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ON THE DISTRIBUTION OF REDUCIBLE POLYNOMIALS

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ABSTRACT. Let $\mathscr{R}_n(t)$ denote the set of all reducible polynomials $p(X)$ over $\mathbb{Z}$ with degree $n \geq 2$ and height $\leq t$. We determine the *true order of magnitude* of the cardinality $|\mathscr{R}_n(t)|$ of the set $\mathscr{R}_n(t)$ by showing that, as $t \to \infty$, $t^2 \log t \ll |\mathscr{R}_2(t)| \ll t^2 \log t$ and $t^n \ll |\mathscr{R}_n(t)| \ll t^n$ for every fixed $n \geq 3$. Further, for $1 < \frac{n}{2} < k < n$ fixed let $\mathscr{R}_{k,n}(t) \subset \mathscr{R}_n(t)$ such that $p(X) \in \mathscr{R}_{k,n}(t)$ if and only if $p(X)$ has an irreducible factor in $\mathbb{Z}[X]$ of degree k. Then, as $t \to \infty$, we always have $t^{k+1} \ll |\mathscr{R}_{k,n}(t)| \ll t^{k+1}$ and hence $|\mathscr{R}_{n-1,n}(t)| \gg |\mathscr{R}_n(t)|$ so that $\mathscr{R}_{n-1,n}(t)$ is the dominating subclass of $\mathscr{R}_n(t)$ since we can show that $|\mathscr{R}_n(t) \setminus \mathscr{R}_{n-1,n}(t)| \ll t^{n-1}(\log t)^2$. On the contrary, if $R_n^s(t)$ is the total number of all polynomials in $\mathscr{R}_n(t)$ which split completely into linear factors over $\mathbb{Z}$, then $t^2(\log t)^{n-1} \ll R_n^s(t) \ll t^2(\log t)^{n-1}$ $(t \to \infty)$ for every fixed $n \geq 2$.

1. Introduction and statement of results

For a fixed integer $n \geq 2$ and a real parameter $t \geq 1$ we consider all polynomials $p(X) = a_n X^n + a_{n-1} X^{n-1} + \cdots + a_2 X^2 + a_1 X + a_0$ with coefficients $a_i \in \mathbb{Z}$ such that $a_n \neq 0$ and $H(p) \leq t$ where $H(p) := \max\{|a_i| : \ i = 0, 1, \ldots, n\}$ is the *height* of $p(X)$. Of course, the total number of all these polynomials equals $[2t] \cdot [2t+1]^n \asymp t^{n+1}$ where $[\,]$ are the Gauss brackets. Let $\mathscr{R}_n(t)$ denote the set of all polynomials $p(X)$ over $\mathbb{Z}$ with degree $n \geq 2$ and height $\leq t$ which are *reducible* over $\mathbb{Q}$. Note that $p(X)$ is reducible in the ring $\mathbb{Q}[X]$ if and only if $p(X)$ can be written as a product of two polynomials in the ring $\mathbb{Z}[X]$ both of less degree than $p(X)$. In the famous exercise book of P o l y a and S z e g ö [3, Example 266] one can find the estimate

$$|\mathscr{R}_n(t)| = O\big(t^n (\log t)^2\big) \qquad (t \to \infty).$$

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By the method used in D ö r g e [2] this estimate can be improved to $|\mathscr{R}_n(t)| = O\big(t^n(\log t)\big)$ which still is not best possible when $n \geq 3$. Indeed, the *true order of magnitude* of the lattice points counting function $t \mapsto |\mathscr{R}_n(t)|$ reads as follows.

Theorem 1. *For every integer $n \geq 3$ there is a constant $C_n > 0$ such that*

$$t^n \leq |\mathscr{R}_n(t)| \leq C_n \cdot t^n \qquad \text{for all} \quad t \geq 1.$$

Theorem 2. *As $t \to \infty$, $t^2 \log t \ll |\mathscr{R}_2(t)| \ll t^2 \log t$.*

There is a natural generalization of Theorem 2. Let $R_n^s(t)$ denote the total number of all polynomials $p(X)$ over $\mathbb{Z}$ with degree n and height $\leq t$ such that $p(X)$ splits completely into linear factors in the ring $\mathbb{Q}[X]$ or, equivalently, in the ring $\mathbb{Z}[X]$. Naturally, $R_2^s(t) = |\mathscr{R}_2(t)|$, so that by Theorem 2 we have $R_2^s(t) \asymp t^2 \log t$ as $t \to \infty$. (We often write $A \asymp B$ iff $B \ll A \ll B$.) Now in general the following estimation holds.

Theorem 3. *For every fixed $n \geq 2$, $t^2(\log t)^{n-1} \ll R_n^s(t) \ll t^2(\log t)^{n-1}$ $(t \to \infty)$.*

Certainly, Theorem 3 is also true in the trivial case $n = 1$ where $R_1^s(t) = [2t] \cdot [2t+1]$ for every $t \geq 1$. (Of course, in general the case $n = 1$ is of no interest since $\mathscr{R}_1(t) = \emptyset$.)

Theorem 3 demonstrates that the totally splitting polynomials contribute only very little to the total number of all reducible polynomials of fixed degree ≥ 3 and bounded height. On the other hand there is a special subclass of $\mathscr{R}_n(t)$ whose contribution to $|\mathscr{R}_n(t)|$ is absolutely dominating. This class lies on top of a hierarchy of pairwise disjoint subclasses of $\mathscr{R}_n(t)$. For $\frac{n}{2} < k < n$ fixed and arbitrary $t \geq 1$ let $\mathscr{R}_{k,n}(t) \subset \mathscr{R}_n(t)$ such that $p(X) \in \mathscr{R}_{k,n}(t)$ if and only if $p(X)$ has an irreducible factor in $\mathbb{Z}[X]$ of degree k. The following theorem shows that $\mathscr{R}_{n-1,n}(t)$ is the mentioned top class.

Theorem 4. *For $1 < \frac{n}{2} < k < n$ fixed we have $t^{k+1} \ll |\mathscr{R}_{k,n}(t)| \ll t^{k+1}$ $(t \to \infty)$. Specifically, $|\mathscr{R}_{n-1,n}(t)| \asymp |\mathscr{R}_n(t)| \asymp t^n$ $(t \to \infty)$ for every $n \geq 3$. Moreover, $|\mathscr{R}_n(t) \setminus \mathscr{R}_{n-1,n}(t)| \ll t^{n-1}(\log t)^2$ $(t \to \infty)$ for every $n \geq 3$ and the factor $(\log t)^2$ can be omitted if and only if $n \geq 4$.*

2. Preparation of the proofs

Since $\mathrm{e}^x > 2^{x-2}\sqrt{x+1}$ for $x \geq 0$, as an immediate consequence of [4, Theorem 4.2.2] we obtain:

Lemma 5. *If p, q are polynomials over $\mathbb{Z}$ with positive degrees $\deg p$ and $\deg q$ such that $n = \deg(pq) = \deg p + \deg q$, then $\mathrm{e}^{-n} H(p)H(q) \leq H(pq) \leq nH(p)H(q)$.*

For $T \geq 1$ consider the hyperbola triangle

$$\mathscr{D}(T) := \big\{(x,y) \in \mathbb{R}^2 : \ x,y \geq 1 \ \wedge \ xy \leq T\big\}$$

and define the integral

$$I(T;a,b) := \iint\limits_{\mathscr{D}(T)} x^a y^b \,\mathrm{d}(x,y)$$

with real exponents $a,b \geq 0$. We compute

$$I(T;a,b) = \frac{1}{(a+1)(b+1)} + \frac{1}{a-b}\left(\frac{T^{a+1}}{a+1} - \frac{T^{b+1}}{b+1}\right) \qquad \text{when} \quad a \neq b$$

and

$$I(T;c,c) = \frac{T^{c+1}\log T}{c+1} - \frac{T^{c+1}-1}{(c+1)^2} \qquad \text{for} \quad c \geq 0$$

and hence we obtain

Lemma 6. *For $a,b \geq 0$ fixed we have*

$$I(T;a,b) \asymp T^{1+\max\{a,b\}}(\log T)^{\nu} \qquad (T \to \infty)$$

with $\nu = 0$ when $a \neq b$, and $\nu = 1$ when $a = b$.

As usual, let $\varphi(\cdot)$ denote the Euler totient function. We will use the following well-known result due to M e r t e n s [1, Theorem 22].

Lemma 7. *As $t \to \infty$, $\sum\limits_{m \leq t} \varphi(m) = \frac{3}{\pi^2} t^2 + O(t \log t)$.*

Further we will need the following lemma which immediately follows from Lemma 3 via partial summation.

Lemma 8. *As $t \to \infty$,*

$$\sum_{m \leq t} \varphi(m) \cdot m^{-2} \asymp \log t$$

and

$$\sum_{m \leq t} \varphi(m) \cdot m^{\alpha} \asymp t^{\max\{0,\alpha+2\}}$$

for every real $\alpha \neq -2$. (The two $\asymp$-constants depend only on α.)

3. Proof of Theorem 1

The lower bound in Theorem 1 is trivial since there are $[2t] \cdot [2t+1]^{n-1}$ polynomials $p(X)$ over $\mathbb{Z}$ with degree n and height $\leq t$ such that $p(0) = 0$.

Let $\mathscr{P}_n(t)$ denote the set of all pairs (p,q) of non-constant polynomials over $\mathbb{Z}$ such that $\deg p+\deg q=n$ and $H(pq)\le t$. Then we obviously have $|\mathscr{P}_n(t)|\ge|\mathscr{R}_n(t)|$ for all $t\ge 1$. In view of Lemma 1 the set

$$\mathscr{P}_n^*(t):=\big\{(p,q)\in(\mathbb{Z}[X]\setminus\mathbb{Z})^2:\ \deg p+\deg q=n\ \wedge\ H(p)\cdot H(q)\le \mathrm{e}^n t\big\}$$

contains the set $\mathscr{P}_n(t)$ and thus we have the estimate $|\mathscr{P}_n^*(t)|\ge|\mathscr{R}_n(t)|$ for all $t\ge 1$. In order to prove Theorem 1 we show

$$|\mathscr{P}_n^*(t)|\ll t^n\qquad(t\to\infty)\tag{3.1}$$

for every $n\ge 3$.

For abbreviation we set $T=\mathrm{e}^n t$. Since obviously

$$\big|\big\{p\in\mathbb{Z}[X]:\ \deg p=k\ \wedge\ H(p)=h\big\}\big|\le 2\cdot(2h+1)^k\cdot(k+1)$$

we get (3.1) by showing

$$\sum_{k=1}^{n-1}\sum_{(x,y)\in\mathscr{G}(T)}2(2x+1)^k(k+1)\cdot 2(2y+1)^{n-k}(n-k+1)\ll T^n\qquad(T\to\infty)$$

where $\mathscr{G}(T):=\mathscr{D}(T)\cap\mathbb{Z}^2$ with $\mathscr{D}(T)=\big\{(x,y)\in\mathbb{R}^2:\ x,y\ge 1\ \wedge\ xy\le T\big\}$. Thus it is enough to verify

$$\sum_{(x,y)\in\mathscr{G}(T)}x^k y^{n-k}\ll T^n\qquad(T\to\infty)\tag{3.2}$$

for $1\le k<n$ and $n\ge 3$ fixed.

Now, (3.2) is true because by Lemma 2 for the corresponding integral we have

$$I(T,k,n-k)\ll T^n$$

provided that $n\ge 3$. (Clearly, the difference between the sum in (3.2) and $I(T,k,n-k)$ is $\ll T^n$ as $T\to\infty$.) Additionally, $I(T;1,1)\ll T^2\log T$ yields $|\mathscr{P}_n^*(t)|\ll t^n\log t$ in the exceptional case $n=2$ and hence we also obtain the upper bound in Theorem 2.

4. Proof of Theorem 2

It remains to verify the lower bound in Theorem 2. As usual, we call a linear polynomial $aX+b$ over $\mathbb{Z}$ *primitive* when a,b are coprime. Then for every quadratic polynomial $q(X)$ over $\mathbb{Z}$ which splits over $\mathbb{Q}$ there exists one and only one set $\{f,g\}$ of primitive linear polynomials $f(X)$, $g(X)$ such that $f(X)g(X)$ divides $q(X)$ in the ring $\mathbb{Z}[X]$. Let $Q(t)$ denote the number of all quadratic polynomials over $\mathbb{Z}$ with height $\le t$ which split over $\mathbb{Z}$ into two primitive linear factors. Then we have $Q(t)\le|\mathscr{R}_2(t)|$ and $2\cdot Q(t)$ is not smaller than the cardinality of the set

$$\big\{(f,g)\in\mathbb{Z}[X]^2:\ \deg f=\deg g=1\ \wedge\ (f,g\ \text{primitive})\ \wedge\ H(fg)\le t\big\},$$

which contains the set

$$\big\{(f,g)\in\mathbb{Z}[X]^2:\ \deg f=\deg g=1\ \wedge\ (f,g\ \text{primitive})\ \wedge\ H(f)\cdot H(g)\le \tfrac{1}{2}t\big\}$$

in view of Lemma 1.

Further, the total number of all primitive linear polynomials in $\mathbb{Z}[X]$ with constant height $h\ge 2$ clearly equals $8\cdot\varphi(h)$. The number is equal to 6 when $h=1$. Therefore with the new parameter $T=\frac{1}{2}t$ we have

$$|\mathscr{R}_2(t)|\ge 18\cdot\sum_{(x,y)\in\mathscr{G}(T)}\varphi(x)\varphi(y)$$

and the proof of Theorem 2 is finished by showing

$$T^2\log T\ll\sum_{(x,y)\in\mathscr{G}(T)}\varphi(x)\varphi(y)\qquad(T\to\infty).\tag{4.1}$$

Note that (4.1) would immediately follow from $m\ll\varphi(m)$ and the fact that the sum in (3.2) with $k=1$ and $n=2$ is $\gg T^2\log T$, but of course $m\ll\varphi(m)$ is false although $m^{1-\varepsilon}\ll\varphi(m)$ is true for every $\varepsilon>0$.

Nevertheless we will reach our goal by applying Lemma 3. As a consequence of Lemma 3 there exists a constant $C>0$ such that $\sum_{m\le t}\varphi(m)\ge C\cdot t^2$ for all $t\ge 1$. (Actually, this estimate is certainly true if we choose $C=\frac{1}{5}$.) Hence we have

$$\sum_{(x,y)\in\mathscr{G}(T)}\varphi(x)\varphi(y)=\sum_{1\le y\le T}\varphi(y)\cdot\sum_{1\le x\le T/y}\varphi(x)\ge\sum_{1\le y\le T}\varphi(y)\cdot C\frac{T^2}{y^2}.$$

Partial summation yields

$$\sum_{1\le y\le T}\varphi(y)\frac{1}{y^2}=\frac{1}{T^2}\sum_{m\le T}\varphi(m)+\int_1^T\frac{2}{u^3}\Big(\sum_{m\le u}\varphi(m)\Big)\,\mathrm{d}u\ge C+2C\log T$$

and we arrive at (4.1) as requested.

5. Proof of Theorem 3

Since the case $n=2$ is already settled by Theorem 2, in order to prove Theorem 3 we may assume $n\ge 3$. Further, by adapting the proof of Theorem 1 it is straightforward to get the upper bound in Theorem 3. Actually, this bound has the same order of magnitude as the integral

$$I_n(T)=\int\cdots\int_{\mathscr{D}_n(T)}x_1\cdots x_n\,\mathrm{d}(x_1,\dots,x_n)$$

with $\mathscr{D}_n(T) := \big\{(x_1,\dots,x_n)\in[1,\infty[^2\colon\ x_1\cdots x_n\le T\big\}$ and it is plain to verify

$$I_n(T)\asymp T^2(\log T)^{n-1}\qquad(T\to\infty)$$

for every $n\ge 2$ by induction starting from $I_2(T)=I(T,1,1)$ and using the estimate $I(T,1,1)\asymp T^2\log T$ $(T\to\infty)$ of Lemma 2.

On the other hand, following the lines of the proof of the lower estimate in Theorem 2 it is plain that

$$n!\cdot R_n^s(t)\ge 6^n\cdot\sum_{(x_1,\dots,x_n)\in\mathscr{G}_n(T)}\varphi(x_1)\cdots\varphi(x_n)$$

with $T=n^{1-n}t$ and $\mathscr{G}_n(T):=\mathscr{D}_n(T)\cap\mathbb{Z}^n$.

Now by applying Lemma 3 and partial summation, induction leads to

$$T^2(\log T)^{n-1}\ll\sum_{(x_1,\dots,x_n)\in\mathscr{G}_n(T)}\varphi(x_1)\cdots\varphi(x_n)\qquad(T\to\infty)$$

for every $n\ge 2$ since

$$\int_1^T\left(\left(\log\frac{T}{u}\right)^{n-2}\frac{2\log(T/u)+n-1}{u^3}\right)\cdot u^2\,\mathrm{d}u=\frac{2}{n}\cdot(\log T)^n+(\log T)^{n-1}$$

for all $T\ge 1$ and every $n\ge 2$. This concludes the proof of Theorem 3

6. Proof of Theorem 4

The following facts, where always $k,h\in\mathbb{Z}$ is assumed, are essential for our proof of Theorem 4.

(F1) *For every $k\ge 2$ and $h\ge 1$ there is at least one irreducible $p(X)\in\mathbb{Z}[X]$ with $\deg p=k$ and $H(p)=h$.*

P r o o f. This is certainly true because, e.g., $X^k-hX^{k-1}-X^{k-2}-\cdots-1$ is irreducible, which follows immediately from [4, Theorem 2.2.6]. □

(F2) *For every $k\ge 2$ and $h\ge 9$ the number of all irreducible $p(X)\in\mathbb{Z}[X]$ with $\deg p=k$ and $H(p)=h$ is greater than $h^k/3$.*

P r o o f. We apply Eisenstein's Irreduciblity Criterion with 2 as the testing prime. If h is odd, then obviously all polynomials $hX^k+2a_{k-1}X^{k-1}+\cdots+2a_1X+2\cdot(2l-1)$ with $l,a_i\in\mathbb{Z}$ and $2|a_i|<h$ and $4|l|<h-2$ are irreducible. If h is even, then all polynomials $(2l-1)X^k+hX^{k-1}+2a_{k-2}X^{k-2}+\cdots+2a_1X+2\cdot(2l'-1)$ with $l,l',a_i\in\mathbb{Z}$ and $2|a_i|\le h$ and $2|l|<h$ and $4|l'|\le h-2$ are irreducible. Hence the requested number is not less than $h^{k-1}(h-3)/2$ when h is odd and not less than $(h+1)^{k-2}(h-1)(h-2)/2$ when h is even. □

Combining (F1) and (F2) we derive:

(F3) *For every* $k \geq 2$ *and* $h \geq 1$ *the number of all irreducible* $p(X) \in \mathbb{Z}[X]$ *with* $\deg p = k$ *and* $H(p) = h$ *is not smaller than* $9^{-k} \cdot h^k$.

On the other hand, since the number of all $p(X) \in \mathbb{Z}[X]$ with $\deg p = k$ and $H(p) = h$ is certainly not greater than $2(k+1)(2h+1)^k$, we have:

(F4) *For every* $k \geq 2$ *and* $h \geq 1$ *the number of all irreducible* $p(X) \in \mathbb{Z}[X]$ *with* $\deg p = k$ *and* $H(p) = h$ *is not greater than* $2(k+1)3^k \cdot h^k$.

As usual, let us call a polynomial over $\mathbb{Z}$ *primitive* when the greatest common divisor all of its coefficients is 1.

(F5) *For all* $m \geq 1$ *and* $h \geq 1$ *the total number of all primitive polynomials* $p(X)$ *over* $\mathbb{Z}$ *with* $\deg p = m$ *and* $H(p) = h$ *is not greater than* $2(m+1)3^m \cdot h^m$ *and not smaller than* $2^{m+1} \cdot \varphi(h) \cdot h^{m-1}$.

The upper bound corresponds to the bound in (F4) and is trivial. The lower bound comes from counting only all polynomials $\pm hX^m + aX^{m-1} + a_{m-2}X^{m-2} + \cdots + a_0$ with $a, a_i \in \mathbb{Z}$ and $|a|, |a_i| \leq h$ where h and a are coprime.

Further, the following statement is obviously true.

(F6) *If* $1 < \frac{n}{2} < k < n$ *then for every* $p(X) \in \mathscr{R}_{k,n}(t)$ *there exists one and only one pair* $(f, g) \in \mathbb{Z}[X]^2$ *such that* $g(X)$ *is irreducible with* $\deg g = k$ *and* $f(X)$ *is primitive and* $f(X) \cdot g(X) = p(X)$.

Now we are ready to prove Theorem 4. Assume $1 < \frac{n}{2} < k < n$. Then by (F6) the mapping $(f, g) \mapsto f(X) \cdot g(X)$ is a bijection from the set

$$\big\{(f,g) \in \mathbb{Z}[X]^2 : \ \deg f = n-k \ \wedge \ \deg g = k \ \wedge \ H(fg) \leq t \\ \wedge \ (f \ \text{primitive}) \ \wedge \ (g \ \text{irreducible})\big\}$$

onto the set $\mathscr{R}_{k,n}(t)$.

Consequently, with $t \ll T \ll t$, in view of Lemma 1 and (F4) and (F5) we have

$$|\mathscr{R}_{k,n}(t)| \ll \sum_{(x,y)\in\mathscr{G}(T)} x^{n-k} \cdot y^k \ll T^{k+1} \qquad (T \to \infty) \tag{6.1}$$

since $I(T; n-k, k) \ll T^{k+1}$ for $\frac{n}{2} < k < n$ by Lemma 2.

On the other hand, again with $t \ll T \ll t$, by Lemma 1 and by (F3) and (F5),

$$|\mathscr{R}_{k,n}(t)| \gg \sum_{(x,y)\in\mathscr{G}(T)} \varphi(x) x^{n-k-1} \cdot y^k \qquad (T \to \infty).$$

By writing

$$\sum_{(x,y)\in\mathscr{G}(T)} \varphi(x) x^{n-k-1} \cdot y^k = \sum_{1\leq x\leq T} \varphi(x) x^{n-k-1} \sum_{1\leq y\leq T/x} y^k$$

and applying the trivial estimate

$$\sum_{1\leq y\leq u} y^k \geq \int_0^u y^k \, \mathrm{d}y - u^k = \frac{1}{k+1} u^{k+1} - u^k (u \geq 1)$$

and Lemma 4 with $\alpha = n-2k-2 < -2$ on the one hand and with $\alpha = n-2k-1 < -1$ on the other, we derive the desired lower estimate

$$T^{k+1} \ll \sum_{(x,y)\in\mathscr{G}(T)} \varphi(x)x^{n-k-1}\cdot y^k \qquad (T\to\infty).$$

Further, the estimate $t^2(\log t)^2 \ll |\mathscr{R}_3(t)\setminus\mathscr{R}_{2,3}(t)| \ll t^2(\log t)^2$ is equivalent to Theorem 3 for $n=3$. In particular, the factor $(\log t)^2$ in the estimate in Theorem 4 cannot be omitted when $n=3$.

In order to verify $|\mathscr{R}_n(t)\setminus\mathscr{R}_{n-1,n}(t)| \ll t^{n-1}$ for $n\geq 4$ we note that

$$\mathscr{R}_n(t)\setminus\mathscr{R}_{n-1,n}(t) \subset \mathscr{R}_n^*(t) \cup \bigcup_{\frac{n}{2}<k\leq n-2} \mathscr{R}_{k,n}(t) \tag{6.2}$$

where $\mathscr{R}_n^*(t)$ is the set of all reducible polynomials $p(X)$ over $\mathbb{Z}$ with degree n and height $\leq t$ such that the degree of every irreducible factor of $p(X)$ is not greater than $\frac{n}{2}$.

Now, for every $p\in\mathscr{R}_n^*(t)$ we can write $p(X)=f(X)\cdot g(X)$ with $f(X),g(X)\in\mathbb{Z}[X]$ such that the degrees of $f(X)$ and $g(X)$ are both not greater than $n-2$. Hence, by following the arguments in Chapter 3 we only have to estimate the sum in (3.2) for $2\leq k\leq n-2$ in order to obtain $|\mathscr{R}_n^*(t)|\ll t^{n-1}$. Thus, in view of (6.1) via (6.2) we arrive at $|\mathscr{R}_n(t)\setminus\mathscr{R}_{n-1,n}(t)|\ll t^{n-1}$ for $n\geq 4$ and this concludes the proof of Theorem 4.

Final Remark. In view of our proofs it is not difficult to find explicit bounds C_n in Theorem 1 and to produce explicit $\ll$-constants for all estimations in Theorems 2, 3, 4 which depend only (and in a simple way) on the degree n.

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