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Double passage of electromagnetic waves through magnetized plasma: approximation of independent normal waves

Research Article

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Abstract: Polarization properties of electromagnetic waves, double-passed through magnetized plasma, are studied. Analyses are performed in the case of non-interacting normal modes, propagating in homogeneous and weakly inhomogeneous plasmas, and for three kinds of reflectors: metallic plane, 2D corner retro-reflector (2D-CR), and cubic corner retro-reflector (CCR). It is shown that an electromagnetic wave, reflected from a metallic plane and from a CCR, contains only "velocity-preserving" channels, whose phases are doubled in comparison with those of a single-passage propagation. At the same time, an electromagnetic wave reflected from a 2D-CR is shown to contain both "velocity-preserving" and "velocity-converting" channels, the latter converting the fast wave into the slow one and vice-versa. One characteristic feature of "velocity-converting" channels is that they reproduce the initial polarization state near the source, which might be of practical interest for plasma interferometry. In the case of circularly polarized modes, "velocity-preserving" channels completely disappear, and only "velocity-converting" channels are to be found.

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1. Introduction

The issue of a double-passage scheme of measurements in plasma polarimetry arises from reducing the number of windows in the walls of large thermonuclear reactors, such as ITER and W-7X. A two-passage scheme of polarimetric measurements, shown in Fig. 1, contains three basic elements: the source of electromagnetic waves in the far infrared (FIR) or microwave band, a retro-reflector, which returns electromagnetic waves back to the source, and an analyzer.

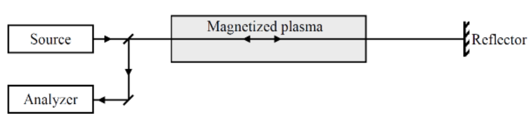

Figure 1. Scheme for double-passage measurements.

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It is intuitively evident that in the case of non-interacting normal modes, the double-passage regime should provide twice as large phase shifts between normal modes in plasma, compared with the single-passage scheme. This fact can be readily proved for a pure Faraday effect, which deals with circularly polarized waves, and for pure a Cotton-Mouton phenomenon, observed for linearly polarized waves. However, it is not an easy matter to derive the phase-doubling phenomenon for electromagnetic waves of arbitrary polarization. To the best of the authors' knowledge, a general analysis has not, so far, been published in the literature.

This paper studies polarization of the electromagnetic beam after double-passage through magnetized plasma. Our analysis is based on the quasi-isotropic approximation (QIA) of the geometric optics method [1–5], which adequately describes electromagnetic waves' propagation in weakly anisotropic media, primarily in plasma, which manifests properties of weakly anisotropic media in millimeter, submillimeter, and FIR ranges, which are used for plasma diagnostics in modern thermonuclear reactors.

The materials of this paper are presented in the following order: Basic equations of quasi-isotropic approach are presented in Sec. 2. QIA equations are applied to analyze forward and backward normal waves both in homogeneous (Sec. 3) and in weakly inhomogeneous plasma (Sec. 4). Secs. 5-7 study polarization of waves, reflected from three kinds of reflectors: a metallic plane, a 2D corner retro-reflector (2D-CR), and a cubic corner retro-reflector (CCR). As will be shown, the slow (fast) incident electromagnetic wave, being reflected from a metallic plane or a CCR, is transformed into a slow (fast) wave, forming "velocity-preserving" channels of reflection. These channels are characterized by phase doubling for every normal mode, compared with the single-passage regime.

At the same time the wave reflected from a 2D corner retro-reflector is shown to contain both "velocity-preserving" and "velocity-converting" channels. The latter convert the slow wave into the fast one and vice-versa. These "velocity-converting" channels, which have not been described so far, might be of practical interest for far infrared (FIR) and microwave interferometers, because these channels reproduce the original polarization state when the wave returns back to the primary source.

2. Waves in weakly anisotropic media: Quasi-isotropic approximation (QIA) of the geometrical optics method

The dielectric permittivity tensor $\varepsilon_{\alpha\beta}$ of weakly anisotropic media consists of two parts: a large isotropic component, where $\delta_{\alpha\beta}$ is a unit tensor, and a small anisotropic component $\nu_{\alpha\beta}$:

$$\varepsilon_{\alpha\beta} = \varepsilon_0 \delta_{\alpha\beta} + \nu_{\alpha\beta}. \tag{1}$$

Intending to emphasize weakness of the anisotropy tensor $\nu_{\alpha\beta}$ compared with the isotropic part ε_0, we involve an "anisotropic" small parameter μ_A as follows:

$$\mu_A = \frac{\max |\nu_{\alpha\beta}|}{\varepsilon_0} \ll 1. \tag{2}$$

The theory of electromagnetic wave propagation in weakly anisotropic media is based on a quasi-isotropic approximation (QIA) of the geometrical optics method [1–3]. A short outline of the QIA is presented in books [4, 5] and recent papers [6, 7]. Along with the "anisotropic" small parameter μ_A (Eq. (3)), QIA also uses a "geometrical" small parameter

$$\mu_{GO} = \frac{1}{k_0 L} = \frac{\lambda_0}{2\pi L}, \tag{3}$$

where k_0 and $\lambda_0 = 2\pi/k_0$ are, respectively, wave number and wavelength in free space, and L is a characteristic scale of the medium's inhomogeneity.

In the lowest (zero) order in a combined small parameter

$$\mu = \max(\mu_A, \mu_{GO}) \tag{4}$$

QIA provides an asymptotic solution to Maxwell's equations in the "quasi-isotropic" form:

$$\mathbf{E} = \boldsymbol{\Gamma} A(\mathbf{r}) \exp[ik_0 \Psi(\mathbf{r})], \tag{5}$$

where $\Psi(\mathbf{r})$ and $A(\mathbf{r})$ are the eikonal and amplitude of the geometrical optics (GO) wave field in the isotropic, inhomogeneous medium, with permittivity $\varepsilon_0(\mathbf{r})$ (time dependence $\exp(-i\omega t)$ is omitted for brevity). Vector $\boldsymbol{\Gamma}$ (polarization vector) describes the evolution of polarization in a weakly anisotropic medium.

The values $\Psi(\mathbf{r})$ and $A(\mathbf{r})$ obey the eikonal and transport equations, respectively [4, 5] and [8]:

$$(\nabla\Psi)^2 = \varepsilon_0, \quad \operatorname{div}\left(A^2 \nabla\Psi\right) = 0. \tag{6}$$

According to the eikonal equation, $\Psi(\mathbf{r})$ can be found by the integration of the refractive index $n_0 = \sqrt{\varepsilon_0}$ along the ray:

$$\Psi = \int \sqrt{\varepsilon_0} d\sigma = \int n_0 d\sigma, \tag{7}$$

where $d\sigma$ is an elementary arc length along the ray. The phase factor $\exp[ik_0\Psi(\mathbf{r})]$ can be represented as

$$\exp[ik_0\Psi(\mathbf{r})] = \exp\left[i\int_0^\sigma N_0(\mathbf{r})\, d\sigma\right], \tag{8}$$

where $N_0 = k_0 n_0$ is a local wave number. In turn the amplitude A obeys the energy conservation law in a ray tube [4, 5, 8].

In the lowest order in small parameter μ, the polarization vector $\mathbf{\Gamma}$ is orthogonal to the ray and can be represented as

$$\mathbf{\Gamma} = \Gamma_1\mathbf{e}_1 + \Gamma_2\mathbf{e}_2, \quad \mathbf{e}_1 \perp \mathbf{e}_2 \perp \mathbf{l}, \tag{9}$$

where $\mathbf{l} = d\mathbf{r}/d\sigma$ is a unit vector, tangent to the ray trajectory $\mathbf{r}(\sigma)$, and $\mathbf{e}_1$ and $\mathbf{e}_2$ are the unit vectors, orthogonal to the ray trajectory.

M. Popov has introduced an orthogonal, curvilinear coordinate system [9], which performs parallel transport of the vector wave field $\mathbf{E}$ along the ray. The parallel-transport coordinate frame has been widely used in the theory of open resonators [10] and in applications. From previous studies [9, 10], the unit vectors $\mathbf{e}_{1,2}$ satisfy the equations

$$\dot{\mathbf{e}}_1 = \mathbf{l}(\mathbf{e}_1 \cdot \nabla \ln(n)), \quad \dot{\mathbf{e}}_2 = \mathbf{l}(\mathbf{e}_2 \cdot \nabla \ln(n)), \tag{10}$$

where the dot over the vector means a derivative in the arc length σ: $\dot{\mathbf{e}}_{1,2} = d\mathbf{e}_{1,2}/d\sigma$.

In this coordinate system, QIA equations for the polarization vector $\boldsymbol{\alpha}$ take the form [1–5]

$$\begin{cases} \dot{\Gamma}_1 = \frac{ik_0}{2\sqrt{\varepsilon_0}}[\nu_{11}\Gamma_1 + \nu_{12}\Gamma_2], \\ \dot{\Gamma}_2 = \frac{ik_0}{2\sqrt{\varepsilon_0}}[\nu_{21}\Gamma_1 + \nu_{22}\Gamma_2], \end{cases} \tag{11}$$

where $\nu_{\alpha\beta}$ are the components of the anisotropy tensor. By choosing the isotropic permittivity of the collisionless magnetized plasma as

$$\varepsilon_0 = 1 - v, v = \frac{\omega_p^2}{\omega^2} \equiv \frac{4\pi e^2 N_e}{m\omega^2}, \tag{12}$$

one can rewrite Eq. (11) as

$$\begin{aligned} \dot{\Gamma}_1 &= -(i/2)(2\Omega_0 - \Omega_\perp - \Omega_1)\Gamma_1 + (1/2)(i\Omega_2 - \Omega_3)\Gamma_2, \\ \dot{\Gamma}_2 &= (1/2)(i\Omega_2 + \Omega_3)\Gamma_1 - (i/2)(2\Omega_0 - \Omega_\perp + \Omega_1)\Gamma_2. \end{aligned} \tag{13}$$

Here $\Omega_{1,2,3}$ are the components of vector $\mathbf{\Omega}$

$$\mathbf{\Omega} = \begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{pmatrix} = \frac{k_0}{2\sqrt{1-v}} \frac{v}{1-u} \begin{pmatrix} u \sin^2\alpha_\parallel \cos 2\alpha_\perp \\ u \sin^2\alpha_\parallel \sin 2\alpha_\perp \\ 2\sqrt{u}\cos\alpha_\parallel \end{pmatrix}, \tag{14}$$

used by Segre to describe the Stokes vector evolution in plasma [11]. In the frame of Stokes vector formalism, $\Omega_\perp$ and $\Omega_0 \equiv \Omega_\perp(\alpha_\parallel = \pi/2)$ are auxiliary parameters

$$\Omega_\perp = \sqrt{\Omega_1^2 + \Omega_2^2} = \frac{k_0}{2\sqrt{1-v}} \frac{uv}{1-u} \sin^2\alpha_\parallel = \Omega_0 \sin^2\alpha_\parallel. \tag{15}$$

Besides, $u = (eB_0/mc\omega)^2$ is the "magnetic" plasma parameter [12, 13]. QIA Eqs. (13) slightly differ from QIA equations, derived in [7], because the isotropic part of the permittivity tensor was chosen in Czyz *et al.* [7] to be $\varepsilon_0 = 1 - v/(1-u) \approx 1 - v - vu$, instead of $\varepsilon_0 = 1 - v$ here.

3. Forward and backward normal waves in homogeneous plasma

Let us apply QIA Eqs. (13) for the analysis of polarization evolution along the ray in the homogeneous plasma. In order to discern parameters related to forward and backward waves, we supply them with subscripts f and b, respectively. Let us study first the forward normal waves, using an orthogonal basis $\mathbf{e}_{1f}, \mathbf{e}_{2f}, \mathbf{l}_f$ and longitudinal variable σ_f, shown in Fig. 2.

The solution of Eq. (13) for the components Γ_{1f} and Γ_{2f} of the polarization vector $\mathbf{\Gamma}_f = \Gamma_{1f}\mathbf{e}_{1f} + \Gamma_{2f}\mathbf{e}_{2f}$ in the homogeneous plasma can be represented in the form of a harmonic wave

$$\Gamma_{1f} = A_{1f}\exp[iN_f(\sigma_f - \sigma_{0f})], \Gamma_{2f} = A_{2f}\exp[iN_f(\sigma_f - \sigma_{0f})], \tag{16}$$

where σ_{0f} is a starting point, and N_f is a propagation constant, answering to one of the normal modes in plasma:

$$\begin{aligned} N_{f\pm} &= -\Omega_0 + \frac{1}{2}\left(\Omega_\perp \pm \sqrt{\Omega_\perp^2 + \Omega_{3f}^2}\right) \\ &= -\Omega_0 + \frac{1}{2}(\Omega_\perp \pm \Omega), \\ \Omega &\equiv |\mathbf{\Omega}| = \sqrt{\Omega_\perp^2 + \Omega_{3f}^2}. \end{aligned} \tag{17}$$

The values Ω_{2f} and Ω_{3f} have subscript f in order to discern them from analogous values for backward waves, which

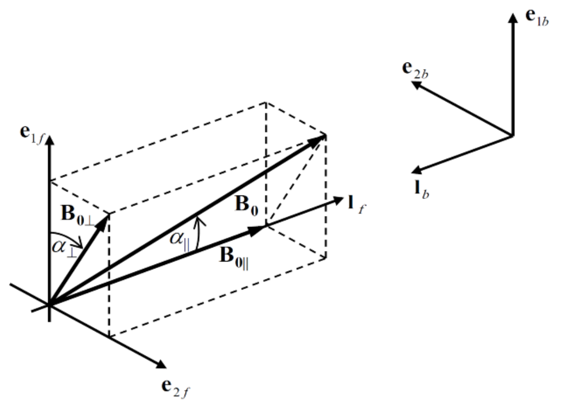

Figure 2. Orientation of the static magnetic field $\mathbf{B}_0$ in the parallel transport coordinate frame $(\mathbf{e}_{f1}, \mathbf{e}_{f2}, \mathbf{l}_f)$, in the case of a forward wave. Coordinate systems $(\mathbf{e}_{b1}, \mathbf{e}_{b2}, \mathbf{l}_b)$, connected with a backward wave is shown in the right part of this figure.

differ from Ω_{2f} and Ω_{3f} by opposite sign. At the same time the signs of parameters $\Omega_\perp$ and Ω_1 are the same both for forward and backward waves. Therefore these parameters do not need any special subscripts.

Total propagation constants $N_{f\pm}^{tot}$, uniting QIA constants (17) with the common geometrical optics propagation constant $N_0 = k_0 n_0$, will be

$$N_{f\pm}^{tot} = N_0 + N_{f\pm} = N_0 - \Omega_0 + (1/2)(\Omega_\perp \pm \Omega). \tag{18}$$

The subscripts $\pm$ in this and the subsequent equations correspond simply to signs of the square root in Eq. (17). The minus sign in Eq. (17) corresponds to the "fast" wave, whose phase velocity $v_{ph-} = \omega/N_{f-}^{tot}$ exceeds the phase velocity of the slow wave $v_{ph+} = \omega/N_{f+}^{tot}$:

$$\begin{aligned} v_{ph-} &= \omega/[N_0 - \Omega_0 + (1/2)(\Omega_\perp - \Omega] > v_{ph+} \\ &= \omega/[N_0 - \Omega_0 + (1/2)(\Omega_\perp + \Omega)]. \end{aligned} \tag{19}$$

In plasma, the electrical vector of the fast wave rotates in the direction of the Larmour rotation of electrons, whereas the slow wave's electrical vector rotates in the opposite direction.

It follows from Eq. (13) that the complex amplitude ratio $\zeta_{f\pm} = (A_{2f}/A_{1f})_\pm$ of the electromagnetic wave can be represented by two equivalent expressions:

$$\zeta_{f\pm} = \frac{-\Omega_1 \pm \Omega}{\Omega_{2f} + i\Omega_{3f}} = \frac{-\Omega_{2f} + i\Omega_{3f}}{-\Omega_1 \mp \Omega}. \tag{20}$$

It can be readily shown that the sign "–" in Eq. (17) corresponds to clockwise rotation of the electric vector $\mathbf{\Gamma}_{f-}$ about the ray (right-hand polarization in optical definition [8]), and sign "+" corresponds to counterclockwise rotation (left-hand rotation). The general criterion that differentiates the right- from the left-hand rotation uses the sign of the imaginary part of the complex amplitude ratio ζ_f:

$$\text{Im}(\zeta_f) > 0 \Leftrightarrow \text{right hand rotation}, \tag{21a}$$
$$\text{Im}(\zeta_f) < 0 \Leftrightarrow \text{left hand rotation}. \tag{21b}$$

As a simple example of these criteria, take circularly polarized waves, corresponding to the pure Faraday effect, which takes place at $|\Omega_3| \gg |\Omega_1|, |\Omega_2|, |\Omega_\perp|$. Assuming that $\Omega_3 > 0$ (corresponding to an acute angle $\alpha_\parallel < \pi/2$) we have

$$\zeta_{f-} = i\,\text{sgn}(\Omega_3) = i \Leftrightarrow \text{fast wave, right - hand polarization}, \tag{22a}$$
$$\zeta_{f+} = -i\,\text{sgn}(\Omega_3) = -i \Leftrightarrow \text{slow wave, left - hand polarization}. \tag{22b}$$

For an opposite sign of Ω_3, the relation between fast/slow and right/left polarization will be opposite, too.

In the case of a real-valued amplitude ratio, we deal with the linear polarization of normal waves, characteristic for the pure Cotton-Mouton effect, when $\Omega_3 = 0$:

$$\zeta_{f-} = -\text{ctg}(\alpha_\perp) \Leftrightarrow \text{fast wave}, \tag{23a}$$
$$\zeta_{f+} = \text{tg}(\alpha_\perp) \Leftrightarrow \text{slow wave}. \tag{23b}$$

Involving complex eigenvectors of unit length

$$\begin{aligned} \mathbf{e}_{f\pm} &= \frac{(\mathbf{e}_1 + \zeta_{f\pm}\mathbf{e}_2)}{\sqrt{1 + |\zeta_{f\pm}|^2}} \equiv p_{f\pm}(\mathbf{e}_1 + \zeta_{f\pm}\mathbf{e}_2), \\ p_{f\pm} &= 1/\sqrt{1 + |\zeta_{f\pm}|^2}, \quad |\mathbf{e}_\pm| = 1 \end{aligned} \tag{24}$$

we may represent the wave field $\mathbf{E}_f$ as a superposition of independent (non-interacting) normal modes:

$$\mathbf{E}_f = a_{f+}\mathbf{e}_{f+}\exp(iN_{f+}\sigma_f) + a_{f-}\mathbf{e}_{f-}\exp(iN_{f-}\sigma_f). \tag{25}$$

The starting point σ_{0f} in Eq. (25) is assumed to be zero: $\sigma_{0f} = 0$. Total phases $S_{tot}^{f\pm} = N_{tot}^{f\pm}\sigma_f$ for slow and fast waves are depicted in Fig. 3 by continuous lines.

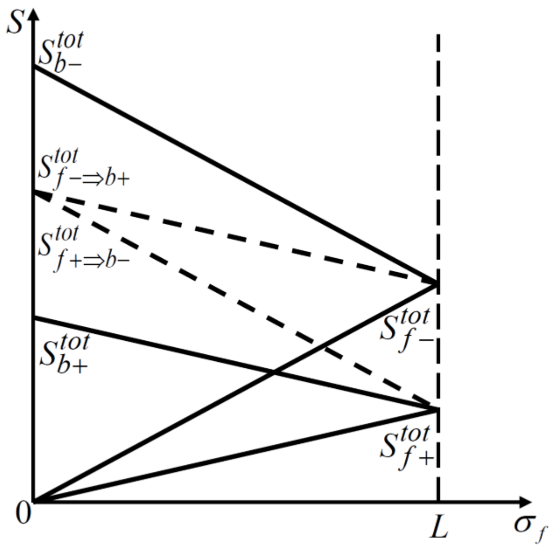

Figure 3. Total phases (continuous lines) for forward ($S^{tot}_{f\pm}$) and backward ($S^{tot}_{b\pm}$) waves, forming "velocity-preserving" channels of reflection from the metallic plane. Total phases $S^{tot}_{f+\Rightarrow b-}$ and $S^{tot}_{f-\Rightarrow b+}$ for "velocity-converting" channels $f+ \Rightarrow b-$ and $f- \Rightarrow b+$, arising under wave reflection from a 2D-CR, are shown by dashed lines.

According to Eq. (20), $\zeta_{f+} = -1/\zeta^*_{f-}$ is equivalent to the relation

$$\zeta_{f+}\zeta^*_{f-} = -1, \tag{26}$$

worked out by Huard [14]. Normal waves in Eq. (25) are orthogonal, because $\mathbf{e}_{f+}\mathbf{e}^*_{f-} = 1 + \zeta_{f+}\zeta^*_{f-} = 1 - 1 = 0$. Therefore we may treat Eq. (26) as the criterion of normal modes' orthogonality.

Considering backward waves, it is convenient to use a "backward" right-handed coordinate system ($\mathbf{e}_{1b}$, $\mathbf{e}_{2b}$, $\mathbf{l}_b$), shown in Fig. 2:

$$\mathbf{e}_{1b} = \mathbf{e}_{1f}, \mathbf{e}_{2b} = -\mathbf{e}_{2f}, \mathbf{l}_b = -\mathbf{l}_f. \tag{27}$$

The relationship between the arc length σ_b along the backward ray and the arc length σ_f along the forward ray is:

$$\sigma_b = const. - \sigma_f. \tag{28}$$

As said earlier, in the backward coordinate system the parameter Ω_{2b} changes its sign compared with the "forward" value $\Omega_{2f}(\Omega_{2b} = -\Omega_{2f})$, because $\mathbf{e}_{2b} = -\mathbf{e}_{2f}$ and $\alpha_{\perp b} = -\alpha_{\perp f}$ (seen in Fig. 2). The parameter Ω_{3b} also changes its sign in comparison with Ω_{3f}, but for another reason: For a backward wave, the angle $\alpha_{b\parallel}$ between the direction of propagation and the static magnetic field is shifted by π from $\alpha_{f\parallel}(\alpha_{b\parallel} = \pi - \alpha_{f\parallel})$, so that $\cos\alpha_{b\parallel} = -\cos\alpha_{f\parallel}$ and $\Omega_{3b} = -\Omega_{3f}$. It agrees with the fact from the electrodynamics of magnetic media [12, 14] that parameters, linear in magnetic field, change their signs when the wave propagates in the opposite direction.

As a result, the solution of Eq. (13) presented by superposition of normal modes acquires a form similar to Eq. (16):

$$\begin{aligned}\Gamma_{1b} &= A_{1b}\exp\left[iN_b\left(\sigma_b - \sigma_{0b}\right)\right],\\ \Gamma_{2b} &= A_{2b}\exp\left[iN_b\left(\sigma_b - \sigma_{0b}\right)\right],\end{aligned} \tag{29}$$

with "backward" propagation constants identical to "forward" ones:

$$\begin{aligned}N_{b\pm} &= -\Omega_0 + \frac{1}{2}\left(\Omega_\perp \pm \sqrt{\Omega_\perp^2 + \Omega_{3f}^2}\right)\\ &= -\Omega_0 + \frac{1}{2}\left(\Omega_\perp \pm \Omega\right) = N_{f\pm}.\end{aligned} \tag{30}$$

The "backward" amplitude ratio $\zeta_{b\pm} = A_{2b}/A_{1b}$ has the opposite sign:

$$\zeta_{b\pm} = \frac{\Omega_1 \mp \Omega}{\Omega_{2f} + i\Omega_{3f}} = \frac{-\Omega_{2f} + i\Omega_{3f}}{\Omega_1 \pm \Omega} = -\zeta_{b\pm}. \tag{31}$$

Involving normalized complex eigenvectors

$$\begin{aligned}\mathbf{e}_{b\pm} &= \frac{(\mathbf{e}_1 + \zeta_{b\pm}\mathbf{e}_2)}{\sqrt{1 + |\zeta_{b\pm}|^2}} \equiv p_{b\pm}\left(\mathbf{e}_1 + \zeta_{b\pm}\mathbf{e}_2\right),\\ p_{b\pm} &= 1/\sqrt{1 + |\zeta_{b\pm}|^2}, \qquad |\mathbf{e}_{b\pm}| = 1,\end{aligned} \tag{32}$$

one can represent the backward wave field $\mathbf{E}_b$ in the form of a normal modes composition similar to Eq. (25):

$$\begin{aligned}\mathbf{E}_b = a_{b+}\mathbf{e}_{b+}\exp\left[iN_{b+}(\sigma_b - \sigma_{0b})\right]\\ + a_{b-}\mathbf{e}_{b-}\exp\left[iN_{b-}(\sigma_b - \sigma_{0b})\right].\end{aligned} \tag{33}$$

4. Forward and backward normal waves in weakly inhomogeneous plasma

Inhomogeneous plasma polarization changes, described by Eqs. (13), are usually studied numerically. An analytic solution can only be found for a limited number of partial cases, such as circular-mode conversion near the orthogonality point [2] between the ray and magnetic field, or in a uniformly sheared plasma [11]. Here we explore a representation of the wave field in the inhomogeneous plasma in the form of independent (non-interacting) normal waves. This representation is of practical interest for many applications in plasma diagnostics. It can be readily obtained by generalization of Eqs. (25) and (33) for a weakly inhomogeneous plasma. As is known, interactions between normal modes will be negligibly weak if the characteristic scale $L_P \sim P/|\nabla_\parallel P| = P/|\partial P/\partial\sigma|$ of the plasma parameter P, essential for electromagnetic wave propagation, significantly exceeds the spatial "beating" length $l_b \sim 2\pi/(N_+ - N_-) = 2\pi/\Omega$ between normal modes [4, 5]:

$$L_P \gg l_b \quad \text{or} \quad |\nabla_\parallel P| = |\partial P/\partial\sigma| \ll P\Omega/2\pi. \tag{34}$$

Assuming inequality (34) to be fulfilled and neglecting normal waves' interaction, we may substitute the phase factors $\exp(iN_{f\pm}\sigma_f)$ and $\exp[iN_{b\pm}(\sigma-\sigma_{0b})]$ with the factors $\exp\left[i\int_0^{\sigma_f} N_{f\pm}d\sigma_f\right]$ and $\exp\left[i\int_{\sigma_{0b}}^{\sigma_b} N_{b\pm}d\sigma_b\right]$, containing the integrals of varying wave numbers $N_{f\pm}$ and $N_{b\pm}$ along the ray. The amplitude ratios ζ_f and ζ_b, given for homogeneous plasma by Equations (20) and (31), also preserve their values for weakly inhomogeneous media, so that the polarization of non-interacting normal waves in the weakly inhomogeneous plasma will be given by the same formulae (20) and (31) as for the homogeneous plasma. As a result, the wave, propagating in the weakly inhomogeneous plasma in a forward direction, takes the following form:

$$\mathbf{E}_f = a_{f+}\mathbf{e}_{f+}\exp\left[i\int_0^{\sigma_f} N_{f+}d\sigma_f\right] + a_{f-}\mathbf{e}_{f-}\exp\left[i\int_0^{\sigma_f} N_{f-}d\sigma_f\right]. \tag{35}$$

Analogously, the backward wave can be given by

$$\mathbf{E}_b = a_{b+}\mathbf{e}_{b+}\exp\left[i\int_{\sigma_{0b}}^{\sigma_b} N_{b+}d\sigma_b\right] + a_{b-}\mathbf{e}_{b-}\exp\left[i\int_{\sigma_{0b}}^{\sigma_b} N_{b-}d\sigma_b\right]. \tag{36}$$

5. "Velocity-preserving" reflection from a metallic plane

Let us consider a reflection of a forward wave (26) from three kinds of reflectors: a metallic plane, a 2D-corner retro-reflector (2D-CR), and a cubic corner retro-reflector (CCR). All three reflectors are placed at distance $\sigma_f = L$ from the starting point $\sigma_{0f} = 0$.
Requiring the tangent component of the total wave field $\mathbf{E}_{tot} = \mathbf{E}_{inc} + \mathbf{E}_{refl}$ to be zero at the metallic surface, which we suppose to be a conductor with high electric conductivity, we have

$$\mathbf{E}_{tot}^{(\tan)} = \mathbf{E}_{inc}^{(\tan)} + \mathbf{E}_{refl}^{(\tan)} = 0. \tag{37}$$

Then the reflected wave can presented as

$$\mathbf{E}_{refl} = 2\mathbf{E}_{inc}^{(n)} - \mathbf{E}_{inc}. \tag{38}$$

At normal incidence of the transverse wave on a metallic plane, the normal component of the electromagnetic field happens to be zero, $\mathbf{E}_{inc}^{(n)} = 0$. Therefore the incident plane wave (26) produces the reflected wave (33) in such a way that the summary wave field $\mathbf{E}_{tot} = \mathbf{E}_{inc} + \mathbf{E}_{refl} = \mathbf{E}_f + \mathbf{E}_b$ would be zero at the reflecting plane $\sigma_f = L$:

$$(\mathbf{E}_f + \mathbf{E}_b)_{\sigma_f = L} = 0. \tag{39}$$

Assume that $\sigma_{0b} = 0$ and that *const* in Eq. (28) equals L. Then the $\sigma_b = L - \sigma_f$ boundary condition (39) takes the following form:

$$a_{f+}\mathbf{e}_{f+}\exp(iN_{f+}L) + a_{f-}\mathbf{e}_{f-}\exp(iN_{f-}L) + a_{b+}\mathbf{e}_{b+} + a_{b-}\mathbf{e}_{b-} = 0. \tag{40}$$

Since

$$\mathbf{e}_{b+} = \mathbf{e}_{f+}, \qquad \mathbf{e}_{b-} = \mathbf{e}_{f-}, \tag{41}$$

Eq. (40) gives

$$\begin{aligned} a_{b+} &= -a_{f+}\exp(iN_{f+}L), \\ a_{b-} &= -a_{f-}\exp(iN_{f-}L). \end{aligned} \tag{42}$$

Taking into account that propagation constants $N_{f\pm}$ (Eq. (17)) and $N_{b\pm}$ (Eq. (30)) are equal to each other, and considering the backward wave at the starting point $\sigma_f = 0$, we obtain the basic relation for the electromagnetic wave, double-passed through magnetized plasma:

$$\mathbf{E}_b|_{\sigma_f=0} = -[a_{f+}\mathbf{e}_{b+}\exp(2iN_{f+}L) + a_{f-}\mathbf{e}_{b-}\exp(2iN_{f-}L)]. \quad (43)$$

For weakly inhomogeneous plasma, satisfying condition (34), Eq. (43) becomes

$$\mathbf{E}_b|_{\sigma_f=0} = -\left[a_{f+}\mathbf{e}_{b+}\exp\left(2i\int_0^L N_{f+}d\sigma_f\right) + a_{f-}\mathbf{e}_{b-}\exp(2i\int_0^L N_{f-}d\sigma_f)\right]. \quad (44)$$

According to Eqs. (43) and (44), the phases of reflected normal waves are twice as big as those in the single-passage regime. The same is true for the phase difference:

$$\Delta S_{\text{double passage}} = 2\int_0^L (N_{f+} - N_{f-})d\sigma_f = 2\int_0^L \Omega d\sigma_f = 2\Delta S_{\text{single passage}}. \quad (45)$$

This fact was known earlier for pure Faraday and pure Cotton-Mouton phenomena at a qualitative rather than at an analytical level. Eqs. (43) and (44) generalize these partial results for the general case of joint action of Faraday and Cotton-Mouton phenomena, when normal modes are of arbitrary elliptical polarization.

Dependence of the total phases

$$S_{b\pm}^{tot} = N_{b\pm}^{tot}(L - \sigma_f) + N_{f\pm}^{tot}L \quad (46)$$

of the backward wave on "forward" distance σ_f is presented in Fig. 3 by solid lines.

The total backward phase $S_{b\pm}^{tot}$ has an initial value $S_{b\pm}^{tot} = N_{b\pm}^{tot}L$ at distance $\sigma_f = L$ and achieves doubled value $S_{b\pm}^{tot} = 2N_{b\pm}^{tot}L$ near the origin $\sigma_f = 0$. Fig. 3 shows that a slow incident wave $\mathbf{e}_{f-}$ gives rise to a slow reflected wave $\mathbf{e}_{b-}$, and that the fast incident wave $\mathbf{e}_{f+}$ generates a fast reflected wave $\mathbf{e}_{b+}$. It is these channels of reflection that we identify with the "velocity-preserving" ones.

It is important that "velocity-preserving" channels of reflection demonstrate the opposite direction of electric vector rotation on the polarization ellipse along the ray as compared with the forward wave. In other words, the right-hand rotation of the forward wave produces the left-hand rotation of the backward wave, following directly from boundary condition (39). Therefore "velocity-preserving" reflection can be also treated as a "rotation-converting" one. Thus, both for forward and backward waves, the electrical vector of the fast wave rotates in the same direction as electrons, performing Larmour rotation. At the same time the electric vector of the slow wave in both cases rotates in the opposite direction to electrons.

6. "Velocity-preserving" and "velocity-converting" channels under reflection from a 2D corner retro-reflector

An analysis of electromagnetic wave reflection from a 2D corner retro-reflector can be performed on the basis of boundary condition (38). Let us suppose that the edge of a 2D corner retro-reflector is oriented along basis vector $\mathbf{e}_{2f}$ and perpendicular to unit vector $\mathbf{e}_{1f}$. Two planes, forming 2D-CR, are characterized by the unit vectors $\mathbf{n}_1$ and $\mathbf{n}_2$, which are mutually orthogonal ($(\mathbf{n}_1 \cdot \mathbf{n}_2) = 0$). By applying Eq. (38) to both planes of 2D-CR, we conclude that the vertical component $\mathbf{e}_{1f}$ of the "forward" eigenvector $\mathbf{e}_{f\pm} = p_{f\pm}(\mathbf{e}_{1f} + \zeta_{f\pm}\mathbf{e}_{2f})$ changes its orientation after reflection from a 2D–CR ($\mathbf{e}_{1f}|_{refl} = -\mathbf{e}_{1f}$). At the same time the horizontal component $\mathbf{e}_{2f}$ becomes unchanged ($\mathbf{e}_{2f}|_{refl} = -\mathbf{e}_{2f}$). As a result, being reflected from 2D–CR, the eigenvector $\mathbf{e}_{f\pm} = p_{f\pm}(\mathbf{e}_{1f} + \zeta_{f\pm}\mathbf{e}_{2f})$ becomes

$$\mathbf{e}_{f\pm}|_{refl} = -p_{f\pm}\mathbf{e}_{1f} + p_{f\pm}\zeta_{f\pm}\mathbf{e}_{2f}. \quad (47)$$

Writing down Eq. (47) in the "backward" basis $[(\mathbf{e}_{1b}, \mathbf{e}_{2b}) = (\mathbf{e}_{1f}, -\mathbf{e}_{2f})]$, and taking into account that $\zeta_{f\pm}\mathbf{e}_{2f} = \zeta_{b\pm}\mathbf{e}_{2b}$, we have

$$\mathbf{e}_{f\pm}|_{refl} = -p_{b\pm}\mathbf{e}_{1b} + p_{b\pm}\zeta_{b\pm}\mathbf{e}_{2b}. \quad (48)$$

The reflected eigenvector (48) does not coincide with any "backward" eigenvectors $\mathbf{e}_{b\pm}$. Polarization state $\mathbf{e}_{f\pm}$ after reflection from a 2D-CR becomes transformed into a superposition of polarization states $\mathbf{e}_{b+}$ and $\mathbf{e}_{b-}$,

$$\mathbf{e}_{f\pm}|_{refl} = \langle f_\pm \mid b_+\rangle \mathbf{e}_{b+} + \langle f_\pm \mid b_-\rangle \mathbf{e}_{b-}, \quad (49)$$

where the matrix element coefficients $\langle f_\pm \mid b_\pm\rangle$ are given by formulae

$$\langle f_\pm \mid b_+\rangle = \mathbf{e}_{f\pm}|_{refl}\,\mathbf{e}^*_{b+} = -p_{b\pm}\left(\mathbf{e}_{1b} - \zeta_{b\pm}\mathbf{e}_{2b}\right)\mathbf{e}^*_{b+} = -p_{b\pm}p_{b+}\left(\mathbf{e}_{1b} - \zeta_{b\pm}\mathbf{e}_{2b}\right)\left(\mathbf{e}_{1b} + \zeta^*_{b+}\mathbf{e}_{2b}\right) = -p_{b\pm}p_{b+}\left(1 - \zeta_{b\pm}\zeta^*_{b+}\right), \quad (50)$$

$$\langle f_\pm \mid b_-\rangle = \mathbf{e}_{f\pm}|_{refl}\,\mathbf{e}^*_{b-} = -p_{b\pm}\left(\mathbf{e}_{1b} - \zeta_{b\pm}\mathbf{e}_{2b}\right)\mathbf{e}^*_{b-} = -p_{b\pm}p_{b-}\left(1 - \zeta_{b\pm}\zeta^*_{b-}\right). \quad (51)$$

By virtue of Eq. (49) the backward wave becomes

$$\mathbf{E}_b = a_+\langle f_+ \mid b_+\rangle\,\mathbf{e}_{b+}\exp[iN_{b+}(L-\sigma_f) + iN_{b+}L] + a_+\langle f_+ \mid b_-\rangle\,\mathbf{e}_{b-}\exp[iN_{b-}(L-\sigma_f) + iN_{b+}L] + a_-\langle f_+ \mid b_-\rangle\,\mathbf{e}_{b+}\exp[iN_{b+}(L-\sigma_f) + iN_{b-}L] + a_-\langle f_- \mid b_-\rangle\,\mathbf{e}_{b-}\exp[iN_{b-}(L-\sigma_f) + iN_{b-}L]. \quad (52)$$

At the initial point $\sigma_f = 0$, the reflected wave field $\mathbf{E}_b$ takes the form

$$\mathbf{E}_b = a_+\langle f_+ \mid b_+\rangle\,\mathbf{e}_{b+}\exp[i2N_{b+}L] + a_+\langle f_+ \mid b_-\rangle\,\mathbf{e}_{b-}\exp[i(N_{b-}+N_{b+})L] + a_-\langle f_- \mid b_+\rangle\,\mathbf{e}_{b+}\exp[i(N_{b+}+N_{b-})L] + a_-\langle f_- \mid b_-\rangle\,\mathbf{e}_{b-}\exp[i2N_{b-}L]. \quad (53)$$

The first and fourth terms in Eq. (53) describe "velocity-preserving" channels of reflection, which are accompanied by doubled phase shifts, such as Eq. (43):

$$\mathbf{E}_b|_{doubled\ phase} = a_+\langle f_+ \mid b_+\rangle\,\mathbf{e}_{b+}\exp[i2N_{b+}L] + a_-\langle f_- \mid b_-\rangle\,\mathbf{e}_{b-}\exp[i2N_{b-}L]. \quad (54)$$

In turn, the second and third terms relate to the "velocity-converting" regime, which transforms the slow wave into the fast one and vice versa. As a result both the fast and slow converted modes arrive at the origin $\sigma_f = 0$ with equal phase changes $[(N_{b-} + N_{b+})L]$:

$$\mathbf{E}_b|_{converted} = [a_+\langle f_+ \mid b_-\rangle\,\mathbf{e}_{b-} + a_-\langle f_- \mid b_+\rangle\,\mathbf{e}_{b+}]\exp[i(N_{b-}+N_{b+})L]. \quad (55)$$

Matrix elements $\langle f_\pm \mid b_\pm\rangle$ generally differ from zero and unity, except for cases when $|\Omega_1| \ll |\Omega_2|, |\Omega_3|$ – for example, when the Faraday effect prevails, that is, when $|\Omega_3| \gg |\Omega_1|, |\Omega_2| \approx 0$. Then $\zeta_{b\pm}\zeta^*_{b\pm} = 1$ and "velocity-preserving" channels completely disappear,

$$\langle f_+ \mid b_+\rangle = \langle f_- \mid b_-\rangle = 0, \quad (56)$$

whereas $\zeta_{b\pm}\zeta^*_{b\mp} = -1$ and the "velocity-converting" channels acquire maximum (in module) values

$$\langle f_+ \mid b_-\rangle = \langle f_- \mid b_+\rangle = -1. \quad (57)$$

As a result the converted component of the reflected wave at the origin $\sigma_f = 0$ will be as follows:

$$\mathbf{E}_b|_{converted} = -[a_+\mathbf{e}_{b-} + a_-\mathbf{e}_{b+}]\exp[i(N_{b-}+N_{b+})L]. \quad (58)$$

Remarkably, polarization of the converted wave (58) at $\sigma_f = 0$ completely coincides with polarization of the initial wave (25), which is a kind of phase-conjugation phenomena in magnetoactive plasma. This property might be of practical interest for plasma interferometry.

Summing the phase shift $(N_{b-} + N_{b+})\,L$ with the "isotropic" phase shift $2N_0L$, we obtain the total phase of the converted component:

$$S^{tot}_{conv}(\sigma_f) = 2N_0L + (N_{b+} + N_{b-})L = (2N_0 + \Omega_\perp)L. \quad (59)$$

The dependence of the total phases $S^{tot}_{b\pm}|_{convert}(\sigma_f)$ of the converted modes on "forward" distance σ_f is shown in Fig. 3 by dashed lines.

The phase (59) contains the term $\Omega_\perp$, which is quadratic in a magnetic field and is not sensitive to the Faraday effect, so that the phase (59) can be treated as

"Faraday-independent" one. Therefore, the "velocity-converted" components can be helpful for reducing the influence of the magnetic field on the wave's phase and can be recommended for interferometers in microwave and FIR bands. These components can be separated from "velocity-preserving" ones by means of a polarization filter having the same polarization as the primary wave.

7. Reflection from a cubic corner retro-reflector

Let $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$ be the unit vectors perpendicular to the facets of a CCR. In the orthogonal coordinate system, formed by mutually orthogonal vectors $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$, the incident wave $\mathbf{E}_{inc}$ can be represented by the expansion

$$\mathbf{E}_{inc} = \mathbf{n}_1(\mathbf{n}_1\mathbf{E}_{inc}) + \mathbf{n}_2(\mathbf{n}_2\mathbf{E}_{inc}) + \mathbf{n}_3(\mathbf{n}_3\mathbf{E}_{inc}). \qquad (60)$$

Applying Eq. (38) sequentially to every facet of a CCR, for the reflected wave field one has

$$\mathbf{E}_{refl} = \mathbf{n}_1(\mathbf{n}_1\mathbf{E}_{inc}) + \mathbf{n}_2(\mathbf{n}_2\mathbf{E}_{inc}) + \mathbf{n}_3(\mathbf{n}_3\mathbf{E}_{inc}). \qquad (61)$$

According to Eq. (61), the electric vector of the reflected wave coincides with that of the incident field, though both waves propagate in opposite directions. It means that the forward wave $\mathbf{E}_{inc} = a_{f+}\mathbf{e}_{f+}\exp(iN_{f+}L) + a_{f-}\mathbf{e}_{f-}\exp(iN_{f-}L)$ produces the backward wave

$$\mathbf{E}_b = a_{f+}\mathbf{e}_{b+}\exp[iN_{f+}(L-\sigma_f)]\exp(iN_{f+}L) + a_{f-}\mathbf{e}_{b-}\exp[iN_{f-}(L-\sigma_f)]\exp(iN_{f-}L). \qquad (62)$$

This differs from the wave field (43), reflected from the metallic plane, only by sign. Thus, a CCR meets the "velocity preserving" regime, such as the metallic plane.

8. Conclusions

The properties of electromagnetic waves, reflected from the target of a magnetized plasma, are studied with a special emphasis on plasma polarimetry in thermonuclear reactors ITER and W-7X. An analysis is carried out for independent normal modes, propagating both in homogeneous and weakly inhomogeneous plasmas. Three kinds of reflectors are considered: metallic plane, 2D-corner retro-reflector (2D-CR) and cubic corner retro-reflector (CCR). An electromagnetic wave, reflected from a metallic plane and from a CCR is shown to contain only "velocity-preserving" channels of scattering, when the fast (slow) normal wave also reproduces, after reflection, the fast (slow) normal wave. As a result, the phases of the reflected waves become doubled in comparison with the single-passage propagation. This fact, known earlier for the pure Faraday effect (circularly polarized waves) and for the pure Cotton-Mouton effect (linearly polarized normal waves) is generalized now for elliptically polarized, non-interacting, normal waves of the general type.

At the same time, the wave reflection from a 2D-corner retro-reflector (2D-CR) is characterized by the appearance of both "velocity-preserving" and "velocity-converting" channels, the latter transforming the fast wave into the slow one and vice-versa. It is shown that in the case of circularly polarized modes, the "velocity-preserving" channels completely disappear, and only "velocity-converting" channels exist. As shown above, for these channels, the initial polarization state of the electromagnetic wave is reconstructed after double passage through magnetized plasma.

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