (L, \mu) \longrightarrow (\operatorname{Coz} \mathcal{H}L, \operatorname{Coz} \mathcal{H}\mu)$ given by $\downarrow$.

3. The completion in SSNσFrm

In this section we show the existence of a completion, unique up to isomorphism, for any super strong nearness σ-frame in the spirit of [18, 19]. A nearness σ-frame L is said to be *super strong* if L is strong and the completion of the nearness frame $\mathcal{H}L$, *viz.* $C(\mathcal{H}L)$, is Lindelöf. In [19] it is shown that the completion of every Lindelöf uniform frame is Lindelöf. Also, for a uniform σ-frame (L, μ), $\mathcal{H}L$ is a Lindelöf uniform frame, hence the underlying frame of its completion $C(\mathcal{H}L)$ is Lindelöf. Thus, a uniform σ-frame is indeed a super strong nearness σ-frame.

We recall that a σ-frame homomorphism $h\colon (L,\mu) \longrightarrow (M,\nu)$ between nearness σ-frames is a *surjection* if h is onto and $\{h(A) : \ A \in \mu\} = h[\mu]$ generates ν. (M,ν) is a *complete* nearness σ-frame if and only if every dense surjection $h\colon (L,\mu) \longrightarrow (M,\nu)$ is an isomorphism. A completion of a nearness σ-frame (M,ν) is a pair $((L,\mu),h)$ with (L,μ) a complete nearness σ-frame and $h\colon (L,\mu) \longrightarrow (M,\nu)$ a dense surjection.

Lemma 3.1. *If* $(L,\mu) \in \mathbf{SSN\sigma Frm}$*, then* $(C(\mathcal{H}L), C(\mathcal{H}\mu)) \in \mathbf{SepSLNFrm}$.

P r o o f. Let (L,μ) be a super strong nearness σ-frame. By [12: Lemma 3.5], the nearness frame $\mathcal{H}L$ is also strong. Since the completion map $\gamma_{\mathcal{H}L}\colon C(\mathcal{H}L) \longrightarrow \mathcal{H}L$ is a dense surjection, the completion $C(\mathcal{H}L)$ is a strong nearness frame by Lemma 2.1(2). Also, if $\mathcal{A}$ is a uniform cover of the completion $C(\mathcal{H}L)$ then $(\gamma_{\mathcal{H}L})_*\gamma_{\mathcal{H}L}(\check{\mathcal{A}}) \le \mathcal{A}$ by Lemma 2.1(1). Since $\gamma_{\mathcal{H}L}(\check{\mathcal{A}})$ is a uniform cover of the separable nearness frame $\mathcal{H}L$, we may find a countable $\mathcal{B} \in \mathcal{H}\mu$ such that $\mathcal{B} \le \gamma_{\mathcal{H}L}(\check{\mathcal{A}})$. Since $(\gamma_{\mathcal{H}L})_*[\mathcal{H}\mu]$ generates the nearness on $C(\mathcal{H}L)$, $(\gamma_{\mathcal{H}L})_*(\mathcal{B})$ is a countable uniform cover of $C(\mathcal{H}L)$ such that $(\gamma_{\mathcal{H}L})_*(\mathcal{B}) \le (\gamma_{\mathcal{H}L})_*\gamma_{\mathcal{H}L}(\check{\mathcal{A}}) \le \mathcal{A}$ showing that the nearness frame $C(\mathcal{H}L)$ is also separable. Since L is super strong, $C(\mathcal{H}L)$ is Lindelöf. □

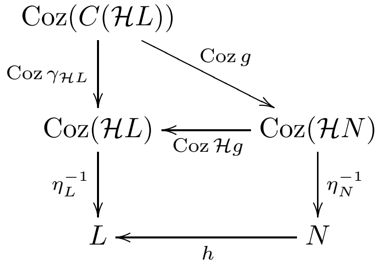
Theorem 3.1. $\big(\mathrm{Coz}(C(\mathcal{H}L)), \mathrm{Coz}(C(\mathcal{H}\mu)), \eta_L^{-1} \circ \mathrm{Coz}\,\gamma_{\mathcal{H}L}\big)$ *is the completion of the super strong nearness* σ*-frame* (L,μ)*, unique up to isomorphism.*

P r o o f. Let (L,μ) be a super strong nearness σ-frame. By the Lemma above, $C(\mathcal{H}L)$ is a complete separable strong Lindelöf nearness frame. Consequently, by [13: Lemma 3.4], $(\mathrm{Coz}(C(\mathcal{H}L)), \mathrm{Coz}(C(\mathcal{H}\mu))$ is a complete strong nearness σ-frame. Moreover, since $C(\mathcal{H}L)$ is Lindelöf, $\mathcal{H}\,\mathrm{Coz}(C(\mathcal{H}L)) \simeq C(\mathcal{H}L)$ so that $\mathrm{Coz}(C(\mathcal{H}L))$ is complete in $\mathbf{SSN\sigma Frm}$. Since the functor Coz preserves dense surjections ([13: Lemma 3.3]), $\mathrm{Coz}\,\gamma_{\mathcal{H}L}$ is a dense surjection between the strong nearness σ-frames $\mathrm{Coz}(C(\mathcal{H}L))$ and $\mathrm{Coz}\,\mathcal{H}L \simeq L$. Thus $\mathrm{Coz}(C(\mathcal{H}L))$ is indeed a completion of L with dense surjection $\eta_L^{-1} \circ \mathrm{Coz}\,\gamma_{\mathcal{H}L}\colon \mathrm{Coz}(C(\mathcal{H}L) \longrightarrow L$.

For the uniqueness of $\eta_L^{-1} \circ \mathrm{Coz}\,\gamma_{\mathcal{H}L}$, let $h\colon (N,\xi) \longrightarrow (L,\mu)$ be any completion of (L,μ) in $\mathbf{SSN\sigma Frm}$. Then $\mathcal{H}h\colon (\mathcal{H}N, \mathcal{H}\xi) \longrightarrow (\mathcal{H}L, \mathcal{H}\mu)$ is a dense surjection in $\mathbf{SepSLNFrm}$ by [13: Lemma 3.2]. Consequently, by [6: Proposition 8], there is a dense surjection $g\colon C(\mathcal{H}L) \longrightarrow \mathcal{H}N$ such that $\mathcal{H}h \circ g = \gamma_{\mathcal{H}L}$. Since $\mathcal{H}N$ is complete, g is an isomorphism. Since η is natural the following is a commutative diagram

$$\begin{array}{ccc} L & \xrightarrow{\ \eta_L\ } & \mathrm{Coz}(\mathcal{H}L) \\ {\scriptstyle h}\uparrow & & \uparrow{\scriptstyle \mathrm{Coz}\,\mathcal{H}h} \\ N & \xrightarrow[\ \eta_N\ ]{} & \mathrm{Coz}(\mathcal{H}N) \end{array}$$

and thus $\eta_L^{-1} \circ \operatorname{Coz} \mathcal{H}h = h \circ \eta_N^{-1}$. We then have the following



Then

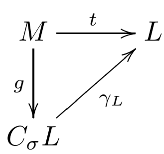
$$h \circ \eta_N^{-1} \circ \operatorname{Coz} g = \eta_L^{-1} \circ \operatorname{Coz} \mathcal{H}h \circ \operatorname{Coz} g = \eta_L^{-1} \circ \operatorname{Coz} \gamma_{\mathcal{H}L}.$$

Since g is an isomorphism, so is $\operatorname{Coz} g$. Consequently, $\bar{g} = \eta_N^{-1} \circ \operatorname{Coz} g$ is an isomorphism. Thus, we have shown that $\eta_L^{-1} \circ \operatorname{Coz} \gamma_{\mathcal{H}L}$ is unique up to isomorphism since given any completion $h\colon (N, \xi) \longrightarrow (L, \mu)$ we can find an isomorphism $\bar{g}\colon \operatorname{Coz}(C(\mathcal{H}L)) \longrightarrow N$ such that $h \circ \bar{g} = \eta_L^{-1} \circ \operatorname{Coz} \gamma_{\mathcal{H}L}$. □

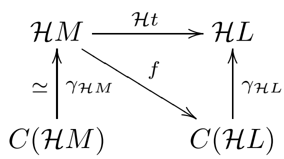
If (L, μ) is a uniform σ-frame, then $\big(\operatorname{Coz}(C(\mathcal{H}L)), \operatorname{Coz}(C(\mathcal{H}\mu)), \eta_L^{-1} \circ \operatorname{Coz} \gamma_{\mathcal{H}L}\big)$ is also its uniform completion. We will denote the completion of the super strong nearness σ-frame (L, μ) by $C_\sigma(L, \mu)$ (for brevity $C_\sigma L$) and γ_L will be the completion map $\eta_L^{-1} \circ \operatorname{Coz} \gamma_{\mathcal{H}L}$. In [3] completion is shown to be a coreflection in the larger category of strong nearness frames. This also transmits to the category of super strong nearness σ-frames as shown below.

Theorem 3.2. *Completion is a coreflection in* **SSNσFrm**.

Proof. Let (L, μ) and (M, ν) be super strong nearness σ-frames and let $t\colon (M, \nu) \longrightarrow (L, \mu)$ with (M, ν) complete. We must show that there is a unique $g\colon (M, \nu) \longrightarrow C_\sigma(L, \mu)$ such that $\gamma_L \circ g = t$, *i.e.* the following triangle commutes:



Consider the completion $C(\mathcal{H}L)$ of the nearness frame $\mathcal{H}L$ with the completion map $\gamma_{\mathcal{H}L}$. We then have the following diagram:



Since $\mathcal{H}$ preserves completeness, by [13: Lemma 3.5], $\mathcal{H}M$ is a complete separable strong Lindelöf nearness frame. Since $\mathcal{H}M$ is complete, $\gamma_{\mathcal{H}M}$ is an isomorphism. Since (L,μ) is a strong nearness σ-frame, [12: Lemma 3.5] implies that $\mathcal{H}L$ is a strong nearness frame. Since completion is a coreflection on strong nearness frames (see [2, 3]) there exists a unique $f\colon \mathcal{H}M \longrightarrow C(\mathcal{H}L)$ such that $\gamma_{\mathcal{H}L} \circ f = \mathcal{H}t$. Now consider

$$C(\mathcal{H}M) \xrightarrow[\simeq]{\gamma_{\mathcal{H}M}} \mathcal{H}M \xrightarrow{f} C(\mathcal{H}L)$$

Applying the functor Coz gives

$$\begin{array}{ccccc} C_\sigma M & \xrightarrow[\simeq]{\operatorname{Coz}\gamma_{\mathcal{H}M}} & \operatorname{Coz}(\mathcal{H}M) & \xrightarrow{\operatorname{Coz} f} & C_\sigma L \\ {\scriptstyle \gamma_M}\downarrow{\scriptstyle \simeq} & & & & \downarrow{\scriptstyle \gamma_L} \\ M & & \xrightarrow[t]{\qquad\qquad} & & L \end{array}$$

Note that since (M,ν) is complete γ_M and $\operatorname{Coz}\gamma_{\mathcal{H}M}$ are isomorphisms. We then have

$$\begin{aligned} \gamma_L \circ (\operatorname{Coz} f \circ \operatorname{Coz}\gamma_{\mathcal{H}M}) &= (\eta_L^{-1} \circ \operatorname{Coz}\gamma_{\mathcal{H}L}) \circ \operatorname{Coz} f \circ \operatorname{Coz}\gamma_{\mathcal{H}M} \\ &= \eta_L^{-1} \circ \operatorname{Coz}(\gamma_{\mathcal{H}L} \circ f \circ \gamma_{\mathcal{H}M}) \\ &= \eta_L^{-1} \circ \operatorname{Coz}(\mathcal{H}t \circ \gamma_{\mathcal{H}M}) \qquad (\because\ \gamma_{\mathcal{H}L} \circ f = \mathcal{H}t) \\ &= \eta_L^{-1} \circ \operatorname{Coz}\mathcal{H}t \circ \operatorname{Coz}\gamma_{\mathcal{H}M} \\ &= (\eta_L^{-1} \circ \operatorname{Coz}\mathcal{H}t \circ \eta_M) \circ (\eta_M^{-1} \circ \operatorname{Coz}\gamma_{\mathcal{H}M}) \\ &= (\eta_L^{-1} \circ \operatorname{Coz}\mathcal{H}t \circ \eta_M) \circ \gamma_M. \end{aligned}$$

By [12: Theorem 3.2], η_L is a natural isomorphism. Thus the following diagram is commutative

$$\begin{array}{ccc} (M,\nu) & \xrightarrow{\eta_M} & (\operatorname{Coz}\mathcal{H}M, \operatorname{Coz}\mathcal{H}\nu) \\ {\scriptstyle t}\downarrow & & \downarrow{\scriptstyle \operatorname{Coz}\mathcal{H}t} \\ (L,\mu) & \xrightarrow[\eta_L]{} & (\operatorname{Coz}\mathcal{H}L, \operatorname{Coz}\mathcal{H}\mu) \end{array}$$

so that $t = \eta_L^{-1} \circ \operatorname{Coz}\mathcal{H}t \circ \eta_M$. Let $g = (\operatorname{Coz} f \circ \operatorname{Coz}\gamma_{\mathcal{H}M}) \circ \gamma_M^{-1}$. We then have

$$\begin{aligned} \gamma_L \circ (\operatorname{Coz} f \circ \operatorname{Coz}\gamma_{\mathcal{H}M}) &= (\eta_L^{-1} \circ \operatorname{Coz}\mathcal{H}t \circ \eta_M) \circ \gamma_M \\ &= t \circ \gamma_M \\ \therefore\ \gamma_L \circ (\operatorname{Coz} f \circ \operatorname{Coz}\gamma_{\mathcal{H}M}) \circ \gamma_M^{-1} &= t \\ \text{i.e.}\quad \gamma_L \circ g &= t. \end{aligned}$$

Now if $g'\colon (M,\nu) \longrightarrow C_\sigma(L,\mu)$ such that $\gamma_L \circ g' = t$ then $\gamma_L \circ g = \gamma_L \circ g'$. Since γ_L is dense it is monic. Thus $g = g'$. Hence g is unique such that $\gamma_L \circ g = t$ proving our result. □

Corollary 3.2.1. *If* $f\colon (M,\nu) \longrightarrow (L,\mu)$ *is a uniform homomorphism in* **SSNσFrm**, *then there is a unique uniform homomorphism* $c\colon C_\sigma(M,\nu) \longrightarrow C_\sigma(L,\mu)$ *such that the following diagram commutes*

$$\begin{array}{ccc} C_\sigma(M,\nu) & \xrightarrow{\;c\;} & C_\sigma(L,\mu) \\ {\scriptstyle \gamma_M}\big\downarrow & & \big\downarrow{\scriptstyle \gamma_L} \\ (M,\nu) & \xrightarrow[\;f\;]{} & (L,\mu) \end{array}$$

P r o o f. Let $f\colon (M,\nu) \longrightarrow (L,\mu)$ be a uniform homomorphism between super strong nearness σ-frames. Since $f \circ \gamma_M\colon C_\sigma M \longrightarrow L$ with $C_\sigma M$ complete, by the previous theorem there is a unique $c\colon C_\sigma M \longrightarrow C_\sigma L$ such that $\gamma_L \circ c = f \circ \gamma_M$. ☐

We also observe that if $f\colon (M,\nu) \longrightarrow (L,\mu)$ is a surjection between super strong nearness σ-frames, then since the functor $\mathcal{H}$ preserves surjections $\mathcal{H}f\colon (\mathcal{H}M,\mathcal{H}\nu) \longrightarrow (\mathcal{H}L,\mathcal{H}\mu)$ is a surjection between strong nearness frames. Then, by [3: Corollary 6.1], $\mathcal{H}h$ lifts to a surjection g between the completions $C(\mathcal{H}M) \longrightarrow C(\mathcal{H}L)$. Following through with the functor Coz, the surjection f lifts to a surjection $c\colon C_\sigma M \longrightarrow C_\sigma L$. Since every uniform σ-frame is a super strong nearness σ-frames, the observation on lifts of surjections to completions and the last two results above are applicable in the category **UσFrm**.

4. The Samuel compactification and completion

In this concluding section we focus on the effects of total boundedness on structured σ-frames. We also revisit the Samuel compactification of a nearness σ-frame depicted in [10]. The formulation in [11] on the coreflective subcategory **TNFrm** of **NFrm** carries through to the σ-frame case with a minor modification to incorporate pseudocomplements since, in general, elements of σ-frames do not have pseudocomplements.

Let (L,μ) be a nearness σ-frame and consider the filter of covers generated by the finite uniform members $\mu_t = \{A \in \mu :\ B \le A \text{ for some finite } B \in \mu\}$. If $x \in L$ then the admissibility of μ gives $x = \bigvee Y$ for some $Y \subseteq_c \{y \in L :\ y \lhd_\mu x\}$. If $y \lhd_\mu x$, we may find $A \in \mu$ such that $Ay \le x$. Let $s = \bigvee\{a \in A :\ a \wedge y = 0\}$. Then $s \wedge y = 0$ and $s \vee x = 1$. Moreover, since $A \le \{s,x\}$, $\{s,x\} \in \mu$. Since $\{s,x\}y = x$, $y \lhd_{\mu_t} x$. Consequently, μ_t is admissible. Then (L,μ_t) is the totally bounded coreflection of (L,μ) with coreflection map, the identity, $\mathrm{id}_L\colon (L,\mu_t) \longrightarrow (L,\mu)$. We denote the resulting category of totally bounded nearness σ-frames by **TNσFrm**.

Lemma 4.1. *If* $(L,\mu) \in \mathbf{TN\sigma Frm}$*, then* $(\mathcal{H}L, \mathcal{H}\mu) \in \mathbf{TNFrm}$.

Proof. Suppose that (L,μ) is a totally bounded nearness σ-frame. Let $\mathcal{A} \in \mathcal{H}\mu$. Then there is $A \in \mu$ such that $\downarrow A \le \mathcal{A}$. Consequently, there is a finite $B \in \mu$ such that $B \le A$. Then $\downarrow B \le \mathcal{A}$ and $\downarrow B \in \mathcal{H}\mu$ is finite so that $(\mathcal{H}L, \mathcal{H}\mu)$ is a totally bounded nearness frame. □

The proof of [7: Lemma 3.3] holds verbatim with "nearness frame" replaced by "nearness σ-frame". We state this below for σ-frames.

Lemma 4.2. *If* $(L,\mu) \in \mathbf{TN\sigma Frm}$*, then* μ *is strong if and only if* $(L,\mu) \in \mathbf{U\sigma Frm}$.

The functors $\mathcal{H}$ and Coz may also be used to achieve the above result, albeit rather circuitously as follows. If $(L,\mu) \in \mathbf{N\sigma Frm}$ is strong and totally bounded, then $(\mathcal{H}L, \mathcal{H}\mu) \in \mathbf{NFrm}$ is also strong ([12: Lemma 3.5]) and totally bounded by Lemma 4.1, hence a (separable) uniform frame ([10: Theorem 6.1]). Then $(\mathrm{Coz}_u\,\mathcal{H}L, \mathrm{Coz}_u\,\mathcal{H}\mu) = (\mathrm{Coz}\,\mathcal{H}L, \mathrm{Coz}\,\mathcal{H}\mu)$ is a uniform σ-frame ([17: Proposition 4.5]). Since $(L,\mu) \simeq (\mathrm{Coz}\,\mathcal{H}L, \mathrm{Coz}\,\mathcal{H}\mu)$ ([12: Lemma 3.7]), $(L,\mu) \in \mathbf{U\sigma Frm}$.

We observe the following based on the discussion above. If (L,μ) is totally bounded and strong then so is the nearness frame $(\mathcal{H}L, \mathcal{H}\mu)$. Since $(\mathcal{H}L, \mathcal{H}\mu)$ is now a totally bounded uniform frame, by [2: Proposition 5], the completion $C(\mathcal{H}L)$ is compact (hence Lindelöf so that L is super strong) and thus $C\mathcal{H}\mu = \mathrm{cov}(\mathcal{H}L)$ *i.e.* the nearness on the completion is fine. Hence, $C_\sigma(L,\mu) = (\mathrm{Coz}(C\mathcal{H}L), \mathrm{Coz}(\mathrm{cov}(\mathcal{H}L))$ the completion of the strong totally bounded nearness σ-frame is compact. Now consider the completion $C_\sigma(L,\mu)$ of a super strong nearness σ-frame. If $C_\sigma(L,\mu)$ is compact, then unpacking the completion from Theorem 3.1 we see that $\mathrm{Coz}(C(\mathcal{H}L))$ is compact. Applying the functor $\mathcal{H}$ makes $\mathcal{H}\,\mathrm{Coz}(C(\mathcal{H}L))$ a compact nearness frame. However, $C(\mathcal{H}L) \simeq \mathcal{H}\,\mathrm{Coz}(C(\mathcal{H}L))$ since $C(\mathcal{H}L)$ is separable strong and Lindelöf ([12: Lemma 3.8]). Since $C(\mathcal{H}L)$ is the completion of $\mathcal{H}L$, compactness renders $\mathcal{H}L$ totally bounded and uniform ([2: Proposition 5]). Consequently, $(\mathrm{Coz}\,\mathcal{H}L, \mathrm{Coz}\,\mathcal{H}\mu) \simeq (L,\mu)$ is a totally bounded uniform σ-frame. We have thus shown the following.

Theorem 4.1. *For any super strong nearness* σ*-frame* L *its completion* $C_\sigma L$ *is compact if and only if* L *is a totally bounded uniform* σ*-frame.*

Lemma 4.1 above shows that the functor $\mathcal{H}$ preserves total boundedness. For any Lindelöf nearness frame (L,μ), $(\mathrm{Coz}\,L, \mathrm{Coz}\,\mu)$ is a nearness σ-frame by [12: Theorem 3.1]. We show next that the functor Coz also preserves total boundedness.

Lemma 4.3. *If* (L,μ) *is a totally bounded Lindelöf nearness frame, then* $(\mathrm{Coz}\,L, \mathrm{Coz}\,\mu)$ *is a totally bounded nearness* σ*-frame.*

P r o o f. Let $A \in \operatorname{Coz}\mu$, then $A \subseteq_c \operatorname{Coz} L$ and $A \in \mu$. We may then find a finite $B \in \mu$ such that $B \leq A$ since (L, μ) is totally bounded. Then for each $b \in B$ there is $a_b \in A$ such that $b \leq a_b$. Then $B \leq \overline{A} = \{a_b : \ b \in B\}$ so that $\overline{A} \in \mu$. Moreover, $\overline{A} \subseteq_f \operatorname{Coz} L$ so that $\overline{A} \in \operatorname{Coz}\mu$. Since $\overline{A} \leq A$, $(\operatorname{Coz} L, \operatorname{Coz}\mu)$ is a totally bounded nearness σ-frame. □

For any nearness σ-frame (L, μ) consider the totally bounded nearness frames $(\mathcal{H}L, \mathcal{H}\mu_t)$ and $(\mathcal{H}L, (\mathcal{H}\mu)_*)$. We show below that the totally bounded coreflection of the nearness frame $(\mathcal{H}L, \mathcal{H}\mu)$ is precisely $(\mathcal{H}L, \mathcal{H}\mu_t)$. For any nearness frame or nearness σ-frame L, let τL denote its totally bounded coreflection. Also, denote the nearness of a nearness frame (or nearness σ-frame) M by $\mathfrak{N}M$.

Theorem 4.2. *For any nearness σ-frame L, $\mathcal{H}(\tau L) = \tau(\mathcal{H}L)$.*

P r o o f. Clearly, the underlying frames of $\mathcal{H}(\tau L)$ and $\tau(\mathcal{H}L)$ coincide. So it remains to show that $\mathfrak{N}(\mathcal{H}(\tau L)) = \tau(\mathfrak{N}(\mathcal{H}L))$. Let $\mathcal{A} \in \mathfrak{N}(\mathcal{H}(\tau L))$. Then there exists a finite uniform cover A of L such that $\downarrow A \leq \mathcal{A}$. Now, $\downarrow A$ is a finite uniform cover of $\mathcal{H}L$, and hence a uniform cover of $\tau(\mathcal{H}L)$. Therefore $\mathcal{A} \in \tau(\mathfrak{N}(\mathcal{H}L))$, and hence $\mathfrak{N}(\mathcal{H}(\tau L)) \subseteq \tau(\mathfrak{N}(\mathcal{H}L))$.

On the other hand, let $\mathcal{B} \in \tau(\mathfrak{N}(\mathcal{H}L))$. Consider any finite uniform cover $\{J_1, \ldots, J_m\}$ of $\mathcal{H}L$ refining $\mathcal{B}$. Then pick a uniform cover B of L such that

$$\downarrow B \leq \{J_1, \ldots, J_m\}.$$

For each $k \in \{1, \ldots, m\}$, let

$$B^{(k)} = \{x \in B : \ \downarrow x \subseteq J_k\},$$

and put $b_k = \bigvee B^{(k)}$. Then the set $\overline{B} = \{b_1, \ldots, b_k\}$ is a uniform cover of L (it is refined by B), and so $\overline{B}$ is a uniform cover of τL such that $\downarrow \overline{B} \leq \mathcal{B}$. Thus, $\mathcal{B} \in \mathfrak{N}(\mathcal{H}(\tau L))$; establishing the other inclusion. □

A direct internal description of the compact regular coreflection of a nearness σ-frame is presented in [10]. For a nearness σ-frame (L, μ), the compact σ-frame $\mathfrak{N}\mathfrak{R}_\sigma L$ of all countably generated uniformly normally regular ideals is established as the Samuel compactification of (L, μ) with coreflection map $\varrho\colon \mathfrak{N}\mathfrak{R}_\sigma L \longrightarrow (L, \mu)$ given by join. An ideal J of L is uniformly normally regular if for each $x \in J$, $x \blacktriangleleft y$ for some $y \in J$ where $x \blacktriangleleft y$ means that there is a normal μ-cover A such that $Ax \leq y$. $A \in \mu$ is normal if there is a sequence $(A_n) \subseteq \mu$ such that $A = A_1$ and $A_{n+1} \leq^* A_n$ for each n. We now provide an external characterization of the Samuel compactification of a nearness σ-frame required for the purpose of our remaining results.

In [1], for a nearness frame (L, μ) its uniform coreflection $(\mathcal{U}L, \mathcal{U}\mu)$ is constructed with coreflection map given by the inclusion $j\colon \mathcal{U}L \longrightarrow L$. Also, the frame $\mathfrak{J}L$ of all ideals of L is considered and the subframe $\mathfrak{N}\mathfrak{R}L$ of all normally regular ideals is shown to be isomorphic to the Samuel compactification of the uniform coreflection of (L,μ) *i.e.* $\mathfrak{N}\mathfrak{R}L \simeq \mathfrak{R}\mathcal{U}L$ ([1: Theorem 3,3]). The description of the Samuel compactification $\mathfrak{R}M$ of a uniform frame M is given by Banaschewski and Pultr in [5]. Subsequent to this, [1] establishes $\mathfrak{R}\mathcal{U}L$ as the Samuel compactification of the nearness frame (L,μ). Now, for a nearness σ-frame (L,μ) we may consider the compact regular coreflection $\mathfrak{R}\mathcal{U}\mathcal{H}L$ of the nearness frame $(\mathcal{H}L,\mathcal{H}\mu)$ with coreflection map $\rho_{\mathcal{U}\mathcal{H}L}\colon \mathfrak{R}\mathcal{U}\mathcal{H}L \longrightarrow \mathcal{U}\mathcal{H}L$ given by join. Consequently, $\operatorname{Coz}\mathfrak{R}\mathcal{U}\mathcal{H}L$ is a compact nearness σ-frame. We can then show that $\operatorname{Coz}\mathfrak{R}\mathcal{U}\mathcal{H}L$ is the Samuel compactification of the nearness σ-frame (L,μ).

Theorem 4.3. *$\operatorname{Coz}\mathfrak{R}\mathcal{U}\mathcal{H}L \simeq \mathfrak{N}\mathfrak{R}_\sigma L$.*

P r o o f. It is shown in [11] that the Samuel compactification of a nearness frame is the same as the completion of the totally bounded coreflection of its uniform coreflection. Thus for the nearness frame $(\mathcal{H}L,\mathcal{H}\mu)$, $\mathfrak{N}\mathfrak{R}\mathcal{H}L \simeq C(\mathcal{U}\mathcal{H}L,(\mathcal{U}\mathcal{H}\mu)_*)$ with the following from [11] for the nearness frame of all σ-ideals of L

$$\begin{array}{ccccc}
C(\mathcal{U}\mathcal{H}L,(\mathcal{U}\mathcal{H}\mu)_*) & \simeq & \mathfrak{R}\mathcal{U}\mathcal{H}L & \simeq & \mathfrak{N}\mathfrak{R}\mathcal{H}L \\
\Big\downarrow{\scriptstyle \gamma_{\mathcal{U}\mathcal{H}L}} & & \Big\downarrow{\scriptstyle \vee} & & \Big\downarrow{\scriptstyle \rho_{\mathcal{H}L}} \\
(\mathcal{U}\mathcal{H}L,(\mathcal{U}\mathcal{H}\mu)_*) & \xrightarrow[\mathrm{id}_{\mathcal{U}\mathcal{H}L}]{} & (\mathcal{U}\mathcal{H}L,\mathcal{U}\mathcal{H}\mu) & \xrightarrow[j_{\mathcal{H}L}]{} & (\mathcal{H}L,\mathcal{H}\mu)
\end{array}$$

Now if (M,ν) is any compact nearness σ-frame and $h\colon (M,\nu) \longrightarrow (L,\mu)$ is uniform then $(\mathcal{H}M,\mathcal{H}\nu)$ is a compact nearness frame so that there is a unique uniform homomorphism $\xi\colon (\mathcal{H}M,\mathcal{H}\nu) \longrightarrow \mathfrak{N}\mathfrak{R}\mathcal{H}L$ such that $\rho_{\mathcal{H}L} \circ \xi = \mathcal{H}h$. Consequently, following the diagram below,

$$\begin{array}{ccc}
\operatorname{Coz}\mathfrak{N}\mathfrak{R}\mathcal{H}L & \xleftarrow{\operatorname{Coz}\xi} & \\
\Big\downarrow{\scriptstyle \operatorname{Coz}\rho_{\mathcal{H}L}} & & \\
(\operatorname{Coz}\mathcal{H}L,\operatorname{Coz}\mathcal{H}\mu) & \xleftarrow[\operatorname{Coz}\mathcal{H}h]{} & (\operatorname{Coz}\mathcal{H}M,\operatorname{Coz}\mathcal{H}\nu) \\
{\scriptstyle \eta_L}\Big\uparrow\Big\downarrow{\scriptstyle \eta_L^{-1}} & & \Big\uparrow{\scriptstyle \eta_M} \\
(L,\mu) & \xleftarrow[h]{} & (M,\nu)
\end{array}$$

$\eta_L^{-1} \circ \operatorname{Coz}\rho_{\mathcal{H}L} \circ \operatorname{Coz}\xi\eta_M = h$ with $\operatorname{Coz}\xi \circ \eta_M$ unique. Thus the uniform homomorphism $\eta_L^{-1} \circ \operatorname{Coz}\rho_{\mathcal{H}L}\colon \operatorname{Coz}\mathfrak{N}\mathfrak{R}\mathcal{H}L \longrightarrow (L,\mu)$ is universal with respect to homomorphisms from compact nearness σ-frames to (L,μ). We have thus established $\operatorname{Coz}\mathfrak{R}\mathcal{U}\mathcal{H}L \simeq \operatorname{Coz}\mathfrak{N}\mathfrak{R}\mathcal{H}L$ as the Samuel compactification of the nearness σ-frame (L,μ). Consequently, $\operatorname{Coz}\mathfrak{R}\mathcal{U}\mathcal{H}L \simeq \mathfrak{N}\mathfrak{R}_\sigma L$. □

We can now show the correlation between the Samuel compactification, total boundedness and the completion of a super strong nearness σ-frame. Our concluding result confirms that in the category of super strong nearness σ-frames, the Samuel compactification of a super strong nearness σ-frame with strong totally bounded coreflection is the same as the completion of its totally bounded coreflection. If μ and μ_t are strong we call (L, μ) a *uniformly normal* nearness σ-frame, the concept and terminology being a transition from [7].

THEOREM 4.4. *Let* $(L, \mu) \in \mathbf{SSN\sigma Frm}$ *be uniformly normal. Then* $\mathfrak{NR}_\sigma L \simeq C_\sigma(L, \mu_t)$.

Proof. If $(L, \mu) \in \mathbf{N\sigma Frm}$ is uniformly normal, then so is the nearness frame $(\mathcal{H}L, \mathcal{H}\mu)$ (Lemma 4.1, Theorem 4.2 and [12: Lemma 3.5]). Then $(\mathcal{H}L, (\mathcal{H}\mu)_*)$ is a totally bounded strong nearness frame, hence uniform. Then by [5: Proposition 3], $\mathfrak{NR}\mathcal{H}L \simeq C(\mathcal{H}L, (\mathcal{H}\mu)_*)$. Since $(\mathcal{H}L, (\mathcal{H}\mu)_*)$ and (L, μ_t) are uniform, $\operatorname{Coz}\mathfrak{NR}\mathcal{H}L \simeq \operatorname{Coz} C(\mathcal{H}L, (\mathcal{H}\mu)_*) = C_\sigma(L, \mu_t)$ (see [14] or [17: Proposition 4.7]). Hence $\mathfrak{NR}_\sigma L \simeq C_\sigma(L, \mu_t)$. □

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