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ON A SYSTEM OF RATIONAL DIFFERENCE EQUATION

Abstract. In this paper, we study local asymptotic stability, global character and periodic nature of solutions of the system of rational difference equations given by $x_{n+1} = \frac{ay_n}{b+cy_n}$, $y_{n+1} = \frac{dy_n}{e+fx_n}$, $n = 0, 1, \ldots$, where the parameters $a,\ b,\ c,\ d,\ e,\ f \in (0, \infty)$, and with initial conditions $x_0,\ y_0 \in (0, \infty)$. Some numerical examples are given to illustrate our results.

1. Introduction and preliminaries

The theory of discrete dynamical systems and difference equations developed greatly during the last twenty five years of the twentieth century. Applications of difference equations also experienced enormous growth in many areas. Many applications of discrete dynamical systems and difference equations have appeared recently in the areas of biology, economics, physics, resource management, and others. The theory of difference equations occupies a central position in applicable Analysis. There is no doubt that the theory of difference equations will continue to play an important role in mathematics as a whole. Nonlinear difference equations, of order greater than one, are of paramount importance in applications. Such equations also appear naturally as discrete analogues and as numerical solutions of differential and delay differential equations which model various diverse phenomena in biology, ecology, physiology, physics, engineering and economics. Recently, there has been great interest in studying difference equation systems. One of the reasons for this is a necessity for some techniques which can be used in investigating equations arising in mathematical models describing real life situations in population biology, economic, probability theory, genetics, psychology, and so forth. There are many papers related to the difference equations system, for example, the following ones.

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C. Cinar [1] investigated the periodicity of the positive solutions of the system of rational difference equations:

$$x_{n+1} = \frac{1}{y_n},\ y_{n+1} = \frac{y_n}{x_{n-1}y_{n-1}}.$$

S. Stević [2] studied the system of two nonlinear difference equation:

$$x_{n+1} = \frac{u_n}{1+v_n},\ y_{n+1} = \frac{w_n}{1+s_n},$$

where $u_n,\ v_n,\ w_n,\ s_n$ are some sequences x_n or y_n, with real initial values x_0 and y_0.

S. Stević [3] studied the system of three nonlinear difference equations:

$$x_{n+1} = \frac{a_1 x_{n-2}}{b_1 y_n z_{n-1} x_{n-2} + c_1},\ y_{n+1} = \frac{a_2 y_{n-2}}{b_2 z_n x_{n-1} y_{n-2} + c_2},$$
$$z_{n+1} = \frac{a_3 z_{n-2}}{b_3 x_n y_{n-1} z_{n-2} + c_3},\ n = 0, 1, \ldots,$$

where the parameters $a_i,\ b_i,\ c_i,\ i \in \{1, 2, 3\}$, and initial values $x_{-j},\ y_{-j},\ z_{-j}$, $j \in \{0, 1, 2\}$ are real numbers.

Ignacio Bajo and Eduardo Liz [4] investigated the global behavior of difference equation:

$$x_{n+1} = \frac{x_{n-1}}{a + b x_n x_{n-1}},\ n = 0, 1, \ldots,$$

for all values of real parameters $a,\ b$ and any initial condition $(x_0, x_{-1}) \in \mathbb{R}^2$.

S. Kalabušić, M. R. S. Kulenović and E. Pilav [5] investigated the global dynamics of the following systems of difference equations:

$$x_{n+1} = \frac{\alpha_1 + \beta_1 x_n}{A_1 + y_n},\ y_{n+1} = \frac{\gamma_2 y_n}{A_2 + B_2 x_n + y_n},$$

where the parameters $\alpha_1,\ \beta_1,\ A_1,\ \gamma_2,\ A_2,\ B_2$ are positive numbers, and the initial conditions x_0 and y_0 are arbitrary nonnegative numbers.

Abdullah Selçuk Kurbanli [6] investigated the behavior of solutions of the system of rational difference equations:

$$x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} - 1},\ y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} - 1},\ z_{n+1} = \frac{1}{y_n z_n},\ n = 0, 1, \ldots,$$

where $x_0,\ x_{-1},\ y_0,\ y_{-1},\ z_0,\ z_{-1}$ are real numbers such that $y_0 x_{-1} \neq 1$, $x_0 y_{-1} \neq 1$ and $y_0 z_0 \neq 0$.

A. S. Kurbanli, C. Çinar, I. Yalçinkaya [7] studied the behavior of positive solutions of the system of rational difference equations:

$$x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} + 1},\ y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} + 1},\ n = 0, 1, \ldots,$$

where $y_0,\ y_{-1},\ x_0,\ x_{-1} \in [0, \infty)$.

E. M. Elabbasy, H. El-Metwally and E. M. Elsayed [8] investigated the periodic solutions of particular cases of the following general system of difference equations:

$$x_{n+1} = \frac{a_1 + a_2 y_n}{a_3 z_n + a_4 x_{n-1} z_n}, \; y_{n+1} = \frac{b_1 z_{n-1} + b_2 z_n}{b_3 x_n y_n + b_4 x_n y_{n-1}},$$
$$z_{n+1} = \frac{c_1 z_{n-1} + c_2 z_n}{c_3 x_{n-1} y_{n-1} + c_4 x_{n-1} y_n + c_5 x_n y_n},$$

where the initial conditions $x_{-1},\ x_0,\ y_{-1},\ y_0,\ z_{-1}$ and z_0 are arbitrary nonzero real numbers and $a_i,\ b_i$ and c_j , for $i = 1,\ 2,\ 3,\ 4$ and $j = 1,\ 2,\ 3,\ 4,\ 5$ are non-negative real numbers.

To be motivated by the above studies, our aim in this paper is to investigate the solutions, stability character and asymptotic behavior of the system of difference equations:

$$(1) \qquad x_{n+1} = \frac{a y_n}{b + c y_n}, \; y_{n+1} = \frac{d y_n}{e + f x_n}, \; n = 0, 1, \ldots,$$

where the parameters $a,\ b,\ c,\ d,\ e,\ f \in (0, \infty)$, and with initial conditions $x_0,\ y_0 \in (0, \infty)$.

We review some results which will be useful in our investigation.

2. Linearized stability

Let F and G are continuously differentiable real-valued functions. Consider the system of difference equations:

$$(2) \qquad x_{n+1} = F(x_n, y_n),\ y_{n+1} = G(x_n, y_n),\ n = 0, 1, \ldots,$$

with $x_0,\ y_0 \in (0, \infty)$. Let $(\tilde{x}, \tilde{y})$ be an equilibrium point of system (2), then linearized system about the equilibrium point $(\tilde{x}, \tilde{y})$ has the form:

$$(3) \qquad \begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = A \begin{pmatrix} x_n \\ y_n \end{pmatrix},$$

where $A = \begin{pmatrix} \frac{\partial F}{\partial x}(\tilde{x}, \tilde{y}) & \frac{\partial F}{\partial y}(\tilde{x}, \tilde{y}) \\ \frac{\partial G}{\partial x}(\tilde{x}, \tilde{y}) & \frac{\partial G}{\partial y}(\tilde{x}, \tilde{y}) \end{pmatrix}$. The characteristic equation of system (3) is given by:

$$(4) \qquad \det(A - \lambda I_2) = 0.$$

Definition 1. A point $(\tilde{x}, \tilde{y})$ is called an equilibrium point of the system (2) if

$$\tilde{x} = F(\tilde{x}, \tilde{y}),\ \tilde{y} = G(\tilde{x}, \tilde{y}).$$

Definition 2. Let $(\tilde{x}, \tilde{y})$ be an equilibrium point of the system (2). Then

(i) An equilibrium point $(\tilde{x}, \tilde{y})$ is said to be stable if for any $\varepsilon > 0$ there is $\delta > 0$ such that for initial point (x_0, y_0) for which
$$\|(\tilde{x}, \tilde{y}) - (x_0, y_0)\| < \delta,$$
the iterates (x_n, y_n) of (x_0, y_0) satisfy $\|(\tilde{x}, \tilde{y}) - (x_n, y_n)\| < \varepsilon$ for all $n > 0$ (where $\|\cdot\|$ is Euclidean norm in $\mathbb{R}^2$).

(ii) An equilibrium point $(\tilde{x}, \tilde{y})$ is said to be unstable if it is not stable.

(iii) An equilibrium point $(\tilde{x}, \tilde{y})$ is said to be asymptotically stable if there exists $\gamma > 0$ such that $(x_n, y_n) \to (\tilde{x}, \tilde{y})$ as $n \to \infty$ for all (x_0, y_0) that satisfies:
$$\|(\tilde{x}, \tilde{y}) - (x_n, y_n)\| < \gamma.$$

(iv) An equilibrium point $(\tilde{x}, \tilde{y})$ is called a global attractor if $(x_n, y_n) \to (\tilde{x}, \tilde{y})$ as $n \to \infty$.

(v) An equilibrium point $(\tilde{x}, \tilde{y})$ is called asymptotic global attractor if it is a global attractor and stable.

Definition 3. A solution $\{(x_n, y_n)\}$ of system (2) is bounded and persists if there exist $\delta_1 > 0$ and $\delta_2 > 0$ such that $x_n \geq \delta_1$ and $y_n \leq \delta_2$ for all $n \geq 0$.

Theorem 1. *If both roots of characteristic equation* (4) *have absolute value less than one, then the equilibrium* $(\tilde{x}, \tilde{y})$ *of system* (2) *is locally asymptotically stable. Moreover, if at least one root of equation* (4) *has absolute value greater than one, then* $(\tilde{x}, \tilde{y})$ *is unstable. The equilibrium point* $(\tilde{x}, \tilde{y})$ *of system* (2) *is a saddle point if equation* (4) *has roots both inside and outside the unit disk.*

3. Main results

In this section, we investigate the dynamical properties of system (1) under the conditions that all parameters in the system (1) are positive and the initial conditions are nonnegative real numbers.

Let $F(x, y) = \frac{ay}{b+cy}$, $G(x, y) = \frac{dy}{e+fx}$ and $(\tilde{x}, \tilde{y})$ be an equilibrium point of system (1). Then $F(\tilde{x}, \tilde{y}) = \tilde{x}$ and $G(\tilde{x}, \tilde{y}) = \tilde{y}$ which implies that $\tilde{x} = \frac{a\tilde{y}}{b+c\tilde{y}}$ and $\tilde{y} = \frac{d\tilde{y}}{e+f\tilde{x}}$. Hence, $O \equiv (0, 0)$ and $E \equiv \left(\frac{d-e}{f}, \frac{b(d-e)}{af+ce-cd}\right)$ be two equilibrium points of system (1). Moreover, $\left(\frac{\partial F}{\partial x}(x, y), \frac{\partial F}{\partial y}(x, y)\right) = \left(0, \frac{-acy}{(b+cy)^2} + \frac{a}{b+cy}\right)$ and $\left(\frac{\partial G}{\partial x}(x, y), \frac{\partial G}{\partial y}(x, y)\right) = \left(\frac{-dfy}{(e+fx)^2}, \frac{d}{e+fx}\right)$.

Theorem 2. *The following statements are true:*

(i) *The equilibrium point* O *is locally asymptotically stable if* $d < e$.

(ii) *The equilibrium point* O *is unstable if* $d > e$.

(iii) *Assume that* $e < d$, *then the equilibrium point* E *is locally asymptotically stable if:*
(1) $\Delta < 0.5$ *such that* Δ *is real valued,*
(2) *and* $|0.5 + \Delta| < 1$ *such that* Δ *is complex valued, where* $\Delta = \sqrt{0.25 + \frac{cd^2-2cde-adf+aef}{adf}}$.

Proof. (i) The linearized equation of system (1) about the equilibrium O is

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & \frac{a}{b} \\ 0 & \frac{d}{e} \end{pmatrix} \begin{pmatrix} x_n \\ y_n \end{pmatrix}. \tag{5}$$

The roots of characteristic equation of the system (5) about O are given by $\lambda_1 = 0 < 1$ and $\lambda_2 = \frac{d}{e} < 1$. Hence, proof of (i) follows from Theorem 1.

The proof of (ii) is similar and is omitted.

(iii) The linearized equation of system (1) about the equilibrium E is

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & \frac{(c(e-d)+af)^2}{abf^2} \\ \frac{bf(e-d)}{cd(e-d)+adf} & 1 \end{pmatrix} \begin{pmatrix} x_n \\ y_n \end{pmatrix}. \tag{6}$$

The roots of characteristic equation of the system (6) about E are given by $\lambda_1 = 0.5 - \Delta$ and $\lambda_2 = 0.5 + \Delta$, where $\Delta = \sqrt{0.25 + \frac{cd^2-2cde-adf+aef}{adf}}$. From Theorem 1, E is locally asymptotically stable if $|\lambda_1| < 1$ and $|\lambda_2| < 1$, i.e.,

$$|0.5 - \Delta| < 1, \tag{7}$$

and

$$|0.5 + \Delta| < 1. \tag{8}$$

Since $|\lambda_1| < |\lambda_2|$, so it is sufficient to prove that $|\lambda_2| < 1$. Now there are two cases:

Case(1) If Δ is real valued then $\Delta > 0$, therefore (8) is satisfied only if $\Delta < 0.5$.

Case(2) If Δ is an imaginary number then E is asymptotically stable if $|0.5 + \Delta| < 1$. ∎

Theorem 3. *Let* $a < b$ *and* $d < e$, *then the equilibrium point* O *is globally asymptotically stable.*

Proof. From Theorem 2, the equilibrium point O is locally asymptotically stable. Let $\{(x_n, y_n)\}$ be a solution of system (1) with initial conditions $x_0, y_0 \in (0, \infty)$ and $a < b,\ d < e$. It suffices to prove that:

$$\lim_{n\to\infty} x_n = 0, \ \lim_{n\to\infty} y_n = 0.$$

Since

$$x_{n+1} = \frac{ay_n}{b + cy_n} < \frac{a}{b} y_n.$$

It follows that:

$$x_1 < \frac{a}{b}y_0,\ x_2 < \frac{a}{b}y_1 < \left(\frac{a}{b}\right)^2 y_0,\ x_3 < \frac{a}{b}y_2 < \left(\frac{a}{b}\right)^3 y_0, \ldots,\ x_n < \left(\frac{a}{b}\right)^n y_0.$$

Similarly,

$$y_{n+1} = \frac{dy_n}{e + fx_n} < \frac{d}{e}y_n.$$

It follows that:

$$y_1 < \frac{d}{e}y_0,\ y_2 < \frac{d}{e}y_1 < \left(\frac{d}{e}\right)^2 y_0,\ y_3 < \frac{d}{e}y_2 < \left(\frac{d}{e}\right)^3 y_0, \ldots,\ y_n < \left(\frac{d}{e}\right)^n y_0.$$

Hence,

$$\lim_{n\to\infty} x_n = 0,\ \lim_{n\to\infty} y_n = 0.\ \blacksquare$$

Theorem 4. *Assume that $f = 0$ and $x_0, y_0 \in (0,\infty)$ in system (1). Then the following hold true:*

(i) *If $\frac{d}{e} < 1$ then $x_n \to 0$ and $y_n \to 0$ as $n \to \infty$.*
(ii) *If $\frac{d}{e} > 1$ then $x_n \to \frac{a}{c}$ and $y_n \to \infty$ as $n \to \infty$.*

Proof. Let $f = 0$ then system (1) becomes:

$$x_{n+1} = \frac{ay_n}{b + cy_n},\ y_{n+1} = \frac{d}{e}y_n. \tag{9}$$

Solution of system (9) is given by:

$$x_n = \frac{a(\frac{d}{e})^{n-1}y_0}{b + c(\frac{d}{e})^{n-1}y_0},\ y_n = \left(\frac{d}{e}\right)^n y_0.$$

It is easy to see that:

If $\frac{d}{e} < 1$ then $x_n \to 0$ and $y_n \to 0$ as $n \to \infty$. Moreover, if $\frac{d}{e} > 1$ then $x_n \to \frac{a}{c}$ and $y_n \to \infty$ as $n \to \infty$. $\blacksquare$

Theorem 5. *The system (1) has no prime period-two solutions.*

Proof. System (1) can be reduced to the following second-order difference equation:

$$y_{n+1} = \frac{dy_n(b + cy_{n-1})}{e(b + cy_{n-1}) + afy_{n-1}}. \tag{10}$$

Now it is sufficient to prove that the equation (10) has no prime period-two solutions. Assume that this is not true for equation (10), that is, that

$$p, q, p, q, \ldots, (p \neq q)$$

is a prime period-two solution of equation (10). Then one has:

$$p = \frac{dq(b + cp)}{e(b + cp) + afp},\ q = \frac{dp(b + cq)}{e(b + cq) + afq}.$$

This implies that

$$p\left(e(b+cp)+afp\right)=dq(b+cp),\ q\left(e(b+cq)+afq\right)=dp(b+cq).$$

By subtraction, we obtain

$$(p-q)[be+bd+(c+af)(p+q)]=0,$$

and this implies that $p=q$, which is a contradiction. ∎

Theorem 6. *Assume that $x_0=y_0=h$ and $d=a,\ e=b,\ f=c$, then the solution of system (1) is $x_n=y_n=\frac{a^n(a-b)h}{b^n(a-b)+c(a^n-b^n)h}$, $n=0,1,\ldots$.*

Proof. Let $d=a,\ e=b,\ f=c$ then system (1) is given by:

$$x_{n+1}=\frac{ay_n}{b+cy_n},\ y_{n+1}=\frac{ay_n}{b+cx_n},\ n=0,1,\ldots.$$

For $x_0=y_0=h$, one has:

$$x_1=\frac{ah}{b+ch},\ y_1=\frac{ah}{b+ch},$$
$$x_2=\frac{a^2h}{b^2+c(a+b)h},\ y_2=\frac{a^2h}{b^2+c(a+b)h},$$
$$x_3=\frac{a^3h}{b^3+c(a^2+ab+b^2)h},\ y_3=\frac{a^3h}{b^3+c(a^2+ab+b^2)h}.$$

For any $n=k\in\mathbb{N}$, suppose that:

$$x_k=y_k=\frac{a^kh}{b^k+c(a^{k-1}+a^{k-2}b+\cdots+b^{k-1})h}.$$

Then from $x_{n+1}=\frac{ay_n}{b+cy_n}$ and $y_{n+1}=\frac{ay_n}{b+cx_n}$, one has:

$$x_{k+1}=y_{k+1}=\frac{a^{k+1}h}{b^{k+1}+c(a^k+a^{k-1}b+\cdots+b^k)h}.\ \blacksquare$$

Remark 1. To find the general solution of system (1) is an open problem.

4. Examples

In order to verify our theoretical results and to support our theoretical discussions, we consider in this section several interesting numerical examples. These examples represent different types of qualitative behavior of solutions to the system of nonlinear difference equations (1). We consider some numerical examples to discuss their stability points with the plots.

Example 1. Let $a=0.35,\ b=2,\ c=0.3,\ d=0.4,\ e=0.25,\ f=0.6,\ x_0=0.1,\ y_0=1$ in system (1). In this case $e<d$ and $\Delta=\sqrt{0.25+\frac{cd^2-2cde-adf+aef}{adf}}=0.2112885636821286\iota$. Moreover, $|0.5+\Delta|=$

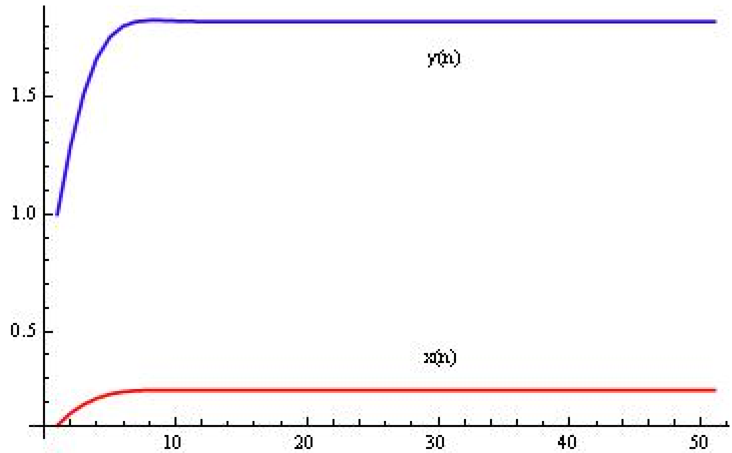

Fig. 1. Plot of system $x_{n+1} = \frac{0.35y_n}{2+0.3y_n}$, $y_{n+1} = \frac{0.4y_n}{0.25+0.6x_n}$

$0.5428101483418092 < 1$ and $\left(\frac{d-e}{f}, \frac{b(d-e)}{af+ce-cd}\right) = (0.25, 1.81818)$ is an equilibrium point of system (1). Figure (1) shows that the equilibrium point E is locally asymptotically stable.

Example 2. Let $a = 0.5$, $b = 0.03$, $c = 0.2$, $d = 0.5$, $e = 0.1$, $f = 0.2$, $x_0 = 0.1$, $y_0 = 0.4$ in system (1). In this case $e < d$ and $\Delta = \sqrt{0.25 + \frac{cd^2-2cde-adf+aef}{adf}} = 0.3$. Moreover, $\Delta = 0.3 \in (0, 0.5)$ and $\left(\frac{d-e}{f}, \frac{b(d-e)}{af+ce-cd}\right) = (2, 0.6)$ is an equilibrium point of system (1). Figure (2) shows that the equilibrium point E is locally asymptotically stable.

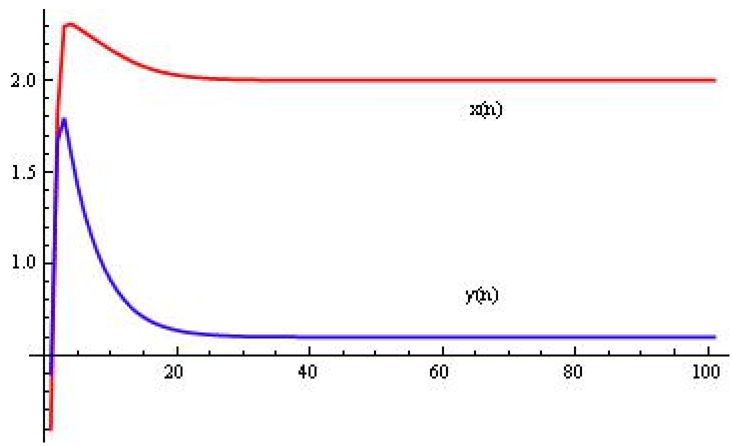

Fig. 2. Plot of system $x_{n+1} = \frac{0.5y_n}{0.03+0.2y_n}$, $y_{n+1} = \frac{0.5y_n}{0.1+0.2x_n}$

Example 3. Let $a = 0.85$, $b = 0.03$, $c = 0.2$, $d = 0.45$, $e = 0.05$, $f = 0.09$, $x_0 = 0.01$, $y_0 = 0.004$ in system (1). In this case $e < d$ but $\Delta = \sqrt{0.25 + \frac{cd^2 - 2cde - adf + aef}{adf}} = 0.539136 > 0.5$. Hence, (iii) of Theorem 2 shows that the equilibrium point E is unstable. Figure (3) shows the unstability of E.

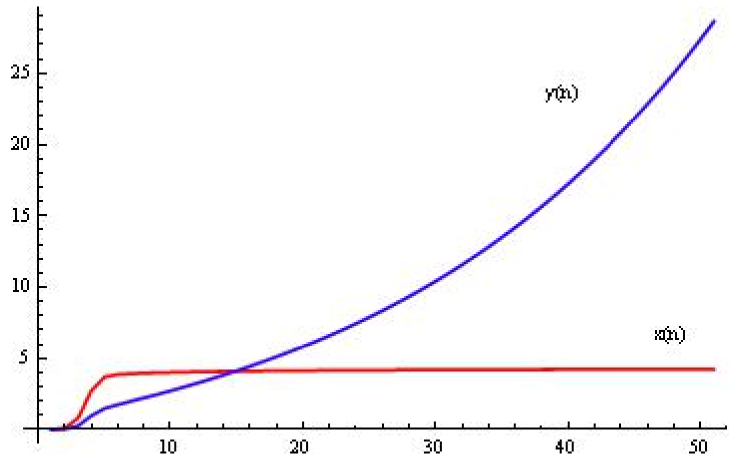

Fig. 3. Plot of system $x_{n+1} = \frac{0.85y_n}{0.03+0.2y_n}$, $y_{n+1} = \frac{0.45y_n}{0.05+0.09x_n}$

Example 4. Let $a = 0.47$, $b = 1.63$, $c = 1.52$, $d = 0.54$, $e = 0.55$, $f = 0.8$, $x_0 = 0.001$, $y_0 = 0.002$ in system (1). In this case $d < e$. Hence, O is a stable equilibrium point. Figure (4) shows the stability of O.

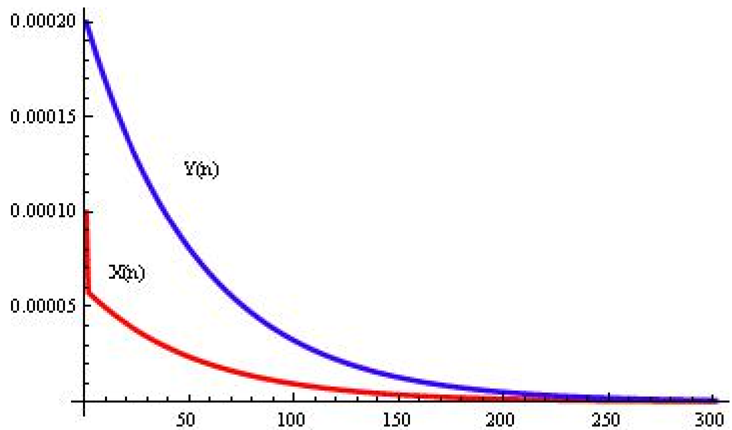

Fig. 4. Plot of system $x_{n+1} = \frac{0.47y_n}{1.63+1.52y_n}$, $y_{n+1} = \frac{0.54y_n}{0.55+0.8x_n}$

Example 5. Let $a = 1$, $b = 2$, $c = 3$, $d = 4$, $e = 3.99$, $f = 6$, $x_0 = 0.001$, $y_0 = 0.002$ in system (1). In this case $e < d$ and $\Delta = \sqrt{0.25 + \frac{cd^2-2cde-adf+aef}{adf}} = 0.497506$. Moreover, $\Delta = 0.497506 \in (0, 0.5)$ and $\left(\frac{d-e}{f}, \frac{b(d-e)}{af+ce-cd}\right) = (0.00166667, 0.00335008)$ is an equilibrium point of system (1). Figure (5) shows that the equilibrium point E is locally asymptotically stable.

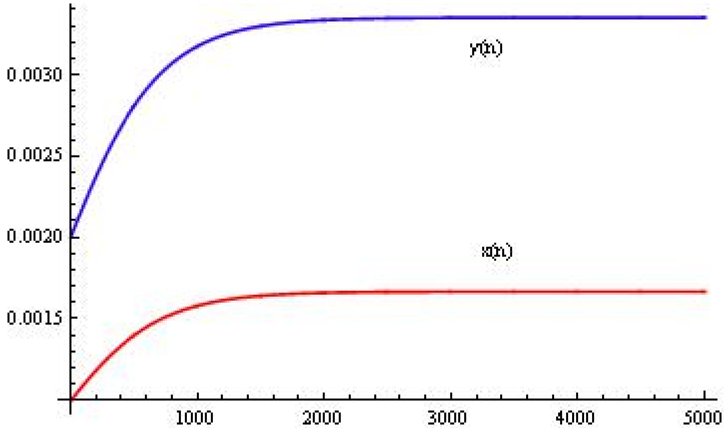

Fig. 5. Plot of system $x_{n+1} = \frac{y_n}{2+3y_n}$, $y_{n+1} = \frac{4y_n}{3.99+6x_n}$

Example 6. Let $a = 0.5$, $b = 1.1$, $c = 13$, $d = 0.018$, $e = 0.0001$, $f = 14$, $x_0 = 0.001$, $y_0 = 0.002$ in system (1). In this case $e < d$ and $\Delta = \sqrt{0.25 + \frac{cd^2-2cde-adf+aef}{adf}} = 0.8434371759895752\iota$. Moreover, $|0.5 + \Delta| =$

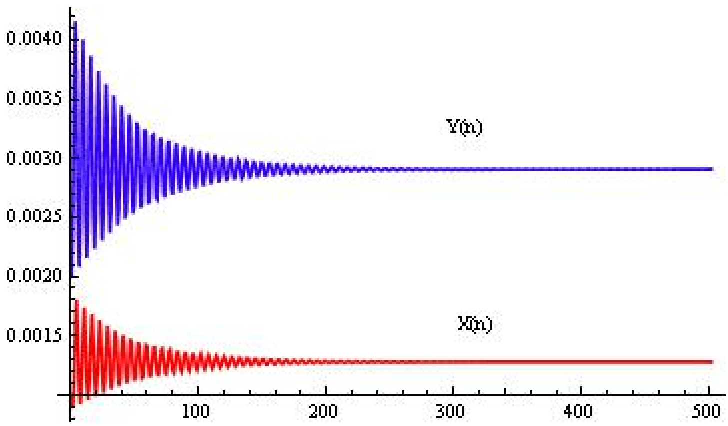

Fig. 6. Plot of system $x_{n+1} = \frac{0.5y_n}{1.1+13y_n}$, $y_{n+1} = \frac{0.018y_n}{0.0001+14x_n}$

$0.9805030697765661 < 1$ and $\left(\frac{d-e}{f}, \frac{b(d-e)}{af+ce-cd}\right) = (0.00127857, 0.00290958))$ is an equilibrium point of system (1). Figure (6) shows that the equilibrium point E is locally asymptotically stable. In this case the parametric plot of the system $x_{n+1} = \frac{0.5y_n}{1.1+13y_n}$, $y_{n+1} = \frac{0.018y_n}{0.0001+14x_n}$ with initial conditions $x_0 = 0.001$, $y_0 = 0.002$ is shown in Figure (7).

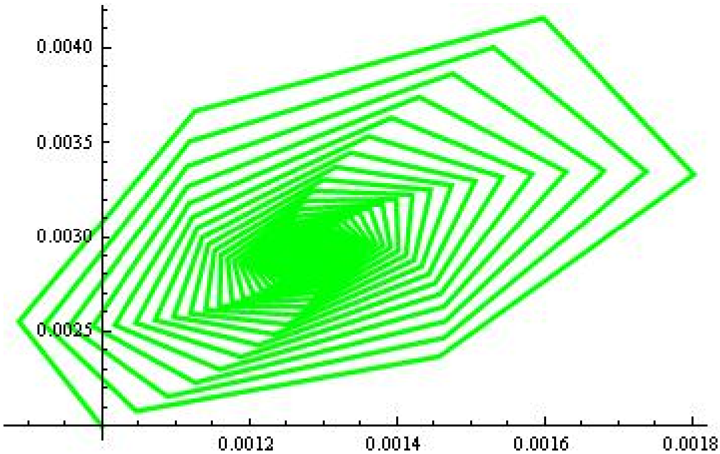

Fig. 7. Parametric plot of system $x_{n+1} = \frac{0.5y_n}{1.1+13y_n}$, $y_{n+1} = \frac{0.018y_n}{0.0001+14x_n}$

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