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CELLULAR COVERS OF TOTALLY ORDERED ABELIAN GROUPS

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Dedicated to Charles Holland on his 75th birthday
(*Communicated by Anatolij Dvurečenskij*)

ABSTRACT. Cellular covers of groups, and in particular, those of divisible abelian groups, were studied in [FARJOUN, E. D.—GÖBEL, R.—SEGEV, Y.: *Cellular covers of groups*, J. Pure Appl. Algebra **208**, (2007), 61–76], [CHACHÓLSKI, W.—FARJOUN, E. D.—GÖBEL, R.—SEGEV, Y.: *Cellular covers of divisible abelian groups*. In: Contemp. Math. 504, Amer. Math. Soc., Providence, RI, 2009, pp. 77–97], and continued in [FUCHS, L.—GÖBEL, R.: *Cellular covers of abelian groups*, Results Math. **53**, (2009), 59–76] for abelian groups in general. In this note we are investigating cellular covers in the category of totally ordered abelian groups (called o-cellular covers; for definition see Section 2). Some results are similar to those on torsion-free abelian groups (unordered), while others are completely different. For instance, though kernels of o-cellular covers can not be non-zero divisible groups (Lemma 3.1), they may contain non-zero divisible subgroups (Example 3.2); however, the divisible part can not be much larger than the reduced part (Theorem 3.4). There are o-groups, even among the additive subgroups of the rationals, whose o-cellular covers form a proper class (Theorem 4.3).

1. Introduction

We continue the investigations on cellular covers of groups initiated by Farjoun, Göbel and Segev [3], and continued by Chachólski, Farjoun, Göbel and Segev [2] for divisible abelian groups. In this note we consider the case where all the groups are totally ordered abelian groups. Our discussion relies heavily on the paper Fuchs and Göbel [7] that gives a systematic study of cellular covers

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of abelian groups; of course, we are now interested only in torsion-free groups. Needless to say, the situation is not the same when the groups are furnished with a linear order. For instance, a major difference is due to the fact that the order-preserving homomorphisms between ordered abelian groups do not form a group.

Just as for unordered abelian groups, the kernels of o-cellular covering maps can not be divisible groups. But contrary to the case of unordered groups, kernels in the ordered case may contain divisible summands (Example 3.2). However, we will show that the divisible part of the kernel can not be much larger than its reduced part (Theorem 3.4).

As for unordered groups (see Farjoun, Göbel, Segev and Shelah [4] and Fuchs and Göbel [7]), it can very well happen that the o-cellular covers of an o-group form a proper class. This phenomenon (which occurs already for rank 1 o-groups) is a convincing evidence that the classification of o-cellular covers of an o-group in general is not a possibility.

For unexplained terminology and results we refer to Fuchs [6] for abelian groups and to Fuchs [5] or Glass [8] for ordered groups. Our standard reference to cellular covers is the article Farjoun, Göbel and Segev [3].

2. Preliminaries

In this note, all groups are totally ordered abelian groups, written additively, unless stated otherwise. For brevity, we will say that they are *o-groups*. All maps considered here between o-groups are order-preserving, i.e. they are order-homomorphisms (o-homomorphisms). $\hom(G, A)$ will stand for the set of o-homomorphisms $G \to A$. This is a commutative monoid under addition.

We say that (G, γ) is an *o-cellular cover* for the o-group A if G is an o-group and $\gamma\colon G \to A$ is an o-homomorphism such that every o-homomorphism $\phi\colon G \to A$ factors *uniquely* through γ; in other words, to a given ϕ there is a *unique* o-homomorphism $\bar{\phi}\colon G \to G$ such that $\phi = \gamma\bar{\phi}$. This is equivalent to saying that the map

$$\hom(G, G) \to \hom(G, A)$$

induced by γ is bijective. We then have a commutative diagram

$$\begin{array}{ccc} G & = & G \\ {\scriptstyle \exists!\bar{\phi}}\downarrow & & \downarrow{\scriptstyle \forall\phi} \\ G & \xrightarrow{\gamma} & A \end{array}$$

of o-groups and o-homomorphisms. The o-group G is called an *o-cellular cover* and the map γ an *o-cellular covering map* of A. We will concentrate on cases where γ is *surjective*; this does not mean any loss of generality, since the non-surjective covering maps are surjective on a fully invariant subgroup of A (which can then be studied instead of A); see Farjoun, Göbel and Segev [3, Lemma 3.5]. In view of this, it will be convenient to focus on exact sequences

$$0 \to K = \operatorname{Ker}\gamma \to G \xrightarrow{\gamma} A \to 0 \tag{1}$$

of o-groups and o-homomorphisms; they will be called *o-cellular exact sequences* for A. Here K may be viewed as a convex subgroup of G, and G as a lexicographic extension of K by A. We keep in mind that, since A is torsion-free, K is a pure subgroup in G.

Sometimes we have to distinguish between cellular covers of an o-group A viewed as an abelian group or as an o-group; in such a situation, we will specify the category $\mathcal{A}b$ of abelian groups or the category $\mathcal{A}b_o$ of abelian o-groups in which the covers are taken. Needless to say, an exact sequence of o-groups that is cellular in $\mathcal{A}b$ is also cellular in $\mathcal{A}b_o$.

Obviously, every o-group A admits *trivial* o-cellular exact sequences; these are of the form: $0 \to 0 \to A \xrightarrow{\gamma} A \to 0$ with γ an o-automorphism of A.

The o-cellular covers (G_1, γ_1) and (G_2, γ_2) of the same o-group A are said to be *equivalent* if there is an o-isomorphism $\alpha\colon G_1 \to G_2$ such that $\gamma_1 = \gamma_2\alpha$. Evidently, all trivial o-cellular covers are equivalent.

From what has been said it should be clear that surjective o-cellular covers are in general not unique, not even up to equivalence; as a matter of fact, they are unique if and only if the o-group admits only the trivial o-cellular covers. At the other extreme, we find that the situation is similar to the case observed in $\mathcal{A}b$: the inequivalent o-cellular covers of an o-group may form a proper class (cf. Theorem 4.2). This can happen — as in $\mathcal{A}b$ — even for certain rank 1 torsion-free o-groups.

An important property of the kernels that is most relevant in $\mathcal{A}b$ holds in $\mathcal{A}b_o$ as well. This is stated in the next lemma.

Lemma 2.1. *If* (1) *is an o-cellular exact sequence for the o-group A, then there exists no non-trivial o-homomorphism $\psi\colon G \to K$ (and hence no non-trivial map $A \to K$).*

P r o o f. If there is such an o-homomorphism ψ, then the map $\bar{\phi}$ in the definition can not be unique, because a non-trivial ψ can be added to every o-endomorphism $G \to G$. $\square$

However, as far as the kernels are concerned, there are basic differences between $\mathcal{A}b$ and $\mathcal{A}b_o$. This will be discussed in Section 3 in more details.

The following observation is most useful; this is the ordered version of [7, Proposition 2.6] (see also [2, Proposition 5.1] which was later found independently).

Lemma 2.2.

(i) *If a monoid S operates on the o-group A (in an order-preserving manner), then it also operates on each o-cellular cover G of A, and the covering maps are S-maps.*

(ii) *If an o-group A is a module over an o-ring R, then both G and K are R-modules and γ is a module map.*

Proof.

(i) Consider the diagram

$$\begin{array}{ccc} G & \xrightarrow{\ \gamma\ } & A \\ {\scriptstyle\bar{\alpha}}\big\downarrow & & {\scriptstyle\alpha}\big\downarrow \\ G & \xrightarrow{\ \gamma\ } & A \end{array}$$

where $\alpha \in S$ is given; it acts as an endomorphism of A, so by the o-cover property there is a unique $\bar{\alpha}\colon G \to G$ making the diagram commute. It is straightforward to see that if we define the action of $\alpha \in S$ on G as given by $\bar{\alpha}$, then S will operate on G and γ will be an S-map.

(ii) If A is a module over an o-ring R (i.e. positive ring elements induce o-endomorphisms on A), then by (i) multiplications by positive elements of R represent o-endomorphisms in G and also in K. The positive cone of R acts on both G and K as group endomorphisms; these determine the action of every element of R on A, so G as well as K is an R-module in the pure module-theoretical sense. □

Example 2.3. If A is an o-vector space over the ordered field of rationals $\mathbb{Q}$ or over the reals $\mathbb{R}$, then its o-cellular cover G has to be a vector space over the same field as well. (This implies that such an A admits only the trivial o-cellular covers; cf. Lemma 3.1 below.)

Example 2.4. For any prime p, let $A \cong J_p$ be equipped with an archimedean total order (this is possible since the additive group J_p of the p-adic integers is embeddable in $\mathbb{R}$ as an object of $\mathcal{A}b$). From the exact sequence $0 \to \mathbb{Z}_p \to J_p \to \oplus\mathbb{Q} \to 0$ (where $\mathbb{Z}_p$ stands for the localization of $\mathbb{Z}$ at the prime p) we obtain the exact sequence

$$0 = \mathrm{Hom}(J_p, \mathbb{Z}_p) \to \mathrm{Hom}(\mathbb{Z}_p, \mathbb{Z}_p) \cong \mathbb{Z}_p \to \mathrm{Ext}(\oplus\mathbb{Q}, \mathbb{Z}_p) \to \mathrm{Ext}(J_p, \mathbb{Z}_p) \to 0.$$

Here the first Ext is a non-trivial torsion-free divisible group, hence the second Ext is $\neq 0$. Let $0 \to \mathbb{Z}_p \to G \to J_p \to 0$ represent a non-splitting exact sequence. Furnish $\mathbb{Z}_p$ with a linear order and G with the lexicographic order, using the assigned total order on J_p. Then the last exact sequence becomes a sequence of o-groups, and we claim that this is an o-cellular sequence for J_p. It is easily seen that, since $\mathrm{Hom}(J_p, \mathbb{Z}_p) = 0$, the induced map $\mathrm{hom}(G, G) \to \mathrm{hom}(G, J_p)$ is bijective. (Observe that the same exact sequence is not cellular in $\mathcal{A}b$; indeed, J_p carries a ring structure, but $\mathbb{Z}_p$ is not a J_p-module.)

The following consequence of Lemma 2.2 will be needed in the proof of Theorem 4.3 below.

Corollary 2.5. *Let R denote a subring of $\mathbb{Q}$ (no ordering). If the o-group A is an R-module, then so are G and K in* (1)*, and the cellular covering map is an R-homomorphism.*

Proof. A subring R of $\mathbb{Q}$ admits a unique total ring order, and if the group A is an R-module, then the o-group A is automatically a module over the o-ring R. The rest follows from (ii) in the preceding lemma. $\square$

3. Kernels of o-cellular covering maps

There are several relevant properties shared by the kernels of o-cellular maps that we wish to explore. We start with the following lemma.

Lemma 3.1.

(i) *An o-cellular exact sequence* (1) *for an o-group A cannot be a split exact sequence pure group-theoretically* (*i.e. in $\mathcal{A}b$*)*.*

(ii) *The kernel K of an o-cellular exact sequence* (1) *cannot be an algebraically compact group.*

Proof.

(i) If K is a summand of G as a group (i.e. in $\mathcal{A}b$), then being a convex subgroup any o-homomorphism α of K induces an o-homomorphism ϕ of G. Then the map $\bar{\phi}$ in the definition can not be unique, because such an α can be added to every o-endomorphism $G \to G$, in particular, to the identity map of G.

(ii) Since A is torsion-free, K is a pure subgroup of G. Pure subgroups that are algebraically compact are summands in $\mathcal{A}b$, so the claim follows from (i). $\square$

Though like in $\mathcal{A}b$, the kernel of an o-cellular cover map can not be a divisible group (by Lemma 3.1(i)), it may very well contain divisible subgroups. Indeed, the next example shows that $\mathbb{Q}$ (or any divisible subgroup of the reals $\mathbb{R}$) can be a subgroup of the kernel (similar example exists where the group J_p of p-adic integers, for any prime p, is a proper summand of the kernel).

Example 3.2. Let A be a rigid torsion-free group (i.e. it has only endomorphisms that are multiplications by rational numbers) of countable rank, and G a non-trivial cellular cover of A with a countable kernel K (also torsion-free), taken in $\mathcal{A}b$. Such groups exist in abundance as it is clear from [7, Lemma 5.2].

Furnish A with a total order in such a way that it has no maximal proper convex subgroup, and equip $K' = K \oplus \mathbb{Q}$ with an archimedean total order. This can be done, since K' is isomorphic to a subgroup of the reals. Now the group $G' = G \oplus \mathbb{Q}$ as a lexicographic extension of K' by A will be an o-cellular cover for A; indeed, the o-endomorphisms of G' are completely determined on G, so also G' will have only o-endomorphisms uniquely determined by the maps $G \to A$.

However, it is important to point out that there is an upper bound on the cardinality of the maximal divisible subgroup of the kernel of an o-cellular map, depending on the reduced part — as it will be shown by Theorem 3.4 below. We will need the following simple lemma.

Lemma 3.3. *Suppose that $C < B$ are convex subgroups of the o-group G such that B/C is a divisible group. If C is a summand of B in $\mathcal{A}b$, then G has an o-endomorphism η with the following properties:*

(i) *η induces multiplication by an integer $n > 1$ on B/C;*

(ii) *$\eta|C = \mathbf{1}_C$;*

(iii) *η induces the identity on G/B.*

P r o o f. B/C may be viewed as a summand of G/C in $\mathcal{A}b$, so multiplication by any $n > 1$ on B/C (while keeping the complementary summand fixed) is a group homomorphism of G. Since this map carries positive elements of G into positive elements, it is an o-endomorphism. It clearly satisfies (ii) and (iii). □

We can now verify:

Theorem 3.4. *Let $K \neq 0$ be the kernel of an o-cellular map $G \to A$, and $K = D \oplus L$ (in $\mathcal{A}b$) where D is divisible and $L \neq 0$ is reduced. Then*

$$|D| \leq 2^{|L|}.$$

P r o o f. Consider a maximal chain of convex subgroups C_α of G. If there is a pair $C < B \leq K$ of convex subgroups as stated in Lemma 3.3, then G would have an o-endomorphism $\neq \mathbf{1}_G$ inducing the identity on A — a contradiction. On the other hand, if there is no such pair $C < B \leq K$ of convex subgroups, then the divisible part D of K is distributed among the links of the chain so that no quotient $C_{\alpha+1}/C_\alpha$ (from K) is disjoint from the reduced part L. Therefore, the chain may contain at most $\operatorname{rk} L \leq |L|$ subgroups. Each quotient has cardinality $\leq 2^{\aleph_0}$, thus $|D| \leq |K| \leq (2^{\aleph_0})^{|L|} = 2^{|L|}$. □

A group A is said to be *cotorsion-free* if it does not contain any subgroup isomorphic to any of the following groups:

1) cyclic group of prime order;
2) the group $\mathbb{Q}$ of rational numbers;
3) the additive group J_p of the p-adic integers, for any prime p.

Thus a cotorsion-free group is torsion-free and reduced as an abelian group. From a result of Buckner and Dugas [1] it follows that every cotorsion-free abelian group is the kernel of a suitably chosen cellular covering map of some torsion-free abelian group (taken in $\mathcal{A}b$). We are making use of this theorem in order to verify:

Theorem 3.5. *Every o-group K that is cotorsion-free as an abelian group appears as the kernel of the o-cellular covering map of some o-group A.*

P r o o f. Let K be an o-group that is cotorsion-free as a group. By Buckner and Dugas [1] there exists in $\mathcal{A}b$ a cellular exact sequence $0 \to K \to G \xrightarrow{\gamma} A \to 0$ of torsion-free abelian groups. It remains to furnish A with any total order and G with the lexicographic order to complete the proof. □

A complete characterization of those groups that can be kernels of cellular covering maps of abelian groups is given in Fuchs and Göbel [7]. According to this result, for torsion-free groups all the kernels have to be cotorsion-free (that follows from [1] already). For o-groups, though the kernels are always torsion-free, the situation is different; see Example 3.2. But there is a limitation: as is shown by Theorem 3.4, the divisible part of the kernel K cannot be too large. This upper bound on the cardinality of the divisible part is not a sufficient condition, because this hypothesis alone will not guarantee that there can not be any o-homomorphism of A into K.

4. Rank 1 torsion-free groups

We wish to prove an analogue of a result by Fuchs and Göbel [7] which is an essentially improved version of a theorem of Farjoun, Göbel, Segev and Shelah [4]. It shows *inter alia* that in general it is impossible to classify the o-cellular covers of torsion-free o-groups, not even for the simplest case: for the rank 1 o-groups. In fact, we will prove that, for those o-groups of rank 1 that are not isomorphic to additive groups of subrings of $\mathbb{Q}$, the o-cellular covers always form a proper class. This is the content of our next theorem.

First, we state a sufficient condition for o-cellularity (condition (ii) is necessary, but (i) is not) which we will need in the proof of the theorem.

Lemma 4.1. *The exact sequence* (1) *is an o-cellular exact sequence for the o-group A provided that*

(i) *it is a cellular sequence for A in $\mathcal{A}b$;*

(ii) *every o-endomorphism $\phi\colon G \to A$ is induced by an o-endomorphism of G.*

P r o o f.

(ii) implies that every o-homomorphism $G \to A$ lifts to an o-endomorphism $G \to G$, while (i) guarantees the uniqueness of such a map. □

The general construction is illustrated by the following typical example. Observe that for rank one torsion-free groups (i.e. subgroups of $\mathbb{Q}$) the order structure is completely determined up to multiplication by -1.

Lemma 4.2. *Let A be an o-group of rank* 1 *and of type $\tau = (1, 1, \dots, 1, \dots)$. For every cardinal $\kappa \geq 1$, there exists a cellular exact sequence* (1) *for A such that K is an o-group of rank κ.*

P r o o f. We refer to [7, Theorem 4.5] for the construction of the groups K and G in $\mathcal{A}b$. As in the proof there, we let K be any (rigid) group of rank κ whose endomorphism ring $\operatorname{End} K$ is isomorphic to $\mathbb{Z}[q^{-\infty}] \subset \mathbb{Q}$ for some prime q and which does not contain any pure subgroup of rank 1 whose type is $\geq \varrho = (1, 0, 1, 0, \dots, 1, 0, \dots)$. Such a K exists in abundance of any prescribed rank, as indicated in [7]. Moreover, we may also assume that K contains a rank 1 pure subgroup $\langle c \rangle_*$ (= smallest pure subgroup containing the element c) whose type is $\not\geq \sigma = (0, 1, 0, 1, 0, \dots, 0, 1, \dots)$. Let B be the subgroup of $\mathbb{Q}$ in which the element 1 has characteristic ϱ, so $\operatorname{End} B \cong \mathbb{Z}$. Let P denote the set of primes at which σ has 1. Define

$$G = \left\langle B \oplus K, \frac{1+c}{p} \mid \text{all } p \in P \right\rangle.$$

Clearly, $G/K \cong A$. As proved in [7], this is a cellular exact sequence for A in $\mathcal{A}b$. Once we have defined these groups, we furnish K with an arbitrary total order, A with one of the two possible archimedean orders, and G with the lexicographic order to complete the construction.

We now check that conditions (i)–(ii) in Lemma 4.1 are satisfied in this situation. (i) is obvious in view of the construction. To see that (ii) too is satisfied, observe that every group endomorphism of G is a multiplication by an integer $n \in \mathbb{Z}$, hence every o-endomorphism of G must be a multiplication by a non-negative integer. The same holds for A, moreover, every multiplication by a non-negative integer induces an o-endomorphism of G and of A alike. This guarantees that condition (ii) in Lemma 4.1 is satisfied, completing the proof. □

Here is the theorem mentioned above.

Theorem 4.3. *Let A be a torsion-free o-group of rank 1. A has either only the trivial o-cellular covers or o-cellular covers of arbitrarily large cardinalities according as A is of an idempotent type or not.*

Proof. First assume that A is of idempotent type, i.e. it is isomorphic to the additive group of a subring R of $\mathbb{Q}$. R may be viewed as an o-ring, so from Corollary 2.5 we conclude that any o-cellular cover G of A is likewise an o-module over R. As A is a free R-module, G splits in $\mathcal{A}b$, and hence by Lemma 3.1, we must have $K = 0$. This means that A admits only the trivial o-cellular covers.

On the other hand, if the type of A is not idempotent, then its type is represented by a characteristic $(k_1, \dots, k_n, \dots)$ such that infinitely many indices n satisfy $0 < k_n < \infty$. The method of Lemma 4.2 works *mutatis mutandis* for this A: since K can be chosen arbitrarily large, we can find arbitrarily large o-cellular covers for such an A. □

Let us point out another interesting fact on o-cellular covers. If A is a torsion-free o-group of rank 1, not of idempotent type, then in the above construction the rigid group K may be chosen from a rigid system of groups of the prescribed rank κ such that every member of this system has the required properties; see e.g. Göbel and Trlifaj [9, Theorem 14.2.12, p. 554]. Such a rigid system consists of 2^κ groups; they are torsion-free, so each of them can be totally ordered, giving rise to an o-cellular covering group of A. These o-covering groups form a rigid system of o-groups (that is rigid even in $\mathcal{A}b$). Consequently, we can state:

Corollary 4.4. *Let A be a torsion-free o-group of rank 1, not of idempotent type. Then it admits, for every infinite cardinal κ, 2^κ o-cellular covers of size κ which form a rigid system, i.e. all of their o-endomorphisms are multiplications by non-negative rational numbers and they do not admit any non-trivial o-homomorphism into each other.*

It is possible to construct a more interesting rigid family of large o-cellular covers. Indeed, we can establish the existence of such a family where the o-groups become isomorphic when the order relations are ignored.

Corollary 4.5. *If A is a torsion-free o-group of rank 1, not of idempotent type, then it admits, for every infinite cardinal κ, a rigid system of 2^κ o-cellular covers such that all the groups in the system are isomorphic in $\mathcal{A}b$.*

Proof. As we saw above, for every κ, there exists a cellular exact sequence $0 \to K \to G \to A \to 0$ where K has rank κ. There are 2^κ different continuous well-ordered ascending chains

$$0 = K_0 < K_1 < \cdots < K_\alpha < \dots \qquad (\alpha < \kappa)$$

of pure subgroups with $K = \bigcup_{\alpha<\kappa} K_\alpha$ such that all the factors $K_{\alpha+1}/K_\alpha$ are torsion-free of rank 1. Indeed, at each step we have more than one possibility to choose the next subgroup; as a matter fact, we can start a chain with K_1 in κ different ways. The lexicographic orders on these chains lead to a desired rigid system, because all the o-homomorphisms between different members must be multiplications by positive rational numbers (as this holds in $\mathcal{A}b$) and obviously, such a map cannot carry a chain to a different chain. $\square$

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