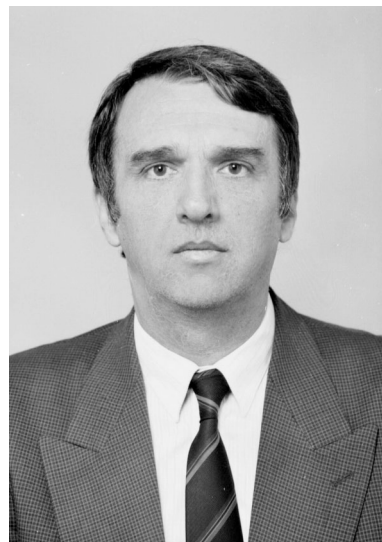


## Boško Jovanović — 60

This year Professor Boško Jovanović will celebrate his 60th birthday. We take this pleasant opportunity to survey the most significant aspects of his rich scientific career.

Born in 1946 in Belgrade, in former Yugoslavia, Boško Jovanović graduated in 1969 from the Department of Mathematics at the Faculty of Natural Sciences and Mathematics in Belgrade. During the academic year of 1972/73 he undertook advanced studies at the Faculty of Numerical Mathematics and Cybernetics of the Lomonosov Moscow State University, under the guidance of the late Nikolay Bakhvalov. Boško Jovanović defended his Ph.D. thesis in 1976 at the University of Belgrade under the supervision of Professor Konstantin Orlov. For many years now, he has been Head of the Department of Numerical Mathematics and Optimization at the Faculty of Mathematics in Belgrade, where he has been a member of the academic staff since 1969.



Boško Jovanović is the author or co-author of over 140 scientific papers and of several monographs. He acted as a referee for many journals, including SIAM Journal on Numerical Analysis, Mathematics of Computation, the IMA Journal of Numerical Analysis, Differential Equations, Computational Methods in Applied Mathematics, and many others.

The largest part of Boško Jovanović's scientific opus is devoted to the analysis of stability and convergence of finite difference schemes for various problems related to PDEs — a field in which he has achieved very impressive results. On the other hand, we also wish to emphasize his significant contributions to maintaining a lively and flourishing scientific environment in the area of numerical analysis of PDEs within Serbia and Montenegro and beyond. The scientific results achieved by Boško Jovanović are a clear evidence in support of this fact as are his commitments to graduate teaching: over the years, he supervised seven doctoral dissertations and eleven M.Sc. theses; also, he had acted as a principal investigator for numerous scientific projects.

At the beginning of his career, among other things, Boško Jovanović worked on multigrid methods; in particular, his doctoral thesis is an early contribution to this relevant field. Based on the results achieved by Fedorenko and Bakhvalov, he considered multigrid methods in the case of nonuniform and curvilinear meshes, equations exhibiting singularities, as well as higher-order equations. Later on, the study of this method flourished in the literature, with further significant contributions from leading numerical analysts, including Achi Brandt, Wolfgang Hackbusch and others.

In the early 1980s, B. Jovanović and his colleague Lav Ivanović became acquainted with the pioneer work of Samarskii, Lazarov and Makarov (Moscow) on the finite difference approximation of generalized solutions to linear constant-coefficient partial differential equations with rough (e.g., locally integrable) forcing terms. The central idea of the approach was to construct difference schemes by mollifying the nonsmooth forcing function in the

equation through local averaging (Steklov-averaging) over the dual meshcells surrounding the mesh points internal to the finite difference grid. For any fixed mesh point, the value of the mollified forcing function at the mesh point then becomes a bounded linear functional of the forcing function over a suitable Sobolev space. Thereby, tools from the theory of finite element methods, such as the Bramble — Hilbert lemma and scalings of Sobolev seminorms, can be used in conjunction with a discrete energy-method to derive optimal-order convergence estimates in discrete Sobolev norms. The beauty of the approach is that it bypasses the demanding continuity and differentiability requirements associated with the use of classical approaches based on Taylor series expansion of the truncation error.

B. Jovanović was quick to realise the potentials of the ideas of Samarskii, Lazarov and Makarov, and initiated a small advanced seminar series in his group in the Faculty of Mathematics at the University of Belgrade, which involved Lav Ivanović and B. Jovanović's young doctoral student Endre Süli. The fruitful collaboration, which continued well after Süli's departure in 1985 to a permanent position at the University of Oxford, led to twelve papers that extended the scope of the original Samarskii — Lazarov — Makarov theory in various directions. These extensions included the finite difference approximation of generalised solutions in fractional-order non-Hilbertian Sobolev spaces, fourth-order elliptic equations, and second-order elliptic and hyperbolic equations with rough coefficients.

Influenced by the ideas of Zlotnik, B. Jovanović applied interpolation of Banach spaces to obtain error estimates for finite difference approximations to initial-boundary value problems. The estimates turned out to be sharper than the standard ones in the case of hyperbolic equations.

Strong and asymptotic stability of differential equations and finite difference schemes is another important topic of B. Jovanović's research. When solving a problem for a differential equation, it may happen that the coefficients are given approximately (by a numerical algorithm or as a result of physical measurements, for example). In such instances it is important to investigate the stability of the solution of the differential equation, not only in the sense of perturbations of the initial conditions and the right-hand side, but also in the sense of perturbations of unbounded operators, which are present in the equation (strong stability).

Analogous problems appear in the corresponding difference schemes approximating the differential problem. A series of papers by B. Jovanović are devoted to this problem. He established strong stability estimates for solutions of first- and second-order differential and operator equations involving unbounded operators in Hilbert spaces. In this context perturbations of the operators appearing in the equations have been estimated in the corresponding norms. A priori estimates for perturbations of solutions to first-order differential and operator equations as well as to two-layer operator and difference schemes have been established in time-integrated norms. These results are particularly important in the study of correctness of problems with integrated solutions.

A number of papers by B. Jovanović are devoted to the asymptotic stability of differential, and operator equations, and operator and difference schemes. While for a first-order (in time) dissipative differential or operator equation an asymptotic behavior of the solution is expected, for a nondissipative second-order equation, such as a linear second-order hyperbolic equation, an asymptotic behavior is not usually observed. For such equations (Cauchy problem), it is frequently stated in the context of global stability, when  $t \rightarrow \infty$ , that the associated energy integral (the square of the norm of the solution in a suitable Sobolev space) is conserved throughout time. In his work, B. Jovanović obtained, for abstract Cauchy

problems associated with first- and second-order differential and operator equations, new a priori estimates of global and asymptotic stability in various energy norms, as well as estimates of global stability for three-level operator and difference schemes consistent with the corresponding estimates for the differential problem.

The numerical solution of differential equations that arise in transmission problems and problems with interfaces is also an important direction in B. Jovanović's research. In these equations the coefficients are discontinuous or unbounded functions or even generalized functions (in particular Dirac-delta functions). Problems of this kind represent considerable challenge for the theory of convergence analysis compatible with the smoothness of solutions to the differential problem. B. Jovanović may be considered as one of the founders of the theory in this field.

In their joint papers, B. Jovanović and L. Vulkov investigated parabolic equations with interfaces corresponding to concentrated heat capacity and singular source terms. These papers include the analytical and numerical analysis of solutions based on special Sobolev spaces that are intrinsic to such problems. The same authors also considered hyperbolic problems that model the vibration of strings and plates under concentrated mass and external/internal concentrated forces. Rates of convergence and error estimates for the difference schemes were established.

A further series of papers of Jovanović and Vulkov are devoted to elliptic problems with standard or nonstandard (e. g., dynamic) boundary conditions. For such problems analytical and numerical results were obtained.

Our brief attempt to present a cross-section of Boško Jovanović's most significant achievements cannot possibly do justice to his many important results. During his successful scientific career spanning almost four decades, Boško Jovanović has made numerous significant contributions to the field of numerical analysis of partial differential equations, and to the theory of finite difference schemes in particular.

We wish him good health and success in his scientific activity.

*V. Jovanović, P. Matus, E. Süli, P. Vabishchevich, and L. Vulkov*

## Selected Publications

1. B. Jovanović, *Additive difference scheme for a non-stationary fourth-order equation in an arbitrary domain*, USSR Comput. Math. Math. Phys., **17** (1977), (1978), pp. 86–92.
2. E. Süli, B. Jovanović, and L. Ivanović, *Finite difference approximations of generalized solutions*, Math. Comput., **45** (1985), pp. 319–327.
3. L. D. Ivanović, B. S. Jovanović, and E. E. Süli, *The convergence of difference schemes for a biharmonic equation*, USSR Comput. Math. Math. Phys., **26** (1986), (1987), pp. 87–90.
4. B. S. Jovanović, L. D. Ivanović, and E. E. Süli, *Convergence of a finite-difference scheme for second-order hyperbolic equations with variable coefficients*, IMA J. Numer. Anal., **7** (1987), pp. 39–45.
5. B. S. Jovanović, L. D. Ivanović, and E. E. Süli, *Convergence of finite-difference schemes for elliptic equations with variable coefficients*, IMA J. Numer. Anal., **7** (1987), pp. 301–305.
6. B. S. Jovanović, *On the convergence of finite-difference schemes for parabolic equations with variable coefficients*, Numer. Math., **54** (1989), pp. 395–404.
7. B. S. Jovanović, *Finite difference method for boundary value problems with weak solutions*, Posebna izdanja Mat. Instituta, knj. 16, Mat. Institut, Beograd, 1993.
8. B. Jovanović, *Multipliers in Sobolev spaces and exact convergence rate estimates for the finite-difference schemes*, Banach Center Publ., **29** (1994), pp. 165–173.
9. A. A. Samarskiĭ, B. S. Jovanović, P. P. Matus, and V. S. Shcheglik, *Finite-difference schemes on adaptive time grids for parabolic equations with generalized solutions*, Differential Equations, **33** (1997), pp. 981–990.

10. B. Jovanović and I. Šestak, *Least-squares mixed finite elements for linear elasticity problem with non-homogeneous boundary conditions*, Z. Angew. Math. Mech., **78** (1998), no. 9, pp. 641–645.
11. B. S. Jovanović and P. P. Matus, *Estimation of the convergence rate of difference schemes for elliptic problems*, Comput. Math. Math. Phys., **39** (1999), pp. 56–64.
12. B. S. Jovanović, P. P. Matus, and V. S. Shcheglik, *On accuracy of difference schemes for nonlinear parabolic equations with generalized solutions*, Comput. Math. Math. Phys., **39** (1999), pp. 1611–1618.
13. B. S. Jovanović and L. G. Vulkov, *On the convergence of finite difference schemes for the heat equation with concentrated capacity*, Numer. Math., **89** (2001), no. 4, pp. 715–734.
14. B. S. Jovanović and P. P. Matus, *Strong stability of operator-differential equations and operator-difference schemes in norms integral with respect to time*, Differential Equations **37** (2001), no. 7, pp. 998–1007.
15. B. S. Jovanović and P. P. Matus, *Coefficient stability of operator-differential equations of second order*, Differential Equations, **38** (2002), no. 10, pp. 1460–1466.
16. B. S. Jovanović and L. G. Vulkov, *On the convergence of difference schemes for hyperbolic problems with concentrated data*, SIAM J. Numer. Anal., **41** (2003), no. 2, pp. 516–538.
17. B. S. Jovanović and P. P. Matus, *Asymptotic stability of first- and second-order operator-differential equations*, Differential Equations, **39** (2003), no. 3, p. 414–425.
18. D. R. Bojović, B. S. Jovanović, and P. P. Matus, *On the strong stability of first-order operator-differential equations*, Differential Equations, **40** (2004), no. 5, pp. 703–710.
19. B. S. Jovanović and L. G. Vulkov, *Stability of difference schemes for parabolic equations with dynamical boundary conditions and conditions on conjugation*, Appl. Math. Comput., **163** (2005), pp. 849–868.
20. B. S. Jovanović and L. G. Vulkov, *Energy stability for a class of two-dimensional interface linear parabolic problems*, J. Math. Anal. Appl., **311** (2005), pp. 120–138.
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