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CROSSED PRODUCT OF C^*-ALGEBRAS BY HYPERGROUPS VIA GROUP COACTIONS

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ABSTRACT. We define the crossed product of a C^*-algebra by a hypergroup via a group coaction. We generalize the results on Hecke C^*-algebra crossed products to our setting.

1. Introduction

A discrete *Hecke pair* is a pair (G, H) where G is a discrete group and H is a subgroup. It is assumed that every double coset HxH contains just finitely many left cosets [2]. Covariant representations (*covariant pairs*) of Hecke pairs [2] and corresponding C^*-algebras [6] and crossed products [2] are studied. The idea behind a Hecke pair is that the group G acts on the homogeneous space G/H by translation and this action is used in the defining condition of covariant pairs.

The objective of this paper is to consider a more general situation where a discrete group G acts on a semi convo K and co-acts on a C^*-algebra B, and to define the crossed product of B and K (via G) in this situation. This is based on the concept of C^*-algebras of generalized Hecke pairs, introduced by the author in [1]. We show that each covariant representation in the sense of [1] could be integrated to a representation of the corresponding crossed product (Theorem 2.3). The cases of maximal coactions (Theorem 2.5) and dual coactions (Theorem 3.2) are dealt with separately.

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2. Covariant representations

Let G be a discrete group, B be a C^*-algebra, and

$$\delta\colon B \to B \otimes M(C^*(G))$$

be a coaction of G on B, where $M(C^*(G))$ is the multiplier algebra of $C^*(G)$. Let K be a discrete semi convo with the corresponding hypergroup $\bar{K}$ of quotients [1] and $\gamma_R\colon G \times K \to K$ be a right action of G on K [1]. Given $t \in G$ and $a \in K$, we write $a \cdot t$ for $\gamma_R(t)(a)$. We use $a \in K$ to denote the characteristic function χ_a of the singleton $\{a\}$ as an element in $c_0(K)$, and δ_a to denote the characteristic function χ_a as an element in $\ell^2(K)$.

Following Echterchoff and Quigg [4], let us consider the spectral subspaces

$$B_t = \big\{x \in B:\ \delta(x) = x \otimes t\big\} \qquad (t \in G),$$

where G is embedded into $M(C^*(G))$ via universal representation. The spectral subspaces form a Fell bundle $\mathfrak{B}$ over G, and the direct product $\mathfrak{B} \times K$ is a Fell bundle over the transformation groupoid $G \times K$. The $*$-algebra $\Gamma_c(\mathfrak{B} \times K)$ of compactly supported sections of the latter bundle could be identified with $span\big\{(x,a):\ x \in B,\ a \in K\big\}$. It has a universal enveloping C^*-algebra [7], which is denoted by $B \rtimes_{\delta,\gamma} K$ and is called the *crossed product* of B by K via the coaction of G.

A pair (π, ν), where $\pi\colon B \to B(\mathcal{H})$ is a non-degenerate $*$-representation of B and $\nu\colon c_0(K) \to B(\mathcal{H})$ is a $*$-representation of the commutative C^*-algebra $c_0(K)$, on the same Hilbert space, is called a *covariant representation* of the system $(B, K, G, \delta, \gamma)$ if

$$\pi(x_t)\nu(\delta_a) = \nu(\delta_{a\cdot t})\pi(x_t) \qquad (t \in G,\ \ a \in K,\ \ x_t \in B_t).$$

Every covariant representation (π, ν) could be integrated into a representation $\pi \times \nu$ of $B \rtimes_{\delta,\gamma} K$ such that

$$(\pi \times \nu)(x, a) = \pi(x)\nu(\delta_a) \qquad (x \in B,\ \ a \in K).$$

When δ is maximal, the enveloping C^*-algebra $C^*(\mathfrak{B})$ of $\Gamma_c(\mathfrak{B})$ is isomorphic to B [5, 4.2] and every representation of $B \rtimes_{\delta,\gamma} K$ comes from a covariant representation (Theorem 2.5 below and [4, 2.7]).

Following Kumjian [7], for a Fell Bundle $\mathfrak{D} = \{D_x\}$ over a discrete groupoid $\mathcal{G}$ with the $*$-algebra $\Gamma_c(\mathfrak{D})$ of finitely supported sections, a *homomorphism* of $\mathfrak{D}$ into a C^*-algebra C is a map $\phi\colon \mathfrak{D} \to C$ which is linear on each fiber, and multiplicative (whenever it makes sense), and preserves adjoints (in [7], the groupoid is r-discrete, and the homomorphisms are also assumed to be continuous in an appropriate sense). A *representation* of $\mathfrak{D}$ on a Hilbert space $\mathcal{H}$ is a homomorphism $\pi\colon \mathfrak{D} \to B(\mathcal{H})$ which is called *non-degenerate* if $span(\pi(\mathfrak{D})\mathcal{H})$ is dense in $\mathcal{H}$. A C^*-algbera B is said to be the enveloping C^*-algebra of an $*$-algebra

B_0 if the supremum of the C^*-seminorms on B_0 is finite and B is the Hausdorff completion of B_0 in this largest C^*-seminorm (which is a norm).

Lemma 2.1. ([4, 2.1, 2.2]) *If $\mathfrak{D}$ is a Fell bundle over an discrete groupoid $\mathcal{G}$, then the $*$-algebra $\Gamma_c(\mathfrak{D})$ of compactly supported sections has an enveloping C^*-algebra $C^*(\mathfrak{D})$. The assignment $\Pi \mapsto \Pi|_{\mathfrak{D}}$ gives a one-one correspondence between homomorphisms of $C^*(\mathfrak{D})$ and homomorphisms of $\mathfrak{D}$, which sends non-degenerate representations $C^*(\mathfrak{D})$ to non-degenerate representations of $\mathfrak{D}$.*

Going back to our setup, we consider the case where $\mathcal{G} = G \times K$ and $\mathfrak{D} = \mathfrak{B} \times K$. We assume that $G \cdot e = K$ and take a minimal set $J \subseteq G$ of representatives such that each $a \in K$ is of the form $t \cdot e$, for a unique $t \in J$. Given $a \in K$, consider the set $J_a = \{t \in J : \ e \cdot t = a\}$, where e is the identity of K [1]. We assume that J_a is finite for each $a \in K$. It is easy to see that $t \cdot Ja = J_{a \cdot t}$, for each $t \in G$, $a \in K$.

Lemma 2.2. *Let (π, u) be a covariant representation of the system (B, G, δ) and define $\nu_e \colon c_0(K) \to B(\mathcal{H})$ by*

$$\nu_e(f) = \sum_{a \in K} \sum_{t \in J_a} f(a) u(a) \qquad (f \in c_0(K)),$$

then (π, ν_e) is a covariant representation of the system $(B, K, G, \delta, \gamma_R)$.

P r o o f. Let $t \in G, a \in K$, and $b_t \in B_t$, then

$$\begin{aligned} \pi(x_t)\nu_e(a) &= \sum_{s \in J_a} \pi(x_t)\nu(s) \\ &= \sum_{s \in J_a} u(ts)\pi(x_t) \\ &= \sum_{s \in J_{a \cdot t}} u(s)\pi(x_t) = \nu_e(a \cdot t)\pi(x_t). \end{aligned}$$

□

Theorem 2.3. *For each covariant representation (π, ν) of $(B, K, G, \delta, \gamma)$ there is a unique non-degenerate representation $\pi \times \nu$ of $B \rtimes_{\delta,\gamma} K$ such that*

$$\pi \times \nu(x, a) = \pi(x)\nu(a) \qquad (x \in B, \quad a \in K),$$

and when $B = C^*(\mathfrak{B})$, every non-degenerate representation of $B \rtimes_{\delta,\gamma} K$ is of this form.

P r o o f. If (π, ν) is a covariant representation of $(B, K, G, \delta, \gamma)$, then $(x, a) \mapsto \pi(x)\nu(a)$ is a non-degenerate representation of the Fell bundle $\mathfrak{B} \times K$. By Lemma 2.1, there is a unique representation $\pi \times \nu$ of $C^*(\mathfrak{B} \times K) = B \rtimes_{\delta,\gamma} K$ satisfying the compatibility condition above.

Conversely, if $B = C^*(\mathfrak{B})$ and Π is a non-degenerate representation of $C^*(\mathfrak{B} \times K) = B \rtimes_{\delta,\gamma} K$, we define $\sigma_a \colon B_1 \to B(\mathcal{H}_\Pi)$ by $\sigma_a(x_1) = \pi(x_1, a)$, for each $x_1 \in B_1$ and $a \in K$, where 1 is the identity of G. Then, for each bounded approximate identity $\{x_{1,i}\}$ of the C^*-algebra B_1, we may assume (passing to a subnet, if necessary) that $\{\sigma_a(x_{1,i})\}$ is WOT-convergent, say to p_a. We claim that $\{p_a : a \in K\}$ is an orthogonal family of projections in $B(\mathcal{H}_\Pi)$, satisfying the following relations:

$$p_{a*b} = p_a p_b, \quad \Pi(x_t, a)p_b = \delta_{a,b}\Pi(x_t, b), \quad p_{a\cdot t}\Pi(x_t, b) = \delta_{a,b}\Pi(x_t, b),$$

for each $t \in G$, $a, b \in K$, where $\delta_{a,b}$ is the Kronecker delta, and

$$p_{a*b} = \sum_{c\in K} (\delta_a * \delta_b)(c) p_c.$$

Define $\nu \colon c_0(K) \to B(\mathcal{H}_\Pi)$ by $\nu(f) = \sum_{a\in K} f(a)p_a$ and $\pi \colon B \to B(\mathcal{H}_\Pi)$ by $\pi(x) = \sum_{a\in K} \Pi(x, a)$, where the latter is defined in the WOT-sense. These are representations of $c_0(K)$ and B, respectively and, for each $t \in G$, $a \in K$, and $x_t \in B_t$,

$$\pi(x_t)\nu(a) = \sum_{b\in K}\sum_{c\in K} \chi_a(c)\Pi(x_t, b)p_c = \sum_{b\in K} \Pi(x_t, b)p_c,$$
$$\nu(a \cdot t)\pi(x_t) = \sum_{b\in K} p_{a\cdot t}\Pi(x_t, b) = \Pi(x_t, a),$$

and $(\pi \times \nu)(x, a) = \pi(x)\nu(a) = \Pi(x, a)$. □

Definition 2.4. Let σ be a representation of B on $\mathcal{H}_\sigma$ and $\lambda \colon G \to B(\ell^2(K))$ be the quasi-regular representation of G, given by

$$\lambda_t(\zeta)(a) = \zeta(a \cdot t^{-1}) \qquad (t \in G, \ \ a \in K, \ \ \zeta \in \ell^2(K)),$$

and M be the multiplication representation of the algebra $B(K)$ of bounded Borel measurable functions on K on $\ell^2(\bar{K})$, then the pair $\big((\sigma \otimes \lambda) \circ \delta, 1 \otimes M\big)$ is a covariant representation of $(B, K, G, \delta, \gamma)$ on $\mathcal{H}_\sigma \otimes \ell^2(\bar{K})$. Indeed, for each $t \in G$, $a \in K$, $\lambda_t(\delta_a) = \delta_{a\cdot t}$, and so for each $b_t \in B_t$,

$$\begin{aligned}((\sigma \otimes \lambda) \circ \delta)(x_t)(1 \otimes M)(\delta_a) &= (\sigma(x_t) \otimes \lambda_t)(1 \otimes M(\delta_a)) \\ &= \sigma(x_t) \otimes (\lambda_t M(\delta_a)\lambda_t^*\lambda_t) \\ &= (1 \otimes M(\delta_{a\cdot t}))(\sigma(x_t) \otimes \lambda_t) \\ &= (1 \otimes M)(\delta_{a\cdot t})(\sigma \otimes \lambda) \circ \delta(x_t).\end{aligned}$$

The corresponding integrated representation $((\sigma \otimes \lambda) \circ \delta) \times (1 \otimes M)$ is called the regular representation of $B \rtimes_{\delta,\gamma} K$ and is denoted by $\mathrm{Ind}(\sigma)$.

Theorem 2.5. *If δ is maximal, then an integrated representation $\pi \times \nu$ on $\mathcal{H}$ is equivalent to a regular representation if and and only if there is a representation V of $M(K)$ on $\mathcal{H}$ such that (ν, V) is a covariant pair for K and range projections of V and π commute.*

Proof. Suppose that $\pi \times \nu$ is equivalent to a regular representation, namely there is a representation $\sigma\colon B \to B(\mathcal{H}_\sigma)$ and a unitary isomorphism $\Psi\colon \mathcal{H} \to \mathcal{H}_\sigma \otimes \ell^2(K)$ which intertwines $\pi \times \nu$ and $\operatorname{Ind}(\sigma)$. Put $V = \operatorname{Ad}\Psi^* \circ (1 \otimes \rho)$, where ρ is the representation of $M(\bar{K})$ on $\ell^2(K)$ by right convolution [1]. Then (ν, V) is a covariant pair, as it is equivalent to $(1 \otimes M, 1 \otimes \rho)$ [1], and the ranges of π and V commute as the same hold for the ranges of $(\sigma \otimes \lambda) \circ \delta$ and $1 \otimes \rho$.

Conversely, suppose that (ν, V) is a covariant pair, for some V, such that ranges of π and V commute. By [1, Theorem 2.5], there is a Hilbert space $\mathcal{K}$ and a unitary isomorphism $\Psi\colon \mathcal{H} \to \mathcal{K} \otimes \ell^2(K)$ which intertwines (ν, V) and $(1 \otimes M, 1 \otimes \rho)$. Put $\bar{\pi} = \operatorname{Ad}\Psi \circ \pi$, $\bar{\nu} = \operatorname{Ad}\Psi \circ \nu = 1 \otimes M$, and $\bar{V} = \operatorname{Ad}\Psi \circ V = 1 \otimes \rho$. We need to find a representation $\sigma\colon B \to B(\mathcal{K})$ such that $(\sigma \otimes \lambda) \circ \delta = \bar{\pi}$. Given $t \in G$ and $x_t \in B_t$, we claim that $(1 \otimes \lambda_t^*)\bar{\pi}(x_t)$ commutes with operators of the form $1 \otimes (\delta_a \otimes \delta_b)$, for $a, b \in K$. By [1, Example 2.4], $1 \otimes (\delta_a \otimes \delta_b) = 1 \otimes (M(a)\rho(\bar{a} * b)M(b)) = \bar{\nu}(a)\bar{V}(\bar{a} * b)\bar{\nu}(b)$, hence by the covariance of $(\bar{\pi}, \bar{\nu})$ for $(B, K, G, \delta, \gamma_R)$ and the covariance of $(\bar{\nu}, 1 \otimes \lambda) = (1 \otimes M, 1 \otimes \lambda)$ for the action of G on $c_0(K)$ induced by γ_L [1, Example 2.4], and the fact that the ranges of $\bar{\pi}$ and $\bar{V}$, and those of $\bar{V} = 1 \otimes \rho$ and $1 \otimes \lambda$ commute, we have

$$\begin{aligned}
(1 \otimes \lambda_t^*)\bar{\pi}(x_t)(1 \otimes (\delta_a \otimes \delta_b)) &= (1 \otimes \lambda_t^*)\bar{\pi}(x_t)\bar{\nu}(a)\bar{V}(\bar{a} * b)\bar{\nu}(b) \\
&= (1 \otimes \lambda_t^*)\bar{\nu}(t \cdot a)\bar{\pi}(x_t)\bar{V}(\bar{a} * b)\bar{\nu}(b) \\
&= (1 \otimes \lambda_t^*)\bar{\nu}(t \cdot a)\bar{V}(\bar{a} * b)\bar{\pi}(x_t)\bar{\nu}(b) \\
&= (1 \otimes \lambda_t^*)\bar{\nu}(t \cdot a)\bar{V}(\bar{a} * b)\bar{\nu}(t \cdot b)\bar{\pi}(x_t) \\
&= \bar{\nu}(a)(1 \otimes \lambda_t^*)\bar{V}(\bar{a} * b)\bar{\nu}(t \cdot b)\bar{\pi}(x_t) \\
&= \bar{\nu}(a)\bar{V}(\bar{a} * b)(1 \otimes \lambda_t^*)\bar{\nu}(t \cdot b)\bar{\pi}(x_t) \\
&= \bar{\nu}(a)\bar{V}(\bar{a} * b)\bar{\nu}(b)(1 \otimes \lambda_t^*)\bar{\pi}(x_t) \\
&= (1 \otimes (\delta_a \otimes \delta_b))(1 \otimes \lambda_t^*)\bar{\pi}(x_t).
\end{aligned}$$

Since the algebra of compact operators is the closed linear span of rank-one operators, $(1 \otimes \lambda_t^*)\bar{\pi}(x_t)$ commutes with $1 \otimes \mathfrak{K}(\ell^2(K))$ and so it is in $B(\mathcal{K}) \otimes 1$, say $(1 \otimes \lambda_t^*)\bar{\pi}(x_t) = \sigma(x_t) \otimes 1$, for some linear map $\sigma\colon \Gamma_c(B) \to B(\mathcal{K})$, satisfying $(\sigma \otimes \lambda) \circ \delta = \bar{\pi}$ on $\Gamma_c(B)$. It is straightforward to check that σ is then a $*$-homomorphism (since $\bar{\pi}$ and λ^* are so) and therefore extends to a representation of $B = C^*(\mathcal{B})$ satisfying $(\sigma \otimes \lambda) \circ \delta = \bar{\pi}$, by the universality of the enveloping C^*-algebra. □

3. Dual coactions

In this section we deal with the situation that G acts on a C^*-algebra A and by α and co-acts on the *-algebra crossed product $B = A \rtimes_\alpha G$ by the dual coaction $\delta = \hat{\alpha}$. It is well known that the coaction $\hat{\alpha}$ is maximal. Lifting the action γ_R of G on K to an action on $c_0(K)$, we have:

Lemma 3.1. *There is an isometric isomorphism from $(A \rtimes_\alpha G) \rtimes_{\hat{\alpha},\gamma_R} K$ onto $(A \otimes c_0(K)) \rtimes_{\gamma_R \otimes \alpha} G$ carrying each representation $(\psi \times W) \times \nu$ of the former to the representation $(\psi \times \nu) \times W$ of the latter, and in particular, carrying $(\phi \times U) \otimes \lambda) \circ \hat{\alpha} \times (1 \otimes M)$ into $(\phi \otimes M) \times (U \otimes \lambda)$, for each integrated representation $\phi \times U$ of $A \rtimes_\alpha G$.*

Proof. Given $t \in G$, the algebra B_t is given in terms of the universal covariant representation $(\iota_A, \iota_G)\colon (A, G) \to M(A \rtimes_\alpha G)$ by $B_t = \{\iota_A(x)\iota_G(t) : x \in A\}$. A triple (ψ, W, ν) gives a covariant representation $(\psi \times W, \nu)$ of $(A \rtimes_\alpha G, K, G, \hat{\alpha}, \gamma_R)$ if and only if

$$\psi(x)W_t\nu(\delta_a) = \nu(\delta_{a\cdot t})\psi(x)W_t \qquad (t \in G, \ \ a \in K, \ \ x \in A)$$

if and only if the ranges of ψ and ν commute and (ν, W) is a covariant representation of $(c_0(K), G, \alpha)$. Hence the canonical embedding $(\kappa_A, \kappa_G, \kappa_{C_0(K)})$ of $(A, G, C_0(K))$ in $M(A \otimes c_0(K)) \rtimes_{\gamma \otimes \alpha} G$ generates a covariant representation $(\kappa_A \times \kappa_G, \kappa_{c_0(K)})$ of $(A \rtimes_\alpha G, K, G, \hat{\alpha}, \gamma)$, and so by Theorem 2.5, induces a homomorphism Λ from $(A \rtimes_\alpha G) \rtimes_{\hat{\alpha},\gamma} K$ into $M(A \otimes c_0(K)) \rtimes_{\gamma \otimes \alpha} G$ satisfying

$$\Lambda(\iota_A(x)\iota_G(t), a) = \kappa_A(x)\kappa_G(t)\kappa_{c_0(K)}(a) \qquad (t \in G, \ \ a \in K, \ \ x \in A).$$

Clearly Λ is surjective. Now each representation of $(A \rtimes_\alpha G) \rtimes_{\hat{\alpha},\gamma} K$ has the form $\sigma \times \nu$, for some covariant pair (σ, ν), where $\sigma = \psi \times W$, for some covariant representation of (A, G, α). Since ranges of ψ and ν commute, $\psi \otimes \nu$ is a representation of $A \otimes c_0(K)$. Also $(\psi \otimes \nu, W)$ is covariant for $\gamma \otimes \alpha$, as (ψ, W) is covariant for α and (ν, W) is covariant for γ. Now one can easily check that $(\psi \otimes \nu) \times W) \circ \Lambda = (\psi \times W) \times \nu$. Finally, since $\hat{\alpha}(\iota_A(x)) = \iota_A(x) \otimes 1$ and $\hat{\alpha}(\iota_G(t)) = \iota_G(t) \otimes u(t)$, where $u\colon G \to M(C^*(G))$ is the canonical map induced by translation representation of G on $\ell^1(G)$,

$$((\phi \otimes U) \otimes \lambda) \circ \hat{\alpha}) = (\phi \otimes 1) \times (U \otimes \lambda)$$

and so

$$(((\phi \otimes U) \otimes \lambda) \circ \hat{\alpha}) \times (1 \otimes M) = ((\phi \otimes 1) \times (U \otimes \lambda) \times (1 \otimes M)),$$

which is carried by Λ into

$$((\phi \otimes 1) \otimes_{\max} (1 \otimes M)) \times (U \otimes \lambda) = (\phi \otimes M) \times (U \otimes \lambda).$$

□

Now $H = \ker(\gamma_R) \leq G$ is a closed subgroup of G and by the classical imprimitivity of group actions, one has an imprimitivity $(A \otimes c_0(K)) \rtimes_{\gamma_R \otimes \alpha} G$-$A \rtimes_\alpha H$-bimodule X, which gives a bijection

$$X_{\mathrm{ind}}\colon \operatorname{Rep}(A \rtimes_\alpha H) \to \operatorname{Rep}((A \otimes c_0(K)) \rtimes_{\gamma_R \otimes \alpha} G).$$

Theorem 3.2. *If (ψ, T) is a covariant representation of (A, H, α) on $\mathcal{H}_\psi$, $B = A \rtimes_\alpha H$, and π and ν are representations of $A \rtimes_\alpha G$ and $C_0(K)$ on $\mathcal{H} = X \otimes_B \mathcal{H}_\psi$ such that $\pi \times \nu$ is the representation of $(A \rtimes_\alpha G) \rtimes_{\hat{\alpha}, \gamma_R} K$ corresponding to $X_{\mathrm{ind}}(\psi \times T)$, then there is a representation $(\psi, \bar{T})$ of (A, G, α) on $\mathcal{H}_\psi$ such that $\bar{T}|_H = T$ and there is a representation V of $M(K)$ on $\mathcal{H}$ such that (ν, V) is a covariant pair for K and the ranges of V and π commute.*

Proof. By [3, Theorem 1], the existence of $(\psi, \bar{W})$ is equivalent to the existence of a representation $\phi \times U$ of $A \rtimes_\alpha G$ such that $X_{\mathrm{ind}}(\psi \times W)$ is unitarily equivalent to $(\phi \otimes M) \times (U \otimes \lambda)$, which is, by Lemma 3.1, the same as $\phi \times U$ in a way that $\pi \times \nu$ is unitarily equivalent to the regular representation $(((\phi \times U) \otimes \lambda) \circ \hat{\alpha}) \times (1 \otimes M)$. Now the result follows from Theorem 2.5 applied to the maximal coaction $\delta = \hat{\alpha}$ of G on $B = A \rtimes_\alpha G$. $\square$

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