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From massive self-dual p-forms towards gauge p-forms

Research Article

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Abstract: Massive self-dual p-forms are quantized through the construction of an equivalent first-class system and then quantizing the resulting first-class system. The construction of the equivalent first-class system is achieved using the gauge unfixing and constraints conversion BF methods. The Hamiltonian path integral of the first-class system takes a manifestly Lorentz-covariant form.

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1. Introduction

The quantization of Hamiltonian systems possessing only second-class constraints using the Dirac method [1] is intricate because the Poisson brackets between the constraints functions may contain canonical variables, and the quantum realization of the Dirac brackets that depend on the canonical variables may be nontrivial and is by no means guaranteed [2]. This issue can be solved through the construction of an equivalent first-class system followed by quantization of the resulting first-class system. The construction of the equivalent first-class system can be achieved using the gauge unfixing [2–6] or constraints conversion [7–11] methods. This quantization procedure has been applied to various models [12–29].

The gauge unfixing (GU) method [3, 4] is based on the possibility of interpreting a second-class constraint set as resulting from gauge-fixation of a first-class constraint set. The construction of the first-class theory using the GU approach involves the following steps: 1. the separation of the original second-class constraints into two subsets, one of them being first-class and the other providing some canonical gauge conditions for the first-class subset ("undo" gauge-fixing); 2. the construction of a first-class Hamiltonian with respect to first-class constraint subset starting from the original canonical Hamiltonian. Step 2 is achieved by use of an operator that projects any smooth function defined on the phase-space into an application that is in strong involution with the first-class subset. A systematic BRST treatment of the gauge unfixing method has been realized in [30, 31]. A constraints conversion method is represented by the Batalin-Fradkin (BF) method [8–11]. The BF approach to the issue of converting a second-class system into a first-class one relies

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on an appropriate extension of the original phase-space through the introduction of new variables. The first-class constraint set and the first-class Hamiltonian are constructed as a power series in the new variables, with the property that they coincide with the original second-class constraints and with the starting canonical Hamiltonian if one sets all the extravariables equal to zero. The terms of the series corresponding to the first-class constraints are determined by the requirement that the constraint functions must be in (strong) involution. The form of the first-class Hamiltonian results from the requirement for it to be in involution with the first-class constraint functions. The advantage of the BF method is that it involves the entire second-class constraint set, while the GU approach requires the separation of the second-class constraints into two subsets (one of them being first-class and the other providing some canonical gauge conditions for the first-class subset) which cannot always be achieved.

The purpose of this paper is to quantize massive self-dual p-forms [32]. Models with p-forms are interesting from the point of view of string and superstring theory, supergravity, and the gauge theory of gravity [33–38]. Moreover, p-forms have a special place in the theory of p-branes [37], where $(p+1)$-forms couple naturally to p-branes. The quantization procedure is based on the construction of an equivalent first-class system using the gauge unfixing and the Batalin–Fradkin methods, and then quantizing the resulting first-class system.

The paper is organized in six sections. In Section 2, starting from the massive self-dual p-forms we construct an equivalent first-class system using the GU approach. Section 3 is dedicated to the construction of the path integrals corresponding to the equivalent first-class system. In Section 4, we exemplify the BF method on massive self-dual p-forms, and in the Section 5 we construct the path integrals associated with the resulting first-class system. We address both the case p odd and the case p even. In the case p odd, based on some appropriate extensions of the phase-space, integrating out the auxiliary fields, and performing some field redefinitions, we discover (for both approaches) the manifestly Lorentz-covariant path integral corresponding to the Lagrangian formulation of the first-class systems. For different kinds of extensions of the phase-space, we identify the Lagrangian path integral for $(p-1)$- and p-forms with Stückelberg-like coupling, or the Lagrangian path integral for two types of p-forms with Chern-Simons-like coupling. In the case p even, for both approaches the Hamiltonian path integral of the first-class system takes a manifestly Lorentz-covariant form – the Lagrangian path integral for a topological Chern-Simons-like theory. Section 6 ends the paper with the main conclusions.

2. The construction of the first-class system – gauge unfixing approach

We start with a bosonic dynamic system with the phase-space locally parameterized by n canonical pairs $z^a = \left(q^i,\ p_i\right)$, endowed with the canonical Hamiltonian H_c, and subject to the second-class constraints

$$\chi_{\alpha_0}\left(z^a\right) \approx 0, \qquad \alpha_0 = \overline{1, 2M_0}. \tag{1}$$

We assume that one can split the second-class constraint set (1) into two subsets

$$\chi_{\alpha_0}\left(z^a\right) \equiv \left(G_{\bar{\alpha}_0}\left(z^a\right), C^{\bar{\beta}_0}\left(z^a\right)\right) \approx 0,\ \ \bar{\alpha}_0, \bar{\beta}_0 = \overline{1, M_0}, \tag{2}$$

such that the subset $G_{\bar{\alpha}_0}\left(z^a\right) \approx 0$ to be first-class

$$\left[G_{\bar{\alpha}_0}, G_{\bar{\beta}_0}\right] = D^{\bar{\gamma}_0}_{\bar{\alpha}_0\bar{\beta}_0} G_{\bar{\gamma}_0}. \tag{3}$$

The second-class behaviour of the overall constraint set ensures that $C^{\bar{\alpha}_0}\left(z^a\right) \approx 0$ may be regarded as some gauge-fixing conditions for the first-class subset.
We introduce an operator $\hat{X}$ [12] that associates an application $\hat{X}F$ with every smooth function F on the original phase-space

$$\hat{X}F = F - C^{\bar{\alpha}_0}\left[G_{\bar{\alpha}_0}, F\right] + \frac{1}{2} C^{\bar{\alpha}_0} C^{\bar{\beta}_0}\left[G_{\bar{\alpha}_0}, \left[G_{\bar{\beta}_0}, F\right]\right] - \cdots, \tag{4}$$

such that it is in strong involution with the functions $G_{\bar{\alpha}_0}$

$$\left[\hat{X}F, G_{\bar{\alpha}_0}\right] = 0. \tag{5}$$

With the help of this operator we construct a first-class Hamiltonian $H_{GU} = \hat{X}H_c$ with respect to the first-class constraints subset.
The original second-class theory and the GU system (subject to the first-class constraints $G_{\bar{\alpha}_0}\left(z^a\right) \approx 0$ and endowed with the first-class Hamiltonian H_{GU}) are classically equivalent since they possess the same number of physical degrees of freedom

$$\mathcal{N}_O = \tfrac{1}{2}\left(2n - 2M_0\right) = \mathcal{N}_{\mathrm{GU}}, \tag{6}$$

and the corresponding algebras of classical observables are isomorphic. Consequently, the two systems also become equivalent at the level of the path integral quantization, and we can replace the Hamiltonian path integral of the original second-class theory with that associated with the GU first-class system.

We next quantize the massive self-dual p-forms in the framework of the GU method. Self-dual p-forms in $D = 2p+1$ space-time dimensions are described by the Lagrangian action [32]

$$S = \int d^{2p+1}x \left(-\alpha\varepsilon_{\mu_1\ldots\mu_{2p+1}} F^{\mu_1\ldots\mu_{p+1}} A^{\mu_{p+2}\ldots\mu_{2p+1}} - \frac{m^2}{2p!} A_{\mu_1\ldots\mu_p} A^{\mu_1\ldots\mu_p} \right), \quad (7)$$

where α is a constant.

The corresponding field equations are

$$-2\alpha\varepsilon_{\mu_1\ldots\mu_p\nu_1\ldots\nu_{p+1}} F^{\nu_1\ldots\nu_{p+1}} - \frac{m^2}{p!} A_{\mu_1\ldots\mu_p} = 0. \quad (8)$$

Setting $\alpha = \frac{-m}{2p!(p+1)!}$ the field equations (8) may be written as self-dual equations for p odd [32]

$$A_{\mu_1\ldots\mu_p} = \frac{1}{m(p+1)!} \varepsilon_{\mu_1\ldots\mu_p\nu_1\ldots\nu_{p+1}} F^{\nu_1\ldots\nu_{p+1}}. \quad (9)$$

For p even, a factor "i" appears in the self-duality equations

$$A_{\mu_1\ldots\mu_p} = \frac{\mathrm{i}}{m(p+1)!} \varepsilon_{\mu_1\ldots\mu_p\nu_1\ldots\nu_{p+1}} F^{\nu_1\ldots\nu_{p+1}}, \quad (10)$$

with the antisymmetric tensor field complex. The corresponding Lagrangian action reads as

$$S = \int d^{2p+1}x \left(-\frac{\mathrm{i}}{m(p+1)!} \varepsilon_{\mu_1\ldots\mu_{2p+1}} F^{\mu_1\ldots\mu_{p+1}} A^{*\mu_{p+2}\ldots\mu_{2p+1}} + A_{\mu_1\ldots\mu_p} A^{*\mu_1\ldots\mu_p} \right). \quad (11)$$

If we decompose $A_{\mu_1\ldots\mu_p}$ into its real and imaginary parts it can be shown that the imaginary part is an auxiliary field. The elimination of the auxiliary field on its own field equation recovers the action (7) for the real part [32]. In the above relations $F_{\mu_1\ldots\mu_{p+1}} (\equiv \partial_{[\mu_1} A_{\mu_2\ldots\mu_{p+1}]})$ denotes the field-strength of the p-forms. In what follows the notation $[\mu\ldots\nu]$ signifies antisymmetry with respect to all the indices between brackets, without normalization factors (i.e. the independent terms appear only once and are not multiplied by overall numerical factors). We work with the Minkowski metric tensor of 'mostly minus' signature $\sigma_{\mu\nu} = \sigma^{\mu\nu} = \mathrm{diag}(+-\ldots-)$.

2.1. The case p odd

The canonical analysis of the model described by the Lagrangian action (7) displays the constraints [16]

$$\chi^{(1)i_1\ldots i_{p-1}} \equiv \pi^{0i_1\ldots i_{p-1}} \approx 0, \quad (12)$$

$$\chi^{(2)}_{i_1\ldots i_{p-1}} \equiv 2p\partial^i \pi_{ii_1\ldots i_{p-1}} - \frac{m^2}{(p-1)!} A_{0i_1\ldots i_{p-1}} \approx 0, \quad (13)$$

$$\chi^{i_1\ldots i_p} \equiv \pi^{i_1\ldots i_p} + \alpha(p+1)\varepsilon^{0i_1\ldots i_p j_1\ldots j_p} A_{j_1\ldots j_p} \approx 0, \quad (14)$$

and the canonical Hamiltonian

$$H_c^{(odd)} = \int d^{2p}x \left(-2pA_{0i_1\ldots i_{p-1}} \partial_i \pi^{ii_1\ldots i_{p-1}} + \frac{m^2}{2p!} A_{\mu_1\ldots\mu_p} A^{\mu_1\ldots\mu_p} \right), \quad (15)$$

where $\pi^{\mu_1\ldots\mu_p}$ are the canonical momenta conjugated to the fields $A_{\mu_1\ldots\mu_p}$. Constraints (12)–(14) are second-class and irreducible [2].

In order to separate the second-class constraint set (12)–(14) into two subsets, one being first-class and the other providing some canonical gauge conditions for the first-class subset, we write the constraints (13) in an equivalent form

$$\chi^{(2)}_{i_1\ldots i_{p-1}} \equiv \frac{1}{m^2} \left[-p!\partial^i \left(\pi_{ii_1\ldots i_{p-1}} - \alpha(p+1)\varepsilon_{0ii_1\ldots i_{p-1}j_1\ldots j_p} A^{j_1\ldots j_p} \right) + m^2 A_{0i_1\ldots i_{p-1}} \right] \approx 0, \quad (16)$$

and we eliminate the second-class constraints (14) (the reduced phase-space is locally parameterized by $A_{i_1\ldots i_p}$, $A_{0i_1\ldots i_{p-1}}$ and $\pi_{0i_1\ldots i_{p-1}}$). We are left with a system subject to the second-class constraints

$$C^{i_1\ldots i_{p-1}} \equiv \pi^{0i_1\ldots i_{p-1}} \approx 0, \quad (17)$$

$$G_{i_1\ldots i_{p-1}} \equiv \frac{1}{m^2} \left(2\alpha p! \varepsilon_{0i_1\ldots i_{p-1}j_1\ldots j_{p+1}} \partial^{[j_1} A^{j_2\ldots j_{p+1}]} + m^2 A_{0i_1\ldots i_{p-1}} \right) \approx 0, \quad (18)$$

while the canonical Hamiltonian (15) takes the form

$$\bar{H}_c^{(odd)} = \int d^{2p}x \left(2\alpha p A_{0i_1\ldots i_{p-1}} \varepsilon^{0i_1\ldots i_{p-1}j_1\ldots j_{p+1}} \partial_{[j_1} A_{j_2\ldots j_{p+1}]} + \frac{m^2}{2p!} A_{\mu_1\ldots\mu_p} A^{\mu_1\ldots\mu_p} \right). \quad (19)$$

Replacing the number of canonical variables $2n = \binom{2p}{p} + 2\binom{2p}{p-1}$ and the number of independent second-class constraints $2M_0 = \binom{2p}{p-1} + \binom{2p}{p-1}$ in (6), the number of physical degrees of freedom per space point is found to be equal to $\mathcal{N}_O^{(odd)} = \frac{1}{2}\binom{2p}{p}$.

The matrix of the Poisson bracket between the constraint functions $\chi_{\alpha_0} \equiv \left(G_{i_1\ldots i_{p-1}}, C^{j_1\ldots j_{p-1}}\right) \approx 0$ becomes

$$\left(\left[\chi_{\alpha_0}, \chi_{\beta_0}\right]\right) = \begin{pmatrix} \mathbf{0} & \frac{1}{(p-1)!}\delta^{j_1}_{[i_1}\ldots\delta^{j_{p-1}}_{i_{p-1}]} \\ -\frac{1}{(p-1)!}\delta^{i_1}_{[j_1}\ldots\delta^{i_{p-1}}_{j_{p-1}]} & \mathbf{0} \end{pmatrix}. \tag{20}$$

According to the GU method we will consider (18) as the first-class constraint set and the remaining constraints (17) as the corresponding canonical gauge conditions (the other choice, namely, (17) as the first-class constraint set and the remaining constraints (18) as the corresponding canonical gauge conditions yields a path integral that cannot be written (after integrating out auxiliary variables) in a manifestly covariant form [17]).

The first-class Hamiltonian with respect to (18) follows from the relation (4)

$$H_{GU}^{(odd)} = \bar{H}_c^{(odd)} - C^{i_1\ldots i_{p-1}}\left[G_{i_1\ldots i_{p-1}}, \bar{H}_c\right] + \frac{1}{2}C^{i_1\ldots i_{p-1}}C^{j_1\ldots j_{p-1}}\left[G_{i_1\ldots i_{p-1}}, \left[G_{j_1\ldots j_{p-1}}, \bar{H}_c\right]\right] - \cdots. \tag{21}$$

The concrete form of the first-class Hamiltonian $H_{GU}^{(odd)}$ is given by

$$H_{GU}^{(odd)} = \bar{H}_c^{(odd)} + \int d^{2p}x \left[\frac{1}{p}\left(A_{i_1\ldots i_p} + \frac{1}{m^2}\frac{(p-1)!}{2}\partial_{[i_1}\pi_{i_2\ldots i_p]0}\right)\partial^{[i_1}\pi^{i_2\ldots i_p]0}\right]. \tag{22}$$

Inserting the number of canonical variables and the number of the independent first-class constraints $M_0 = \binom{2p}{p-1}$ in (6), the number of physical degrees of freedom is found to be equal to $\mathcal{N}_{GU}^{(odd)} = \frac{1}{2}\binom{2p}{p}$.

2.2. The case p even

The canonical analysis of the model described by the Lagrangian action (7) in the case of p even displays the constraints [16]

$$\chi^{(1)i_1\ldots i_{p-1}} \equiv \pi^{0i_1\ldots p-1} \approx 0, \tag{23}$$

$$\chi^{(1)}_{i_1\ldots i_p} \equiv \pi_{i_1\ldots i_p} + \alpha\,(p+1)\,\varepsilon_{0i_1\ldots i_p j_1\ldots j_p}A^{j_1\ldots j_p} \approx 0, \tag{24}$$

$$\chi^{(2)}_{i_1\ldots i_{p-1}} \equiv -\frac{m^2}{(p-1)!}A_{0i_1\ldots i_{p-1}} \approx 0, \tag{25}$$

$$\chi^{(2)}_{i_1\ldots i_p} \equiv \pi_{i_1\ldots i_p} - \alpha\,(p+1)\,\varepsilon_{0i_1\ldots i_p j_1\ldots j_p}A^{j_1\ldots j_p} \approx 0, \tag{26}$$

and the canonical Hamiltonian

$$H_c^{(even)} = \int d^{2p}x \left(\frac{m^2}{2p!}A_{i_1\ldots i_p}A^{i_1\ldots i_p} + \frac{m^2}{2\,(p-1)!}A_{0i_1\ldots i_{p-1}}A^{0i_1\ldots i_{p-1}}\right). \tag{27}$$

Constraints (23)–(26) are second-class and irreducible. The matrix of the Poisson bracket between the constraint functions becomes

$$\left(\left[\chi_{\alpha_0}, \chi_{\beta_0}\right]\right) = \begin{pmatrix} \mathbf{0} & \mathbf{0} & X^{i_1\ldots i_{p-1}}_{j_1\ldots j_{p-1}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & Y^{i_1\ldots i_p j_1\ldots j_p} \\ -X^{j_1\ldots j_{p-1}}_{i_1\ldots i_{p-1}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -Y_{i_1\ldots i_p j_1\ldots j_p} & \mathbf{0} & \mathbf{0} \end{pmatrix}, \tag{28}$$

where $X^{i_1\ldots i_{p-1}}_{j_1\ldots j_{p-1}} = \frac{m^2}{(p-1)!}\frac{1}{(p-1)!}\delta^{i_1}_{[j_1}\ldots\delta^{i_{p-1}}_{j_{p-1}]}$ and $Y^{i_1\ldots i_p j_1\ldots j_p} = 2\alpha\,(p+1)\,\varepsilon^{0i_1\ldots i_p j_1\ldots j_p}$. We observe that the constraints (23) and (25) generate a second-class constraints subset. Eliminating the second-class constraints (23) and (25) (the coordinates of the reduced phase-space are $A_{i_1\ldots i_p}$ and $\pi^{i_1\ldots i_p}$) we are left with a system subject to the second-class constraints (24) and (26) while the canonical Hamiltonian takes the form

$$\bar{H}_c^{(even)} = \int d^{2p}x \left(\frac{m^2}{2p!}A_{i_1\ldots i_p}A^{i_1\ldots i_p}\right). \tag{29}$$

Replacing the number of canonical variables $2n = 2\binom{2p}{p}$ and the number of independent second-class constraints $2M_0 = 2\binom{2p}{p}$ in (6), one obtains that the number of physical degrees of freedom is equal to $\mathcal{N}_O^{(even)} = 0$.

According to the GU method we consider (24) as the first-class constraint set, the remaining constraints (26) as the corresponding canonical gauge conditions, and redefine the first-class constraints (24) as

$$G_{i_1\ldots i_p} \equiv \frac{1}{2\alpha p!\,(p+1)!}\varepsilon_{0i_1\ldots i_p j_1\ldots j_p}\left(\pi^{j_1\ldots j_p} + \alpha\,(p+1)\,\varepsilon^{0j_1\ldots j_p k_1\ldots k_p}A_{k_1\ldots k_p}\right) \approx 0. \tag{30}$$

The first-class Hamiltonian with respect to (30) follows from the relation

$$H_{GU}^{(even)} = \bar{H}_c^{(even)} - \chi^{(2)i_1\ldots i_p}\left[G_{i_1\ldots i_p}, \bar{H}_c^{(even)}\right] + \frac{1}{2}\chi^{(2)i_1\ldots i_p}\chi^{(2)j_1\ldots j_p}\left[G_{i_1\ldots i_p}, \left[G_{j_1\ldots j_p}, \bar{H}_c^{(even)}\right]\right] - \cdots . \quad (31)$$

The concrete form of the first-class Hamiltonian is given by

$$H_{GU}^{(even)} = \int d^{2p}x \left[\frac{m^2}{8p!}\left(A_{i_1\ldots i_p} + \frac{1}{\alpha p!\,(p+1)!}\varepsilon_{0i_1\ldots i_p j_1\ldots j_p}\pi^{j_1\ldots j_p}\right) \times \left(A^{i_1\ldots i_p} + \frac{1}{\alpha p!\,(p+1)!}\varepsilon^{0i_1\ldots i_p k_1\ldots k_p}\pi_{k_1\ldots k_p}\right)\right]. \quad (32)$$

Inserting the number of canonical variables and the number of independent first-class constraints $M_0 = \binom{2p}{p}$ in (6), the number of physical degrees of freedom is found to be equal to $\mathcal{N}_{GU}^{(even)} = 0$.

3. Covariant path integral for the GU system

3.1. The case p odd

In Ref. [39] the massive self-dual 3-forms in $D = 7$ were analyzed from the point of view of the Hamiltonian path integral quantization. The quantization procedure was based on the construction of a first-class system equivalent with the original second-class theory, and then quantizing the resulting first-class system. In the second section of Ref. [39] the equivalent first-class system was constructed in the framework of the GU approach and it was obtained the manifestly Lorentz-covariant path integral corresponding to the Lagrangian formulation of the equivalent first-class system.

In order to obtain a Lorentz-covariant path integral we start from first-class system derived in the above (subject to the first-class constraints (18) and whose evolution is governed by the first-class Hamiltonian (22)), and then consider another first-class system (a reducible one) which is equivalent to the GU system at both classical and path integral levels. After an appropriate extension of the phase-space (we enlarge the phase-space with the Lagrange multipliers), some field redefinitions, and performing some partial integrations over the auxiliary fields, we find that the argument of the exponential from the Hamiltonian path integral of the first-class system takes the form

$$S_{GU}^{(odd)} = \int d^{2p+1}x\left[-\alpha\varepsilon^{\mu_1\ldots\mu_{2p+1}}\bar{F}_{\mu_1\ldots\mu_{p+1}}\bar{A}_{\mu_{p+2}\ldots\mu_{2p+1}} - \frac{1}{2p!}\left(\partial_{[\mu_1}B_{\mu_2\ldots\mu_p]} - m\bar{A}_{\mu_1\ldots\mu_p}\right)\left(\partial^{[\mu_1}B^{\mu_2\ldots\mu_p]} - m\bar{A}^{\mu_1\ldots\mu_p}\right)\right]. \quad (33)$$

The functional (33) describes a (Lagrangian) Stückelberg-like coupling between the $(p-1)$-form $B_{\mu_1\ldots\mu_{p-1}}$ and the p-form $\bar{A}_{\mu_1\ldots\mu_p}$ [40]. The $(p-1)$-form $B_{\mu_1\ldots\mu_{p-1}}$ acts like the Stückelberg field, thus ensuring the gauge invariance of the above functional.

Starting from the GU system constructed in the above we consider the following field combinations

$$\mathcal{F}_{i_1\ldots i_p} = A_{i_1\ldots i_p} + \frac{(p-1)!}{m^2}\partial_{[i_1}\pi_{i_2\ldots i_p]0},\quad \mathcal{F}_{0i_1\ldots i_{p-1}} = A_{0i_1\ldots i_{p-1}}. \quad (34)$$

which are in (strong) involution with first-class constraints (18)

$$\left[\mathcal{F}_{i_1\ldots i_p}, G_{j_1\ldots j_{p-1}}\right] = \left[\mathcal{F}_{0i_1\ldots i_{p-1}}, G_{j_1\ldots j_{p-1}}\right] = 0. \quad (35)$$

By direct computation we obtain that $\mathcal{F}_{\mu_1\ldots\mu_{p+1}} \equiv \left\{\mathcal{F}_{0i_1\ldots i_{p-1}}, \mathcal{F}_{i_1\ldots i_p}\right\}$ satisfy the equation

$$\partial^\nu\partial_{[\nu}\mathcal{F}_{\mu_1\ldots\mu_p]} = -\frac{m^4}{4\alpha^2(p!)^2[(p+1)!]^2}\mathcal{F}_{\mu_1\ldots\mu_p} + \mathcal{O}\left(G_{i_1\ldots i_{p-1}}\right). \quad (36)$$

and it is divergenceless

$$\partial^\nu\mathcal{F}_{\nu\mu_1\ldots\mu_{p-1}} = 0, \quad (37)$$

on the first-class surface $G_{i_1\ldots i_{p-1}} \approx 0$. Based on the gauge invariance and divergenceless of the $\mathcal{F}_{\mu_1\ldots\mu_p}$ we introduce a p-form $V_{\mu_1\ldots\mu_p}$ through the relation

$$\mathcal{F}_{\mu_1\ldots\mu_p} = \frac{1}{(p+1)!}\varepsilon_{\mu_1\ldots\mu_p\nu_1\ldots\nu_{p+1}}\partial^{[\nu_1}V^{\nu_2\ldots\nu_{p+1}]}. \quad (38)$$

Consequently, we enlarge the phase-space by adding the bosonic fields/momenta $\{V^{\nu_1\ldots\nu_p}, P_{\nu_1\ldots\nu_p}\}$. From the gauge transformation of the quantity $\partial_{[i_1}\pi_{i_2\ldots i_p]0}$ we obtain that

$$\partial_{[i_1}\pi_{i_2\ldots i_p]0} = -\frac{1}{(p-1)!}\varepsilon_{0i_1\ldots i_p j_1\ldots j_p}P^{j_1\ldots j_p}. \quad (39)$$

If we replace (38) in (18), then the constraint set takes the form

$$\frac{1}{m^2}\varepsilon_{0i_1\ldots i_{p-1}j_1\ldots j_{p+1}}\left(2\alpha p!\partial^{[j_1}A^{j_2\ldots j_{p+1}]} + \frac{m^2}{(p+1)!}\partial^{[j_1}V^{j_2\ldots j_{p+1}]}\right) \approx 0, \quad (40)$$

and remains first-class, and becomes reducible of order $(p-1)$. The number of physical degrees of freedom is conserved if we impose the supplementary constraints

$$P_{0i_1\ldots i_{p-1}} \approx 0, \qquad -p\partial^j P_{ji_1\ldots i_{p-1}} \approx 0. \tag{41}$$

The Hamiltonian path integral of the reducible first-class systems reads

$$Z'_{GU} = \int \mathcal{D}\left(A_{i_1\ldots i_p}, V_{\mu_1\ldots\mu_p}, P^{\mu_1\ldots\mu_p}, \lambda^{i_1\ldots i_{p-1}}, \lambda^{(1)i_1\ldots i_{p-1}}, \lambda^{(2)i_1\ldots i_{p-2}}\right) \mu\left([A_{i_1\ldots i_p}],[V_{\mu_1\ldots\mu_p}]\right) \exp\left(\mathrm{i}S'^{(\mathrm{odd})}_{\mathrm{GU}}\right). \tag{42}$$

where

$$\begin{aligned} S'^{(odd)}_{GU} = & \int d^{2p+1}x \Big[-\alpha(p+1)\left(\partial_0 A_{i_1\ldots i_p}\right)\varepsilon^{0i_1\ldots i_p j_1\ldots j_3}A_{j_1\ldots j_3} + \left(\partial_0 V_{\mu_1\ldots\mu_p}\right)P^{\mu_1\ldots\mu_p} - \mathcal{H}'^{(odd)}_{GU} \\ & -\lambda^{(1)i_1\ldots i_{p-1}}P_{0i_1\ldots i_{p-1}} + p\lambda^{(2)i_1\ldots i_{p-1}}\partial^j P_{ji_1\ldots i_{p-1}} - \frac{1}{m^2}\lambda^{i_1\ldots i_{p-1}}\varepsilon_{0i_1\ldots i_{p-1}j_1\ldots j_{p+1}}\left(2\alpha p!\partial^{[j_1}A^{j_2\ldots j_{p+1}]} + \frac{m^2}{(p+1)!}\partial^{[j_1}V^{j_2\ldots j_{p+1}]}\right)\Big], \end{aligned} \tag{43}$$

and '$\mu\left([A_{i_1\ldots i_p}],[V_{\mu_1\ldots\mu_p}]\right)$' signifies the integration measure associated with the model subject to the reducible first-class constraints (40) and (41). The first-class Hamiltonian $\mathcal{H}'^{(odd)}_{GU}$ is obtained from (22) using (34), (38) and (39). Proceeding further in a similar manner to Ref. [39] it can be shown that the argument of the exponential from the Hamiltonian path integral of the first class system takes the form

$$\begin{aligned} S'^{(odd)}_{GU} = & \int d^{2p+1}x \\ & \Big(-\alpha\varepsilon^{\mu_1\ldots\mu_{2p+1}}\bar{F}_{\mu_1\ldots\mu_{p+1}}\bar{A}_{\mu_{p+2}\ldots\mu_{2p+1}} + \frac{1}{2(p+1)!}\tilde{F}_{\mu_1\ldots\mu_{p+1}}\tilde{F}^{\mu_1\ldots\mu_{p+1}} \\ & -\frac{m}{p!(p+1)!}\varepsilon^{\mu_1\ldots\mu_{2p+1}}\tilde{F}_{\mu_1\ldots\mu_{p+1}}\bar{A}_{\mu_{p+2}\ldots\mu_{2p+1}}\Big) \end{aligned} \tag{44}$$

where $\tilde{F}_{\mu_1\ldots\mu_{p+1}} = \partial_{[\mu_1}\bar{V}_{\mu_2\ldots\mu_{p+1}]}$. This describes a Chern-Simons-like coupling between the p-forms $\bar{A}_{\mu_1\ldots\mu_p}$ and $\bar{V}_{\mu_1\ldots\mu_p}$ [41]. This result incorporates (for $p=1$) the equivalence between the self-dual 1-form [32] and the gauge invariant topologically massive electrodynamics [42–44] proved in [45]. In [46], starting from a master Lagrangian (similar for $p=1$ with (44)), the common origin of self-dual model and the Maxwell-Chern-Simons theory was revealed, and also was put into evidence the interplay between gauge invariance and self-duality.

The two path integrals corresponding to (33) and (44) coincide as both theories represent first-class extensions of the same second-class model. The aforementioned first-class theories correspond to different extensions of the phase-space associated to the original second-class theory. In view of this the field spectra of the gauge theories are different.

3.2. The case p even

Based on the equivalence between the first-class system and the original second-class theory, we replace the Hamiltonian path integral of self-dual p-forms with that of the first-class system. Imposing some suitable gauge-fixing conditions $C_{i_1\ldots i_p} \approx 0$, the Hamiltonian path integral for the above first-class system, subject to the first-class constraints (30) and with evolution governed by the first-class Hamiltonian (32), takes the form

$$\begin{aligned} Z^{(even)}_{GU} = & \int \mathcal{D}\left(A_{i_1\ldots i_p}, \pi^{i_1\ldots i_p}, \lambda^{i_1\ldots i_p}\right)\delta\left(C_{i_1\ldots i_p}\right) \\ & \left(\det\left(\left[G_{j_1\ldots j_p}, C_{k_1\ldots k_p}\right]\right)\right)\exp\left(\mathrm{i}S^{(even)}_{GU}\right) \end{aligned} \tag{45}$$

with

$$\begin{aligned} S^{(even)}_{GU} = & \int d^{2p+1}x \Big[\partial_0\left(A_{i_1\ldots i_p}\right)\pi^{i_1\ldots i_p} - \mathcal{H}^{(even)}_{GU} - \\ & \frac{1}{2}\lambda^{i_1\ldots i_p}\left(A_{i_1\ldots i_p} + \frac{1}{\alpha p!(p+1)!}\varepsilon_{0i_1\ldots i_p j_1\ldots j_p}\pi^{j_1\ldots j_p}\right)\Big]. \end{aligned} \tag{46}$$

Performing partial integrations over the Lagrange multipliers $\lambda^{i_1\ldots i_p}$ and the momenta $\pi^{i_1\ldots i_p}$, we discover the manifestly Lorentz covariant path integral corresponding to the Lagrangian formulation of the first-class system

$$S_{GU}^{(even)} = \int d^{2p+1}x \left(-\alpha\varepsilon_{\mu_1\ldots\mu_{2p+1}} F^{\mu_1\cdots\mu_{p+1}} A^{\mu_{p+2}\cdots\mu_{2p+1}}\right). \tag{47}$$

The functional (47) describes a topological Chern-Simons-like theory.

4. The construction of the first-class system – BF method

Construction of a first-class system equivalent to the initial second-class one (subject to the second-class constraints (1)) using the BF method includes the following three steps: 1. we enlarge the original phase-space with the variables $(\zeta^{\alpha})_{\alpha=\overline{1,2M}}$, $(M \geq M_0)$ and extend the Poisson bracket to the newly added variables; 2. we construct a set of independent, smooth, real functions defined on the extended phase-space, $(G_A(z,\zeta))_{A=\overline{1,M_0+M}}$, such that

$$G_{\alpha_0}(z,0) \equiv \chi_{\alpha_0}(z), \; G_{\bar{A}}(z,0) \equiv 0, \; [G_A, G_B] = 0, \tag{48}$$

where $\bar{A} = \overline{2M_0+1, M_0+M}$; 3. we generate a smooth, real function $H_{BF}(z,\zeta)$, defined on the extended phase-space, with the properties

$$H_{BF}(z,0) \equiv H_c(z), \qquad [H_{BF}, G_A] = V_A^B G_B. \tag{49}$$

These steps unravel a dynamic system subject to the first-class constraints $G_A(z,\zeta) \approx 0$, whose evolution is governed by the first-class Hamiltonian $H_{BF}(z,\zeta)$. The first-class system constructed by the BF method is classically equivalent to the original second-class theory since both display the same number of physical degrees of freedom

$$\mathcal{N}_{BF} = \frac{1}{2}\left[2n + 2M - 2(M_0 + M)\right] = \frac{1}{2}(2n - 2M_0) = \mathcal{N}_O \tag{50}$$

and the corresponding algebras of classical observables are isomorphic. Consequently, the two systems also become equivalent at the level of the path integral quantization, and we can to replace the Hamiltonian path integral of the original second-class theory with that of the BF first-class system.

4.1. The case p odd

In order to apply the BF method to the case of self-dual p-forms we consider the second-class system constructed in the subsection (2.1) (with phase-space locally parametrized by $\{A_{i_1\ldots i_p}, A_{0i_1\ldots i_{p-1}}, \pi^{0i_1\ldots i_{p-1}}\}$ subject to the second-class constraints 17)–(18), and endowed with the canonical Hamiltonian (19)) and we enlarge the phase-space by adding the bosonic fields/momenta $\{B^{\mu_1\ldots\mu_{p-1}}, \Pi_{\mu_1\ldots\mu_{p-1}}\}$. The constraints $G_A(z,\zeta) \approx 0$ gain in the case of self-dual p-forms the concrete form

$$G^{(1)}_{i_1\ldots i_{p-1}} \equiv \chi^{(1)}_{i_1\ldots i_{p-1}} + mB_{i_1\ldots i_{p-1}} \approx 0, \tag{51}$$

$$G^{(2)}_{i_1\ldots i_{p-1}} \equiv \bar{\chi}^{(2)}_{i_1\ldots i_{p-1}} - \frac{m}{(p-1)!}\Pi_{i_1\ldots i_{p-1}} \approx 0, \tag{52}$$

$$G_{i_1\ldots i_{p-2}} \equiv \Pi_{0i_1\ldots i_{p-2}} \approx 0, \tag{53}$$

where

$$\bar{\chi}^{(2)}_{i_1\ldots i_{p-1}} \equiv -2\alpha p\varepsilon_{0i_1\ldots i_{p-1}j_1\ldots j_{p+1}}\partial^{[j_1}A^{j_2\cdots j_{p+1}]} - \frac{m^2}{(p-1)!}A_{0i_1\ldots i_{p-1}} \approx 0. \tag{54}$$

Constraints (51)–(53) form an Abelian and irreducible first-class constraint set. The first-class Hamiltonian complying with the general requirements (49) is expressed (in terms of the first-class constraint functions) by

$$H_{BF}^{(odd)} = \bar{H}_c^{(odd)} + \int d^{2p}x \left[-\frac{1}{2(p-1)!}\Pi^{i_1\ldots i_{p-1}}\Pi_{i_1\ldots i_{p-1}} - \frac{1}{p}\left(mA^{i_1\ldots i_p} - \frac{(p-1)!}{2}\partial^{[i_1}B^{i_2\ldots i_p]}\right)\partial_{[i_1}B_{i_2\ldots i_p]} - \frac{1}{m}\Pi^{i_1\ldots i_{p-1}}G^{(2)}_{i_1\ldots i_{p-1}} + \frac{(p-1)!}{m}B^{0i_1\ldots i_{p-2}}\partial^j G^{(2)}_{ji_1\ldots i_{p-2}}\right]. \tag{55}$$

We remark that in $H_{BF}^{(odd)}$ and in the first-class Hamiltonian $H_{GU}^{(odd)}$ (22) appear the same terms, apart from the quadratic term in momenta and the terms proportional to the first-class constraint functions, via the identification $B_{i_1\ldots i_{p-1}} = -\frac{1}{m}\pi_{0i_1\ldots i_{p-1}}$.

The Hamiltonian gauge algebra relations (49) are given by

$$\left[H_{BF}^{(odd)}, G^{(1)}_{i_1\ldots i_{p-1}}\right] = 0, \quad \left[H_{BF}^{(odd)}, G^{(2)}_{i_1\ldots i_{p-1}}\right] = 0,$$

$$\left[H_{BF}^{(odd)}, G_{i_1\ldots i_{p-2}}\right] = \frac{(p-1)!}{m}\partial^j G^{(2)}_{ji_1\ldots i_{p-2}}. \tag{56}$$

Replacing the number of canonical variables $2n + 2M = \binom{2p}{p} + 2\binom{2p}{p-1} + 2\binom{2p+1}{p-1}$ and the number of independent first-class constraints $M_0 + M = \binom{2p}{p-1} + \binom{2p}{p-1} + \binom{2p}{p-2}$ in (50), one obtains that the number of physical degrees of freedom is equal to $\mathcal{N}_{BF}^{(odd)} = \frac{1}{2}\binom{2p}{p}$.

4.2. The case p even

In this case, starting from the second-class system with the phase-space locally parameterized by $\{A_{i_1\ldots i_p}, \pi^{i_1\cdots i_p}\}$, endowed with the canonical Hamiltonian (29) , and subject to the second-class constraints (24) and (26), we construct an equivalent first-class system using the BF method. In order to construct the equivalent first-class system we enlarge the phase-space with the variables $\{B_{i_1\ldots i_p}, \Pi^{i_1\cdots i_p}\}$. In the case of self-dual p-forms with p even the constraints $G_A(z,\zeta)\approx 0$ take the form

$$G^{(1)}_{i_1\ldots i_p} \equiv \pi_{i_1\ldots i_p} + \alpha\,(p+1)\,\varepsilon_{0i_1\ldots i_p j_1\ldots j_p}\left(A^{j_1\cdots j_p} + 2B^{j_1\cdots j_p}\right) \approx 0, \tag{57}$$

$$G^{(2)}_{i_1\ldots i_p} \equiv \pi_{i_1\ldots i_p} - \alpha\,(p+1)\,\varepsilon_{0i_1\ldots i_p j_1\ldots j_p}A^{j_1\cdots j_p} - \Pi_{i_1\ldots i_p} \approx 0. \tag{58}$$

Constraints (57)–(58) form an Abelian and irreducible first-class constraint set. The first-class Hamiltonian complying with the general requirements (49) is expressed by

$$H^{(even)}_{BF} = \int d^{2p}x \left[\frac{m^2}{2p!}\left(A_{i_1\ldots i_p} + B_{i_1\ldots i_p} + \frac{1}{2\alpha p!\,(p+1)!}\varepsilon_{0i_1\ldots i_p j_1\ldots j_p}P^{j_1\cdots j_p}\right)\left(A^{i_1\cdots i_p} + B^{i_1\cdots i_p} + \frac{1}{2\alpha p!\,(p+1)!}\varepsilon^{0i_1\ldots i_p k_1\ldots k_p}P_{k_1\ldots k_p}\right)\right]. \tag{59}$$

The Hamiltonian gauge algebra relations (49) are given by

$$\left[H^{(even)}_{BF}, G^{(1)}_{i_1\ldots i_p}\right] = 0, \quad \left[H^{(even)}_{BF}, G^{(2)}_{i_1\ldots i_p}\right] = 0. \tag{60}$$

Replacing the number of canonical variables $2n + 2M = 2\binom{2p}{p} + 2\binom{2p}{p}$ and the number of independent first-class constraints $M_0 + M = \binom{2p}{p} + \binom{2p}{p}$ in (50), one obtains the number of physical degrees of freedom equal to $\mathcal{N}^{(even)}_{BF} = 0$.

5. Covariant path integral for the BF system

5.1. The case p odd

In the third section of Ref. [39], starting from a second-class theory (the massive self-dual 3-forms in $D=7$) an equivalent first-class system was constructed using the BF method, and the manifestly Lorentz-covariant path integral corresponding to the Lagrangian formulation of the equivalent first-class system was obtained. Starting from the BF system constructed in the subsection 4.1, and proceeding in a similar manner we discover the manifestly Lorentz-covariant path integral corresponding to the Lagrangian formulation of the first-class system. This reduces for different kinds of extensions of the phase-space to the Lagrangian path integral for $(p-1)$- and p-forms with Stückelberg-like coupling (33), or the Lagrangian path integral for two kinds of p-forms with Chern-Simons-like coupling (44).

5.2. The case p even

We finally consider the Hamiltonian path integral for the BF system constructed in the above, subject to the first-class constraints (57)–(58), whose evolution is governed by the first-class Hamiltonian (59). Imposing some suitable gauge-fixing conditions

$$C_A \equiv \left(C^{(1)}_{i_1\ldots i_p} \equiv A_{i_1\ldots i_p}, C^{(2)}_{i_1\ldots i_p} \equiv B_{i_1\ldots i_p}\right) \approx 0, \tag{61}$$

the Hamiltonian path integral takes the form

$$Z^{(even)}_{BF} = \int\mathcal{D}\left(A_{i_1\ldots i_p}, B_{i_1\ldots i_p}, \pi^{i_1\cdots i_p}, \Pi^{i_1\cdots i_p}, \lambda^{(1)i_1\ldots i_p}, \lambda^{(2)i_1\ldots i_p}\right) \times\left(\prod_A \delta\left(C_A\right)\right)\left(\det\left(\left[G_{A'}, C_{B'}\right]\right)\right)\exp\left(iS^{(even)}_{BF}\right), \tag{62}$$

with

$$S^{(even)}_{BF} = \int d^{2p+1}x\Big\{\left(\partial_0 A_{i_1\ldots i_p}\right)\pi^{i_1\cdots i_p} + \left(\partial_0 B_{i_1\ldots i_p}\right)\Pi^{i_1\cdots i_p} - \mathcal{H}^{(even)}_{BF} - \lambda^{(1)i_1\ldots i_p}\left[\pi_{i_1\ldots i_p} + \alpha\,(p+1)\,\varepsilon_{0i_1\ldots i_p j_1\ldots j_p}\left(A^{j_1\cdots j_p} + 2B^{j_1\cdots j_p}\right)\right] - \lambda^{(2)i_1\ldots i_p}\left(\pi_{i_1\ldots i_p} - \alpha\,(p+1)\,\varepsilon_{0i_1\ldots i_p j_1\ldots j_p}A^{j_1\cdots j_p} - \Pi_{i_1\ldots i_p}\right)\Big\}. \tag{63}$$

Performing in the path integral partial integrations over $\pi^{i_1\ldots i_p}$, $\Pi^{i_1\ldots i_p}$, $\lambda^{(1)}_{i_1\ldots i_p}$, $\lambda^{(2)i_1\ldots i_p}$ and $B_{i_1\ldots i_p}$ the functional (63) associated with the first-class system takes a manifestly Lorentz-covariant form

$$S_{BF}^{(even)} = \int d^{2p+1}x \left(-\alpha\varepsilon^{\mu_1\ldots\mu_{2p+1}}A_{\mu_1\ldots\mu_p}F_{\mu_{p+1}\ldots\mu_{2p+1}}\right), \tag{64}$$

and describes a topological Chern-Simons-like theory.

6. Conclusion

In this paper the massive self-dual p-forms have been analyzed from the point of view of the Hamiltonian path integral quantization, in the framework of GU and BF methods. The first step of this approaches is represented by the construction of an equivalent first-class system. The construction of the equivalent first-class system in the GU approach does not require an extension of the original phase-space, while the construction of the equivalent first-class system using the BF method demands an appropriate extension of the original phase-space. The second step involves the construction of the Hamiltonian path integral corresponding to the equivalent first-class system. The Hamiltonian path integral of the first-class system takes a manifestly Lorentz-covariant form after integrating out the auxiliary fields and performing some field redefinitions. In the case p odd, we note that in order to obtain a manifestly covariant path integral, both approaches requires some extensions of the phase-space. For appropriate phase-space extensions we identify the Lagrange path integral for $(p-1)$- and p- forms with Stückelberg-like coupling, or the Lagrangian path integral for two kinds of p-forms with Chern-Simons-like coupling. In the case p even, it is not necessary to further enlarge the phase-space in order to obtain a manifestly covariant path integral. Both methods finally output the manifestly Lorentz covariant path integral which describe a topological Chern-Simons-like theory.

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