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Blast waves propagation in magnetogasdynamics: power series method

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Abstract: Blast waves are produced when there is a sudden deposition of a substantial amount of energy into a confined region. It is an area of pressure moving supersonically outward from the source of the explosion. Immediately after the blast, the fore-end of the blast wave is headed by the shock waves, propagating in the outward direction. As the considered problem is highly nonlinear, to find out its solution is a tough task. However, few techniques are available in literature that may give us an approximate analytic solution. Here, the blast wave problem in magnetogasdynamics involving cylindrical shock waves of moderate strength is considered, and approximate analytic solutions with the help of the power series method (or Sakurai's approach [1]) are found. The magnetic field is supposed to be directed orthogonally to the motion of the gas particles in an ideal medium with infinite electrical conductivity. The density is assumed to be uniform in the undisturbed medium. Using power series method, we obtain approximate analytic solutions in the form of a power series in $(a_0/U)^2$, where a_0 and U are the velocities of sound in an undisturbed medium and shock front, respectively. We construct solutions for the first-order approximation in closed form. Numerical computations have been performed to determine the flow-field in an ideal magnetogasdynamics. The numerical results obtained in the absence of magnetic field recover the existing results in the literature. Also, these results are found to be in good agreement with those obtained by the Runge–Kutta method of fourth-order. Further, the flow variables are illustrated through figures behind the shock front under the effect of the magnetic field. The interesting fact about

the present work is that the solutions to the problem are obtained in the closed form.

Keywords: blast waves; ideal gas; magnetogasdynamics; Rankine–Hugoniot conditions.

1 Introduction

The phenomena of blast waves are very common in the surrounding of Earth. It results from sudden release of an extensive amount of energy, for example, lightning and nuclear explosions. It is very well known that after an explosion the front of the blast waves is headed by the shock waves that propagate in the outward direction. After World War II, it becomes quite necessary to enhance the understanding about the dynamics of explosion, and find out the way to tackle such kind of problems. Motivated by this, Sedov [2] gave an idea on the similarity solution for the explosion problem in an ideal gas. Exact solution of the equations governing the motion in a gas generated by a point explosion was obtained by Taylor [3]. Thus, Sedov and Taylor showed a way to estimate the effects of nuclear or supernova explosions. Furthermore, Sakurai investigated the point explosion problem in an ideal gas for the planar and cylindrical symmetries, and obtained the first [1] and second [4] approximations for the propagation of the blast wave. Murata [5] applied an analytic approach to obtain the solution of the blast wave problem in an ideal gas, which has been further extended by Singh et al. [6] to the real gas. So, a considerable number of research articles on the shock wave propagation in gas dynamics are available in the literature.

Magnetic field is spread throughout the universe, and has many applications in the field of astrophysics, oceanography, atmospheric sciences and hypersonic aerodynamics, etc. Many interesting problems involve magnetic field. The shock waves in the presence of magnetic field in conducting perfect gas may be important for interpretation of the phenomena encountered in astrophysics and supernova explosion. Complex filamentary structures in molecular clouds, shapes and the shaping of planetary nebulae, synchrotron radiation from supernova remnants, magnetized stellar winds,

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galactic winds, gamma-ray bursts, dynamo effects in stars, galaxies, and galaxy clusters are some interesting astrophysical situations that involve magnetic field. The applications of studying cylindrical shock waves in the presence of magnetic field may also include explosion of long thin wire, experiments on pinch effect, exploding wire, some axially symmetric hypersonic problem such as the shock envelope behind fast meteor or missile [7], etc. Among the industrial applications involving applied external magnetic fields are drag reduction in duct flows, design of efficient coolant blankets in tokamak fusion reactors, control of turbulence of immersed jets in the steel casting process, and advanced propulsion and flow control schemes for hypersonic vehicles [8, 9]. Many researchers have worked to better understand the dynamics of shock waves with magnetic field effects. The works of Arora et al. [10], Jena [11], Menon and Sharma [12], Nath and Singh [13, 14], Pandey et al. [15], Siddiqui et al. [16], Sahu [17], Singh et al. [18], Singh and Arora [19], Bira et al. [20], Kuila and Sekhar [21], Bira and Sekhar [22], Sekhar and Sharma [23], Singh et al. [24, 25], and Vishwakarma and Yadav [26] are worth mentioning in this context.

In this paper, we study the propagation of one-dimensional, unsteady and adiabatic flow of cylindrical shock waves produced due to strong explosion in an ideal gas under the influence of transverse magnetic field. The magnetic field is either axial or azimuthal in cylindrically symmetric motion. Using power series method [1], we obtain the approximate analytic solutions by expanding the flow variables in the power series of $(a_0/U)^2$, where a_0 is the sound speed in the undisturbed medium and U is the shock velocity. The present work is an extension of Sakurai's work [1] in the case of cylindrical shock waves by considering the effects of the magnetic field in an ideal gas. As per the authors' knowledge, the problem under consideration has not been solved by the power series method [1]. The benefit of this approach is that we obtain the solutions in the analytic form so that it can be used to classify and understand the non-linear phenomena involved. With the aid of Sakurai's blast wave analysis [1], solutions of any desired degree of accuracy can be obtained; however there is a great deal of work involved in carrying out the required computations. In our problem, the first-order approximation to the solutions is constructed in closed form with the help of the said method, and distributions of the flow variables are shown

graphically behind the shock. Also, the effect of shock Cowling number on the flow variables is discussed.

We have summarized the paper as follows: Section 1 includes a brief introduction about the earlier studies of the topic. Section 2 presents the basic equations governing the conservation laws together with the R-H jump conditions across the shock front. Section 3 contains the transformation of the basic equations in the form of non-dimensional functions using the similarity analysis. In Section 4, we obtain the solutions of the considered problem in the form of power series in $(a_0/U)^2$. In Section 5, we construct first-order approximate solutions in closed form. Further, Section 6 contains the results and discussion. At last, Section 7 is the conclusion about the whole study of the present work.

2 Fundamental equations with R-H jump conditions

The fundamental equations governing the one-dimensional, unsteady and cylindrically symmetric adiabatic flow of an ideal gas under the influence of magnetic field can be expressed as [15]

$$\begin{aligned}\rho_t + v\rho_x + \rho v_x + \frac{\rho v}{x} &= 0, \\ v_t + vv_x + \frac{1}{\rho} \left[p_x + h_x + \frac{2jh}{x} \right] &= 0, \\ p_t + vp_x + a^2\rho \left[v_x + \frac{v}{x} \right] &= 0, \\ h_t + vh_x + 2h \left[v_x + \frac{(1-j)v}{x} \right] &= 0,\end{aligned}\tag{2.1}$$

where ρ , v and p are the gas density, velocity and pressure, respectively, and $h = \frac{\mu H^2}{2}$ is the magnetic pressure with μ being the magnetic permeability and H being the transverse magnetic field, which is either axial ($j = 0$) or azimuthal ($j = 1$) in the cylindrically symmetric flow; the independent variables x and t are the distance from the axis and time, respectively. Also, a is the velocity of sound, given by $a = \sqrt{\frac{\gamma p}{\rho}}$, where γ is the adiabatic index. Viscosity, thermal conductivity and electrical resistivity are not considered in the present problem. For motion in an ideal gas, the equation of state is taken as follows:

$$p = \rho RT,\tag{2.2}$$

where R and T are the gas constant and temperature, respectively.

Let $\chi = \chi(t)$ be the position of the shock front at any time t . Then, the propagation velocity of the shock front is given by $U = \frac{d\chi}{dt}$. The flow variables immediately ahead of the shock front are characterized by

$$v = 0, \quad p = p_0, \quad \rho = \rho_0, \quad h = h_0(\chi) = h_c \chi^{-2}, \quad (2.3)$$

where p_0 , ρ_0 and h_c are the appropriate constants, and suffix 0 refers to the conditions just ahead of the shock front. For the self-similar solutions, shock velocity is supposed to vary as [13, 27]

$$U^2 = B^2 \chi^{-\alpha}, \quad (2.4)$$

where B and α are constants. The R-H conditions across the shock front ($x = \chi(t)$) can be written as follows [28]:

$$\begin{aligned} (\rho)_{x=\chi} &= \frac{\gamma+1}{\gamma-1} \rho_0 \left[1 + \frac{2}{\gamma-1} \left(\frac{a_0}{U} \right)^2 \right]^{-1}, \\ (v)_{x=\chi} &= \frac{2}{\gamma+1} U \left[1 - \left(\frac{a_0}{U} \right)^2 \right], \\ (p)_{x=\chi} &= \frac{2}{\gamma+1} \rho_0 U^2 \left[1 - \frac{\gamma-1}{2} \left(\frac{a_0}{U} \right)^2 \right] \\ &\quad - \frac{1}{2} \left(\frac{\gamma+1}{\gamma-1} \right)^2 C_0 \rho_0 U^2 \left[1 + \frac{2}{\gamma-1} \left(\frac{a_0}{U} \right)^2 \right]^2, \\ (h)_{x=\chi} &= \frac{1}{2} \left(\frac{\gamma+1}{\gamma-1} \right)^2 C_0 \rho_0 U^2 \left[1 + \frac{2}{\gamma-1} \left(\frac{a_0}{U} \right)^2 \right]^2, \end{aligned} \quad (2.5)$$

where $C_0 = 2h_0/(\rho_0 U^2)$ is the shock Cowling number, and $a_0^2 = \frac{\gamma p_0}{\rho_0}$ is the square of sound velocity in the undisturbed medium. The necessary condition for C_0 to be constant is $\alpha = 2$. As the total energy E carried by the blast wave is equal to the energy released by the explosive and thus assumed to be constant. Therefore, we have

$$E = \int_0^\chi \left[\frac{1}{2} v^2 + \frac{1}{\gamma-1} \left(\frac{p}{\rho} - \frac{p_0}{\rho_0} \right) + \left(\frac{h}{\rho} - \frac{h_0}{\rho_0} \right) \right] \rho x dx, \quad (2.6)$$

where E denotes the explosion energy per unit area of the surface of the shock front for cylinder of unit length. From Lagrangian equation of continuity, we obtain the following relation [1]:

$$\int_0^\chi \frac{\rho}{\rho_0} x dx = \frac{\chi^2}{2}. \quad (2.7)$$

Using (2.3), (2.6) and (2.7), we get the following expression for E :

$$E = \int_0^\chi \left[\frac{1}{2} \rho v^2 + \frac{1}{\gamma-1} p + h \right] x dx - \frac{p_0}{\gamma-1} \frac{\chi^2}{2} - h_0 \frac{\chi^2}{2}. \quad (2.8)$$

3 Transformation of the basic equations in non-dimensional functions

We introduce r and s as the new independent variables in place of x and t , which are defined as follows:

$$\frac{x}{\chi} = r, \quad \left(\frac{a_0}{U} \right)^2 = s. \quad (3.1)$$

Now, we write the unknown functions ρ , v , p and h as follows:

$$\begin{aligned} \rho &= \rho_0 \Lambda(r, s), \\ v &= U \Phi(r, s), \\ p &= p_0 \left(\frac{U}{a_0} \right)^2 \Psi(r, s) = p_0 \frac{\Psi(r, s)}{s}, \\ h &= p_0 \left(\frac{U}{a_0} \right)^2 \Omega(r, s) = p_0 \frac{\Omega(r, s)}{s}, \end{aligned} \quad (3.2)$$

where Λ, Φ, Ψ and Ω are the non-dimensional functions. From Eqs. (3.1) and (3.2), we obtain

$$\frac{\partial}{\partial x} = \frac{1}{\chi} \frac{\partial}{\partial r}, \quad (3.3)$$

$$\frac{D}{Dt} = \frac{U}{\chi} \left[(\Phi - r) \frac{\partial}{\partial r} + s \bar{\lambda} \frac{\partial}{\partial s} \right], \quad (3.4)$$

where $\bar{\lambda} = \chi \frac{(ds/d\chi)}{s}$ is a function of s only. Now substituting Eqs. (3.1)–(3.4) into the basic equations (2.1), we obtain

$$\begin{aligned} (\Phi - r) \Lambda_r + s \bar{\lambda} \Lambda_s + \Lambda \left(\Phi_r + \frac{\Phi}{r} \right) &= 0, \\ \Lambda \left\{ -\frac{\bar{\lambda}}{2} \Phi + (\Phi - r) \Phi_r + s \bar{\lambda} \Phi_s \right\} + \frac{1}{\gamma} \left(\Psi_r + \Omega_r + \frac{2j\Omega}{r} \right) &= 0, \\ -\bar{\lambda} \Psi + (\Phi - r) \Psi_r + s \bar{\lambda} \Psi_s + \gamma \Psi \left(\Phi_r + \frac{\Phi}{r} \right) &= 0, \\ -\bar{\lambda} \Omega + (\Phi - r) \Omega_r + s \bar{\lambda} \Omega_s + 2\Omega \left(\Phi_r + \frac{(1-j)\Phi}{r} \right) &= 0. \end{aligned} \quad (3.5)$$

Now, Eq. (2.8) becomes

$$s \left(\frac{\chi_0}{\chi} \right)^2 = \int_0^1 \left(\frac{\gamma}{2} \Lambda \Phi^2 + \frac{\Psi}{\gamma-1} + \Omega \right) r dr - \frac{s}{2(\gamma-1)} - \frac{\gamma C_0}{4}, \quad (3.6)$$

where

$$\chi_0 = \left(\frac{E}{p_0} \right)^{\frac{1}{2}}. \quad (3.7)$$

Also, the R-H conditions (2.5) assume the form

$$\begin{aligned} \Phi(1,s) &= \frac{2}{\gamma+1} (1-s), \\ \Lambda(1,s) &= \frac{\gamma+1}{\gamma-1} \left(1 + \frac{2s}{\gamma-1} \right)^{-1}, \\ \Psi(1,s) &= \frac{2\gamma}{\gamma+1} \left\{ 1 - \left(\frac{\gamma-1}{2} \right) s \right\} - \frac{\gamma C_0}{2} \left(\frac{\gamma+1}{\gamma-1} \right)^2 \left\{ 1 + \left(\frac{2}{\gamma-1} \right) s \right\}^2, \\ \Omega(1,s) &= \frac{\gamma C_0}{2} \left(\frac{\gamma+1}{\gamma-1} \right)^2 \left\{ 1 + \left(\frac{2}{\gamma-1} \right) s \right\}^2. \end{aligned} \quad (3.8)$$

Now differentiating Eq. (3.6) with respect to s , we obtain the following expression for $\bar{\lambda}$:

$$\bar{\lambda} = \frac{2J - \frac{s}{\gamma-1} - \frac{\gamma C_0}{2}}{J - s \left(\frac{dJ}{ds} \right) - \frac{\gamma C_0}{4}}, \quad (3.9)$$

where

$$J = \int_0^1 \left(\frac{\gamma}{2} \Lambda \Phi^2 + \frac{1}{(\gamma-1)} \Psi + \Omega \right) r dr. \quad (3.10)$$

4 Power series formation of the solutions

For the strong shock waves, the velocity of the shock front U is larger than a_0 , so the quantity s is considered to be very small there. Therefore, we can expand the non-dimensional functions Φ, Ψ, Λ and Ω in the form of convergent series in powers of s as follows:

$$\begin{aligned} \Phi &= \Phi^{(0)} + s\Phi^{(1)} + s^2\Phi^{(2)} + \dots, \\ \Psi &= \Psi^{(0)} + s\Psi^{(1)} + s^2\Psi^{(2)} + \dots, \\ \Lambda &= \Lambda^{(0)} + s\Lambda^{(1)} + s^2\Lambda^{(2)} + \dots, \\ \Omega &= \Omega^{(0)} + s\Omega^{(1)} + s^2\Omega^{(2)} + \dots, \end{aligned} \quad (4.1)$$

where $\Phi^{(n)}, \Psi^{(n)}, \Lambda^{(n)}$ and $\Omega^{(n)}$ ($n = 0, 1, 2, \dots$) are the functions of r only. Now, using the power series expansion (4.1) in Eq. (3.10), we obtain

$$J = J_0 (1 + \beta_1 s + \beta_2 s^2 + \dots), \quad (4.2)$$

where

$$\begin{aligned} J_0 &= \int_0^1 \left(\frac{\gamma}{2} \Lambda^{(0)} (\Phi^{(0)})^2 + \frac{1}{\gamma-1} \Psi^{(0)} + \Omega^{(0)} \right) r dr, \\ \beta_1 J_0 &= \int_0^1 \left\{ \frac{\gamma}{2} \Lambda^{(1)} (\Phi^{(0)})^2 + \gamma \Lambda^{(0)} \Phi^{(0)} \Phi^{(1)} + \frac{1}{\gamma-1} \Psi^{(1)} + \Omega^{(1)} \right\} r dr, \\ \beta_2 J_0 &= \int_0^1 \left\{ \frac{\gamma}{2} \Lambda^{(2)} (\Phi^{(0)})^2 + \gamma \Lambda^{(0)} \Phi^{(0)} \Phi^{(2)} + \frac{1}{\gamma-1} \Psi^{(2)} + \Omega^{(2)} \right\} r dr \\ &\quad + \frac{\gamma}{2} \int_0^1 \left\{ \Lambda^{(0)} (\Phi^{(1)})^2 + 2\Lambda^{(1)} \Phi^{(1)} \Phi^{(0)} \right\} r dr, \\ &\dots \end{aligned} \quad (4.3)$$

Using (4.2) in (3.6), we get

$$s \left(\frac{\chi_0}{\chi} \right)^2 = J_0 \left\{ \left(1 - \frac{\gamma C_0}{4J_0} \right) + \left(\beta_1 - \frac{1}{2(\gamma-1)J_0} \right) s + \beta_2 s^2 + \dots \right\}. \quad (4.4)$$

In view of Eq. (3.1), Eq. (4.4) becomes

$$\begin{aligned} \left(\frac{a_0}{U} \right)^2 \left(\frac{\chi_0}{\chi} \right)^2 &= J_0 \left\{ \left(1 - \frac{\gamma C_0}{4J_0} \right) + \left(\beta_1 - \frac{1}{2(\gamma-1)J_0} \right) \right. \\ &\quad \left. \left(\frac{a_0}{U} \right)^2 + \beta_2 \left(\frac{a_0}{U} \right)^4 + \dots \right\}. \end{aligned} \quad (4.5)$$

Eq. (4.5) provides a relation between the position of the shock front χ and the velocity of the shock front U , if the quantities J_0 and β_i are known. Further, we expand $\bar{\lambda}$ by using Eqs. (3.9) and (4.2) as follows:

$$\bar{\lambda} = 2 \left[1 + \beta'_1 s + 4\beta'_2 s^2 + 6\beta'_3 s^3 \dots \right], \quad (4.6)$$

where

$$\begin{aligned} \beta'_1 &= \frac{\beta_1 - \frac{1}{2J_0(\gamma-1)}}{1 - \frac{\gamma C_0}{4J_0}}, \\ \beta'_2 &= \frac{\beta_2}{1 - \frac{\gamma C_0}{4J_0}}, \end{aligned} \quad (4.7)$$

$$\begin{aligned} \beta'_3 &= \frac{3\beta_3}{1 - \frac{\gamma C_0}{4J_0}} + \frac{\left(\beta_1 - \frac{1}{2J_0(\gamma-1)} \right) \beta_2}{\left(1 - \frac{\gamma C_0}{4J_0} \right)^2}, \\ &\dots \end{aligned}$$

For simplification, we use the expressions $\bar{\lambda}_1 = \beta'_1$, $\bar{\lambda}_2 = 4\beta'_2$, $\bar{\lambda}_3 = 6\beta'_3$, ..., then the Eq. (4.6) becomes

$$\bar{\lambda} = 2[1 + \bar{\lambda}_1 s + \bar{\lambda}_2 s^2 + \dots]. \quad (4.8)$$

To obtain the ODEs governing the functions $\Phi^{(n)}, \Psi^{(n)}, \Lambda^{(n)}$ and $\Omega^{(n)}$ ($n = 0, 1, 2, \dots$), we use the Eqs. (4.1) and (4.8) in Eq. (3.5) and compare the coefficients of like powers of s on both the sides. On comparing the terms free from s , we get

$$\begin{aligned} (\Phi^{(0)} - r)\Lambda_r^{(0)} + \Lambda^{(0)}\left(\Phi_r^{(0)} + \frac{\Phi^{(0)}}{r}\right) &= 0, \\ (\Phi^{(0)} - r)\Lambda^{(0)}\Phi_r^{(0)} + \frac{1}{\gamma}\left(\Psi_r^{(0)} + \Omega_r^{(0)} + \frac{2j\Omega^{(0)}}{r}\right) - \Lambda^{(0)}\Phi^{(0)} &= 0, \\ -2\Psi^{(0)} + (\Phi^{(0)} - r)\Psi_r^{(0)} + \gamma\Psi^{(0)}\left(\Phi_r^{(0)} + \frac{\Phi^{(0)}}{r}\right) &= 0, \\ -2\Omega^{(0)} + (\Phi^{(0)} - r)\Omega_r^{(0)} + 2\Omega^{(0)}\left(\Phi_r^{(0)} + \frac{(1-j)\Phi^{(0)}}{r}\right) &= 0. \end{aligned} \quad (4.9)$$

Now, comparing the coefficients of first power of s , we get

$$\begin{aligned} (\Phi^{(0)} - r)\Lambda_r^{(1)} + \Phi^{(1)}\Lambda_r^{(0)} + 2\Lambda^{(1)} + \Lambda^{(0)}\left(\Phi_r^{(1)} + \frac{\Phi^{(1)}}{r}\right) \\ + \Lambda^{(1)}\left(\Phi_r^{(0)} + \frac{\Phi^{(0)}}{r}\right) &= 0, \\ -\Lambda^{(0)}(\Phi^{(1)} + \bar{\lambda}_1\Phi^{(0)}) - \Lambda^{(1)}\Phi^{(0)} + \Lambda^{(0)}\{(\Phi^{(0)} - r)\Phi_r^{(1)} + \Phi^{(1)}\Phi_r^{(0)}\} \\ + \Lambda^{(1)}(\Phi^{(0)} - r)\Phi_r^{(0)} + 2\Lambda^{(0)}\Phi^{(1)} + \frac{1}{\gamma}\left(\Psi_r^{(1)} + \Omega_r^{(1)} + \frac{2j\Omega^{(1)}}{r}\right) &= 0, \\ -2(\Psi^{(1)} + \bar{\lambda}_1\Psi^{(0)}) + (\Phi^{(0)} - r)\Psi_r^{(1)} + \Phi^{(1)}\Psi_r^{(0)} + 2\Psi^{(1)} \\ + \gamma\Psi^{(0)}\left(\Phi_r^{(1)} + \frac{\Phi^{(1)}}{r}\right) + \gamma\Psi^{(1)}\left(\Phi_r^{(0)} + \frac{\Phi^{(0)}}{r}\right) &= 0, \\ -2\bar{\lambda}_1\Omega^{(0)} + (\Phi^{(0)} - r)\Omega_r^{(1)} + \Phi^{(1)}\Omega_r^{(0)} + 2\Omega^{(0)}\left(\Phi_r^{(1)} + \frac{(1-j)\Phi^{(1)}}{r}\right) \\ + 2\Omega^{(1)}\left(\Phi_r^{(0)} + \frac{(1-j)\Phi^{(0)}}{r}\right) &= 0. \end{aligned} \quad (4.10)$$

From Eqs. (3.8) and (4.1), we obtain

$$\begin{aligned} \Phi^{(0)}(1) &= \frac{2}{\gamma+1}, \quad \Psi^{(0)}(1) = \frac{2\gamma}{\gamma+1} - \frac{\gamma C_0}{2}\left(\frac{\gamma+1}{\gamma-1}\right)^2, \\ \Lambda^{(0)}(1) &= \frac{\gamma+1}{\gamma-1}, \quad \Omega^{(0)}(1) = \frac{\gamma C_0}{2}\left(\frac{\gamma+1}{\gamma-1}\right)^2, \end{aligned} \quad (4.11)$$

and

$$\begin{aligned} \Phi^{(1)}(1) &= -\frac{2}{\gamma+1}, \quad \Psi^{(1)}(1) = -\frac{\gamma(\gamma-1)}{\gamma+1} - \frac{2C_0\gamma(\gamma+1)^2}{(\gamma-1)^3}, \\ \Lambda^{(1)}(1) &= -\frac{2(\gamma+1)}{(\gamma-1)^2}, \quad \Omega^{(1)}(1) = 2\gamma C_0 \frac{(\gamma+1)^2}{(\gamma-1)^3}. \end{aligned} \quad (4.12)$$

To obtain the first-order approximation to the solutions of the problem, we solve the system of non-linear ODEs (4.9) together with the boundary conditions (4.11) at the shock, and determine the functions $\Phi^{(0)}, \Psi^{(0)}, \Lambda^{(0)}$ and $\Omega^{(0)}$. Hence, the first-order approximations to the solutions of the blast wave problem are obtained in the following form:

$$\begin{aligned} v &= U\Phi^{(0)}(r), \\ p &= p_0\left(\frac{U}{a_0}\right)^2 \Psi^{(0)}(r), \\ \rho &= \rho_0\Lambda^{(0)}(r), \\ h &= p_0\left(\frac{U}{a_0}\right)^2 \Omega^{(0)}(r). \end{aligned} \quad (4.13)$$

Obtaining the second-order approximations to the solutions of the problem is quite complicated. For this we put the first-order approximations $\Phi^{(0)}, \Psi^{(0)}, \Lambda^{(0)}$ and $\Omega^{(0)}$ in Eq. (4.10) to obtain the system of linear inhomogeneous differential equations. These equations contain an undetermined parameter $\bar{\lambda}_1$ which is related to β_1 . The solutions $\Phi^{(1)}, \Psi^{(1)}, \Lambda^{(1)}$ and $\Omega^{(1)}$ to be obtained include the parameter $\bar{\lambda}_1$ in a linear form. Substituting these solutions in Eq. (4.3)₂, we determine the value of $\bar{\lambda}_1$, and hence the second approximations to the solutions.

5 The first-order approximation

Re-writing Eq. (4.9) as follows:

$$\begin{aligned} \frac{\Lambda_r^{(0)}}{\Lambda^{(0)}} &= \frac{\left(\Phi_r^{(0)} + \frac{\Phi^{(0)}}{r}\right)}{(r - \Phi^{(0)})}, \\ (\Phi^{(0)} - r)\Lambda^{(0)}\Phi_r^{(0)} + \frac{1}{\gamma}\left(\Psi_r^{(0)} + \Omega_r^{(0)} + \frac{2j\Omega^{(0)}}{r}\right) - \Lambda^{(0)}\Phi^{(0)} &= 0, \\ \frac{\Psi_r^{(0)}}{\Psi^{(0)}} &= \frac{\left(\gamma\Phi_r^{(0)} + \frac{\gamma\Phi^{(0)}}{r} - 2\right)}{(r - \Phi^{(0)})}, \\ \frac{\Omega_r^{(0)}}{\Omega^{(0)}} &= \frac{\left(2\Phi_r^{(0)} + \frac{2(1-j)\Phi^{(0)}}{r} - 2\right)}{(r - \Phi^{(0)})}. \end{aligned} \quad (5.1)$$

Substituting Eqs. (5.1)₃ and (5.1)₄ in Eq. (5.1)₂, we obtain

$$\Phi_r^{(0)} = \frac{\left[\frac{2}{\gamma} - \frac{\Phi^{(0)}}{r} + \left\{ \frac{2}{\gamma} - \frac{2(1-j)\Phi^{(0)}}{\gamma r} \right\} \frac{\Omega^{(0)}}{\Psi^{(0)}} + \frac{(r-\Phi^{(0)})\Phi^{(0)}\Lambda^{(0)}}{\Psi^{(0)}} - \frac{2j(r-\Phi^{(0)})\Omega^{(0)}}{\gamma r} \frac{\Omega^{(0)}}{\Psi^{(0)}} \right]}{\left[\frac{2\Omega^{(0)}}{\gamma\Psi^{(0)}} + 1 - \frac{\Lambda^{(0)}(r-\Phi^{(0)})^2}{\Psi^{(0)}} \right]}. \quad (5.2)$$

From Eqs. (4.11) and (5.2), we have

$$\Psi^{(0)}(r) = \left\{ \frac{2\gamma}{\gamma+1} - \frac{\gamma C_0}{2} \left(\frac{\gamma+1}{\gamma-1} \right)^2 \right\} \left[\frac{\gamma}{\gamma+1-r^{m-1}} \right]^{\left[\frac{(m+1)\gamma}{(m-1)} \right]}, \quad (5.7)$$

$$\Omega^{(0)}(r) = \frac{\gamma C_0}{2} \left(\frac{\gamma+1}{\gamma-1} \right)^2 \left[\frac{\gamma}{\gamma+1-r^{m-1}} \right]^{\left[\frac{2(2-j-\gamma)+2(m+1-j)(\gamma-1)}{(m-1)(\gamma-1)} \right]} r^{\frac{2(2-j-\gamma)}{\gamma-1}}.$$

$$\Phi_r^{(0)}(1) = \frac{\left[\frac{2}{\gamma(\gamma+1)} + \left\{ \frac{2(\gamma+1)-4(1-j)}{4\gamma(\gamma-1)^2-\gamma C_0(\gamma+1)^3} \right\} C_0(\gamma+1)^2 + \frac{4(\gamma-1)^2}{4\gamma(\gamma-1)^2-\gamma C_0(\gamma+1)^3} - \frac{2C_0j(\gamma-1)(\gamma+1)^2}{4\gamma(\gamma-1)^2-\gamma C_0(\gamma+1)^3} \right]}{\left[\frac{2C_0(\gamma+1)^3}{4\gamma(\gamma-1)^2-\gamma C_0(\gamma+1)^3} + 1 - \frac{2(\gamma-1)^3}{4\gamma(\gamma-1)^2-\gamma C_0(\gamma+1)^3} \right]}. \quad (5.3)$$

Now, we construct the first approximation $\Phi^{(0)}$ following the work of Taylor [3]. That is, the solution of $\Phi^{(0)}$ is supposed to be of the following form:

$$\Phi^{(0)}(r) = \frac{r}{\gamma} + Ar^m, \quad (5.4)$$

where A and m are constants. Using Eqs. (5.4) and (4.11), we obtain the value of A as follows:

$$A = \frac{\gamma-1}{\gamma(\gamma+1)}. \quad (5.5)$$

Using Eqs. (5.3), (5.4) and (5.5) we determine the value of m as

$$m = \left[\frac{\left[\frac{2}{\gamma(\gamma+1)} + \left\{ \frac{2(\gamma+1)-4(1-j)}{4(\gamma-1)^2-\gamma C_0(\gamma+1)^3} \right\} C_0(\gamma+1)^2 + \frac{4(\gamma-1)^2}{4(\gamma-1)^2-\gamma C_0(\gamma+1)^3} - \frac{2C_0j(\gamma-1)(\gamma+1)^2}{4(\gamma-1)^2-\gamma C_0(\gamma+1)^3} \right]}{\left\{ \frac{2C_0(\gamma+1)^3}{4\gamma(\gamma-1)^2-\gamma C_0(\gamma+1)^3} + 1 - \frac{2(\gamma-1)^3}{4\gamma(\gamma-1)^2-\gamma C_0(\gamma+1)^3} \right\}} - 1 \right] \left(\frac{\gamma+1}{\gamma-1} \right). \quad (5.6)$$

Substituting the Eqs. (5.4), (5.5) and (5.6) into the Eqs. (5.1)₁; (5.1)₃; (5.1)₄ Eqs. (5.1)₁, (5.1)₃, (5.1)₄ and after integrating it together with boundary conditions (4.11), we obtain the following expressions for $\Lambda^{(0)}$, $\Psi^{(0)}$ and $\Omega^{(0)}$, respectively,

$$\Lambda^{(0)}(r) = \left(\frac{\gamma+1}{\gamma-1} \right) \left[\frac{\gamma}{\gamma+1-r^{m-1}} \right]^{\left[\frac{2}{(m-1)(\gamma-1)} + \frac{m+1}{m-1} \right]} r^{\frac{2}{\gamma-1}},$$

6 Results and discussion

For the first-order approximation, the profiles of the flow variables such as density $\Lambda^{(0)}$, velocity $\Phi^{(0)}$, pressure $\Psi^{(0)}$ and magnetic pressure $\Omega^{(0)}$ are plotted using the Eqs. (5.4) and (5.7), which are shown graphically in Figures 1–3. We have calculated numerical values of the functions $\Phi^{(0)}$, $\Lambda^{(0)}$, $\Psi^{(0)}$ and $\Omega^{(0)}$ from the approximate analytic solutions (5.4) and (5.7) which are depicted in Tables 1 and 2, which also exhibit numerical values of $\Phi^{(0)}$, $\Lambda^{(0)}$, $\Psi^{(0)}$ and $\Omega^{(0)}$ evaluated by integrating Eq. (5.1) numerically along with boundary conditions (4.11) by Runge–Kutta

method of fourth-order (RK4 method). For calculations we have taken the values of constant parameters as $C_0 = 0, 0.02, 0.04$, $j = 0, 1$ and $\gamma = 1.4$. It may be noted that the effects of magnetic field enter through the parameter C_0 , and the value $C_0 = 0$ corresponds to the nonmagnetic case. From Tables 1 and 2, it is found that the obtained results are in good agreement with those obtained by Runge–Kutta method of fourth-order. Also, these results recover

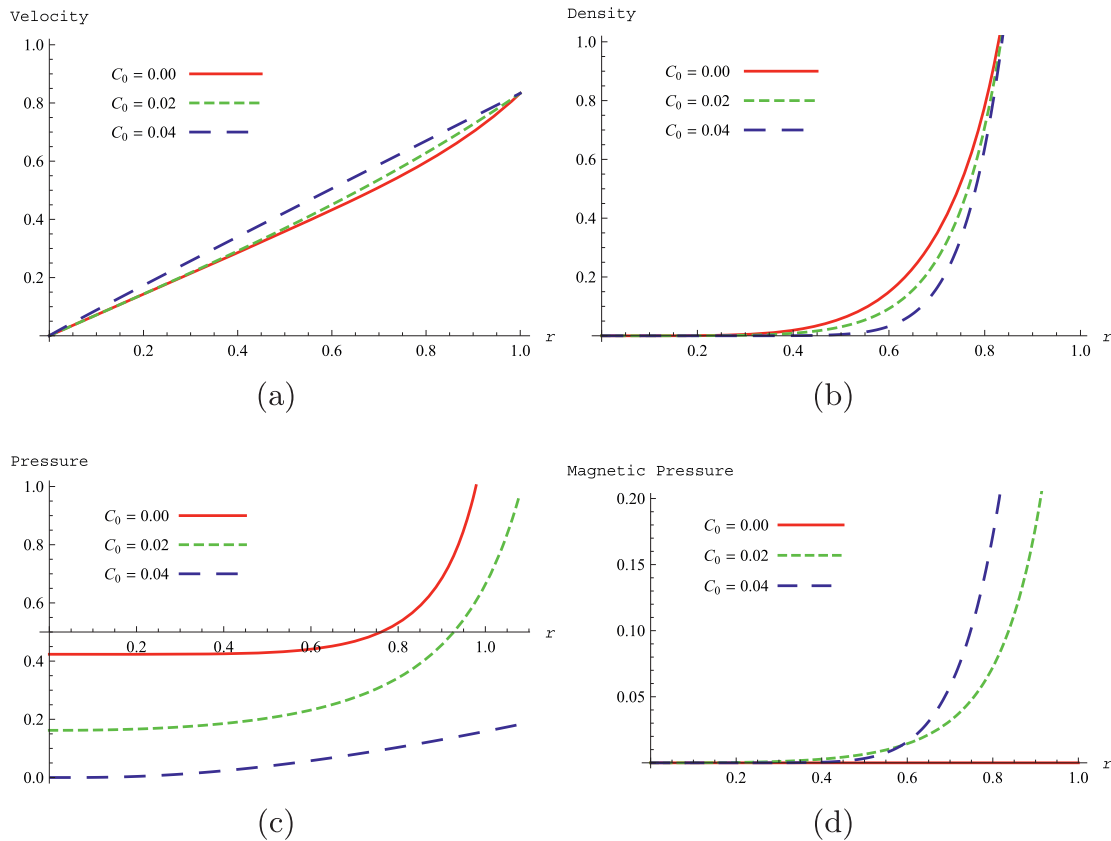


Figure 1: Flow profiles of (a) velocity $\Phi^{(0)}$, (b) density $\Lambda^{(0)}$, (c) pressure $\Psi^{(0)}$ and (d) magnetic pressure $\Omega^{(0)}$ for different values of C_0 , $j=0$ and $\gamma=1.4$.

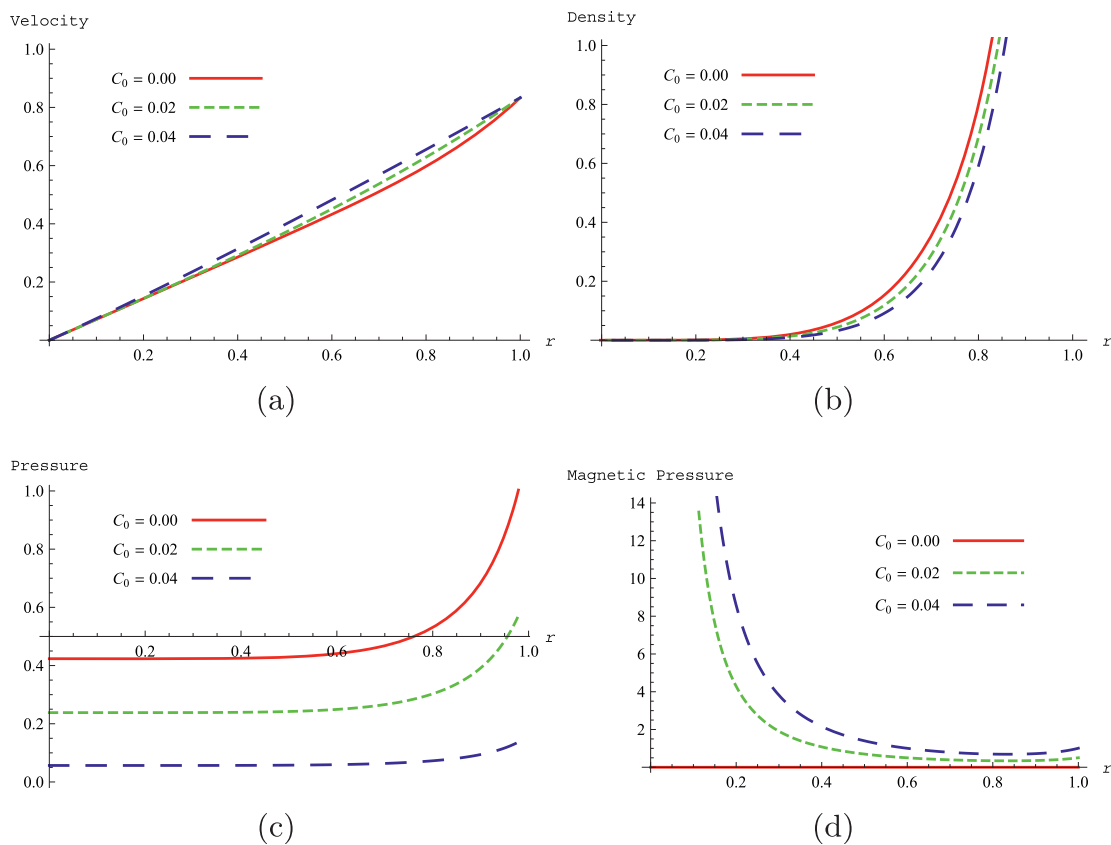


Figure 2: Flow profiles of (a) velocity $\Phi^{(0)}$, (b) density $\Lambda^{(0)}$, (c) pressure $\Psi^{(0)}$ and (d) magnetic pressure $\Omega^{(0)}$ for different values of C_0 , $j=1$ and $\gamma=1.4$.

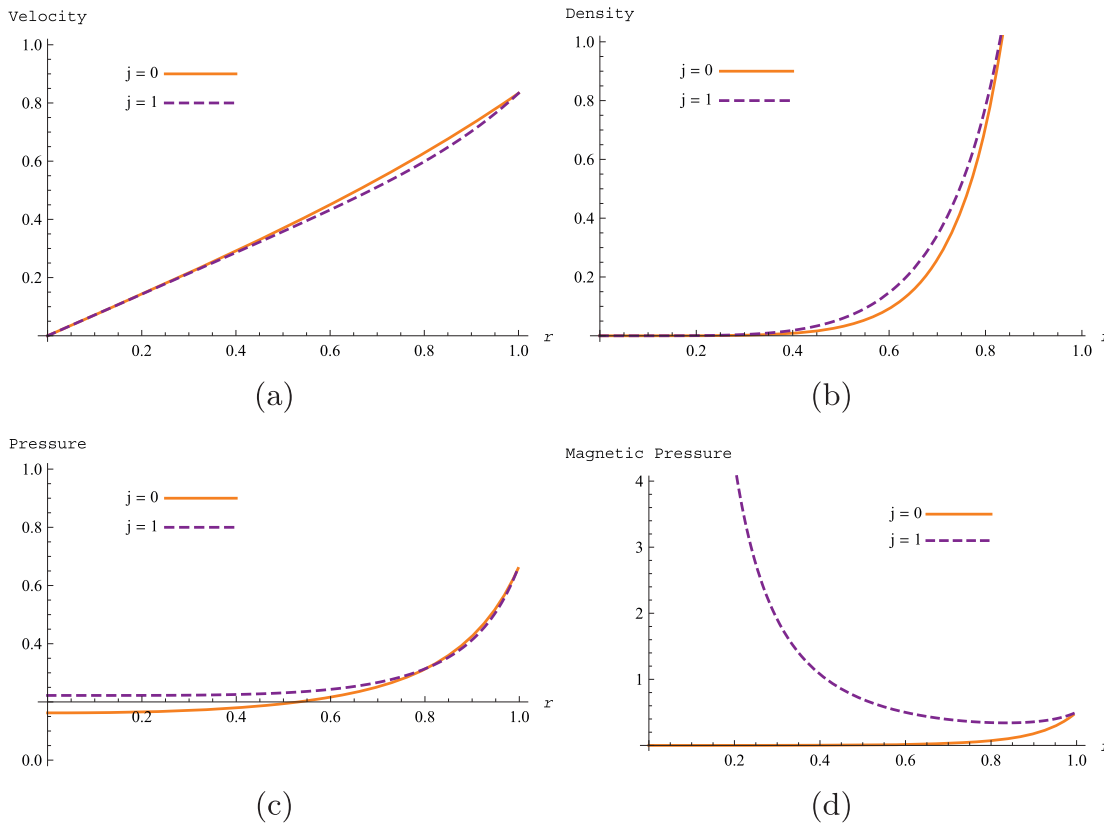


Figure 3: Flow profiles of (a) velocity $\Phi^{(0)}$, (b) density $\Lambda^{(0)}$, (c) pressure $\Psi^{(0)}$ and (d) magnetic pressure $\Omega^{(0)}$ for $C_0 = 0.02$, $\gamma = 1.4$ and $j = 0, 1$.

Sakurai's results [1] very well in the absence of the magnetic field.

Figures 1(a)–(c), 2(a)–(c) and 3(a)–(c) show that behind the shock front the velocity, density and pressure decrease monotonically, while Figures 1(d), 2(d) and 3(d) show that the magnetic pressure decreases for axial magnetic field ($j = 0$) and increases for azimuthal magnetic field ($j = 1$) monotonically as we move toward the axis of symmetry from the shock front. Also, from Figures 1 and 2, we observe the effect of C_0 on the profiles of the flow variables for $\gamma = 1.4$, $j = 0$ and $j = 1$. As C_0 increases, the density and pressure decrease (see Figures 1(b) and (c), 2(b) and (c)), while the velocity and magnetic pressure increase (see Figures 1(a) and (d), 2(a) and (d)) behind the shock. It is much expected result as the charged particles will be transported away very quickly from the shock front with an increase in C_0 . It results in the dropping of the density of the particles after the blast. Further, the decrement in the pressure with the increased value of C_0 enables the gas particles to move more freely, which causes to increase the velocity of the flow behind the shock. Figure 3 shows the behavior of the flow variables in the presence of axial magnetic field ($j = 0$) and azimuthal magnetic field ($j = 1$) for $C_0 = 0.02$ and $\gamma = 1.4$. From Figures 3(b)–(d), it is

obtained that under the effect of azimuthal magnetic field ($j = 1$) the strength of the density, pressure and magnetic pressure is greater than that of axial magnetic field ($j = 0$). Figure 3(a) shows the decay of the velocity in the presence of azimuthal magnetic field ($j = 1$) in comparison with axial magnetic field ($j = 0$) behind the shock.

7 Conclusion

In the present work, we have studied the propagation of cylindrical shock waves produced due to strong explosion in an ideal magnetogasdynamics using power series method [1]. The first-order approximate analytic solutions for the flow variables such as velocity, density, pressure and magnetic pressure are obtained, and shown graphically to elucidate the effects of varying shock Cowling number C_0 . To verify our results, we have compared the obtained numerical values of $\Phi^{(0)}$, $\Lambda^{(0)}$, $\Psi^{(0)}$ and $\Omega^{(0)}$ with the Sakurai's results [1] as well as with the results obtained by Runge–Kutta method of fourth-order, which are listed in Tables 1 and 2. Our results recover the results of Sakurai's published work in the absence of magnetic field, and match well with the results obtained by Runge–Kutta

Table 1: Numerical values of $\Phi^{(0)}$, $\Lambda^{(0)}$, $\psi^{(0)}$ and $\Omega^{(0)}$ for different values of C_0 , $\gamma = 1.4$ and $j = 0$.

r	C_0	$\Phi^{(0)}$ Computed	$\Lambda^{(0)}$ Computed	$\psi^{(0)}$ Computed	$\Omega^{(0)}$ Computed	$\Phi^{(0)}$ Sakurai [1]	$\Lambda^{(0)}$ Sakurai [1]	$\psi^{(0)}$ Sakurai [1]	$\Phi^{(0)}$ RK4 method	$\Lambda^{(0)}$ RK4 method	$\psi^{(0)}$ RK4 method	$\Omega^{(0)}$ RK4 method
1	0	0.8333	6.000	1.167	0	0.8333	6.000	1.167	0.8333	6.000	1.167	0
	0.02	0.8333	6.000	0.663	0.504	—	—	—	0.8333	6.000	0.663	0.504
	0.04	0.8333	6.000	0.159	1.008	—	—	—	0.8333	6.000	0.159	1.008
	0	0.7008	1.898	0.685	0	0.7008	1.898	0.685	0.6980	1.884	0.673	0
0.9	0.02	0.7269	1.751	0.456	0.179	—	—	—	0.7354	1.809	0.485	0.1938
	0.04	0.7517	1.632	0.130	0.441	—	—	—	0.7680	1.778	0.150	0.529
	0	0.5973	0.783	0.531	0	0.5973	0.783	0.531	0.5934	0.786	0.522	0
	0.02	0.6283	0.710	0.342	0.072	—	—	—	0.6626	0.744	0.435	0.094
0.8	0.04	0.6699	0.639	0.104	0.172	—	—	—	0.7320	0.642	0.199	0.330
	0	0.5104	0.347	0.468	0	0.5104	0.347	0.468	0.5076	0.356	0.466	0
	0.02	0.5366	0.259	0.275	0.032	—	—	—	0.5978	0.218	0.440	0.045
	0.04	0.5879	0.162	0.080	0.058	—	—	—	0.5323	—	0.070	—
0.6	0	0.4322	0.149	0.441	0	0.4322	0.153	0.441	0.4308	0.157	0.445	0
	0.02	0.4505	0.092	0.232	0.014	—	—	—	0.5538	0.028	0.463	0.014
	0.04	0.5057	0.032	0.058	0.016	—	—	—	0.4554	—	0.009	—
	0	0.3582	0.058	0.429	0	0.3582	0.058	0.429	0.3577	0.061	0.438	0
0.5	0.02	0.3691	0.030	0.204	0.007	—	—	—	0.4585	—	0.251	0.002
	0.04	0.4232	0.005	0.039	0.003	—	—	—	—	—	—	—
	0	0.2859	0.019	0.425	0	0.2859	0.019	0.425	0.2858	0.020	0.436	0
	0.02	0.2915	0.009	0.186	0.003	—	—	—	0.4091	—	—	—
0.4	0.04	0.3404	0.000	0.023	0.001	—	—	—	—	—	—	—
	0	0.2143	0.005	0.424	0	0.2143	0.005	0.424	0.2143	0.005	0.435	0
	0.02	0.2165	0.002	0.174	0.001	—	—	—	0.3121	—	—	—
	0.04	0.2571	0.000	0.011	0.000	—	—	—	—	—	—	—
0.2	0	0.1429	0.001	0.424	0	0.1429	0.001	0.424	0.1429	0.001	0.435	0
	0.02	0.1434	0.000	0.167	0.000	—	—	—	0.2181	—	—	—
	0.04	0.1732	0.000	0.004	0.000	—	—	—	—	—	—	—
	0	0.0714	0.000	0.424	0	0.0714	0.000	0.424	0.0714	0.000	0.435	0
0.1	0.02	0.0715	0.000	0.163	0.000	—	—	—	0.1351	—	—	—
	0.04	0.0882	0.000	0.000	0.000	—	—	—	—	—	—	—

Table 2: Numerical values of $\Phi^{(0)}$, $\Lambda^{(0)}$, $\psi^{(0)}$ and $\Omega^{(0)}$ for different values of C_0 , $\gamma = 1.4$ and $j = 1$.

r	C_0	$\Phi^{(0)}$ Computed	$\Lambda^{(0)}$ Computed	$\psi^{(0)}$ Computed	$\Omega^{(0)}$ Computed	$\Phi^{(0)}$ Sakurai [1]	$\Lambda^{(0)}$ Sakurai [1]	$\psi^{(0)}$ Sakurai [1]	$\Phi^{(0)}$ RK4 method	$\Lambda^{(0)}$ RK4 method	$\psi^{(0)}$ RK4 method	$\Omega^{(0)}$ RK4 method
1	0	0.8333	6.000	1.167	0	0.8333	6.000	1.167	0.8333	6.000	1.167	0
	0.02	0.8333	6.000	0.663	0.504	—	—	—	0.8333	6.000	0.663	0.504
	0.04	0.8333	6.000	0.159	1.008	—	—	—	0.8333	6.000	0.159	1.008
0.9	0	0.7008	1.898	0.685	0	0.7008	1.898	0.685	0.6980	1.884	0.673	0
	0.02	0.7021	1.871	0.392	0.357	—	—	—	0.7041	1.862	0.397	0.365
	0.04	0.7030	1.853	0.094	0.722	—	—	—	0.7074	1.818	0.097	0.755
0.8	0	0.5973	0.783	0.531	0	0.5973	0.783	0.531	0.5934	0.786	0.522	0
	0.02	0.5986	0.779	0.303	0.345	—	—	—	0.6074	0.791	0.319	0.378
	0.04	0.5995	0.776	0.073	0.696	—	—	—	0.6132	0.789	0.079	0.803
0.7	0	0.5104	0.347	0.468	0	0.5104	0.347	0.468	0.5076	0.356	0.466	0
	0.02	0.5112	0.342	0.265	0.393	—	—	—	0.5266	0.338	0.288	0.466
	0.04	0.5118	0.339	0.064	0.791	—	—	—	0.5323	0.326	0.070	0.995
0.6	0	0.4322	0.149	0.441	0	0.4322	0.153	0.441	0.4308	0.157	0.445	0
	0.02	0.4326	0.147	0.249	0.499	—	—	—	0.4513	0.133	0.269	0.632
	0.04	0.4329	0.145	0.060	1.003	—	—	—	0.4554	0.122	0.064	1.340
0.5	0	0.3582	0.058	0.429	0	0.3582	0.058	0.429	0.3577	0.061	0.438	0
	0.02	0.3584	0.057	0.242	0.698	—	—	—	0.3770	0.044	0.251	0.725
	0.04	0.3585	0.056	0.058	1.398	—	—	—	0.3795	0.039	0.058	1.930
0.4	0	0.2859	0.019	0.425	0	0.2859	0.019	0.425	0.2858	0.020	0.436	0
	0.02	0.2860	0.018	0.239	1.077	—	—	—	0.3025	0.011	0.229	1.472
	0.04	0.2861	0.018	0.057	2.156	—	—	—	0.3037	0.010	0.052	2.321
0.3	0	0.2143	0.005	0.424	0	0.2143	0.005	0.424	0.2143	0.005	0.435	0
	0.02	0.2143	0.004	0.239	1.907	—	—	—	0.2274	0.002	0.201	1.655
	0.04	0.2143	0.004	0.057	3.815	—	—	—	0.2279	0.002	0.045	3.381
0.2	0	0.1429	0.001	0.424	0	0.1429	0.001	0.424	0.1429	0.001	0.435	0
	0.02	0.1429	0.001	0.238	4.288	—	—	—	0.1581	0.001	0.164	4.030
	0.04	0.1429	0.001	0.057	8.576	—	—	—	0.1519	0.000	0.036	8.120
0.1	0	0.0714	0.000	0.424	0	0.0714	0.000	0.424	0.0714	0.000	0.435	0
	0.02	0.0714	0.000	0.238	17.15	—	—	—	0.0759	0.000	0.115	17.23
	0.04	0.0714	0.000	0.057	34.30	—	—	—	0.0759	0.000	0.025	34.85

method of fourth-order. The considered gas dynamical model for cylindrical geometry under the effects of axial or azimuthal magnetic field in an ideal medium might be fruitful for the study of experiments on pinch effect, exploding wires and so forth. Study of cylindrical shock waves is not only associated with the explosion of a long thin wire but also to some axially symmetric hypersonic flow problems such as the shock envelope behind a fast meteor or missile. From the present study, the following can be concluded:

- (i) Figures 1–3 depict that the flow variables such as the velocity, density and pressure decrease monotonically, while the magnetic pressure decreases for axial magnetic field ($j = 0$) and increases for azimuthal magnetic field ($j = 1$) as we move toward the axis of symmetry from the shock front.
- (ii) For axial magnetic field ($j = 0$) or azimuthal magnetic field ($j = 1$), increase in the parameter C_0 causes density and pressure to decrease, and velocity and magnetic pressure to increase behind the shock (see Figures 1 and 2). It happens because the charged gas particles are carried away very quickly from the shock front with the increase in the value of C_0 . Hence, this enables the gas particles to move more freely, which causes an increase in the velocity of the flow behind the shock.
- (iii) Figure 3 shows the behavior of the flow variables in the presence of axial magnetic field ($j = 0$) and azimuthal magnetic field ($j = 1$) for the fixed value of C_0 . From Figure 3, it is observed that under the effect of azimuthal magnetic field ($j = 1$) the strength of the density, pressure and magnetic pressure is greater than that of axial magnetic field ($j = 0$), whereas the velocity decreases in the presence of azimuthal magnetic field ($j = 1$) in comparison with axial magnetic field ($j = 0$) behind the shock.

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