VERSITA

Mathematica Slovaca

DOI: 10.2478/s12175-010-0014-x
Math. Slovaca **60** (2010), No. 3, 319–326

ON THE SPARSE SET TOPOLOGY

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(*Communicated by David Buhagiar*)

ABSTRACT. In this paper we examine the regularity and pseudo-completeness of the sparse set topology introduced in [EAMES, W.: *Local property of measurable sets*, Canad. J. Math. **12** (1960), 632–640].

1. Introduction

The idea of sparse sets and proximal continuous mappings were first introduced by Sarkhel and De [13] in the real numbers. Subsequently in [2], the concept of sparse sets was studied in a topological group where a topology was generated with the help of these sets which was named the sparse set topology.

In 2002 the idea of sparse sets was considered by Das and Rashid [3] in a metric space where the density function (Eames [4]) is different from the others in the sense that it happens to exceed one sometimes. It was observed in [3] that the corresponding sparse set topology is also different from that of [2] in many respects.

In this paper we first examine some further properties of this topology and primarily show that the sparse set topology is regular and pseudo-complete under some general conditions.

2. Preliminaries

Let (X, ρ) be a metric space. Let $\mathcal{C}$ be a class of closed sets from (X, ρ) and τ be a non-negative real valued function on $\mathcal{C}$. We assume that the empty set $\emptyset$ and all the singleton sets are in $\mathcal{C}$, finite union of members of $\mathcal{C}$ is in $\mathcal{C}$ and that $\tau(I) = 0$ if and only if I contains at most one point. For each $A \subset X$,

2000 Mathematics Subject Classification: Primary 28D99, 54C35, 54E99.
Keywords: sparse set topology, regularity, pseudo-completeness.

let $\mu(A)$, $0 \leq \mu(A) \leq \infty$, be defined by $\mu(A) = \lim_{\varepsilon \to 0+} \left[\inf \sum_{n=1}^{\infty} \tau(I_n)\right]$ where the infimum is taken over all possible countable collection of sets I_n from $\mathcal{C}$ such that $A \subset \bigcup_{n=1}^{\infty} I_n$ and the diameter of I_n, $\operatorname{diam}(I_n) < \varepsilon$ for all n. As in Eames [5] we assume that such a countable collection of sets from $\mathcal{C}$ exists for each set A and for every $\varepsilon > 0$. Then μ is an outer measure function [12, p. 35]. A set A is measurable if $\mu(B) = \mu(A \cap B) + \mu(A^c \cap B)$ for every $B \in \mathcal{C}$ where c stands for the complement. All Borel sets of (X, ρ) are measurable, cf. [12, pp. 102–106]. For every set A in X there is a measurable set B, called a measurable cover for A such that $A \subset B$ and $\mu(A) = \mu(B)$ ([12, pp. 107–108]) and so μ is a regular outer measure function.

Definition 1. ([4]) Let $A \subset X$ and $p \in X$. Then the number $D(A, p)$, $0 \leq D(A, p) \leq \infty$, called the density of A at p, is defined by

$$D(A, p) = \lim_{\varepsilon \to 0+} \left[\sup \frac{\mu(A \cap I)}{\tau(I)}\right]$$

where the supremum is taken over all sets I from $\mathcal{C}$ such that $p \in I$ and $\operatorname{diam}(I) < \varepsilon$. When $\tau(I) = 0$ or ∞ we take $\frac{\mu(A \cap I)}{\tau(I)} = 0$.

In [4] it was shown that if the sets in $\mathcal{C}$ satisfy certain regularity conditions and $\mu(A)$ is finite then

(i) $D(A, p) = 1$ for almost all $p \in A$,

(ii) $D(A, p) = 0$ for almost all $p \in A^c$ if and only if A is measurable.

The set function $D(\cdot, p)$ for a fixed $p \in X$ is monotone nondecreasing, finitely subadditive. Further if E, F are measurable then $D(E, p) + D(F, p) = D(E \cup F, p)$ a.e. in X provided $E \cap F = \emptyset$ and also $D(E, p) + D(E^c, p) = D(X, p)$ a.e. in X ([8]).

We now recall the definition of density topology on X.

Definition 2. ([7]) Let $\mathcal{D} = \{U \subset X : D(X - U, p) = 0 \text{ for all } p \in U\}$. Then $\mathcal{D}$ is a topology on X called the density topology (in short d-topology) and thus $(X, \mathcal{D})$ is a topological space. Sets in $\mathcal{D}$ are called d-open.

Open sets of (X, ρ) are d-open ([7]).

The following results of density topology will be needed.

Theorem 1. ([7]) *If E is measurable then the set $\{x \in E : D(X - E, x) = 0\}$ is the d-interior of E.*

Theorem 2. ([7]) *For any set E, $x \in X$ is a d-limit point of E if and only if $D(E, x) > 0$.*

Theorem 3. ([7]) *$\mu(E) = 0$ if and only if E is closed and discrete.*

The definition of sparse sets in a metric space was given as follows.

Definition 3. ([3]) A set $E \subset X$ is said to be sparse at a point $x \in X$ where $D(X,x) \neq 0$ if for every set $F \subset X$ with $D(F,x) < D(X,x)$ we have $D(E \cup F, x) < D(X,x)$. If $x \in X$ is such that $D(X,x) = 0$ then any set $E \subset X$ is said to be sparse at x. The collection of all sets sparse at x will be denoted by $S(x)$.

From the definition it is clear that $D(E,x) = 0$ implies $E \in S(x)$ and for any $x \in X, S(x)$ is a hereditary ring i.e., $E_1, E_2 \in S(x)$ implies $E_1 \cup E_2 \in S(x)$ and $E \in S(x)$, $A \subset E$ implies $A \in S(x)$ ([4]). Also every measurable cover of $E \in S(x)$ belongs to $S(x)$ ([4]).

Definition 4. ([3]) Let $\mathcal{T} = \{E : \ E \subset X \text{ and } E^c \in S(x) \text{ for all } x \in E\}$. Then $\mathcal{T}$ is a topology on X (see [3, Theorem 4]). The topology thus obtained is called the sparse set topology or in short s-topology. The sets of $\mathcal{T}$ are called s-open.

The s-topology is finer than d-topology and if E is s-open, $x \in E$ then $D(E,x) = D(X,x)$. If for some $x \in X$ with $D(X,x) \neq 0$, $D(E^c,x) < D(X,x)$ then x is a s-limit point of E. Also compact sets in s-topology are finite ([3]).

Throughout the paper, as in Definition 4, in $D(X,p) = \lim_{n\to\infty} \sup \frac{\mu(I_n)}{\tau(I_n)}$, the supremum will be taken over all closed sets I_n from $\mathcal{C}$ with $p \in I_n$ and $\operatorname{diam}(I_n) < \frac{1}{n}$ unless otherwise mentioned.

3. Regularity of $(X, \mathcal{T})$

In this section we shall show that under certain conditions, $(X, \mathcal{T})$ is regular. We first prove the following results which will be needed in Theorem 4.

Lemma 1. *Open sets in (X, ρ) are s-open.*

The proof is omitted.

Lemma 2. *If E is measurable then the set $H = \{x \in E : \ E^c \in S(x)\}$ is the s-interior of E.*

Proof. Clearly $H \subset E$. Let $H_1 = \{x \in E : \ D(X - E, x) = 0\}$. Since E is measurable, $\mu(E - H_1) = 0$. Now $H_1 \subset H$ and so $E - H \subset E - H_1$ which implies $\mu(E - H) = 0$. We first show that H is s-open. Let $x \in H$, and $F \subset X$ be such that $D(F,x) < D(X,x)$. Since $x \in H$, $E^c \in S(x)$ and so $D(F \cup E^c, x) < D(X,x)$. Now $H^c = E^c \cup (E - H)$. Therefore

$$\begin{aligned} D(F \cup H^c, x) &= D(F \cup E^c \cup (E - H), x) \\ &\leq D(F \cup E^c, x) + D(E - H, x) \\ &= D(F \cup E^c, x) \qquad (\text{since } \mu(E-H) = 0, \ D(E-H,x) = 0) \\ &< D(X,x) \end{aligned}$$

This shows that $H^c \in S(x)$ and consequently H is s-open.

Next let U be any s-open set contained in E. From definition $U^c \in S(x)$ for all $x \in U$. Since $E^c \subset U^c$ and $S(x)$ is hereditary, $E^c \in S(x)$ for all $x \in U$. Consequently $U \subset H$. Thus H is the s-interior of E. □

We now deduce the main result of this section.

Theorem 4. (cf. [7, Theorem 12]) $(X, \mathcal{T})$ *is regular provided*

(A) $D(X, p) < \infty$ *for all* $p \in X$ *and*

(B) $A \in S(p)$ *if there exists a sequence* $\{I_n\}$ *from* $\mathcal{C}$, $p \in I_n$ *and* $\operatorname{diam}(I_n) < \frac{1}{n}$ *for all* n *such that* $\frac{\mu(A \cap I_n)}{\tau(I_n)} \to 0$ *as* $n \to \infty$.

Proof. Let $p \in X$ and F be a s-closed set such that $p \notin F$. Then $p \in X - F = U$ (say) where U is s-open. Let $D(X, p) = l$.

Case I.

If $l = 0$ then any set is sparse at p and so $X - \{p\} \in S(p)$. Hence $\{p\}$ is s-open. Also since $\{p\}$ is closed in (X, ρ), $\{p\}$ is s-closed. Therefore, $X - \{p\}$ is s-open. Thus $p \in \{p\}$, $F \subset X - \{p\}$ and $\{p\} \cap X - \{p\} = \emptyset$ and so $\mathcal{T}$ is regular.

Case II.

Let $l > 0$. Now since $U = X - F$ is s-open and $p \in U$, $D(U, p) = D(X, p) = l$. Now $D(X, p) = l$ implies $\lim_{n\to\infty} \sup \frac{\mu(I_n)}{\tau(I_n)} = l$. For each fixed n, put $\sup_{\operatorname{diam}(I_n) < \frac{1}{n}} \frac{\mu(I_n)}{\tau(I_n)} = b_n$.

Then $b_n \to l$ as $n \to \infty$. Since $\{b_n\}$ is non-increasing and (A) holds, we can assume without any loss of generality that $l \le b_n < \infty$ for all n.

Again since $D(U, p) = l$, if we write

$$\sup_{\operatorname{diam}(I_n) < \frac{1}{n}} \frac{\mu(U \cap I_n)}{\tau(I_n)} = a_n$$

where $I_n \in \mathcal{C}$ and $p \in I_n$ for all n, then $a_n \to l$ as $n \to \infty$. So for each $n \in \mathbb{N}$, we can find a closed set I_n in $\mathcal{C}$ with $p \in I_n$ and $\operatorname{diam}(I_n) < \frac{1}{n}$ such that $\mu(U \cap I_n) > (a_n - \frac{1}{n^2})\tau(I_n)$.

Since μ is regular, there is a compact set C_n in (X, ρ) such that $C_n \subset U \cap I_n$ and $\mu(U \cap I_n) < \mu(C_n) + \frac{1}{n^2}\tau(I_n)$, i.e. $(a_n - \frac{1}{n^2})\tau(I_n) < \mu(U \cap I_n) < \mu(C_n) + \frac{1}{n^2}\tau(I_n)$, i.e.

$$\mu(C_n) > \left(a_n - \frac{2}{n^2}\right)\tau(I_n). \tag{1}$$

Let $C = \bigcup_{n=1}^{\infty} C_n$ and $E = C \cup \{p\}$. Now C_n being a compact subset is closed and so $C_n \cap I_n$, $(X - C_n) \cap I_n$ are all Borel sets which are measurable.

Then

$$\mu(C_n \cap I_n) + \mu\left((X - C_n) \cap I_n\right) = \mu(I_n)$$

i.e.,

$$\frac{\mu(C_n \cap I_n)}{\tau(I_n)} + \frac{\mu((X - C_n) \cap I_n)}{\tau(I_n)} = \frac{\mu(I_n)}{\tau(I_n)} \le b_n$$

i.e.,

$$\frac{\mu(C_n)}{\tau(I_n)} + \frac{\mu((X - C_n) \cap I_n)}{\tau(I_n)} \le b_n \qquad (\text{since} \quad C_n \subset I_n)$$

Using (1) we now have

$$\left(a_n - \frac{2}{n^2}\right) + \frac{\mu((X - C_n) \cap I_n)}{\tau(I_n)} < b_n$$

i.e.

$$\frac{\mu((X - C_n) \cap I_n)}{\tau(I_n)} < b_n - a_n + \frac{2}{n^2}$$

for all $n \in \mathbb{N}$.

This gives

$$\lim_{n \to \infty} \frac{\mu((X - C_n) \cap I_n)}{\tau(I_n)} = 0$$

and so

$$\lim_{n \to \infty} \frac{\mu((X - C) \cap I_n)}{\tau(I_n)} = 0 \tag{2}$$

since $X - C \subset X - C_n$ for all $n \in \mathbb{N}$. From (2) and condition (B) it follows that $X - C \in S(p)$. So $X - E \in S(p)$, since $S(p)$ is hereditary. So by Lemma 2, p is an s-interior point of E and we can find an s-open set G such that $p \in G \subset E$.

We now prove that E^c is s-open. Let $y \notin E$. Since $p \neq y$, we can find a $\delta > 0$ such that $B(p, \delta) \cap B(y, \delta) = \emptyset$ where $B(p, \delta)$ and $B(y, \delta)$ denote the open balls centered at p and y respectively and radius δ. Let $k \in \mathbb{N}$ be such that $\frac{1}{k} < \delta$. Then $C_n \subset I_n \subset B(p, \delta)$ for all $n \ge k$ (since $\operatorname{diam}(I_n) < \frac{1}{n}$). We write $C = C_1' \cup C_2'$ where $C_1' = \bigcup_{i=1}^{k-1} C_i$ and $C_2' = \bigcup_{i=k}^{\infty} C_i$, C_1' being a finite union of closed sets is closed in (X, ρ) and so is d-closed. Since $y \notin C_1'$, y is not a d-limit point C_1'. Also clearly $D(C_2', y) = 0$. So by Theorem 2, y is not a d-limit point of C_2'. Hence y is not a d-limit point of C which implies that $D(C, y) = 0$ (by Theorem 2). Since $\mu(\{p\}) = 0$, $D(\{p\}, y) = 0$ and so $D(E, y) \le D(C, y) + D(\{p\}, y) = 0$ i.e. $D(E, y) = 0$ for all $y \in E^c$ which implies that E^c is d-open and so s-open. Clearly $E \subset U$ and so $E^c \supset F = X - U$ and $p \in G$ such that $G \cap E^c = \emptyset$. This completes the proof. □

Definition 5. ([12]) A topological space is quasi-regular if every non empty open set contains the closure of some non-void open set.

Corollary 1. *Under the conditions* (A) *and* (B) *of* Theorem 4, $(X, \mathcal{T})$ *is quasi-regular.*

4. Pseudo-completeness of $(X, \mathcal{T})$

In this section we investigate the pseudo-completeness of $(X, \mathcal{T})$. The following definitions are needed.

Definition 6. ([12]) A pseudo-base for a topological space (X, τ') is a subset $\mathcal{B}$ of τ' such that every non void element of τ' contains a non-void element of $\mathcal{B}$.

Definition 7. ([9]) A topological space X is pseudo-complete if

(i) it is quasi-regular,

(ii) there exists a sequence $\{P_i\}$ of pseudo-bases for X such that for every sequence of sets $\{U_i\}$ where $U_i \in P_i$ and $\overline{U}_{i+1} \subset U_i$ for all i, we have $\bigcap\limits_i U_i \neq \emptyset$, where bar denotes the closure.

For the next theorem we note that every proper subset of X has a measurable cover $F \neq X$.

Theorem 5. *For a compact metric space* (X, ρ), $(X, \mathcal{T})$ *is pseudo-complete if in addition to the conditions* (A) *and* (B) *of* Theorem 4, τ *is monotone non-decreasing and* $D(X, p) > 0$ *for all* $p \in X$.

Proof.

Step 1. We first construct a sequence of pseudo-bases $\{\mathcal{B}_n\}$ of $(X, \mathcal{T})$. Let $U\ (\neq X) \in \mathcal{T}$. Then $U^c \in S(x)$ for all $x \in U$. Let F be a measurable cover of U^c (if U^c is measurable we take $F = U^c$). Then $F \in S(x)$ for all $x \in U$ and so for all $x \in F^c$ (since $F^c \subset U$). Hence $F^c \in \mathcal{T}$ and $F^c \subset U$. Let $x \in F^c$. For any $n \in \mathbb{N}$ let $B(x, \frac{1}{6n})$ be the ρ-open ball centred at x with radius $\frac{1}{6n}$. $B(x, \frac{1}{6n})$ is obviously $\mathcal{T}$-open. Let $V = B(x, \frac{1}{6n}) \cap F^c$. Then V is $\mathcal{T}$-open, $V \subset U$ and $\operatorname{diam}(V) < \frac{1}{2n}$. For each fixed n let $\mathcal{B}_n$ consists of all such V's as U runs over $\mathcal{T}$. Then $\mathcal{B}_n$ is a pseudo-base of $(X, \mathcal{T})$.

Step 2. Let $V \in \mathcal{B}_n$. Then $D(V, x) = D(X, x) = l$ (say) where x is the point used in step 1.

Now $D(X, x) = \lim\limits_{k\to\infty} \sup \frac{\mu(I_k)}{\tau(I_k)} = l$. We can choose a positive integer m such that

$$\sup \frac{\mu(I_k)}{\tau(I_k)} < l + \frac{1}{2n^2}$$

for all $k \geq m$.

Again let $a_k = \sup \frac{\mu(V \cap I_k)}{\tau(I_k)}$. Clearly we can find a $I_k \in \mathcal{C}$ such that $x \in I_k$, $\operatorname{diam}(I_k) < \frac{1}{k}$ and $\frac{\mu(V \cap I_k)}{\tau(I_k)} > a_k - \frac{1}{k^2}$.

Now for $k \geq m$, we have both

$$\frac{\mu(V \cap I_k)}{\tau(I_k)} > a_k - \frac{1}{k^2} \tag{3}$$

and

$$\frac{\mu(I_k)}{\tau(I_k)} \le \sup \frac{\mu(I_k)}{\tau(I_k)} < l + \frac{1}{2n^2}. \tag{4}$$

Now since V is measurable,

$$\mu(V \cap I_k) + \mu\left[(X - V) \cap I_k\right] = \mu(I_k)$$

i.e.

$$\frac{\mu(V \cap I_k)}{\tau(I_k)} + \frac{\mu\left[(X - V) \cap I_k\right]}{\tau(I_k)} = \frac{\mu(I_k)}{\tau(I_k)}$$

i.e.

$$\frac{\mu[(X - V) \cap I_k]}{\tau(I_k)} = \frac{\mu(I_k)}{\tau(I_k)} - \frac{\mu(V \cap I_k)}{\tau(I_k)} < l + \frac{1}{2n^2} - a_k + \frac{1}{k^2}.$$

As $k \to \infty$ we get $\lim\limits_{k\to\infty} \frac{\mu[(X-V)\cap I_k]}{\tau(I_k)} \le \frac{1}{2n^2} < \frac{1}{n^2}$. Thus we can find a sequence of closed sets $\{I_k\}$ from $\mathcal{C}$ with $\mathrm{diam}(I_k) < \frac{1}{k}$ such that $\lim\limits_{k\to\infty} \frac{\mu[(X-V)\cap I_k]}{\tau(I_k)} < \frac{1}{n^2}$ and consequently we can find a closed set A with $\mathrm{diam}(A) < \frac{1}{6n}$ from $\mathcal{C}$ such that

$$\frac{\mu[(X - V) \cap A]}{\tau(A)} < \frac{1}{n^2}. \tag{5}$$

Step 3. Let $\{U_n\}$ be a sequence of non void sets such that $U_n \in \mathcal{B}_n$ for each n and s-$\mathrm{cl}(U_{n+1}) \subset U_n$ for all n. So $U_{n+1} \subset U_n$ for all $n \in \mathbb{N}$. Now $\overline{U}_1$ being a closed subset of a compact metric space is compact. Also $\{\overline{U}_n\}$ is a collection of closed subsets of $\overline{U}_1$ with finite intersection property. So $\bigcap\limits_{n=1}^{\infty} \overline{U}_n \ne \emptyset$. Since $\mathrm{diam}(\overline{U}_n) \to 0$ as $n \to \infty$, $\bigcap\limits_{n=1}^{\infty} \overline{U}_n = \{x_0\}$ (say). Now by (5) for each n there is a set A_n in $\mathcal{C}$ such that $A_n \cap \overline{U}_n \ne \emptyset$, $\mathrm{diam}(A_n) < \frac{1}{6n}$ and $\frac{\mu[(X-U_n)\cap A_n]}{\tau(A_n)} < \frac{1}{n^2}$.

Let $B_n = A_n \cup \{x_0\}$. Then $\{B_n\}$ is a sequence of closed sets of $\mathcal{C}$ containing x_0. Since for every $y \in A_n$,

$$\rho(y, x_0) \le \rho(y, x) + \rho(x, x_0) < \frac{1}{6n} + \frac{1}{2n} < \frac{1}{n}$$

where $x \in A_n \cap \overline{U}_n$, we have $\mathrm{diam}(B_n) < \frac{1}{n}$. Now for each fixed k,

$$\begin{aligned}\frac{\mu[(X - U_k) \cap B_n]}{\tau(B_n)} &\le \frac{\mu[(X - U_k) \cap A_n]}{\tau(A_n)} \\ &\le \frac{\mu[(X - U_n) \cap A_n]}{\tau(A_n)} < \frac{1}{n^2} \qquad \text{for all} \quad n \ge k\end{aligned}$$

Hence

$$\frac{\mu[(X - U_k) \cap B_n]}{\tau(B_n)} \to 0 \qquad \text{as} \quad n \to \infty$$

and so by (B), $X - U_k \in S(x_0)$. Then $D(X - U_k, x_0) < D(X, x_0)$ which implies $x_0 \in s\text{-cl}(U_k)$ for all $k \in \mathbb{N}$. Since $s\text{-cl}(U_{k+1}) \subset U_k$ for all k, $x_0 \in \bigcap_{k=1}^{\infty} U_k$ and this completes the proof. □

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Received 1. 9. 2008
Accepted 11. 12. 2008

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