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“WEIGHTED AVERAGES OF RANDOM VARIABLES  
AND EXACT WEIGHTS”

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During preparation of the manuscript, the authors were not aware of a number of papers devoted to so-called exact strong laws of large numbers. Professor André Adler turned our attention to his results on this problem (see [1], [2], [3], where further references may be found). It appears that our Proposition 1 has been already known (see Introduction in [1]), as well as Theorem 1, which can be deduced from the results of [1]. Nevertheless, our proofs and motivations were different. The results of our paper were related to finding examples of weights in the strong law of large numbers obtained by Jajte [4] and Martikainen and Petrov [6]. Our initial motivation was to generalize the results on the convergence of products of sums to the case when the first moment does not exist (see [5]), while the exact strong laws are rooted in the St. Petersburg game problem.

### References

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