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Left-handed chiral metamaterials

Research Article

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Abstract: In this work, the concept of left-handed chiral metamaterials is analyzed by emphasizing their optical ability on the rotation of the plane of polarization of a wave. The possibilities of a negative phase velocity, negative and positive propagation constants, and basic electromagnetic properties of this novel medium are also presented. After the characterization of left-handed chiral metamaterial, we provide a reflection and transmission study for two planar boundaries of nonchiral - left-handed chiral metamaterial for normal angles of incidence. Some numerical results are also provided to validate the formulation found in the analysis and to show the role of the chirality in the propagation constants, phase velocities, reflection and transmission.

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Keywords: left-handed metamaterial • chiral medium • negative phase velocity • reflection, and transmission

1. Introduction

The phenomenon of chiral media began to attract attention of the electromagnetics' community with simple but illuminating microwave experiments of Lindman in 1920-1922 and Pickering in 1945, which were analogous to optical experiments performed in the nineteenth century [1], [2]. Several theoretical and experimental works have been provided about chiral media and their applications since then [3–10]. On the other hand, the concept of left-handed metamaterial (MTM), which have a simultaneous negative permittivity and permeability over a certain frequency band, started with the proposal of V. Veselago in the 1960s when he theoretically investigated various electromagnetic properties of left-handed MTM [11]. So far, left-handed MTMs have occupied important positions in science, technology and our daily life and the topic continues to be of great interest and practical importance due to a variety of potential applications [12–18]. Recently, the optical properties of left-handed MTMs have attracted widespread attention of the research community. Since these novel media can offer revolutionary solutions in a number of practical problems of great importance, the number of works which analyze their properties and potential applications has been growing exponentially for several years. In this sense, the electromagnetic behaviors of chiral metamaterial (CMTM) have been investigated by many researchers in order to understand the fundamental mechanisms of this artificial structure. With regard to that, the concept of a chiral nihility medium was introduced and an extraordinary wave, a backward wave, in this medium was presented by Tretyakov et. al. in 2003 [19]. Pendry discussed the effects of chirality in 2004 and he showed that it offers an alternative to the present routes to negative refraction. As a result of his work, he produced a practical design that is chiral, has many advantages, and

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exhibits novel properties [20]. Some of the most recent work in this area includes the following:

- the possibilities to realize a negative refraction in chiral composites in dual-phase mixtures of chiral and dipole particles by Tretyakov et. al. [21];
- polarization changes of light diffracted on a planar chiral structure by Prosvirnin and Zheludev [22];
- refractive properties of a plane wave incident from free space to the uniaxially anisotropic chiral media by Cheng and Cui [23], and
- a novel procedure for predicting the effective constitutive parameters of three dimensional periodic materials with chiral component phases by Ouchetto et al. [24].

Following the above, magnetoelectric materials which couple electric fields with magnetic fields for physical realization and potential synthesis of left-handed materials were presented by Qiu et al. [25]. Then, Qiu and Zouhdi presented their studies on the possibility of realizing negative refraction with gyrotropically chiral media in 2007 [26]. After that, propagation of eigenwaves in a chiral medium, a special interest in chiral nihility and the effects of chirality on energy transmission are studied by Qiu et al. in 2008 [27]. Shortly thereafter, the characterization and analysis of left-handed chiral materials has been studied by Sabah to observe their electromagnetic properties [28]. As a result of these works, the investigation of negative phase velocity (NPV) and unconventional properties of chiral medium has achieved significant importance [29–36] and it is found that NPV can occur in CMTM. This means that backward wave propagation and/or negative refraction can be realized using CMTM.

In this paper, some electromagnetic properties of CMTM and the possibility of NPV in CMTM is analyzed and presented. An analytical solution of wave vectors in chiral media for both positive phase velocity (PPV) and NPV is investigated. Then, reflection and transmission between two different semi-infinite media (nonchiral - left-handed CMTM) are studied. Also, some numerical examples are given to support the results found theoretically and to illustrate the effect of chirality in the propagation constants, phase velocities, reflection and transmission.

2. Characterization of left-handed chiral metamaterial

The electromagnetic field vectors related by chiral constitutive factors can be described in the following form [2]:

$$D = \varepsilon\, E - j\xi\, B \tag{1}$$

$$H = -j\xi\, E + (1/\mu)\, B \tag{2}$$

where ε, μ, and ξ are real constants that represent permittivity, permeability, and chirality admittance, respectively. In this study, $\exp(j\omega t)$ is assumed to be time- dependent and it is suppressed. Note that the quantity ξ indicates the degree of chirality and there is a bound for this quantity given by $|\xi| \leq (\varepsilon/\mu)^{1/2}$ [5]. Using the constitutive relations given in eqs. (1) and (2), together with Maxwell's equations, the following source-free chiral Helmholtz equation is obtained:

$$\nabla^2 C + 2\omega\mu\, \xi(\nabla \times C) + \omega^2\varepsilon\, \mu\, C = 0 \tag{3}$$

where $\boldsymbol{C} = \boldsymbol{E}, \boldsymbol{H}, \boldsymbol{D}, \boldsymbol{B}$.

For any arbitrary polarized monochromatic plane wave, the solution to Eq. (3) can be expressed as a sum of circularly polarized waves of either left or right handedness. Therefore, left- and right-circularly polarized (LCP and RCP, respectively) waves interact with chiral media. These solutions yield two propagation constants β_L and β_R given by

$$\beta_L = -\omega\mu\, \xi + \sqrt{\omega^2\mu\varepsilon + (\omega\mu\xi)^2} \tag{4}$$

$$\beta_R = \omega\mu\, \xi + \sqrt{\omega^2\mu\varepsilon + (\omega\mu\xi)^2}. \tag{5}$$

Note that the subscript L and R refer to LCP and RCP plane waves with phase velocities $v_L = \omega/\beta_L$ and $v_R = \omega/\beta_R$, respectively. Here, the signs of ε, μ and ξ are very important to state PPV and NPV. For example, if ε, μ and ξ are all positive then RCP and LCP waves have PPV when $\sqrt{\mu^2\xi^2 + \varepsilon\mu} > \mu\, \xi$. If, however, the condition $\mu\, \xi > \sqrt{\mu^2\xi^2 + \varepsilon\mu}$ is satisfied, the LCP wave will have NPV and backward wave propagation will occur. Thus, several conditions can be written to obtain LCP and RCP plane waves with PPV and NPV by arranging the signs and values of ε, μ and ξ [28–33]. In the case of left-handed CMTM, in which the permittivity and permeability are simultaneously negative, the propagation constants β_L and β_R have to be modified as [28–33].

$$\beta_L = -\omega\mu\, \xi - \sqrt{\omega^2\, |\mu|\, |\varepsilon| + (\omega\, |\mu|\, \xi)^2} \tag{6}$$

$$\beta_R = \omega\mu\ \xi - \sqrt{\omega^2\,|\mu|\,|\varepsilon| + (\omega\,|\mu|\,\xi)^2}. \qquad (7)$$

The expressions of $\varepsilon = |\varepsilon|\exp(j\pi)$ and $\mu = |\mu|\exp(j\pi)$ are used in Eqs. (4) and (5) to obtain the modified propagation constants given above for left-handed CMTM. Under these permitted expressions of Eqs. (6) and (7), left-handed CMTM supports two backward waves and negative refraction will occur. When $\xi = 0$, left-handed CMTM turns to be left-handed MTM, in which the propagation constants β_L and β_R become identical, as studied in several works [11–18]. Note that Eqs. (6) and (7) state that there are two waves propagating with different negative phase velocity inside the left-handed CMTM. In addition, it is important to mention that the left-handed CMTMs now occupy a new place in the family of optically active materials which rotate the plane of the linearly polarized incident wave since they have similar optical properties as in the conventional chiral materials.

Now, the computations for the wave numbers and phase velocities using the formulations given above are presented numerically. The propagation constants β_L and β_R , for the conventional chiral and left-handed CMTM media, are plotted as a function of frequency. In addition, the phase velocities v_L and v_R are calculated for each case. For the conventional chiral medium, Eqs. (4) and (5) are used with $\varepsilon = 9\varepsilon_o$, $\mu = \mu_o$, and $\xi = \pm0.0067$ (Siemens). Note that the host medium for the conventional chiral medium is selected to be micaglass (titanium dioxide) which has $\varepsilon = 9\varepsilon_o$, $\mu = \mu_o$ and the chirality chosen from [29]. In turn, Eqs. (6) and (7) are used with ε = -9ε_o, μ = -μ_o, and $\xi = \pm0.0067$ (Siemens) for left-handed CMTM. Fig. 1 shows the propagation constants β_L and β_R versus frequency for the conventional chiral and left-handed CMTM media. The solid lines correspond to the wave numbers inside the left-handed CMTM medium while the dotted lines correspond to the wave numbers inside the conventional chiral medium. From Fig. 1, the wave numbers are negative for the left-handed CMTM medium and they are positive for the conventional chiral medium as expected. Comparing Fig. 1(a) with Fig. 1(b), it can be seen that β_L (β_R) turns to β_R (β_L) when the sign of the chirality parameter exchanges. It is also numerically observed (but not included here), that the left-handed CMTM becomes left-handed MTM as in [11–18] when the chirality parameter is zero. Also, the corresponding phase velocities for LCP and RCP plane waves are given in Table 1. Negative phase velocity and the effect of chirality on the phase velocities of LCP and RCP plane waves can easily be seen from Table 1. In addition, it can be said that there are two propagating LCP and RCP plane waves inside the left-handed CMTM medium and they have negative and different phase velocities.

In the second example, it is intended to show the response

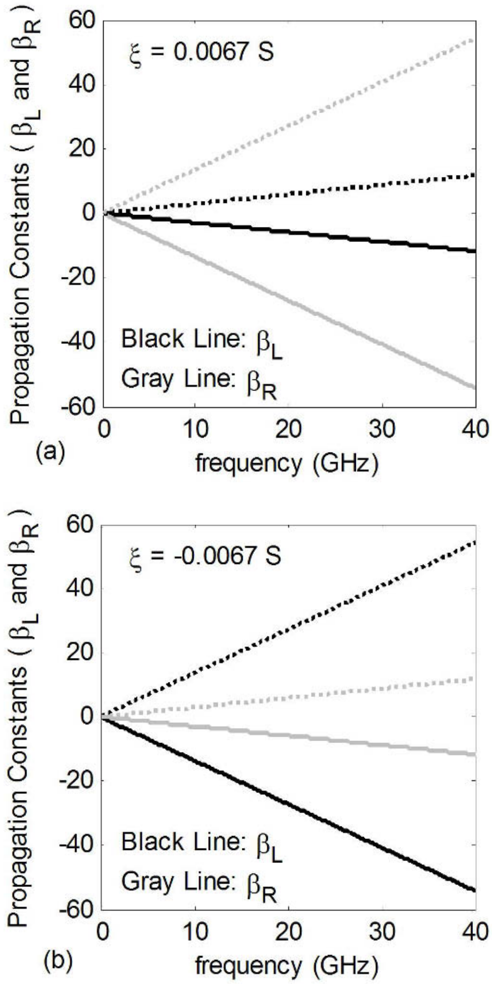

Figure 1. Propagation constants β_L and β_R as a function of the frequency for the conventional chiral and left-handed CMTM media. Solid lines correspond to the CMTM medium and the dotted lines correspond to the conventional chiral medium.

of the propagation constants (β_L / ω and β_R / ω) when the chirality admittance changes. The variations of β_L / ω and β_R / ω are presented in Fig. 2. The propagation constant for the LCP wave shows increasing properties in the left-handed CMTM while it shows decreasing properties in the chiral medium. The propagation constant for the RCP wave, however, shows opposite properties of the LCP wave in both, the left-handed CMTM and chiral media. Also, the propagation constants are negative in the left-handed CMTM and they are positive in the chiral medium as it is seen from Figs. 2(a) and 2(b).

Table 1. Phase velocities for LCP and RCP plane waves for the left-handed CMTM and chiral media.

	$\xi = 0.0067$ S		$\xi = -0.0067$ S	
	CMTM	chiral	CMTM	chiral
v_L[m/s]	-2.1492×10^{10}	2.1492×10^{10}	-4.6529×10^{9}	4.6529×10^{9}
v_R[m/s]	-4.6529×10^{9}	4.6529×10^{9}	-2.1492×10^{10}	2.1492×10^{10}

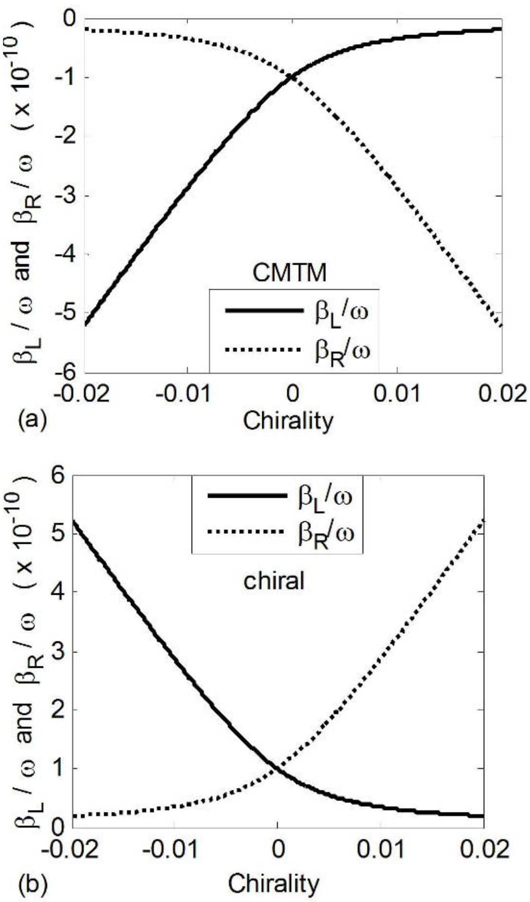

Figure 2. Propagation constants (β_L / ω and β_R / ω) versus the chirality admittance for left-handed CMTM and conventional chiral media.

3. Reflection and transmission

In this section, we will discuss the reflection and transmission coefficients for a monochromatic plane wave with normal incidence on a semi infinite left-handed CMTM. In the analysis, a standard procedure will be followed to obtain the reflection and transmission coefficients. First of all, two planar boundaries of nonchiral - left-handed CMTM media are considered by assuming the incident wave is traveling from a nonchiral medium to a left-handed CMTM medium. The incident wave is also assumed to be perpendicular (normal incidence) to the planar interface formed by two semi-infinite media (nonchiral and left-handed CMTM media). Then, the plane wave reflection and transmission coefficients of a planar interface for normal incidence will be formulated by imposing boundary conditions at the interface. Therefore, the incident electric field with the propagation constant β_i $(= \omega\sqrt{\mu_i \varepsilon_i})$ can be written as follows:

$$E_i = a_x\ E_o \exp(-j\beta_i\ z) \tag{8}$$

where E_o is the magnitude of the incident electric filed. The reflected electric field ($\boldsymbol{E}_r$) can be written by assuming the RCP and LCP plane waves as:

$$E_r = A_R(a_x + ja_y)\exp(j\beta_i\ z) + A_L(a_x - ja_y)\exp(j\beta_i\ z) \tag{9}$$

where A_R and A_L are the magnitudes of the RCP and LCP plane waves of the reflected electric filed. In the left-handed CMTM medium there must be two backward waves propagating toward the nonchiral medium. So the transmitted electric field ($\boldsymbol{E}_t$) can be expressed as:

$$E_t = C_R(a_x + ja_y)\exp(j\beta_R\ z) + C_L(a_x - ja_y)\exp(j\beta_L\ z) \tag{10}$$

where C_R and C_L are the magnitudes of the RCP and LCP plane waves of the transmitted electric filed; β_L and β_R are the propagation constants inside the left-handed CMTM medium.

To find the direction and magnitudes of reflected and transmitted waves, the continuity of electric and magnetic fields, both in phase and magnitude, must be applied. By means of this convention, matching the tangential components of the fields across the nonchiral - left-handed CMTM interface, four unknowns, A_R, A_L, C_R, and C_L, can be found. Consequently, the relationships among the fields in two regions can easily be obtained. After some calculation, it is found that

$$A_R = A_L = \frac{E_o}{2}\left(\frac{k_o + k_c}{k_o - k_c}\right) \tag{11}$$

$$C_R = C_L = E_o\left(\frac{k_o}{k_o - k_c}\right) \tag{12}$$

where $k_c = \beta_R - \omega\mu\xi = \beta_L + \omega\mu\xi = -\sqrt{\omega^2\,|\mu|\,|\varepsilon| + (\omega\,|\mu|\,\xi)^2}$. Note that the permeabilities of the nonchiral and left-handed CMTM media are assumed to be $\mu_i = -\mu_c = \mu_o$ in the calculations and following numerical examples. After that, if Eqs. (11) and (12) are substituted into the reflected and transmitted electric fields, they turn to

$$E_r = a_x\ E_o \left(\frac{k_o + k_c}{k_o - k_c}\right) \exp(j\beta_i\ z) \qquad (13)$$

$$E_t = E_o \left(\frac{k_o}{k_o - k_c}\right)(a_x + ja_y)\exp(j\beta_R\ z) + E_o \left(\frac{k_o}{k_o - k_c}\right)(a_x - ja_y)\exp(j\beta_L\ z) \qquad (14)$$

Finally, from the above equations, the reflection and transmission coefficients can be written as:

$$\rho = \left(\frac{k_o + k_c}{k_o - k_c}\right) \qquad (15)$$

$$\tau = \left(\frac{k_o}{k_o - k_c}\right). \qquad (16)$$

It can be concluded that at normal incidence the reflected wave is polarized in the same direction as the incident wave. In the left-handed CMTM medium, there are left and right circularly polarized backward waves with equal amplitudes but traveling with different velocities.

Furthermore, there is another method to check the correctness of the obtained results. The reflection and transmission coefficients can be found for a monochromatic plane wave with normal incidence traveling from a semi-infinite nonchiral medium to a semi-infinite conventional chiral medium. Then, the results can be tailored for two planar boundaries of nonchiral - left-handed CMTM media by replacing the appropriate propagation constants, permittivities, and permeabilities. The same results given in Eqs. (15) and (16) are achieved by using the above method.

At present, the numerical validations of the reflection and transmission coefficients are demonstrated using the formulation given above. The aim of this numerical study is to show the reflection and transmission characteristics of the left-handed CMTM. In all numerical computations, the conservation of power is satisfied. Note that the permeabilities are arranged as $\mu_i = \mu_o$ for free space, $\mu_c = -\mu_o$ for left-handed CMTM, and $\mu_c = \mu_o$ for the conventional chiral medium in the numerical examples. In the following example, the behavior of the magnitude of reflection and transmission coefficients as a function of relative permittivity is shown in Fig. 3. The first semi-infinite medium (nonchiral medium) is free space and the second medium is assumed to be the left-handed CMTM and/or conventional chiral, in order. The left side of each figure shows the results when the second semi-infinite medium is the left-handed CMTM and the right side shows when the conventional chiral medium is used as the second semi-infinite medium. Fig. 3(a) displays the magnitude of reflection and transmission coefficients when chirality admittance is zero. It means that the left-handed CMTM is the left-handed MTM and the chiral medium is the conventional dielectric medium. The reflection and transmission coefficients show the ordinary increasing and decreasing behavior when the relative permittivity changes. It should be good to mention that there are two transmission coefficients (for RCP and LCP waves) inside the second semi-infinite medium and they overlap with each other due to the normal angle of incidence. As it is seen from Fig. 3(a), the reflection coefficient is zero and the transmission coefficient is 0.5 when $\varepsilon_c = -\varepsilon_o$ for left-handed CMTM or $\varepsilon_c = \varepsilon_o$ for conventional chiral medium. Note that there are two waves propagating with equal amplitude in the second semi-infinite medium. If the chirality admittance is zero, there will be one wave. Thus, the total transmission coefficient becomes 1.0 for $\varepsilon_c = -\varepsilon_o$ in the left-handed CMTM and $\varepsilon_c = \varepsilon_o$ in the conventional chiral medium when $\xi = 0$. This is satisfied by the concept of a left-handed MTM and conventional dielectric media. From Fig. 3(b), $|\rho|$ ($|\tau|$) decreases (increases) when the relative permittivity of the left-handed CMTM becomes more negative. For the conventional chiral medium, $|\rho|$ ($|\tau|$) increases (decreases) when the relative permittivity of the left-handed CMTM becomes more positive. Also, it is observed that the sign of chirality admittance does not affect the behavior of $|\rho|$ and $|\tau|$. Mathematically, this can be seen from the equation of k_c given after Eq. (12) and its non-influence comes into play due to the square of the chirality admittance.

Fig. 4 points out the magnitude of the reflection and transmission coefficients against the chirality admittance. Fig. 4(a) shows the behavior of these coefficients when the relative permittivity of the left-handed CMTM is set to -1.0. When $\xi = 0$, $|\rho| = 0$ and $|\tau| = 0.5$ and there is one wave in the left-handed CMTM. Therefore, the total transmission coefficient becomes 1.0 as it is mentioned in the previous example. From Fig. 4, the general response of the reflection and transmission coefficients can be concluded when the chirality admittance varies. The reflection coefficient decreases till the chirality admittance is zero and after that it increases with increasing values of the chirality admittance. The transmission coefficient shows the opposite response as seen in the reflection coefficient. The changes in the relative permittivity only affect the values of the reflection and transmission coefficients.

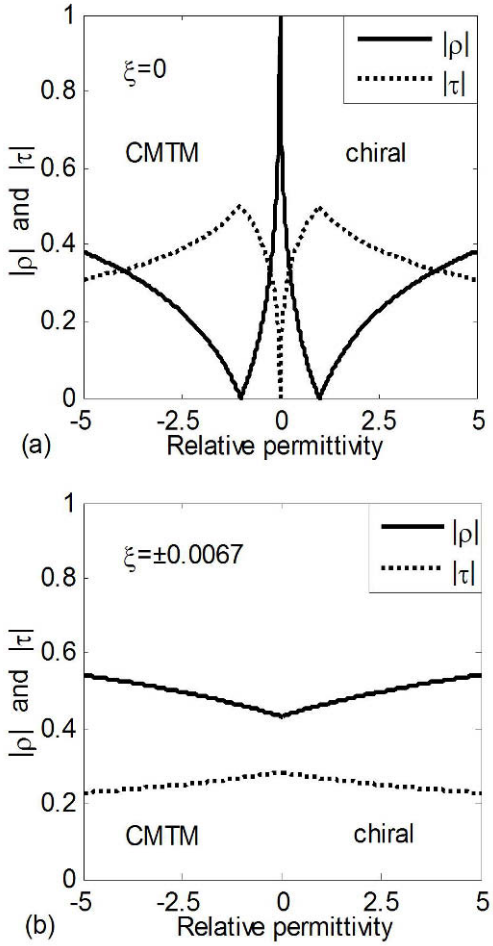

Figure 3. Reflection and transmission coefficients versus the relative permittivity.

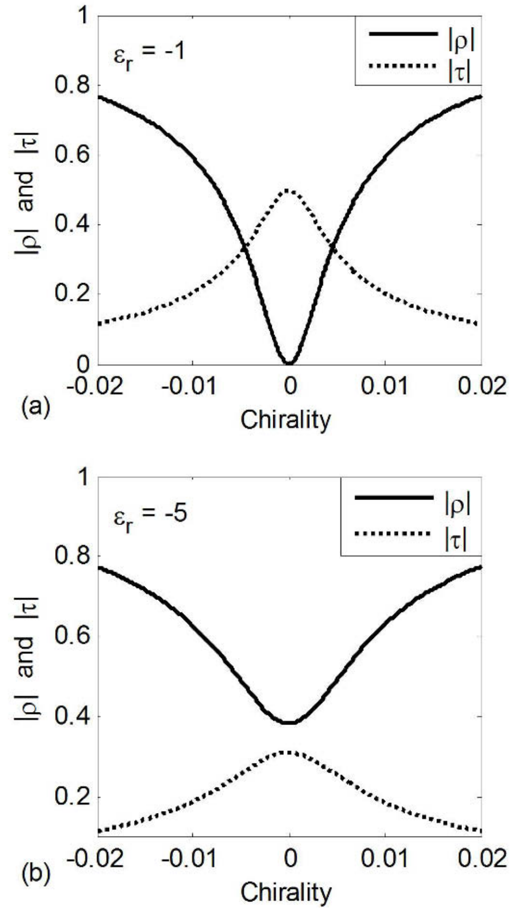

Figure 4. Reflection and transmission coefficients against the chirality of left-handed CMTM.

4. Conclusion and discussion

In this study, left-handed chiral metamaterials are examined with a consideration of their optical properties. In particular, it is shown that a negative phase velocity is possible in these new media. In addition, some characteristic features of these media are pointed out and the possibility of negative propagation constants is explained. Then, the reflection and transmission between planar boundaries of two semi-infinite nonchiral and left-handed CMTM are analyzed and discussed by assuming the plane wave traveling at normal incidence. Some numerical results are also given for validation purposes to examine the reflection and transmission characteristics of the studied medium. The effects of chirality and relative permittivity on the reflection and transmission coefficients and on the propagation constants are shown in the numerical results. It can be concluded that a construction of the left-handed CMTM is achievable according to the theoretical and numerical results obtained here. Many research groups have been working on the possibility and feasibility of the left-handed CMTM. Thus, these results can open a way to the design and manufacture of the left-handed CMTM and they can be used to fabricate and integrate them in various structures. Furthermore, case studies for reflection and transmission at oblique incidence, wave propagation through a single slab of the left-handed CMTM, and multilayer left-handed CMTM are currently being studied to observe the scattering characteristics of the mentioned novel medium. Moreover, a new set of innovative components can be envisaged from these studies.

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