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ON ONE TWOEPOCH LINEAR MODEL WITH THE NUISANCE PARAMETERS

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ABSTRACT. The optimum linear estimators of the useful mean value parameters within a linear regression model with the stable and variable parameters and with the nuisance parameters are derived including their characteristics of accuracy. Some verification of theoretical results is presented.

1. Motivation and notations

The theory of the linear regression models is one of the established statistical disciplines and it may seem that nearly all has been investigated there. But this is valid only for the simplest structures of the linear models. In practice we need to solve more and more complicated problems and investigation of corresponding structures of the models is at the beginning. The formulas are quite complicated there, but easy programmable and it enables us to get the estimators of unknown parameters in the linear models.

The estimation procedures in the certain multiepoch (and specially twoepoch) linear regression models with the nuisance parameters and its application in geodesy and microeconomics were described in [3, Chapter 9] and [2]. We now propose to deal with another type of the models that occur in many fields, for instance in microeconomics, medicine, geography and others.

In this paper we derive optimum estimators and their confidence intervals of the useful mean value within a linear twoepoch model with the stable and variable (nonstable) parameters. The data are also affected by influence which can be involved to a linear model and thus our model includes the parameters

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called nuisance, which are estimable from results of the measurement. The subject of an investigation are changes of the useful parameters in the single epochs and their characteristics of accuracy.

Sometimes the dimension of the useful mean value parameter is essentially smaller than that one of the nuisance parameter. In connection with this fact the problem occurs how to determine the optimum estimators of the useful parameters and their accuracy without evaluating in each epoch the large vector of the nuisance parameters. At the same time, the total number of parameters does not have to be necessarily in a hundred or in a thousand, the *ratio* of numbers of the useful and nuisance parameters can be also the reason for our work. In this case, the question why we should estimate parameters that we do not need for the conclusion of an experiment is an interesting phenomenon worth investigating.

Let us describe an example from medicine. A patient with mild lasting hypertensis is hospitalized, two times in some longer time period, with an illness that isn't connected to the hypertensis but it is necessary to lower the blood pressure during the treatment. Doctor's assumption is that the trend of decrease of the pressure after application of a medicine and the constant pressure (with some periodic noise) in the time before both the hospitalizations are typical for the patient. The doctor wants to know the efficiency of the medicine, i.e. how the pressure decreases per hour after application of the medicine, including both the hospitalizations. It is known that the pressure changes with 24-hour and 16-hour periods. Values of parameters of functions which describe these periods are variable. So if we suppose that the pressure, after application of the medicine, lowers linearly with some periodic noise, we will investigate the linear trend for both the hospitalizations together and the periods separately. Thus the linear term parameter is the useful stable parameter (in expression of the entire linear model modelling the situation), the absolute term parameter is the nuisance stable parameter and the periods parameters are nonstable and nuisance also. It is necessary to stress that the linear term parameter changes its value after interruption to serve the medicine from the negative to the positive one and this parameter moves into zero value after some time period.

The result of the measurement at the ith time point in the first epoch could be described as

$$Y_{1i} = \beta t_{1i} + \kappa + \eta_{11}\cos\lambda_{11}t_{1i} + \eta_{12}\sin\lambda_{11}t_{1i} + \eta_{13}\cos\lambda_{12}t_{1i} + \eta_{14}\sin\lambda_{12}t_{1i} + \varepsilon_{1i},$$

$i = 1, \dots, n_1$, and

$$Y_{2i} = \beta t_{2i} + \kappa + \eta_{21}\cos\lambda_{21}t_{2i} + \eta_{22}\sin\lambda_{21}t_{2i} + \eta_{23}\cos\lambda_{22}t_{2i} + \eta_{24}\sin\lambda_{22}t_{2i} + \varepsilon_{2i},$$

$i = 1, \dots, n_2$, in the second epoch λ_{11}, $\lambda_{12}\lambda_{21}$ and λ_{22} are known from periodogram, see [7, p. 92] or are determined expertly). Here $\beta t_{ji} + \kappa$ describes the linear trend and

$$\eta_{j1}\cos\lambda_{j1}t_{ji} + \eta_{j2}\sin\lambda_{j1}t_{ji} + \eta_{j3}\cos\lambda_{j2}t_{ji} + \eta_{j4}\sin\lambda_{j2}t_{ji}, \qquad j = 1, 2,$$

the period terms in the first and second epoch, respectively.

Let us consider the observation vector $\mathbf{Y} = (\mathbf{Y}_1', \mathbf{Y}_2')'$. The model described above could be rewritten in the form

$$\begin{pmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{X}_1 & \mathbf{S}_1 & \mathbf{Z}_1 & \mathbf{0} \\ \mathbf{X}_2 & \mathbf{S}_2 & \mathbf{0} & \mathbf{Z}_2 \end{pmatrix} \begin{pmatrix} \boldsymbol{\beta} \\ \boldsymbol{\kappa} \\ \boldsymbol{\eta}_1 \\ \boldsymbol{\eta}_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}, \tag{1}$$

where

$$\mathbf{X}_1 = (t_{11}, \ldots, t_{1n_1})', \quad \mathbf{X}_2 = (t_{21}, \ldots, t_{2n_2})',$$

$$\mathbf{S}_1 = (\overbrace{1, \ldots, 1}^{n_1\text{-times}})' = \mathbf{1}_{n_1}, \quad \mathbf{S}_2 = (\overbrace{1, \ldots, 1}^{n_2\text{-times}})' = \mathbf{1}_{n_2},$$

$$\mathbf{Z}_1 = \begin{pmatrix} \cos\lambda_{11}t_{11} & \sin\lambda_{11}t_{11} & \cos\lambda_{12}t_{11} & \sin\lambda_{12}t_{11} \\ \vdots & \vdots & \vdots & \vdots \\ \cos\lambda_{11}t_{1n_1} & \sin\lambda_{11}t_{1n_1} & \cos\lambda_{12}t_{1n_1} & \sin\lambda_{12}t_{1n_1} \end{pmatrix},$$

$$\mathbf{Z}_2 = \begin{pmatrix} \cos\lambda_{21}t_{21} & \sin\lambda_{21}t_{21} & \cos\lambda_{22}t_{21} & \sin\lambda_{22}t_{21} \\ \vdots & \vdots & \vdots & \vdots \\ \cos\lambda_{21}t_{2n_2} & \sin\lambda_{21}t_{2n_2} & \cos\lambda_{22}t_{2n_2} & \sin\lambda_{22}t_{2n_2} \end{pmatrix},$$

$$\boldsymbol{\beta} = \beta, \quad \boldsymbol{\kappa} = \kappa, \quad \boldsymbol{\eta}_1 = (\eta_{11}, \eta_{12}, \eta_{13}, \eta_{14})', \quad \boldsymbol{\eta}_2 = (\eta_{21}, \eta_{22}, \eta_{23}, \eta_{24})'.$$

The matrices $\mathbf{X}_1, \mathbf{X}_2, \mathbf{S}_1, \mathbf{S}_2, \mathbf{Z}_1, \mathbf{Z}_2$ are known, the vector $\boldsymbol{\beta}$ is a vector of the useful stable parameters, $\boldsymbol{\kappa}$ is a vector of the nuisance stable parameters and $\boldsymbol{\eta}$ is a vector of the nuisance variable parameters.

We can observe, that the ratio of numbers of the useful and nuisance parameters here is 1:9. So even in this basic situation (the trend can be complicated and with more periods), the number of the nuisance parameters is $9\times$ higher than the number of the useful parameters. See an example at the end of the paper.

The following notation will be used throughout the paper:

$\mathbb{R}^n$	the space of all n-dimensional real vectors;
$\mathbf{u}$, $\mathbf{A}$	the real column vector, the real matrix;
$\mathbf{A}'$, $r(\mathbf{A})$	the transpose, the rank of the matrix $\mathbf{A}$;
$\mathscr{M}(\mathbf{A})$, $\mathrm{Ker}(\mathbf{A})$	the range, the null space of the matrix $\mathbf{A}$;
$\mathbf{A}^-$	a generalized inverse of a matrix $\mathbf{A}$ (satisfying $\mathbf{A}\mathbf{A}^-\mathbf{A} = \mathbf{A}$);
$\mathbf{A}^+$	the Moore-Penrose generalized inverse of a matrix $\mathbf{A}$ (satisfying $\mathbf{A}\mathbf{A}^+\mathbf{A} = \mathbf{A}$, $\mathbf{A}^+\mathbf{A}\mathbf{A}^+ = \mathbf{A}^+$, $(\mathbf{A}\mathbf{A}^+)' = \mathbf{A}\mathbf{A}^+$, $(\mathbf{A}^+\mathbf{A})' = \mathbf{A}^+\mathbf{A}$);
$\mathbf{P}_A$	the orthogonal projector onto $\mathscr{M}(\mathbf{A})$ (in Euclidean sense);
$\mathbf{M}_A = \mathbf{I} - \mathbf{P}_A$	the orthogonal projector onto $\mathscr{M}^\perp(\mathbf{A}) = \mathrm{Ker}(\mathbf{A}')$;
$\mathbf{I}_k$	the $k \times k$ identity matrix;
$\mathbf{0}_{m,n}$	the $m \times n$ null matrix;
$\mathbf{1}_k$	$= (1, \dots, 1)' \in \mathbb{R}^k$;
u	random variable with normalized normal distribution;
$u(1 - \frac{\alpha}{2})$	$(1 - \frac{\alpha}{2})$-quantile of normalized normal distribution;
χ^2_r	random variable with chi squared distribution with r degrees of freedom;
$\chi^2_r(1-\alpha)$	$(1-\alpha)$-quantile of chi squared distribution with r degrees of freedom.

If $\mathscr{M}(\mathbf{A}) \subset \mathscr{M}(\mathbf{S})$, $\mathbf{S}$ positive semidefinite (p.s.d.), then the symbol $\mathbf{P}_A^{S^-}$ denotes the projector projecting vectors in $\mathscr{M}(\mathbf{S})$ onto $\mathscr{M}(\mathbf{A})$ along $\mathscr{M}(\mathbf{S}\mathbf{A}^\perp)$. A general representation of all such projectors $\mathbf{P}_A^{S^-}$ is given by $\mathbf{A}(\mathbf{A}'\mathbf{S}^-\mathbf{A})^-\mathbf{A}'\mathbf{S}^- + \mathbf{B}(\mathbf{I} - \mathbf{S}\mathbf{S}^-)$, where $\mathbf{B}$ is arbitrary, (see [5, (2.14)]). $\mathbf{M}_A^{S^-} = \mathbf{I} - \mathbf{P}_A^{S^-}$.

Lemma 1. *Inverse of partitioned p.d. matrix*

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{D} \\ \mathbf{B}' & \mathbf{C} & \mathbf{0} \\ \mathbf{D}' & \mathbf{0} & \mathbf{E} \end{pmatrix}$$

is equal to

$$\begin{pmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} & \mathbf{Q}_{13} \\ \mathbf{Q}_{21} & \mathbf{Q}_{22} & \mathbf{Q}_{23} \\ \mathbf{Q}_{31} & \mathbf{Q}_{32} & \mathbf{Q}_{33} \end{pmatrix} =$$

$$= \begin{pmatrix} \mathbf{Q}_{11} & -\mathbf{Q}_{11}\mathbf{B}\mathbf{C}^{-1} & -\mathbf{Q}_{11}\mathbf{D}\mathbf{E}^{-1} \\ -\mathbf{C}^{-1}\mathbf{B}'\mathbf{Q}_{11} & \mathbf{C}^{-1} + \mathbf{C}^{-1}\mathbf{B}'\mathbf{Q}_{11}\mathbf{B}\mathbf{C}^{-1} & -\mathbf{Q}_{21}\mathbf{D}\mathbf{E}^{-1} \\ -\mathbf{E}^{-1}\mathbf{D}'\mathbf{Q}_{11} & -\mathbf{E}^{-1}\mathbf{D}'\mathbf{Q}_{12} & \mathbf{E}^{-1} + \mathbf{E}^{-1}\mathbf{D}'\mathbf{Q}_{11}\mathbf{D}\mathbf{E}^{-1} \end{pmatrix},$$

where

$$\mathbf{Q}_{11} = (\mathbf{A} - \mathbf{B}\mathbf{C}^{-1}\mathbf{B}' - \mathbf{D}\mathbf{E}^{-1}\mathbf{D}')^{-1}.$$

P r o o f. See [2, Lemma 2]. $\square$

2. Best linear unbiased estimators

Let us consider the linear model (1), called the twoepoch model with the stable useful parameters and with the stable and nonstable nuisance parameters, where

- $(\mathbf{Y}_1', \mathbf{Y}_2')'$ is an $(n_1 + n_2)$-dimensional random observation vector after the second epoch of measurement,
- $\boldsymbol{\beta} \in \mathbb{R}^k$ is a vector of the useful stable parameters, the same in both epochs,
- $\boldsymbol{\kappa} \in \mathbb{R}^s$ is a vector of the nuisance stable parameters, the same in both epochs,
- $\boldsymbol{\eta} = (\boldsymbol{\eta}_1', \boldsymbol{\eta}_2')' \in \mathbb{R}^{t_1+t_2}$ is a vector of the nuisance nonstable parameters in first and second epoch of measurement,
- $\mathbf{X}_1$, $\mathbf{X}_2$ are $n_1 \times k$, $n_2 \times k$ design matrices belonging to the vector $\boldsymbol{\beta}$,
- $\mathbf{S}_1$, $\mathbf{S}_2$ are $n_1 \times s$, $n_2 \times s$ design matrices belonging to the vector $\boldsymbol{\kappa}$,
- $\mathbf{Z}_1$ is a $n_1 \times t_1$ design matrix belonging to the vector $\boldsymbol{\eta}_1$,
- $\mathbf{Z}_2$ is a $n_2 \times t_2$ design matrix belonging to the vector $\boldsymbol{\eta}_2$.

We suppose that

1. $\forall \boldsymbol{\beta} \in \mathbb{R}^k, \forall \boldsymbol{\kappa} \in \mathbb{R}^s, \forall \boldsymbol{\eta}_1 \in \mathbb{R}^{t_1}, \forall \boldsymbol{\eta}_2 \in \mathbb{R}^{t_2}$:
   $\mathrm{E}(\mathbf{Y}_1) = \mathbf{X}_1\boldsymbol{\beta} + \mathbf{S}_1\boldsymbol{\kappa} + \mathbf{Z}_1\boldsymbol{\eta}_1$, $\mathrm{E}(\mathbf{Y}_2) = \mathbf{X}_2\boldsymbol{\beta} + \mathbf{S}_2\boldsymbol{\kappa} + \mathbf{Z}_2\boldsymbol{\eta}_2$;
2. $\mathrm{var}\left[\begin{pmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{pmatrix}\right] = \begin{pmatrix} \boldsymbol{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_2 \end{pmatrix}$,
3. the matrix $\boldsymbol{\Sigma}_i$ is not a function of the vector $(\boldsymbol{\beta}', \boldsymbol{\kappa}', \boldsymbol{\eta}_i')'$ for $i = 1, 2$.

If the matrix $\begin{pmatrix} \boldsymbol{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_2 \end{pmatrix}$ is p.d. and $r\left[\begin{pmatrix} \mathbf{X}_1 & \mathbf{S}_1 & \mathbf{Z}_1 & \mathbf{0} \\ \mathbf{X}_2 & \mathbf{S}_2 & \mathbf{0} & \mathbf{Z}_2 \end{pmatrix}\right] = k + s + t_1 + t_2 < n_1 + n_2$, the model is said to be *regular* (see [3, p. 13]).

The described model arises by sequential realizations of the linear partial regression models,

$$\mathbf{Y}_1 = (\mathbf{X}_1, \mathbf{S}_1, \mathbf{Z}_1) \begin{pmatrix} \boldsymbol{\beta} \\ \boldsymbol{\kappa} \\ \boldsymbol{\eta}_1 \end{pmatrix} + \boldsymbol{\varepsilon}_1, \quad \mathrm{var}(\mathbf{Y}_1) = \boldsymbol{\Sigma}_1 \tag{2}$$

and

$$\mathbf{Y}_2 = (\mathbf{X}_2, \mathbf{S}_2, \mathbf{Z}_2)\begin{pmatrix} \boldsymbol{\beta} \\ \boldsymbol{\kappa} \\ \boldsymbol{\eta}_2 \end{pmatrix} + \boldsymbol{\varepsilon}_2, \quad \mathrm{var}(\mathbf{Y}_2) = \boldsymbol{\Sigma}_2, \tag{3}$$

representing the model of the measurement within the first and second epoch, respectively.

Theorem 1. *The* BLUE, *i.e. the best linear unbiased estimator, of the parameters $\boldsymbol{\beta}$, $\boldsymbol{\kappa}$, $\boldsymbol{\eta}_i$, $i = 1, 2$, in the single first and second epoch modeled by* (2) *and* (3)*, respectively, is*

$$\begin{aligned}
\widehat{\boldsymbol{\beta}}^{(i)} &= (\mathbf{X}_i'\boldsymbol{\Sigma}_i^{-1}\mathbf{M}_{Z_i}^{\Sigma_i^{-1}}\mathbf{M}_{S_i}^{\Sigma_i^{-1}M_{Z_i}^{\Sigma_i^{-1}}}\mathbf{X}_i)^{-1}\mathbf{X}_i'\boldsymbol{\Sigma}_i^{-1}\mathbf{M}_{Z_i}^{\Sigma_i^{-1}}\mathbf{M}_{S_i}^{\Sigma_i^{-1}M_{Z_i}^{\Sigma_i^{-1}}}\mathbf{Y}_i, \\
\widehat{\boldsymbol{\kappa}}^{(i)} &= (\mathbf{S}_i'\boldsymbol{\Sigma}_i^{-1}\mathbf{M}_{Z_i}^{\Sigma_i^{-1}}\mathbf{S}_i)^{-1}\mathbf{S}_i'\boldsymbol{\Sigma}_i^{-1}\mathbf{M}_{Z_i}^{\Sigma_i^{-1}}(\mathbf{Y}_i - \mathbf{X}_i\widehat{\boldsymbol{\beta}}^{(i)}), \\
\widehat{\boldsymbol{\eta}_i}^{(i)} &= (\mathbf{Z}_i'\boldsymbol{\Sigma}_i^{-1}\mathbf{Z}_i)^{-1}\mathbf{Z}_i'\boldsymbol{\Sigma}_i^{-1}(\mathbf{Y}_i - \mathbf{X}_i\widehat{\boldsymbol{\beta}}^{(i)} - \mathbf{S}_i\widehat{\boldsymbol{\kappa}}^{(i)}),
\end{aligned}$$

where

$$\boldsymbol{\Sigma}_i^{-1}\mathbf{M}_{Z_i}^{\Sigma_i^{-1}} = \boldsymbol{\Sigma}_i^{-1} - \boldsymbol{\Sigma}_i^{-1}\mathbf{Z}_i(\mathbf{Z}_i'\boldsymbol{\Sigma}_i^{-1}\mathbf{Z}_i)^{-1}\mathbf{Z}_i'\boldsymbol{\Sigma}_i^{-1}, \qquad i = 1, 2.$$

The variance matrix of the useful vector parameter estimator $\widehat{\boldsymbol{\beta}}^{(i)}$ equals

$$\mathrm{var}(\widehat{\boldsymbol{\beta}}^{(i)}) = (\mathbf{X}_i'\boldsymbol{\Sigma}_i^{-1}\mathbf{M}_{Z_i}^{\Sigma_i^{-1}}\mathbf{M}_{S_i}^{\Sigma_i^{-1}M_{Z_i}^{\Sigma_i^{-1}}}\mathbf{X}_i)^{-1}.$$

Proof. See [2, Theorem 3] with $\mathbf{W}_i$ instead of $\mathbf{S}_i$, $i = 1, 2$. The expression of variance matrix can be obtained in a standard way. □

Notation 1. *The model* (1) *can be rewritten as*

$$\mathbf{Y} = (\mathbf{W}, \mathbf{Z})\begin{pmatrix} \boldsymbol{\beta} \\ \boldsymbol{\nu} \end{pmatrix} + \boldsymbol{\varepsilon}, \tag{4}$$

where

$$\mathbf{Y} = \begin{pmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{pmatrix}, \quad \mathbf{W} = \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{pmatrix}, \quad \mathbf{Z} = \begin{pmatrix} \mathbf{S}_1 & \mathbf{Z}_1 & \mathbf{0} \\ \mathbf{S}_2 & \mathbf{0} & \mathbf{Z}_2 \end{pmatrix},$$

$$\boldsymbol{\nu} = \begin{pmatrix} \boldsymbol{\kappa} \\ \boldsymbol{\eta} \end{pmatrix}, \quad \boldsymbol{\varepsilon} = \begin{pmatrix} \boldsymbol{\varepsilon}_1 \\ \boldsymbol{\varepsilon}_2 \end{pmatrix} \quad \text{and} \quad \boldsymbol{\Sigma} = \mathrm{var}(\mathbf{Y}) = \begin{pmatrix} \boldsymbol{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_2 \end{pmatrix},$$

so we get the linear model with nuisance parameters.

Proposition 1. *In the regular model* (2) *the* BLUE *of the parameter* $(\boldsymbol{\beta}', \boldsymbol{\nu}')'$ *is given as*

$$\begin{pmatrix} \widehat{\boldsymbol{\beta}} \\ \widehat{\boldsymbol{\nu}} \end{pmatrix} = \begin{pmatrix} (\mathbf{W}'\boldsymbol{\Sigma}^{-1}\mathbf{M}_Z^{\Sigma^{-1}}\mathbf{W})^{-1}\mathbf{W}'\boldsymbol{\Sigma}^{-1}\mathbf{M}_Z^{\Sigma^{-1}} \\ (\mathbf{Z}'\boldsymbol{\Sigma}^{-1}\mathbf{Z})^{-1}\mathbf{Z}'\boldsymbol{\Sigma}^{-1}\mathbf{M}_W^{\Sigma^{-1}M_Z^{\Sigma^{-1}}} \end{pmatrix} \mathbf{Y}. \tag{5}$$

P r o o f. See [4, Theorem 1]. □

Theorem 2. *In the regular model* (1) *the* BLUE*s of the parameters* $\boldsymbol{\beta}$, $\boldsymbol{\kappa}$, $\boldsymbol{\eta}_1$, $\boldsymbol{\eta}_2$ *are given as*

$$\begin{aligned}
\widehat{\boldsymbol{\beta}} \;&=\; [\mathbf{X}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}}\mathbf{X}_1 + \mathbf{X}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}}\mathbf{X}_2 - (\mathbf{X}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}}\mathbf{S}_1 \\
&\quad + \mathbf{X}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}}\mathbf{S}_2)\mathbf{Q}_{11}(\mathbf{S}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}}\mathbf{X}_1 + \mathbf{S}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}}\mathbf{X}_2)]^{-1} \times \\
&\quad \times [\mathbf{X}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}}\mathbf{Y}_1 + \mathbf{X}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}}\mathbf{Y}_2 - (\mathbf{X}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}}\mathbf{S}_1 \\
&\quad + \mathbf{X}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}}\mathbf{S}_2)\mathbf{Q}_{11}(\mathbf{S}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}}\mathbf{Y}_1 + \mathbf{S}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}}\mathbf{Y}_2)], \\
\widehat{\boldsymbol{\kappa}} \;&=\; \mathbf{Q}_{11}[\mathbf{S}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}}\mathbf{Y}_1 + \mathbf{S}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}}\mathbf{Y}_2 - (\mathbf{S}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}}\mathbf{X}_1 \\
&\quad + \mathbf{S}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}}\mathbf{X}_2)\widehat{\boldsymbol{\beta}}], \\
\widehat{\boldsymbol{\eta}_1} \;&=\; (\mathbf{Z}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{Z}_1)^{-1}\mathbf{Z}_1'\boldsymbol{\Sigma}_1^{-1}(\mathbf{Y}_1 - \mathbf{X}_1\widehat{\boldsymbol{\beta}} - \mathbf{S}_1\widehat{\boldsymbol{\kappa}}), \\
\widehat{\boldsymbol{\eta}_2} \;&=\; (\mathbf{Z}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{Z}_2)^{-1}\mathbf{Z}_2'\boldsymbol{\Sigma}_2^{-1}(\mathbf{Y}_2 - \mathbf{X}_2\widehat{\boldsymbol{\beta}} - \mathbf{S}_2\widehat{\boldsymbol{\kappa}}),
\end{aligned}$$

where $\mathbf{Q}_{11} = (\mathbf{S}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}}\mathbf{S}_1 + \mathbf{S}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}}\mathbf{S}_2)^{-1}$.

P r o o f. With respect to Notation 1, it is convenient to use Proposition 1 to prove the asserted theorem. In getting

$$\boldsymbol{\Sigma}^{-1}\mathbf{M}_Z^{\Sigma^{-1}} = \boldsymbol{\Sigma}^{-1} - \boldsymbol{\Sigma}^{-1}\mathbf{Z}(\mathbf{Z}'\boldsymbol{\Sigma}^{-1}\mathbf{Z})^{-1}\mathbf{Z}'\boldsymbol{\Sigma}^{-1}$$

we have to invert

$$\mathbf{Z}'\boldsymbol{\Sigma}^{-1}\mathbf{Z} = \begin{pmatrix} \mathbf{S}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{S}_1 + \mathbf{S}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{S}_2 & \mathbf{S}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{Z}_1 & \mathbf{S}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{Z}_2 \\ \mathbf{Z}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{S}_1 & \mathbf{Z}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{Z}_1 & \mathbf{0} \\ \mathbf{Z}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{S}_2 & \mathbf{0} & \mathbf{Z}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{Z}_2 \end{pmatrix}$$

using Lemma 1. Thus

$$(\mathbf{Z}'\boldsymbol{\Sigma}^{-1}\mathbf{Z})^{-1} \overset{\mathrm{L.1}}{=} \begin{pmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} & \mathbf{Q}_{13} \\ \mathbf{Q}_{21} & \mathbf{Q}_{22} & \mathbf{Q}_{23} \\ \mathbf{Q}_{31} & \mathbf{Q}_{32} & \mathbf{Q}_{33} \end{pmatrix},$$

where

$$
\begin{aligned}
\mathbf{Q}_{11} &= (\mathbf{S}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}}\mathbf{S}_1 + \mathbf{S}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}}\mathbf{S}_2)^{-1},\\
\mathbf{Q}_{12} &= -\mathbf{Q}_{11}\mathbf{S}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{Z}_1(\mathbf{Z}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{Z}_1)^{-1},\\
\mathbf{Q}_{13} &= -\mathbf{Q}_{11}\mathbf{S}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{Z}_2(\mathbf{Z}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{Z}_2)^{-1},\\
\mathbf{Q}_{21} &= -(\mathbf{Z}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{Z}_1)^{-1}\mathbf{Z}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{S}_1\mathbf{Q}_{11},\\
\mathbf{Q}_{22} &= (\mathbf{Z}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{Z}_1)^{-1} + (\mathbf{Z}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{Z}_1)^{-1}\mathbf{Z}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{S}_1\mathbf{Q}_{11}\mathbf{S}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{Z}_1 \times\\
&\quad \times (\mathbf{Z}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{Z}_1)^{-1},\\
\mathbf{Q}_{23} &= (\mathbf{Z}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{Z}_1)^{-1}\mathbf{Z}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{S}_1\mathbf{Q}_{11}\mathbf{S}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{Z}_2(\mathbf{Z}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{Z}_2)^{-1},\\
\mathbf{Q}_{31} &= -(\mathbf{Z}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{Z}_2)^{-1}\mathbf{Z}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{S}_2\mathbf{Q}_{11},\\
\mathbf{Q}_{32} &= (\mathbf{Z}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{Z}_2)^{-1}\mathbf{Z}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{S}_2\mathbf{Q}_{11}\mathbf{S}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{Z}_1(\mathbf{Z}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{Z}_1)^{-1},\\
\mathbf{Q}_{33} &= (\mathbf{Z}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{Z}_2)^{-1} + (\mathbf{Z}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{Z}_2)^{-1}\mathbf{Z}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{S}_2\mathbf{Q}_{11}\mathbf{S}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{Z}_2 \times\\
&\quad \times (\mathbf{Z}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{Z}_2)^{-1}.
\end{aligned}
$$

Consequently $\boldsymbol{\Sigma}^{-1}\mathbf{M}_Z^{\Sigma^{-1}} = \begin{pmatrix} \mathbf{D}_{11} & \mathbf{D}_{12} \\ \mathbf{D}_{21} & \mathbf{D}_{22} \end{pmatrix}$ with

$$
\begin{aligned}
\mathbf{D}_{11} &= \boldsymbol{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}} - \boldsymbol{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}}\mathbf{S}_1\mathbf{Q}_{11}\mathbf{S}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}},\\
\mathbf{D}_{12} &= -\boldsymbol{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}}\mathbf{S}_1\mathbf{Q}_{11}\mathbf{S}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}},\\
\mathbf{D}_{21} &= -\boldsymbol{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}}\mathbf{S}_2\mathbf{Q}_{11}\mathbf{S}_1'\boldsymbol{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}},\\
\mathbf{D}_{22} &= \boldsymbol{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}} - \boldsymbol{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}}\mathbf{S}_2\mathbf{Q}_{11}\mathbf{S}_2'\boldsymbol{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}}
\end{aligned}
$$

and the estimation of the useful parameter $\boldsymbol{\beta}$ is the result of an obvious and simple calculations. To obtain the BLUEs of the nuisance parameters $\boldsymbol{\kappa}$, $\boldsymbol{\eta}_1$, $\boldsymbol{\eta}_2$ it is sufficient to show that

$$
\begin{aligned}
(\mathbf{Z}'\boldsymbol{\Sigma}^{-1}\mathbf{Z})^{-1}\mathbf{Z}'\boldsymbol{\Sigma}^{-1} &= \begin{pmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} & \mathbf{Q}_{13} \\ \mathbf{Q}_{21} & \mathbf{Q}_{22} & \mathbf{Q}_{23} \\ \mathbf{Q}_{31} & \mathbf{Q}_{32} & \mathbf{Q}_{33} \end{pmatrix} \begin{pmatrix} \mathbf{S}_1'\boldsymbol{\Sigma}_1^{-1} & \mathbf{S}_2'\boldsymbol{\Sigma}_2^{-1} \\ \mathbf{Z}_1'\boldsymbol{\Sigma}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}_2'\boldsymbol{\Sigma}_2^{-1} \end{pmatrix}\\
&= \begin{pmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} \\ \mathbf{R}_{21} & \mathbf{R}_{22} \\ \mathbf{R}_{31} & \mathbf{R}_{32} \end{pmatrix},
\end{aligned}
$$

where

$$\begin{aligned}
\mathbf{R}_{11} &= \mathbf{Q}_{11}\mathbf{S}_1'\mathbf{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}},\\
\mathbf{R}_{12} &= \mathbf{Q}_{11}\mathbf{S}_2'\mathbf{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}},\\
\mathbf{R}_{21} &= (\mathbf{Z}_1'\mathbf{\Sigma}_1^{-1}\mathbf{Z}_1)^{-1}(\mathbf{Z}_1'\mathbf{\Sigma}_1^{-1} - \mathbf{Z}_1'\mathbf{\Sigma}_1^{-1}\mathbf{S}_1\mathbf{Q}_{11}\mathbf{S}_1'\mathbf{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}}),\\
\mathbf{R}_{22} &= -(\mathbf{Z}_1'\mathbf{\Sigma}_1^{-1}\mathbf{Z}_1)^{-1}\mathbf{Z}_1'\mathbf{\Sigma}_1^{-1}\mathbf{S}_1\mathbf{Q}_{11}\mathbf{S}_2'\mathbf{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}},\\
\mathbf{R}_{31} &= -(\mathbf{Z}_2'\mathbf{\Sigma}_2^{-1}\mathbf{Z}_2)^{-1}\mathbf{Z}_2'\mathbf{\Sigma}_2^{-1}\mathbf{S}_2\mathbf{Q}_{11}\mathbf{S}_1'\mathbf{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}},\\
\mathbf{R}_{32} &= (\mathbf{Z}_2'\mathbf{\Sigma}_2^{-1}\mathbf{Z}_2)^{-1}(\mathbf{Z}_2'\mathbf{\Sigma}_2^{-1} - \mathbf{Z}_2'\mathbf{\Sigma}_2^{-1}\mathbf{S}_2\mathbf{Q}_{11}\mathbf{S}_2'\mathbf{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}}),
\end{aligned}$$

and

$$\begin{aligned}
\mathbf{M}_W^{\Sigma^{-1}M_Z^{\Sigma^{-1}}}\mathbf{Y} &= \mathbf{Y} - \mathbf{W}(\mathbf{W}'\mathbf{\Sigma}^{-1}\mathbf{M}_Z^{\Sigma^{-1}}\mathbf{W})^{-1}\mathbf{W}'\mathbf{\Sigma}^{-1}\mathbf{M}_Z^{\Sigma^{-1}}\mathbf{Y}\\
&= \begin{pmatrix}\mathbf{Y}_1\\ \mathbf{Y}_2\end{pmatrix} - \begin{pmatrix}\mathbf{X}_1\\ \mathbf{X}_2\end{pmatrix}\widehat{\boldsymbol{\beta}} = \begin{pmatrix}\mathbf{Y}_1 - \mathbf{X}_1\widehat{\boldsymbol{\beta}}\\ \mathbf{Y}_2 - \mathbf{X}_2\widehat{\boldsymbol{\beta}}\end{pmatrix}.
\end{aligned}$$

$\square$

Remark 1. Regarding that $\mathbf{\Sigma}_1$ and $\mathbf{\Sigma}_2$ are supposed to be positive definite, we can write (see [3, p. 441, Lemma 10.1.35])

$$\begin{aligned}
\mathbf{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}} &= \mathbf{\Sigma}_1^{-1} - \mathbf{\Sigma}_1^{-1}\mathbf{Z}_1(\mathbf{Z}_1'\mathbf{\Sigma}_1^{-1}\mathbf{Z}_1)^{-1}\mathbf{Z}_1'\mathbf{\Sigma}_1^{-1} = (\mathbf{M}_{Z_1}\mathbf{\Sigma}_1\mathbf{M}_{Z_1})^{+},\\
\mathbf{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}} &= \mathbf{\Sigma}_2^{-1} - \mathbf{\Sigma}_2^{-1}\mathbf{Z}_2(\mathbf{Z}_2'\mathbf{\Sigma}_2^{-1}\mathbf{Z}_2)^{-1}\mathbf{Z}_2'\mathbf{\Sigma}_2^{-1} = (\mathbf{M}_{Z_2}\mathbf{\Sigma}_2\mathbf{M}_{Z_2})^{+},
\end{aligned}$$

respectively.

Theorem 3. *Let the vector* $(\mathbf{Y}_1', \mathbf{Y}_2')'$ *in the regular model* (1) *be normally distributed. Then the* BLUE *of the useful vector parameter* $\boldsymbol{\beta}$ *is normally distributed vector* $\widehat{\boldsymbol{\beta}} \sim \mathrm{N}_k(\boldsymbol{\beta}, \mathrm{var}(\widehat{\boldsymbol{\beta}}))$, *where*

$$\begin{aligned}
\mathrm{var}(\widehat{\boldsymbol{\beta}}) = {}&[\mathbf{X}_1'\mathbf{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}}\mathbf{X}_1 + \mathbf{X}_2'\mathbf{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}}\mathbf{X}_2 - (\mathbf{X}_1'\mathbf{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}}\mathbf{S}_1\\
&+ \mathbf{X}_2'\mathbf{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}}\mathbf{S}_2)\mathbf{Q}_{11}(\mathbf{S}_1'\mathbf{\Sigma}_1^{-1}\mathbf{M}_{Z_1}^{\Sigma_1^{-1}}\mathbf{X}_1 + \mathbf{S}_2'\mathbf{\Sigma}_2^{-1}\mathbf{M}_{Z_2}^{\Sigma_2^{-1}}\mathbf{X}_2)]^{-1}
\end{aligned}$$

and $\mathbf{Q}_{11}$ *is the matrix from the proof of* Theorem 2.

P r o o f. The assertion is a consequence of the fact that $\boldsymbol{\beta}$ is a linear transform of the normally distributed random vector $(\mathbf{Y}_1', \mathbf{Y}_2')'$. $\square$

Lemma 2. *The confidence intervals for the single useful parameters* $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_k)'$ *are*

$$\mathscr{I}_{1-\alpha}(\beta_i) = \left\langle \widehat{\beta}_i - u\left(1 - \frac{\alpha}{2}\right)\sqrt{\mathbf{e}_i' \operatorname{var}(\widehat{\boldsymbol{\beta}})\mathbf{e}_i}, \widehat{\beta}_i + u\left(1 - \frac{\alpha}{2}\right)\sqrt{\mathbf{e}_i' \operatorname{var}(\widehat{\boldsymbol{\beta}})\mathbf{e}_i} \right\rangle,$$

$i = 1, \ldots, k$, *i.e.*

$$(\forall i = 1, \ldots, k)\big(\mathrm{P}\left[\beta_i \in \mathscr{I}_{1-\alpha}(\beta_i)\right] = 1 - \alpha\big),$$

where $\mathbf{e}_i = (0, \ldots, 0, \overset{i\text{-}th}{1}, 0, \ldots, 0)'$ *for all* $i = 1, \ldots, k$.

Proof. It is obvious. □

3. Numerical example

Let us introduce now an illustrative example of the theory presented above. The efficiency of a drug against hypertensis on a patient during two hospitalizations (I and II) is checked. The pressure is everyhour measured for 48 hours (i.e. $n_1 = n_2 = n = 48$) after giving the medicine with results

$$\begin{aligned}\mathbf{Y}_1 \quad = \quad & (158, 160, 157, 154, 154, 152, 147, 137, 134, 122, 119, 118\\ & 119, 115, 123, 123, 133, 134, 135, 133, 139, 144, 142, 141\\ & 144, 141, 141, 142, 142, 141, 139, 136, 130, 123, 120, 120\\ & 122, 113, 108, 108, 110, 114, 117, 120, 119, 127, 130, 135)'\end{aligned}$$

during I and

$$\begin{aligned}\mathbf{Y}_2 \quad = \quad & (153, 158, 158, 153, 157, 148, 138, 132, 127, 122, 117, 119\\ & 124, 124, 121, 125, 129, 127, 139, 145, 141, 147, 139, 136\\ & 136, 129, 135, 134, 134, 136, 134, 135, 134, 129, 127, 129\\ & 118, 118, 111, 111, 109, 121, 122, 111, 120, 130, 131, 144)'\end{aligned}$$

during the hospitalization II. It is expertly known that $\lambda_{11} = \lambda_{21} = \frac{2\pi}{24}$ and $\lambda_{12} = \lambda_{22} = \frac{2\pi}{16}$. Let us remark, that these data were obtained from the simulations, given by

$$Y_{1i} = -\tfrac{1}{3}i + 140 + 9\cos\tfrac{2\pi}{24}i + 12\sin\tfrac{2\pi}{24}i + 3\cos\tfrac{2\pi}{16}i + 4\sin\tfrac{2\pi}{16}i + \varepsilon_{1i},$$

in the first epoch (I) and

$$Y_{2i} = -\tfrac{1}{3}i + 140 + 8\cos\tfrac{2\pi}{24}i + 11\sin\tfrac{2\pi}{24}i + 4\cos\tfrac{2\pi}{16}i + 5\sin\tfrac{2\pi}{16}i + \varepsilon_{2i},$$

in the second epoch (II), $i = 1, \dots, n$.

We suppose that the elements of the variance matrices $\mathbf{\Sigma}_1$, $\mathbf{\Sigma}_2$ of the random vectors $\boldsymbol{\varepsilon}_1$, $\boldsymbol{\varepsilon}_2$ fulfil the relations $\{\mathbf{\Sigma}_1\}_{ij} = R_1(i-j)$, $\{\mathbf{\Sigma}_2\}_{ij} = R_2(i-j)$, $i, j = 1, \dots, n$, for the covariance functions $R_1(k) = 9\mathrm{e}^{-k}$ and $R_2(k) = 16\mathrm{e}^{-4k}$, where $k = 0, \dots, n-1$, respectively. In practice, the estimation of parameters σ^2 and γ of the covariance function $R(t) = \sigma^2\mathrm{e}^{-\gamma t}$ is not so easy (see [7] and [1]) and requires a little bit practice. In some cases, it is sufficient to use the variance matrix $\sigma^2\mathbf{I}$ instead of $\mathbf{\Sigma}$, $\{\mathbf{\Sigma}\}_{ij} = R(i-j)$, $i, j = 1, \dots, n$. Here, for the simplicity of verification of the results, we will suppose that the values of the functions R_1, R_2 were known.

Then, using the model described in Section 1 and Theorem 2, the estimated parameter $\widehat{\beta} = -0.3302$. This is the only useful parameter for us and expresses that the pressure decreases, utilizing the information from the hospitalizations I and II, approximately 0.33 mmHg per hour.

The other useful information is $(1-\alpha)$-confidence interval of $\widehat{\beta}$ (Corollary 1). It tells us, in case of normality, that the probability of covering of the parameter β equals $1-\alpha$, so describes the accuracy of the estimation $\widehat{\beta}$. Here, $\mathrm{var}(\widehat{\beta}) = 0.0012$ (Theorem 3) and choosing $\alpha = 0.1$ and $\alpha = 0.05$, we get the 0.90-confidence interval $\mathscr{I}_{0.90}(\beta_i) = \langle -0.3870, -0.2735\rangle$ $(u(0.95) = 1.64485)$ and the 0.95-confidence interval $\mathscr{I}_{0.95}(\beta_i) = \langle -0.3979, -0.2626\rangle$ $(u(0.975) = 1.95996)$, respectively (we suppose that the random vectors $\mathbf{Y}_1$ and $\mathbf{Y}_2$ are normally distributed).

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