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Theory of nonlinear Sagnac effect

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A nonlinear wave interaction is considered as a technique to increase the rotation sensitivity of the ring laser. The gyroscopic scale factor K is calculated in an active laser gyro with a dispersive medium. K is in reverse proportion to the group index $n^* = n + \nu dn/d\nu$ of the medium. In a monolithically-integrated GaAs ring laser, the value K ~5000 is obtainable (radius ~1 cm) in a linear case. In the presence of a strong wave, the dynamic nonlinear anomalous dispersion can provide an increase of K by 10–100 times in the vicinity of critical points where n passes zero. An expression of K is derived for the nonlinear Sagnac effect. The nonlinear dispersion is discussed in terms of "slow/fast" light.*

Keywords: Sagnac effect, ring laser, semiconductor laser, gyroscopic scale factor, nonlinear dispersion.

1. Introduction

The Sagnac effect in lasers [1–4] is a sensitivity of resonant optical frequencies in a ring-type cavity to the rotation. It is used for navigation-grade gyros based on ring lasers (active optical gyros). As the rotation sensitivity is in proportion with the size of the ring laser, the miniaturization of the laser gyro leads to a decrease in the rotation sensitivity. Semiconductor ring lasers (SRLs) are known in two versions, i.e., external-cavity type [5–7] and monolithically-integrated type [8,9]. An SRL with fiber external ring has been shown experimentally to have some rotation sensitivity [7]. As to the second type, there are not yet demonstration of such sensitivity as these lasers are quite small. We consider here a principal possibility to increase the sensitivity of small-size active laser gyros by usage of nonlinear properties of a medium filling the ring cavity (the nonlinear Sagnac effect). The semiconductor medium provides significant nonlinearity, including a substantial effect of carrier density on the dielectric permittivity.

A possibility to enhance the Sagnac response $\Delta\nu$ by nonlinear effects was predicted firstly by Kaplan and Meystre [10] on the basis of the intensity-induced nonreciprocity. What is considered here is another nonlinear effect associated with mode interaction [11] and with intensity-induced anomalous dispersion [12]. Comparison is discussed below.

Realizing that there is some discrepancy in the related literature on the rotation sensitivity of the ring laser with a dispersive medium in the cavity, we show the expression for the frequency splitting $\Delta\nu$ of the counter-propagating waves to be dependent on the rotation angular velocity Ω and on parameters of the medium, it is the refractive index n and the dispersion $dn/d\nu$.

2. Sagnac effect in the dispersive medium

Most theoretical works on laser gyros relate to gas media with $n \approx 1$. As to media with $n > 1$, there are discrepancies in the data. To illustrate the conflicting theoretical results on $\Delta\nu$, we can indicate the following reports. According to Refs. 2, 3, 4, 13, and 14, it is predicted $\Delta\nu \propto 1/n$. But it is stated $\Delta\nu \propto 1/n^2$ in Ref. 15 and $\Delta\nu \propto n$ in Ref. 16. In Ref. 17, for a monolithically-integrated SRL it has been obtained $\Delta\nu \propto 1/n^*$, where $n^* = n + \nu dn/d\nu$ is the group index that is not equal to n if $dn/d\nu \neq 0$. Thus, the theoretical values for $K = \Delta\nu/\Omega$ (the scale factor or gyro-factor) using formulas of different authors vary by ~50 times for GaAs-based SRL. This is a quite intolerable situation that should be cleared up. Also there is no common agreement on the influence of the dispersion. It was considered in Refs. 2, 3, 14, and 17, the dispersion is a parameter involved in the effect. However, in the most frequently quoted (Ref. 4), it is stated that the dispersion in the first order does not influence the splitting $\Delta\nu$.

We consider the ring cavity filled uniformly with a dispersive medium. Other assumptions are, the Sagnac splitting is small, $\Delta\nu << \nu$ (ν is the optical frequency) and rotation rate corresponds to a non-relativistic range. A general consideration in Ref. 14 has led to an expression

$$\Delta\nu/\nu = -(1/n)[(4\Omega A/Lc) + \Delta], \tag{1}$$

where n is the mean value of the refractive index around the ring and Δ is the mean value, around the ring, of the difference between the two values of refractive index for counter-propagating waves, A is the area of the ring, and L

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is its perimeter. This expression (Eq. 30 of Ref. 14) is not an ultimate formula because Δ depends on $\Delta\nu$. It is an equation that should be solved. For the uniform case, we assume an approximation accounting for the second order dispersion

$$\Delta = \Delta\nu dn/d\nu + (1/2)\Delta\nu^2 d^2n/d\nu^2 , \qquad (2)$$

and we obtain a solution of Eq. (1)

$$\Delta\nu = \frac{n^*}{\nu d^2n/d\nu^2}\left\{1-\left[1-\frac{8\Omega A\nu^2 d^2n/d\nu^2}{n^{*2}Lc}\right]^{1/2}\right\} \qquad (3)$$

From this, if an inequality

$$8\Omega A\nu^2 d^2n/d\nu^2/(n^{*2}Lc) < 1. \qquad (4)$$

is valid, it is easy to obtain the Sagnac response in the first order approximation of dispersion

$$|\Delta\nu| = 4\Omega A/(L\lambda n^*), \qquad (5)$$

that is the same as obtained in Ref. 17. Here λ is the wavelength in vacuum. The inequality in Eq. (4) appears to be valid for static dispersion in typical semiconductors like GaAs and for moderate rotation rate. Therefore, in these cases Eq. (5) seems to be sufficiently correct. However, in the nonlinear case it could be necessary to take into account a correction associated with second-order dispersion according to Eq. (3). Notice that the latter is nonlinear in respect to the rotation rate in contrast to linear Eq. (5).

Generally, the normal dispersion leads to a decrease in the Sagnac splitting $\Delta\nu$, according to Eq. (3), because it increases the value of the group index n^*. Most of semiconductor lasers work in the range of normal dispersion as is seen from the empirical rule $n^* > n$ for linear parameters (in GaAs $n = 3.618$, $n^* = 4.77$ at $\lambda = 880$ nm). On the other hand, in order to increase $\Delta\nu$, the anomalous (negative) dispersion is desirable.

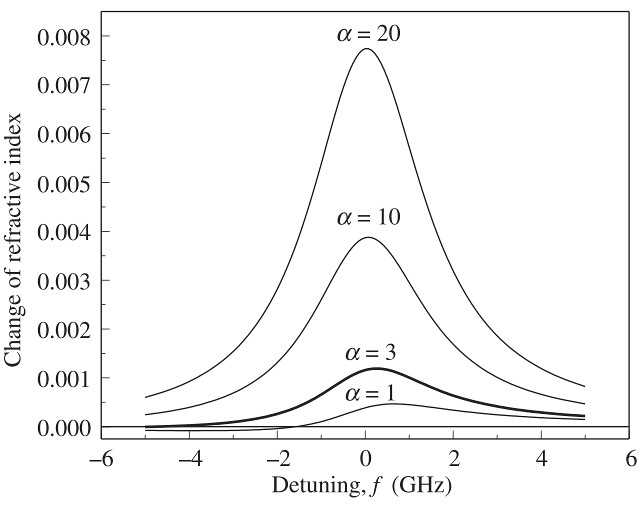

Fig. 1. Calculated spectra of perturbed refractive index (intensity-induced part) in vicinity of a strong wave frequency at different values of the factor α. The relaxation rate is taken $\gamma = 10^{10}$ s^{-1}. The differential gain is assumed to be constant.

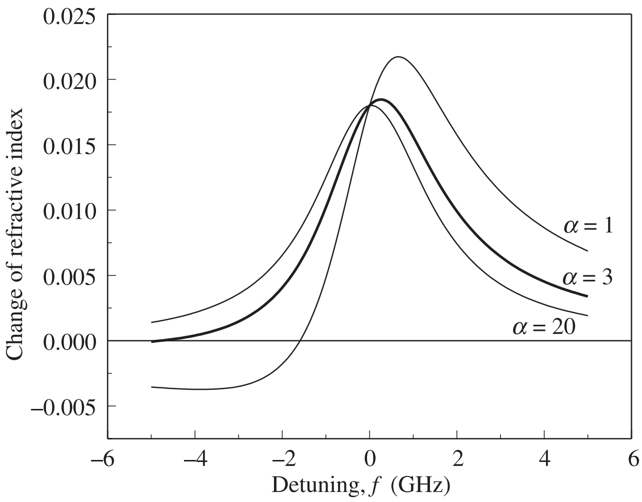

Fig. 2. Calculated spectra of perturbed refractive index (intensity-induced part) in vicinity of a strong mode frequency at different values of the factor α. The relaxation rate is taken $\gamma = 10^{10}$ s^{-1}. The differential index is assumed to be constant.

3. Optical drag coefficient

Equation (1) is derived in the rotating frame, therefore, the medium is assumed to be resting and there is no optical drag. The derivation implies accurate calculation of the phase velocity of light in the rotating frame accounting for an influence of the dispersive medium. In contrast to this, if the Sagnac effect is treated in the rest frame, the optical drag coefficient α_d can be introduced. Corresponding derivations are shown in Refs. 4 and 17. As applied to the uniformly filled ring cavity, the result is

$$\Delta\nu = 4A\Omega n^2(1-\alpha_d)/(L\lambda n^*). \qquad (6)$$

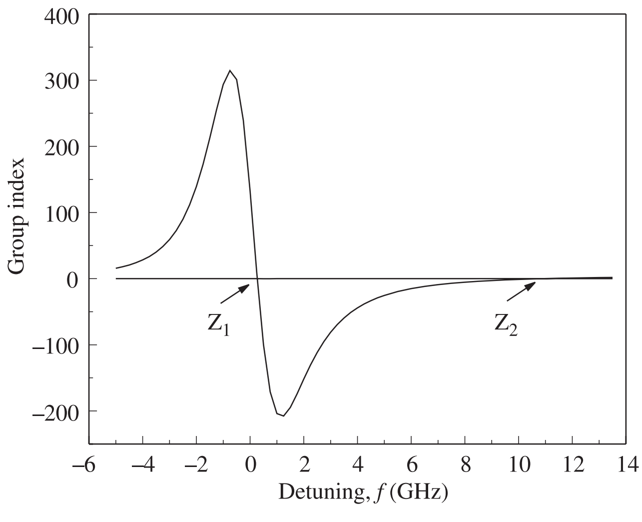

Fig. 3. Calculated spectrum of the perturbed group index in a vicinity of the strong wave frequency. The critical points Z_1 and Z_2 are shown where total group index passes zero value.

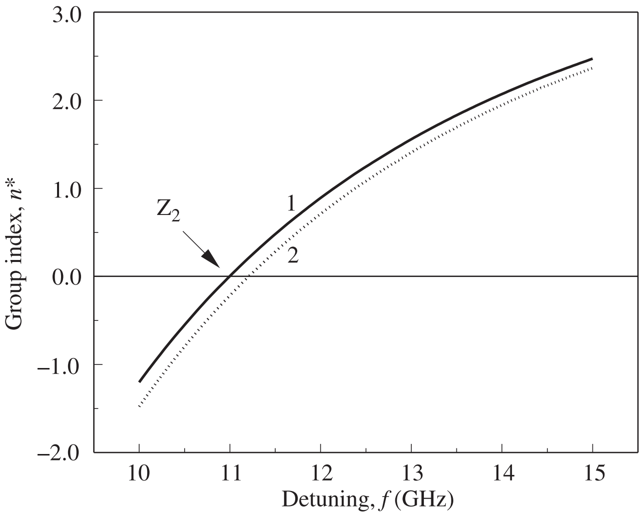

Fig. 4. Perturbed group index in vicinity of the critical point Z_2 (curve 1). Curve 2 is plotted for the same numerical parameters except the intensity of the strong wave that is assumed to be higher by 5%.

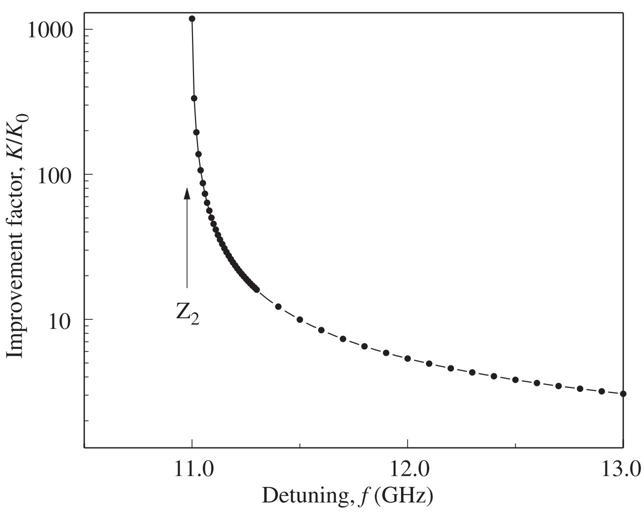

Fig. 5. Factor K/K_0 of the rotation sensitivity improvement by the use of the nonlinear Sagnac effect in the probe wave in function of its detuning from the strong wave frequency. The position of the critical point Z_2 is shown by an arrow.

Considering a non-relativistic case, we expect identical results in both frames with an accuracy of a relative magnitude of the order of υ/c, where υ is the linear velocity of rotation. Comparison with Eq. (5) gives

$$\alpha_d = 1 - 1/n^2. \tag{7}$$

This is a primary expression for the optical drag coefficient as derived by Fresnel in 1818 using a theory of elastic ether. According to Eq. (7), the quantity α_d is not much sensitive to dispersion as it is an averaged value for both opposite directions of light propagation.

4. Estimate of nonlinear anomalous dispersion

In a strong electromagnetic field, the optical parameters of a nonlinear medium are perturbed in some vicinity of the frequency of the strong field. For a localized index perturbation, both signs of the dispersion will appear, positive ("normal" one) and negative ("anomalous" one). Thus, we are interested in a nonlinear mechanism of a perturbation of optical parameters of a medium in a narrow frequency range. We consider below the nonlinear mode (wave) interaction treated in Ref. 11 for the theoretical explanation of the experimentally observed asymmetric suppression of spectral modes in the semiconductor laser. It provides perturbations of the group index in vicinity of a strong wave frequency. This perturbation can be treated in terms of "fast/slow" light, as it is shown recently [12,17]. The spectral range $\delta\nu$ of the perturbation is determined by the rate of the relaxation. The main contribution into a giant nonlinearity comes from the influence of free carriers on the refractive index of semiconductors. An estimate of the index variation δn is

$$\delta n \approx |dn/dN|\ \delta N, \tag{8}$$

where δN is the variation of the carrier density, and dn/dN is the differential index that is about -3.6×10^{-21} cm^3 in bulky-GaAs lasers [18]. Taking δN ~3×10^{18} cm^3 we get an estimate of δn ~10^{-2}. This appears to be sufficient to cause nonlinear phenomena like self-focusing, mode coupling, bistability, etc. A direct influence of δn on $\Delta\nu$ is rather small. However, an influence of dispersion can be substantial, if the index variation δn is localized in a narrow spectral domain. A magnitude of the variation of the dispersion $dn/d\nu$ can be estimated as $|\delta n/\delta\nu|$. Taking $\delta\nu$ as ~10^{11} s^{-1} we obtain an upper estimate ~3.6×10^{-13} Hz^{-1} for the dispersion. It seems to be substantially large, namely we estimate the term $|\nu dn/d\nu|$ ~122 that is much larger than linear refractive index 3.618. It is seen that the nonlinear group index can vary in a wide range including zero points and negative values. It follows from an alternate variation of the dispersion term around the strong wave frequency in a narrow spectral range. Therefore the dynamic dispersion can provide a negative (anomalous) sign of the total dispersion.

5. Nonlinear wave interaction

The nonlinear wave interaction occurs due to a formation of the dynamic grating in the nonlinear medium. The grating of the carrier density appears in accordance with the interference pattern of waves. It can produce substantial perturbation of the complex permittivity, so waves are subjected to a nonlinear scattering with a photon exchange between them. This mechanism was shown to be involved in asymmetric suppression of spectral modes in semiconductor lasers [11], in splitting of lines in the mode-beating spectra of semiconductor lasers, and also in the multi-wave mixing and other nonlinear phenomena in semiconductors. The mechanism has been shown to provide optical perturbations leading to the slow or fast light phenomena [12]. In Ref. 19, the same mechanism was calculated for the light slowing in optical amplifiers.

The nonlinear component δn of the refractive index at the frequency ν can be expressed as [12]

$$\delta n = C|E_0|^2(\alpha + x)/[\gamma(1 + x^2)], \tag{9}$$

where $C = -B(N - N_0)(d\varepsilon''/dN)/(2n)$, N is the carrier density, N_0 is N at the inversion threshold, ε'' is the imaginary part of the dielectric constant, E_0 is the field amplitude of the strong wave, γ is the total rate of relaxation of the population at working levels, B is the stimulated recombination coefficient (in units of m^2/V^2s), α is the linewidth enhancement factor, and $x = 2\pi(\nu_s - \nu)/\gamma$ is the normalized detuning with respect to a strong wave frequency ν_s. As it is shown in Fig. 1, the perturbed index gets a bump in vicinity of ν_s that grows along with an increase in the parameter α. These calculations are performed assuming the parameters N, $d\varepsilon''/dN$, γ and the strong wave intensity to be constant. It is important to mention that an increase in the factor α by a decrease in the differential gain and of the parameter $d\varepsilon''/dN$ (for example, by detuning into the wing of the gain spectrum) does not give a rise of δn. This is shown in Fig. 2 where an assumption is used of a constant differential index. In that case, there is no increase of δn with an increase of α.

The peak value of δn is rather small in the real range of intensities, but the dynamic dispersion appears not to be small because the perturbation is localized in a narrow spectral range. The total group index is

$$n^* = n^*_{\rm lin} - (2\pi\nu C|E_0|^2/\gamma^2)(1 - 2\alpha x - x^2)/(1 + x^2)^2, \tag{10}$$

where n^*_{lin} is the linear component of the group index. As applied to GaAs, the quantity of Eq. (9) has zero points at a sufficient optical power (>10 kW/cm^2). At these critical points, a formal group velocity of the probe wave goes to infinity. The perturbed group index is shown in Figs. 3 and 4. The total value passes zero value in the points Z_1 and Z_2. The first is very close to the strong wave frequency and it is not suitable for usage. Another one, Z_2 seems to be useful as it is detuned from ν_s by about 11 GHz.

6. Nonlinear Sagnac effect. Numerical estimates of gyro-factor

Taking the first-order term of the nonlinear dispersion, we obtain from Eq. (9) the following expression for the nonlinear Sagnac effect

$$\Delta\nu = \frac{4A\Omega}{L}\left\{\lambda n^*_{lin} - \frac{\left(2\pi c C\left|E_0\right|^2\right)\left(1-2\alpha x - x^2\right)}{\left[\gamma^2\left(1+x^2\right)\right]^2}\right\}^{-1} \tag{11}$$

Formally, one can expect an infinite response in the critical points. Actually, the improvement is limited by instability of strong wave intensity. The critical point is not suitable for practical use. The value of K passes infinity and changes a sign. Therefore, an exact critical point corresponds to a strong instability of the Sagnac response. The operation point should be distanced somewhat from the critical point. The position of the critical point Z_2 is shown in Fig. 4 corresponding to the same numerical parameters as in Fig. 3 and taking $\alpha = 3$. It is shown there by the dotted line a shift produced by 5% increase in the strong wave intensity. The sensitivity of the position of Z_2 to the intensity leads to a strict requirement on the stabilization of the optical power in the ring laser. The calculated ratio K/K_0, where K_0 is the gyroscopic scale factor in the linear case, is shown in Fig. 5 for the same numerical parameters. By our estimation, it is possible to increase K by 10–100 times with no change of the ring size.

Two types of semiconductor ring lasers (SRLs) are known [1–5], the external cavity version and the monolithically-integrated version. In GaAs, the background (linear) parameters are $dn/d\nu = 3.41\times10^{-15}$ Hz^{-1} and $d^2n/d\nu^2 = 4.76\times10^{-29}$ Hz^{-2} ($\lambda = 880$ nm). The inequality in Eq. (4) is fulfilled for linear parameters of GaAs and Eq. (5) is valid to calculate the initial gyro scale factor in absence of nonlinear enhancement. Taking the radius $R = 1$ cm for a monolithically integrated circular ring cavity, we obtain an estimate of $K_0 \approx 5000$, that seems to be not sufficient for navigation applications. Therefore, the nonlinear enhancement could be desirable.

Consider quantitatively the illustrative case of group velocity variation in the vicinity of the strong wave frequency $\nu_s = 340.672$ THz (wavelength is ~880 nm). This is illustrated in Fig. 3. The perturbed group index n^* changes a sign. There are ranges of slow light (large n^*). Points of $n^* = 0$ are labelled Z_1 and Z_2. The group velocity goes to infinity and one can expect enhanced rotation sensitivity in these points. The point Z_1 is very close to ν_s and the function $n^*(\nu)$ appears to be very steep. The inequality in Eq. (4) is not valid there. This means that the Sagnac splitting becomes a nonlinear function of the rotation rate. Estimation indicates that there is no substantial improvement in the rotation sensing because of the influence of high-order dispersion terms. The point Z_2 is shifted by ~11 GHz from ν_s. In the vicinity of this point, the inequality in Eq. (4) is fulfilled and Eq. (5) is valid.

7. Discussion

7.1. The group velocity variation

The nonlinear wave interaction produces a substantial variation of the group velocity $v_g = c/n^*$ in vicinity of the strong wave frequency, whereas variations of phase velocity are quite small. There is a possibility to obtain slow ($v_g << c$) or fast ("superluminal", $v_g > c$) light, depending on detuning of the probe wave from the strong wave frequency. There are questions of interpretation of varied group index as it is seen in Fig. 3. The answers are as follows. First, this is the case of propagation of two interacting waves, and the total optical energy is carried by the strong wave in the positive direction. Second, the negative group velocity is in the range where the probe wave is suppressed by the strong one. Therefore the strong wave generates photons at the frequency of probe wave but propagating in the opposite direction (this is the way to suppress the positive flux of photons). These photons correspond to wave packets propagating in the opposite direction, i.e., to waves having a negative group velocity. The critical points are on edges of this frequency range. At these points the wave packets do not exist because frequency components propagate in opposite directions.

7.2. A comparison with other nonlinear mechanisms

The considered approach to a nonlinear enhancement of the Sagnac splitting $\Delta\nu$ is applicable to an active gyro scheme and it is quite different from the induced non-reciprocity approach, proposed in Ref. 10 mainly for the passive scheme. Here we considered the strong wave only as a source of the nonlinear perturbation with an induced anomalous dispersion. The Sagnac effect is calculated for a frequency within the anomalous dispersion domain, being distant from the frequency of the strong wave. The mechanism of the wave interaction includes generally all dynamic gratings, appearing in the nonlinear medium by an interference of the interacting waves. In principle, it includes the grating produced by counter-propagating waves, namely, the standing waves. As it was indicated in Ref. 10, in the semiconductor medium with a nonlinearity provided by mobile carriers, the short-distance variations of carrier density are suppressed by the carrier diffusion. Therefore, these short-distance variations are not important.

In general, the wave interaction via the differential index is sensitive to the diffusion smoothening of the spatial variations of the carrier density. If the period of these variations is produced by mode beating, the spatial component has characteristic length $2\pi/\Delta k$, where Δk is the difference of wave numbers (in a medium), $k = 2\pi n/\lambda$. The criterion of effective smoothening is

$$(1/2\pi)\Delta k L_D \geq 1, \qquad (12)$$

where L_D is the diffusion length of excess carriers. Let us estimate this criterion in two important cases taking the following parameters $\lambda = 880$ nm, $L_D = 1$ µm, $n = 3.6$, and the cavity length $L = 500$ µm:

- variation is produced by a standing wave. In this case $\Delta k = 2k = 4\pi n/\lambda$. The left side of Eq. (11) is $2nL_D/\lambda = 8.2$. This is an indication of a smoothening of $\delta\nu$ variation and of a suppression of the non-reciprocity effect. The intensity-induced non-reciprocity [10] is based on the standing wave effect. According to Ref. 20, it is suppressed in proportion with a factor of $\exp(-4k^2L_D^2)$. We obtain $k^2L_D^2$ ~660. This means that the induced non-reciprocity is totally washed out.
- variation is produced by co-propagating waves, for example, by adjacent longitudinal modes of the laser. In this case $\Delta k = \pi/nL$, and left side of Eq. (10) is $L_D/(2nL) = 2.7\times10^{-4}$. This indicates there is no smoothening. Therefore, within the typical range of intermode frequency spacing the diffusion effect can be neglected.

The considered nonlinear effect is based on the wave interaction via a long-period interference pattern, therefore it has no relation to the induced non-reciprocity. It is related to the concept of a fast light as the anomalous dispersion reduces the group index (even to zero and to negative values).

7.3. A possibility of nonlinear Sagnac effect observation

The enhanced frequency splitting can be achieved in some modes of the ring cavity that fall into the detuning range in respect of a strong mode. The detuning is discussed above. Therefore, the cavity length should be chosen in a manner favourable to an existence and an excitation of such modes. Actually the nonlinear Sagnac response can be observed simultaneously with a normal one (in the strong modes). In other words, the spectrum of mode beating would contain both the regular Sagnac beating signal (from the strong modes) and a signal at enhanced beating frequency (from detuned modes). The difference of these beating frequencies can be also used as a measure of the rotation rate.

The resonant-like enhancement near a critical point produces a strict requirement to the power stability of the strong mode because fluctuations of the critical frequency would be associated with some additional noise. Probably this type of noise (instability of the gyro-factor K) is the most serious difficulty for practical applications of the nonlinear effect.

8. Conclusions

The gyroscopic scale factor in semiconductor ring lasers is shown to be sensitive to both the refractive index and its dispersion. Expressions are given for the Sagnac splitting $\Delta\nu$ accounting for one or two orders of the dispersion. With the linear optical parameters of GaAs the factor K can be as large as ~5000, and it can be, in principle, increased by 10–100 times using the nonlinear negative dispersion. Equation (9) is given for an enhanced nonlinear Sagnac effect. The enhancement is based on the anomalous induced dispersion (as a result of nonlinear mode interaction) and has no relation to the nonlinear non-reciprocity. The mode interaction is shown to provide a substantial perturbation of the group velocity of the probe wave in vicinity of the strong wave frequency, including a generation of "fast" and "slow" light.

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