VERSITA

Mathematica Slovaca

DOI: 10.2478/s12175-011-0007-4
Math. Slovaca **61** (2011), No. 2, 227–234

ON SOME NEW GENERALIZED DIFFERENCE SEQUENCE SPACES ON SEMINORMED SPACES DEFINED BY A SEQUENCE OF ORLICZ FUNCTIONS

Çiğdem A. Bektaş

(*Communicated by Ľubica Holá*)

ABSTRACT. In this paper we define the sequence space $\ell_{\mathbf{M}}(\Delta_v^m, p, q, s)$ on a seminormed complex linear space, by using a sequence of Orlicz functions. We study some algebraic and topological properties. We prove some inclusion relations involving $\ell_{\mathbf{M}}(\Delta_v^m, p, q, s)$.

1. Introduction

Let ω be the set of all sequences of real or complex numbers and ℓ_∞, c and c_0 be the linear spaces of bounded, convergent and null sequences $x = (x_k)$ with complex terms, respectively, normed by $\|x\|_\infty = \sup_k |x_k|$ where $k \in \mathbb{N}$, the set of positive integers.

Kızmaz [5] defined the sequence spaces

$$Z(\Delta) = \{x = (x_k) : \ (\Delta x_k) \in Z\}$$

where Z is any of the sets ℓ_∞, c or c_0. The notion of difference sequence spaces was generalized by Et and Çolak [2] as follows:

$$Z(\Delta^m) = \{x = (x_k) : \ (\Delta^m x_k) \in Z\}$$

for $Z = \ell_\infty$, c or c_0, where $m \in \mathbb{N}$, $(\Delta^m x_k) = (\Delta^{m-1} x_k - \Delta^{m-1} x_{k+1})$ and so that

$$\Delta^m x_k = \sum_{i=0}^{m} (-1)^i \binom{m}{i} x_{k+i}.$$

2010 Mathematics Subject Classification: Primary 40C05; Secondary 46A45.
Keywords: sequence of Orlicz functions, sequence spaces, difference sequence spaces.

Let $v = (v_k)$ be any fixed sequence of non-zero complex numbers. Et and Esi [3] generalized the above sequence spaces to the following sequence spaces

$$Z(\Delta_v^m) = \left\{x = (x_k) : \ (\Delta_v^m x_k) \in Z\right\}$$

where $(\Delta_v^m x_k) = (\Delta_v^{m-1} x_k - \Delta_v^{m-1} x_{k+1})$, $\Delta_v^m x_k = \sum\limits_{i=0}^{m} (-1)^i \binom{m}{i} v_{k+i} x_{k+i}$ for all $k \in \mathbb{N}$.

The sequence spaces $Z(\Delta_v^m)$ are Banach spaces normed by

$$\|x\|_\Delta = \sum_{i=1}^{m} |v_i x_i| + \|\Delta_v^m x\|_\infty .$$

An Orlicz function is a function $M\colon [0,\infty) \to [0,\infty)$, which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$.

If M is a convex function and $M(0) = 0$, then $M(\lambda x) \le \lambda M(x)$ for $0 \le \lambda \le 1$.

Lindenstrauss and Tzafriri [6] used the idea of Orlicz function to construct the sequence space

$$\ell_M = \left\{x \in \omega : \ \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty \ \text{ for some } \ \rho > 0\right\}.$$

The space ℓ_M becomes a Banach space, with the norm

$$\|x\| = \inf\left\{\rho > 0 : \ \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \le 1\right\}$$

which is called an Orlicz sequence space. The space ℓ_M is closely related to the space ℓ_p which is an Orlicz sequence space with $M(x) = x^p$ for $1 \le p < \infty$.

Later on different types of sequence spaces were introduced by using an Orlicz function by Bektaş and Altın [1], Tripathy [8], Tripathy et al [9], Tripathy et al [10], Tripathy and Mahanta [11], [12] and many others.

Let X be a complex linear space with zero element θ and $X = (X, q)$ be a seminormed space with the seminorm q. By $S(X)$ we denote the linear space of all sequences $x = (x_k)$ with $(x_k) \in X$ and the usual coordinatewise operations: $\alpha x = (\alpha x_k)$ and $x + y = (x_k + y_k)$, for each $\alpha \in \mathbb{C}$ where $\mathbb{C}$ denotes the set of complex numbers. If $\lambda = (\lambda_k)$ is a scalar sequence and $x \in S(X)$ then we shall write $\lambda x = (\lambda_k x_k)$.

Let $\mathbf{M} = (M_k)$ be a sequence of Orlicz functions, X be a seminormed space with seminorm q, $p = (p_k)$ be a sequence of positive real numbers and $s \ge 0$ a real number. Then we define

$$\ell_{\mathbf{M}}(\Delta_v^m, p, q, s) = \left\{x \in S(X) : \ \sum_{k=1}^{\infty} k^{-s} \left[M_k\left(q\left(\frac{\Delta_v^m x_k}{\rho}\right)\right)\right]^{p_k} < \infty \right.$$
$$\left. \text{for some } \ \rho > 0\right\}.$$

We get the following sequence spaces from $\ell_{\mathbf{M}}(\Delta_v^m, p, q, s)$ giving particular values to p and s. Taking $p_k = 1$ for all $k \in \mathbb{N}$ we have

$$\ell_{\mathbf{M}}(\Delta_v^m, q, s) = \Big\{ x \in S(X) : \sum_{k=1}^{\infty} k^{-s} \Big[M_k \Big(q \Big(\frac{\Delta_v^m x_k}{\rho} \Big) \Big) \Big] < \infty \quad \text{for some } \ \rho > 0 \Big\}.$$

If we take $s = 0$, then we have

$$\ell_{\mathbf{M}}(\Delta_v^m, p, q) = \Big\{ x \in S(X) : \sum_{k=1}^{\infty} \Big[M_k \Big(q \Big(\frac{\Delta_v^m x_k}{\rho} \Big) \Big) \Big]^{p_k} < \infty \ \text{ for some } \ \rho > 0 \Big\}.$$

If we take $p_k = 1$ for all $k \in \mathbb{N}$ and $s = 0$, then we have

$$\ell_{\mathbf{M}}(\Delta_v^m, q) = \Big\{ x \in S(X) : \sum_{k=1}^{\infty} \Big[M_k \Big(q \Big(\frac{\Delta_v^m x_k}{\rho} \Big) \Big) \Big] < \infty \ \text{ for some } \ \rho > 0 \Big\}.$$

In addition to the above sequence spaces, we have $\ell_{\mathbf{M}}(\Delta_v^m, p, q, s) = \ell_M(p)$ due to Parashar and Choudhary [7], taking $m = 0$, $s = 0$, $q(x) = |x|$, $(M_k) = M$ for all $k \in \mathbb{N}$, $v_k = 1$ for all $k \in \mathbb{N}$ and $X = \mathbb{C}$.

Definition 1.1. ([4])

(i) A sequence space E is said to be symmetric if $(x_{\pi(k)}) \in E$ whenever $(x_k) \in E$, where π is permutation of $\mathbb{N}$.

(ii) A sequence space E is said to be convergence free if $(y_k) \in E$ whenever $(x_k) \in E$ and $y_k = \theta$ when $x_k = \theta$.

(iii) A sequence space E is said to be solid (or normal) if $(\alpha_k x_k) \in E$ whenever $(x_k) \in E$ for all sequences (α_k) of scalars with $|\alpha_k| \le 1$ for all $k \in \mathbb{N}$.

(iv) A sequence space E is said to be monotone if it contains the canonical preimages of all its step-spaces.

For a subsequence J of $\mathbb{N}$ and a sequence space λ, we define λ_J by

$$\lambda_J = \Big\{ x = (x_i) : \big(\exists (y_i) \in \lambda\big)(\forall j \in \mathbb{N})\big((n_j \in J) \implies (x_j = y_{n_j})\big) \Big\}$$

and call λ_J the J-stepspace or J-sectional subspace of λ. If $x_J \in \lambda_J$, then the canonical preimage of x_J is the sequence $\overline{x_J}$ which agrees with x_J on the indices in J and is zero elsewhere. The canonical preimage of λ_J is the space $\overline{\lambda_J}$ containing the canonical preimages of the elements of λ_J ([4]).

It is well known that a sequence space E is normal implies that E is monotone.

The following inequality and $p = (p_k)$ sequence will be used frequently throughout this paper.

$$|a_k + b_k|^{p_k} \le D\big\{ |a_k|^{p_k} + |b_k|^{p_k} \big\}, \tag{1}$$

where $a_k, b_k \in \mathbb{C}$, $0 < p_k \le \sup_k p_k = G$, $D = \max\{1, 2^{G-1}\}$.

2. Main results

In this section we will prove the results of this article involving the sequence space $\ell_{\mathbf{M}}(\Delta_v^m, p, q, s)$.

Theorem 2.1. *The sequence space $\ell_{\mathbf{M}}(\Delta_v^m, p, q, s)$ is a linear space over the field $\mathbb{C}$ of complex numbers.*

Proof. Let $x, y \in \ell_{\mathbf{M}}(\Delta_v^m, p, q, s)$ and $\alpha, \beta \in \mathbb{C}$. Then there exist some positive numbers ρ_1 and ρ_2 such that

$$\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{\Delta_v^m x_k}{\rho_1} \right) \right) \right]^{p_k} < \infty$$

and

$$\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{\Delta_v^m y_k}{\rho_2} \right) \right) \right]^{p_k} < \infty.$$

Define $\rho_3 = \max\{2|\alpha|\rho_1, 2|\beta|\rho_2\}$. Since M_k are non-decreasing convex functions, q is a seminorm and Δ_v^m is linear

$$\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{\Delta_v^m(\alpha x_k + \beta y_k)}{\rho_3} \right) \right) \right]^{p_k}$$
$$\leq D \sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{\Delta_v^m x_k}{\rho_1} \right) \right) \right]^{p_k} + D \sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{\Delta_v^m y_k}{\rho_2} \right) \right) \right]^{p_k} < \infty.$$

This proves that $\ell_{\mathbf{M}}(\Delta_v^m, p, q, s)$ is a linear space. □

Theorem 2.2. *Let $p = (p_k) \in \ell_\infty$ and $s > 1$. The sequence space $\ell_{\mathbf{M}}(\Delta_v^m, p, q, s)$ is a paranormed (not necessarily totally paranormed) space, paranormed by*

$$g_\Delta(x) = \sum_{k=1}^{m} q(x_k) + \inf \left\{ \rho^{p_n/H} : \sup_k \left[M_k \left(q \left(\tfrac{\Delta_v^m x_k}{\rho} \right) \right) \right] \leq 1, \;\; n \in \mathbb{N}, \;\; \rho > 0 \right\}$$

where $H = \max \left\{ 1, \sup_k p_k \right\}$.

Proof. Clearly $g_\Delta(x) = g_\Delta(-x)$. Let $x \in \ell_{\mathbf{M}}(\Delta_v^m, p, q, s)$. Then there exist $\rho_1 > 0$, $\rho_2 > 0$ such that

$$\sup_k \left[M_k \left(q \left(\frac{\Delta_v^m x_k}{\rho_1} \right) \right) \right] \leq 1$$

and

$$\sup_k \left[M_k \left(q \left(\frac{\Delta_v^m y_k}{\rho_2} \right) \right) \right] \leq 1.$$

Let $\rho = \rho_1 + \rho_2$. Then we get the triangle inequality from the following inequality

$$\begin{aligned}
&\sup_k \left[M_k \left(q \left(\frac{\Delta_v^m (x_k + y_k)}{\rho} \right) \right) \right] \\
&\leq \frac{\rho_1}{\rho} \sup_k \left[M_k \left(q \left(\frac{\Delta_v^m x_k}{\rho_1} \right) \right) \right] + \frac{\rho_2}{\rho} \sup_k \left[M_k \left(q \left(\frac{\Delta_v^m y_k}{\rho_2} \right) \right) \right] \leq 1.
\end{aligned}$$

Hence

$$\begin{aligned}
&g_\Delta(x+y) \\
&= \sum_{k=1}^m q(x_k + y_k) + \inf \left\{ \rho^{p_n/H} : \sup_k \left[M_k \left(q \left(\tfrac{\Delta_v^m (x_k+y_k)}{\rho} \right) \right) \right] \leq 1, \ n \in \mathbb{N} \right\} \\
&\leq \sum_{k=1}^m q(x_k) + \sum_{k=1}^m q(y_k) + \inf \Bigg\{ (\rho_1 + \rho_2)^{p_n/H} : \sup_k \left[M_k \left(q \left(\tfrac{\Delta_v^m x_k}{\rho_1} \right) \right) \right] \leq 1, \\
&\qquad\qquad \sup_k \left[M_k \left(q \left(\frac{\Delta_v^m y_k}{\rho_2} \right) \right) \right] \leq 1 \Bigg\} \\
&\leq g_\Delta(x) + g_\Delta(y).
\end{aligned}$$

Since $q(\theta) = 0$ and $M_k(0) = 0$ for all $k \in \mathbb{N}$, we get $\inf\{\rho^{p_n/H}\} = 0$ for $x = \theta$.

Finally, we prove that scalar multiplication is continuous. Let λ be any number. From the linearity of Δ_v^m and the definition,

$$\begin{aligned}
g_\Delta(\lambda x) &= \sum_{k=1}^m q(\lambda x_k) + \inf \left\{ \rho^{p_n/H} : \sup_k \left[M_k \left(q \left(\tfrac{\lambda \Delta_v^m x_k}{\rho} \right) \right) \right] \leq 1, \ n \in \mathbb{N} \right\} \\
&= |\lambda| \sum_{k=1}^m q(x_k) + \inf \left\{ (|\lambda|\, r)^{p_n/H} : \sup_k \left[M_k \left(q \left(\tfrac{\Delta_v^m x_k}{r} \right) \right) \right] \leq 1, \ n \in \mathbb{N} \right\} \\
&= \lambda g_\Delta(x)
\end{aligned}$$

where $r = \rho / |\lambda|$.

Now it can be easily verified that $\lambda \to 0$ and x fixed implies $g_\Delta(\lambda x) \to 0$; λ fixed and $x \to \theta$ implies $g_\Delta(\lambda x) \to 0$ in $\ell_{\mathbf{M}}(\Delta_v^m, p, q, s)$; $\lambda \to 0$ and $x \to \theta$ implies $g_\Delta(\lambda x) \to 0$ in $\ell_{\mathbf{M}}(\Delta_v^m, p, q, s)$. □

Theorem 2.3. *Let $\mathbf{M} = (M_k)$ and $\mathbf{T} = (T_k)$ be any two sequences of Orlicz functions and s, s_1, s_2 be non-negative real numbers. Then we have*

(i) $\ell_{\mathbf{M}}(\Delta_v^m, p, q, s) \cap \ell_{\mathbf{T}}(\Delta_v^m, p, q, s) \subseteq \ell_{\mathbf{M}+\mathbf{T}}(\Delta_v^m, p, q, s)$.

(ii) *If $s_1 \leq s_2$, then* $\ell_{\mathbf{M}}(\Delta_v^m, p, q, s_1) \subseteq \ell_{\mathbf{M}}(\Delta_v^m, p, q, s_2)$.

P r o o f.

(i) From (1) we have

$$\begin{aligned}
&k^{-s}\left[(M_k+T_k)\left(q\left(\frac{\Delta_v^m x_k}{\rho}\right)\right)\right]^{p_k}\\
&= k^{-s}\left[M_k\left(q\left(\frac{\Delta_v^m x_k}{\rho}\right)\right)+T_k\left(q\left(\frac{\Delta_v^m x_k}{\rho}\right)\right)\right]^{p_k}\\
&\le Dk^{-s}\left[M_k\left(q\left(\frac{\Delta_v^m x_k}{\rho}\right)\right)\right]^{p_k}+Dk^{-s}\left[T_k\left(q\left(\frac{\Delta_v^m x_k}{\rho}\right)\right)\right]^{p_k}.
\end{aligned}$$

Let $x \in \ell_{\mathbf{M}}(\Delta_v^m, p, q, s) \cap \ell_{\mathbf{T}}(\Delta_v^m, p, q, s)$; when adding the above inequality from $k = 1$ to ∞, we get $x \in \ell_{\mathbf{M}+\mathbf{T}}(\Delta_v^m, p, q, s)$.

(ii) Let $s_1 \le s_2$ and $x \in \ell_{\mathbf{M}}(\Delta_v^m, p, q, s_1)$. Since $k^{-s_2} \le k^{-s_1}$, we have $x \in \ell_{\mathbf{M}}(\Delta_v^m, p, q, s_2)$. □

Theorem 2.4. *Let $m \ge 1$, then the inclusion $\ell_{\mathbf{M}}(\Delta_v^{m-1}, q, s) \subset \ell_{\mathbf{M}}(\Delta_v^m, q, s)$ is strict. In general $\ell_{\mathbf{M}}(\Delta_v^i, q, s) \subset \ell_{\mathbf{M}}(\Delta_v^m, q, s)$ for all $i = 0, 1, 2, \ldots, m-1$ and the inclusion is strict.*

P r o o f. Let $x \in \ell_{\mathbf{M}}(\Delta_v^{m-1}, q, s)$. Then we have

$$\sum_{k=1}^{\infty} k^{-s}\left[M_k\left(q\left(\frac{\Delta_v^{m-1}x_k}{\rho}\right)\right)\right] < \infty \tag{2}$$

for some $\rho > 0$. Since M_k are non-decreasing convex functions and q is a seminorm, we have

$$\begin{aligned}
&\sum_{k=1}^{\infty} k^{-s}\left[M_k\left(q\left(\frac{\Delta_v^m x_k}{2\rho}\right)\right)\right]\\
&= \sum_{k=1}^{\infty} k^{-s}\left[M_k\left(q\left(\frac{\Delta_v^{m-1}x_k-\Delta_v^{m-1}x_{k+1}}{2\rho}\right)\right)\right]\\
&\le \sum_{k=1}^{\infty} k^{-s}\left[M_k\left(q\left(\frac{\Delta_v^{m-1}x_k}{\rho}\right)\right)\right]+\sum_{k=1}^{\infty} k^{-s}\left[M_k\left(q\left(\frac{\Delta_v^{m-1}x_{k+1}}{\rho}\right)\right)\right]\\
&< \infty \qquad \text{by (2)}.
\end{aligned}$$

Thus $\ell_{\mathbf{M}}(\Delta_v^{m-1}, q, s) \subset \ell_{\mathbf{M}}(\Delta_v^m, q, s)$. Proceeding in this way one will have $\ell_{\mathbf{M}}(\Delta_v^i, q, s) \subset \ell_{\mathbf{M}}(\Delta_v^m, q, s)$ for all $i = 0, 1, 2, \ldots, m-1$. The inclusion is strict. Indeed, let $X = \mathbb{C}$, $M_k(x) = x^p$ for all $k \in \mathbb{N}$, $q(x) = |x|$, $s = 0$ and $v_k = 1$ for all $k \in \mathbb{N}$. Consider the sequence $(x_k) = (k^{m-1})$. Then $(x_k) \in \ell_{\mathbf{M}}(\Delta_v^m, q, s)$, but $(x_k) \notin \ell_{\mathbf{M}}(\Delta_v^{m-1}, q, s)$ since $\Delta_v^m x_k = 0$ and $\Delta_v^{m-1}x_k = (-1)^{m-1}(m-1)!$ for all $k \in \mathbb{N}$. □

Theorem 2.5. *The space $\ell_{\mathbf{M}}(\Delta_v^m, p, q, s)$ is not convergence free and symmetric.*

Proof. It can be proved as in [9]. □

Theorem 2.6. *Let $0 < p_k \le t_k < \infty$ for each $k \in \mathbb{N}$. Then $\ell_{\mathbf{M}}(\Delta_v^m, p, q) \subseteq \ell_{\mathbf{M}}(\Delta_v^m, t, q)$.*

Proof. Let $x \in \ell_{\mathbf{M}}(\Delta_v^m, p, q)$. Then there exists some $\rho > 0$ such that

$$\sum_{k=1}^{\infty} \left[M_k \left(q \left(\frac{\Delta_v^m x_k}{\rho} \right) \right) \right]^{p_k} < \infty.$$

This implies that $M_k \left(q \left(\frac{\Delta_v^m x_k}{\rho} \right) \right) \le 1$ for sufficiently large values of k, say $k \ge k_0$ for some fixed $k_0 \in \mathbb{N}$. Since $p_k \le t_k$ for each $k \in \mathbb{N}$, we get

$$\left[M_k \left(q \left(\frac{\Delta_v^m x_k}{\rho} \right) \right) \right]^{t_k} \le \left[M_k \left(q \left(\frac{\Delta_v^m x_k}{\rho} \right) \right) \right]^{p_k}$$

for all $k \ge k_0$ and therefore

$$\sum_{k \ge k_0} \left[M_k \left(q \left(\frac{\Delta_v^m x_k}{\rho} \right) \right) \right]^{t_k} \le \sum_{k \ge k_0} \left[M_k \left(q \left(\frac{\Delta_v^m x_k}{\rho} \right) \right) \right]^{p_k} < \infty.$$

Hence $x \in \ell_{\mathbf{M}}(\Delta_v^m, t, q)$. □

The following result is a consequence of Theorem 2.6.

Corollary 2.7.

(i) *If $0 < p_k \le 1$ for each $k \in \mathbb{N}$, then $\ell_{\mathbf{M}}(\Delta_v^m, p, q) \subseteq \ell_{\mathbf{M}}(\Delta_v^m, q)$.*

(ii) *If $p_k \ge 1$ for all $k \in \mathbb{N}$, then $\ell_{\mathbf{M}}(\Delta_v^m, q) \subseteq \ell_{\mathbf{M}}(\Delta_v^m, p, q)$.*

Theorem 2.8. *The sequence space $\ell_{\mathbf{M}}(p, q, s)$ is solid.*

Proof. Let $(x_k) \in \ell_{\mathbf{M}}(p, q, s)$, i.e.

$$\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{x_k}{\rho} \right) \right) \right]^{p_k} < \infty.$$

Let (α_k) be a sequence of scalars such that $|\alpha_k| \le 1$ for all $k \in \mathbb{N}$. Then the result follows from the following inequality

$$\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{\alpha_k x_k}{\rho} \right) \right) \right]^{p_k} \le \sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{x_k}{\rho} \right) \right) \right]^{p_k}.$$

□

Corollary 2.9. *The sequence space $\ell_{\mathbf{M}}(p, q, s)$ is monotone.*

REFERENCES

[1] BEKTAŞ, Ç. A.—ALTIN, Y.: *The sequence space $\ell_M(p,q,s)$ on seminormed spaces*, Indian J. Pure Appl. Math. **34** (2003), 529–534.

[2] ET, M.—ÇOLAK, R.: *On some generalized difference sequence spaces*, Soochow J. Math. **21** (1995), 377–386.

[3] ET, M.—ESI, A.: *On Köthe-Toeplitz duals of generalized difference sequence spaces*, Bull. Malays. Math. Sci. Soc. (2) **23** (2000), 25–32.

[4] KAMTHAN, P. K.—GUPTA, M.: *Sequence Spaces and Series*, Marcel Dekker, New York, 1981.

[5] KIZMAZ, H.: *On certain sequence spaces*, Canad. Math. Bull. **24** (1981), 169–176.

[6] LINDENSTRAUSS, J.—TZAFRIRI, L.: *On Orlicz sequence spaces*, Israel J. Math. **10** (1971), 379–390.

[7] PARASHAR, S. D.—CHOUDHARY, B.: *Sequence spaces defined by Orlicz functions*, Indian J. Pure Appl. Math. **25** (1994), 419–428.

[8] TRIPATHY, B. C.: *On generalized difference paranormed statistically convergent sequences*, Indian J. Pure Appl. Math. **35** (2004), 655–663.

[9] TRIPATHY, B. C.—ALTIN, Y.—ET, M.: *Generalized difference sequence spaces on seminormed space defined by Orlicz functions*, Math. Slovaca **58** (2008), 315–324.

[10] TRIPATHY, B. C.—ET, M.—ALTIN, Y.: *Generalized difference sequence spaces defined by Orlicz function in a locally convex space*, J. Anal. Appl. **1** (2003), 175–192.

[11] TRIPATHY, B. C.—MAHANTA, S.: *On a class of sequences related to the ℓ^p spaces defined by Orlicz functions*, Soochow J. Math. **29** (2003), 379–391.

[12] TRIPATHY, B. C.—MAHANTA, S.: *On a class of generalized lacunary difference sequence spaces defined by Orlicz functions*, Acta Math. Appl. Sinica (English Ser.) **20** (2004), 231–238.

Received 27. 11. 2008
Accepted 23. 1. 2009

Department of Mathematics
Firat University
TR–23119, Elazig
TURKEY
E-mail: cigdemas78@hotmail.com