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Gap solitons supported by an optical lattice in biased photorefractive crystals having both the linear and quadratic electro-optic effect

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Abstract: We investigate the existence and stability of gap solitons supported by an optical lattice in biased photorefractive (PR) crystals having both the linear and quadratic electro-optic effect. Such PR crystals have an interesting interplay between the linear and quadratic nonlinearities. Gap solitons are predicted for the first time in such novel PR media. Taking a relevant example (PMN-0.33PT), we find that the gap solitons in the first finite bandgap are single humped, positive and symmetric solitons while those in the second finite band gap are anti-symmetric and double humped. The power of the gap soliton depends upon the value of the axial propagation constant. We delineate three power regimes and study the gap soliton profiles in each region. The gap solitons in the first finite band gap are not linearly stable while those in the second finite band gap are found to be stable against small perturbations. We study their stability properties in detail throughout the finite band gaps. The interplay between the linear and quadratic electro-optic effect is studied by investigating the spatial profiles and stability of the gap solitons for different ratios of the linear and quadratic nonlinear coefficients.

Keywords: gap solitons; optical lattice; photorefractive materials.

1 Introduction

Optical spatial solitons manifesting in photorefractive (PR) materials exhibit a rich diversity and have been quite attractive for research [1–5]. PR materials exhibit a

change in the refractive index with increasing intensity. Inhomogeneous illumination results in photo generated charge carriers which then drift or diffuse to set up a space charge field within the PR crystal. The self-trapping occurs because of the creation of an index waveguide resulting in dynamic equilibrium of the propagating light beam. PR materials present an ideal medium for studying optical spatial solitons as they have a saturable nonlinearity and the solitons can be formed at low laser powers. Depending upon the type of PR crystal used, many different types of optical spatial solitons have been investigated till now. Screening solitons have been observed in PR materials where the external electric field modulates the drift of the charge carriers [6–9]. Photovoltaic solitons have been found to manifest in PR materials with a finite photovoltaic coefficient [10, 11]. The bulk photovoltaic field modulates the drift of the charge carriers in such media instead of the external electric field. Screening photovoltaic solitons result from the interplay between the external electric field and the photovoltaic field [12, 13]. In addition, PR solitons have been observed in centrosymmetric PR crystals [14] where the Kerr nonlinearity predominates while another interesting discovery is that of optical spatial solitons in novel PR crystals where both the Kerr and Pockel nonlinearities are present simultaneously near the phase transition temperature [15]. There is an interaction between the linear and quadratic electro-optic effect in this case which leads to many variations in the self-trapping mechanism.

Photonic crystals or nonlinear optical crystals having a periodically modulated refractive index have also been studied for existence of optical spatial solitons. The periodic refractive index leads to a band structure which results in a photonic band gap analogous to the energy band gap in solids. Light beams having frequency within the photonic band gap cannot traverse through the photonic crystal. It is here that we can understand the significance of gap solitons. Gap solitons exist as defect nonlinear modes inside the photonic band gap of the Floquet Bloch lattice spectrum. Hence, nonlinearity is essential for the observation of gap solitons. Photonic lattice may be prefabricated or induced optically. Optical

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induction of the periodic lattice pattern is preferable as it is reversible. It can be done in two ways, by interference of two coherent optical beams inside the PR crystal, or using an amplitude modulation mask along with spatially coherent light.

The study of gap solitons supported by photonic lattices has been studied extensively in recent times [16–23], particularly in different types of PR crystals [24–31]. The aforementioned investigations have been performed considering various types of optical lattices in diverse configurations of nonlinear optical crystals, including non-centrosymmetric PR crystals. Recently, Zhan and Hou have studied the existence and properties of gap solitons supported by optical lattices in biased centrosymmetric PR crystals [32].

In this paper, we shall investigate theoretically, the existence and characteristics of gap solitons due to an embedded optical lattice in PR crystals exhibiting the linear and quadratic electro-optic effect simultaneously. We shall first discuss the band gap structure of the corresponding linear system and then investigate the existence of gap solitons in the first and second finite band gap. The stability of the gap solitons will also be studied in detail by the perturbation theory in addition to the Vakhitov Kolokolov (VK) criterion. A relevant crystal having both the linear and quadratic electro-optic effect will be considered (0.67PMN-0.33PT) to illustrate our results.

2 Theoretical analysis

2.1 Foundation

We shall consider a PR crystal having both the linear and quadratic nonlinearity with an optical lattice imprinted upon it in the transverse direction. Let an optical beam propagate along the longitudinal, or z -direction. It is allowed to diffract only in the x -direction. The c -axis of the crystal is aligned along the x -direction and the optical beam is linearly polarized along the same direction. The electric field of the incident beam is expressed as, $\vec{E} = \hat{x}A(x, z)\exp(ikz)$ where $k = k_0n_e = (2\pi/\lambda_0)n_e$. n_e is the unperturbed index of refraction and λ_0 is the free space wavelength. Under the above conditions, the optical beam is governed by the equation [32],

$$\left[i \frac{\partial}{\partial z} + \frac{1}{2k} \frac{\partial^2}{\partial x^2} + \frac{k}{n_e} (\Delta n_{PR} + \Delta n_G) \right] A(x, z) = 0 \quad (1)$$

where,

$\Delta n_{PR} = -\frac{1}{2} [n_e^2 r_{eff} E_{sc} + n_e^2 g_{eff} \epsilon_0^2 (\epsilon_r - 1)^2 E_{sc}^2]$ is the change in refractive index due to the PR effect and Δn_G represents the lattice pattern. E_{sc} is the space charge field induced in the PR crystal due to the beam, g_{eff} is the quadratic electro-optic coefficient, r_{eff} is the linear electro-optic coefficient, ϵ_0 is the vacuum dielectric constant, ϵ_r is the relative dielectric constant. For broad optical beams and relatively large external bias fields, the space charge field can be written as [6],

$$E_{sc} = E_0 \frac{I_\infty + I_d}{I + I_d} \quad (2)$$

where I_d is the dark irradiance and $I = (n_e/2\eta_0)|A|^2$ is the total intensity of the beam.

Using the dimensionless co-ordinates, $s = x/x_0$, $\xi = z/kx_0^2$, $A = (2\eta_0 I_d/n_e)^{1/2} U$, the envelope U now satisfies the following dynamical evolution equation,

$$i \frac{\partial U}{\partial \xi} + \frac{1}{2} \frac{\partial^2 U}{\partial s^2} + pR(s)U - \beta_1 \frac{U}{(1 + |U|^2)} - \beta_2 \frac{U}{(1 + |U|^2)^2} = 0 \quad (3)$$

where,

$\beta_1 = (k_0 x_0)^2 n_e^4 r_{eff} E_0/2$, $\beta_2 = (k_0 x_0)^2 n_e^4 g_{eff} \epsilon_0^2 (\epsilon_r - 1)^2 E_0^2/2$. p is the scaling of the lattice depth and $R(s) = \cos(2\pi s/T)$ is the lattice pattern with the modulation period T . We can clearly infer many conserved quantities admitted by Eq. (3). One of them is the energy flow per unit time, or simply the power $P = \int_{-\infty}^{\infty} U U^* dx$.

Now, we shall look for gap soliton solutions to Eq. (3) in the form $U(s, \mu) = w(s, \mu)\exp(i\mu\xi)$ where $w(s, \mu)$ is the real transverse wave profile and μ is the axial propagation constant. Substituting this ansatz in Eq. (3) gives us,

$$\frac{1}{2} \frac{\partial^2 w}{\partial s^2} + pR(s)w - \beta_1 \frac{w}{(1 + w^2)} - \beta_2 \frac{w}{(1 + w^2)^2} - \mu w = 0 \quad (4)$$

In the limit of small power, there exist soliton solutions which are linear modes and can be calculated numerically. So, we shall first study the linear properties of the optical lattice, through the linearized Eq. (4). The optical lattice is characterized by a modulation period T , lattice depth p , and the total power P . A well-known consequence of wave propagation in periodic media is the formation of multiple forbidden band gaps in the transmission spectrum. We take $p = 5$, $T = 2$, and vary the value of the propagation constant μ to derive the band structure of the optical lattice by solving the differential eigenvalue problem numerically. Taking into account the Floquet Bloch theory, linearized Eq. (4) admits periodic solutions with the same period T . The dispersion relation contains

an infinite number of branches in the first Brillouin zone. We get Bloch wave solutions for Eq. (3) in each branch while the band gap between adjoining branches ensures no periodic solutions exist there. One semi-infinite gap along with infinite no. of finite gaps constitutes the Bloch band spectrum. For illustration, we consider the crystal 0.67PMN-0.33PT's parameters [15], $n_e = 2.562$, $x_0 = 20 \mu\text{m}$, $\lambda_0 = 632.8 \text{ nm}$, $r_{\text{eff}} = 182 \times 10^{-12} \text{ m/V}$, $g_{\text{eff}} = 1.38 \times 10^{-16} \text{ m}^2/\text{V}^2$, bias $V = 1000 \text{ V}$, crystal width $W = 1 \text{ cm}$. With these parameters, we get, $\beta_1 = 15.46$ and $\beta_2 = 1.17$. Now, let us consider a general case of $T = 2$. Figure 1 shows the bandgap structure of the periodic lattice as a function of the scaled lattice depth p . Figure 2 shows the bandgap structure in terms of the propagation constant at $p = 5$. The semi-infinite gap extends starting from $\mu \geq 1.91$, the first finite band gap is between $-2.945 \leq \mu \leq 1.761$, and the second finite band gap is $-6.410 \leq \mu \leq -4.520$. The presence of nonlinear terms in Eqs. (3) and (4) is the cause of existence of solitary wave solutions inside these band gaps and hence these are known as gap solitons.

2.2 Gap soliton solutions

We shall solve Eq. (4) numerically for soliton solutions in both the finite band gaps. For a soliton solution in the first finite band gap, we shall consider the three cases of high, moderate and low power. We find that the power of the solitons increases with an increase in the propagation constant. The spatial profiles for high, moderate, and low power, i.e., $\mu = 1, -0.4, -2.7$ are shown in Figure 3a–c respectively. We infer that the solitons are single humped and symmetric with different propagation constants.

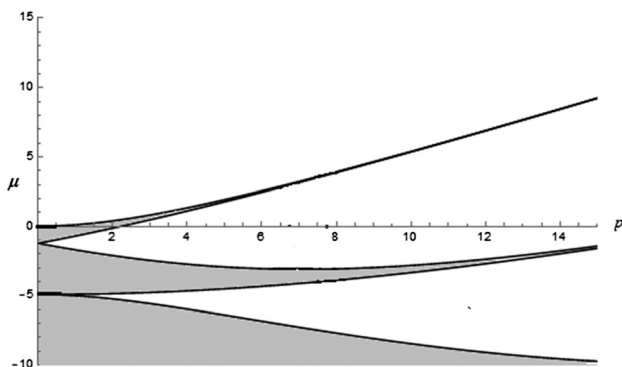


Figure 1: Bandgap structure of the periodic lattice as a function of the lattice depth p (the shaded regions show the allowed frequencies while the non-shaded regions show the photonic band gap).

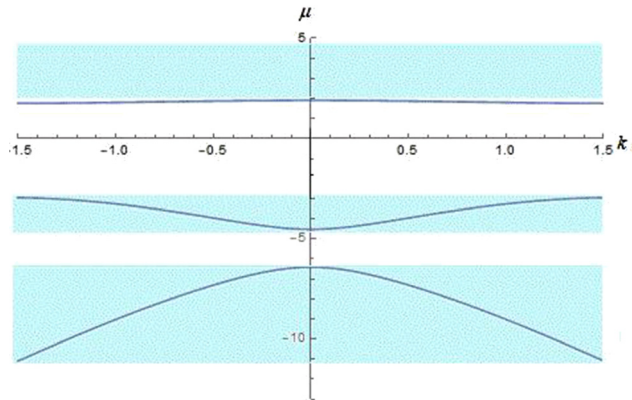


Figure 2: Bandgap structure of the periodic lattice in terms of the propagation constant (the shaded regions show the allowed frequencies while the non-shaded regions show the photonic band gap).

In the second finite band gap, we solve for gap soliton solutions and find that the solitons are now multipole (double humped) and antisymmetric. As in the case of gap solitons in the first finite band gap, the powers of these solitons increase with an increase in the propagation constant. The spatial profiles for high, moderate and low power, i.e., $\mu = -4.9, -5.7, -6.2$ are shown in Figure 4a–c respectively.

2.3 Stability of gap solitons

The stability of gap solitons is an issue worth considering since stable solitons can be observed easily through experiments and have potential applications. We shall study the stability of these gap solitons by the VK stability criterion. The VK stability criterion proposes that the gap solitons formed in the bandgaps of lattices with a uniform nonlinearity are stable if $dP/d\mu > 0$. VK stability criterion is a necessary but not sufficient condition for stability of solitons governed by Nonlinear Schrodinger (NLS) or the modified NLS equation. We shall first study stability by the VK criterion. Figure 5a, b show the dependence of power of the gap solitons on the propagation constant μ for the first and second finite band gaps. We can see that the slope $\partial P/\partial \mu > 0$. Hence these families of gap solitons are VK stable in their existence region. Since the VK stability criterion is a necessary but not sufficient condition for stability, we shall further assess the stability of the gap solitons by the perturbation theory. Small perturbations $u(s)$ and $v(s)$ in the steady state solution are considered,

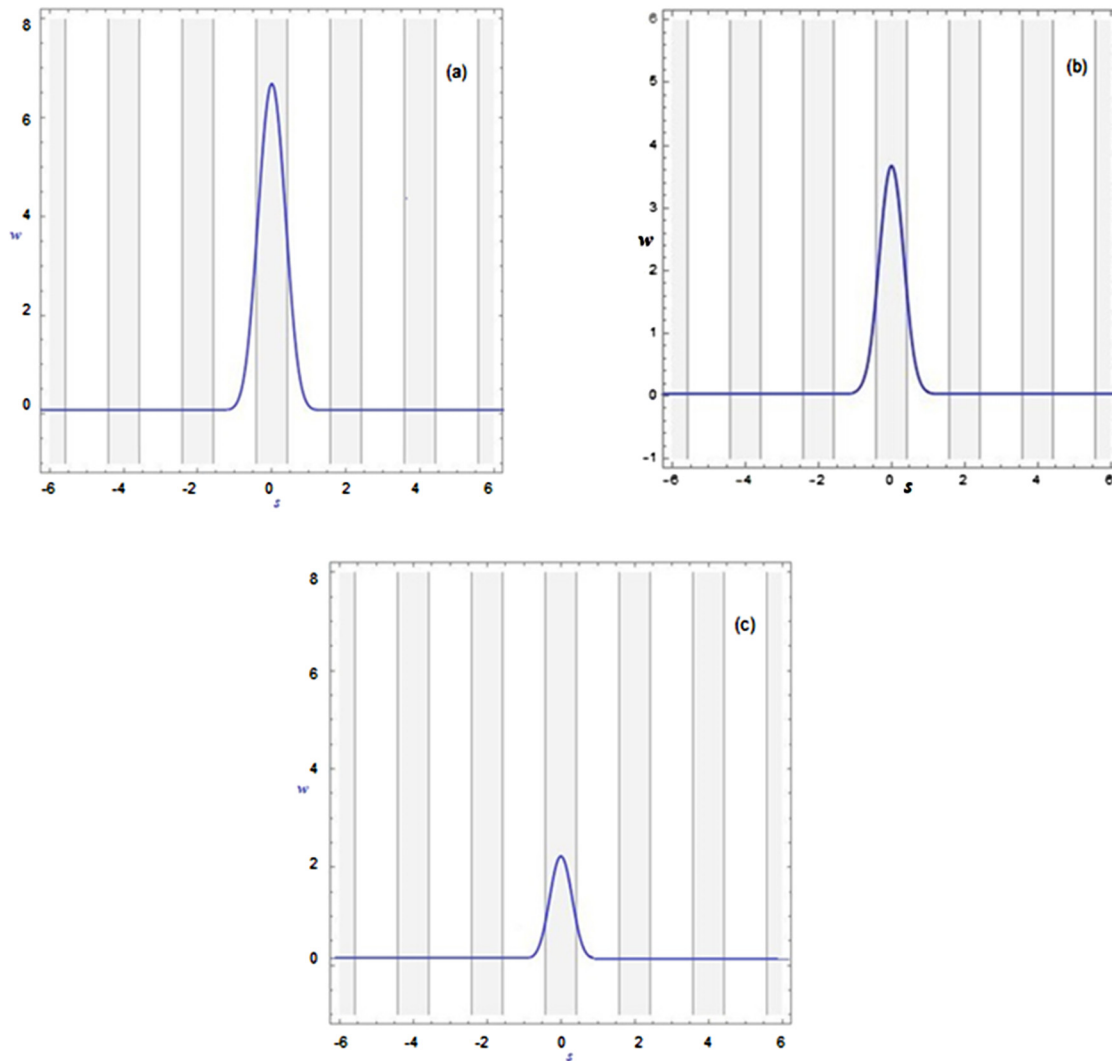


Figure 3: Spatial profile of the gap soliton in the first finite band gap for the case of (a) high power ($\mu = 1$), (b) moderate power ($\mu = -0.4$), (c) low power ($\mu = -2.7$).

$$U(s, \xi) = (w(s, \xi) + [u(s) - v(s)]\exp[i\delta\xi] + [u^*(s) + v^*(s)]\exp[-i\delta^*\xi])\exp[i\mu\xi] \quad (5)$$

The asterisk signifies a complex conjugation; u and v are the perturbation components and $\delta = \delta_r + i\delta_i$ is the complex growth rate of the perturbation. Substituting Eq. (5) in Eq. (3) and linearizing the resulting equation, we get an eigenvalue problem for the perturbation components u , v , and the growth rate δ ,

$$\begin{bmatrix} 0 & L_1 \\ L_2 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \delta \begin{bmatrix} u \\ v \end{bmatrix} \quad (6)$$

with,

$$L_1 = -\frac{1}{2} \frac{d^2}{ds^2} - pR + \frac{\beta_1}{(1+w^2)} + \frac{\beta_2}{(1+w^2)^2} + \mu \quad (7)$$

$$L_2 = -\frac{1}{2} \frac{d^2}{ds^2} - pR + \frac{\beta_1(1-w^2)}{(1+w^2)^2} + \frac{\beta_2(1-3w^2)}{(1+w^2)^3} + \mu \quad (8)$$

The two eigenvalue problems in Eq. (6) can be clearly reduced to a single equation,

$$L_1 L_2 u = \delta^2 u \quad (9)$$

From Eqs. (7) and (8), Eq. (9) becomes,

$$\left(-\frac{1}{2} \frac{d^2}{ds^2} - pR + \frac{\beta_1}{(1+w^2)} + \frac{\beta_2}{(1+w^2)^2} + \mu \right) \left(-\frac{1}{2} \frac{d^2}{ds^2} - pR + \frac{\beta_1(1-w^2)}{(1+w^2)^2} + \frac{\beta_2(1-3w^2)}{(1+w^2)^3} + \mu \right) u = \delta^2 u \quad (10)$$

From Eq. (5), we infer that the gap solitons will be linearly unstable if δ has an imaginary component,

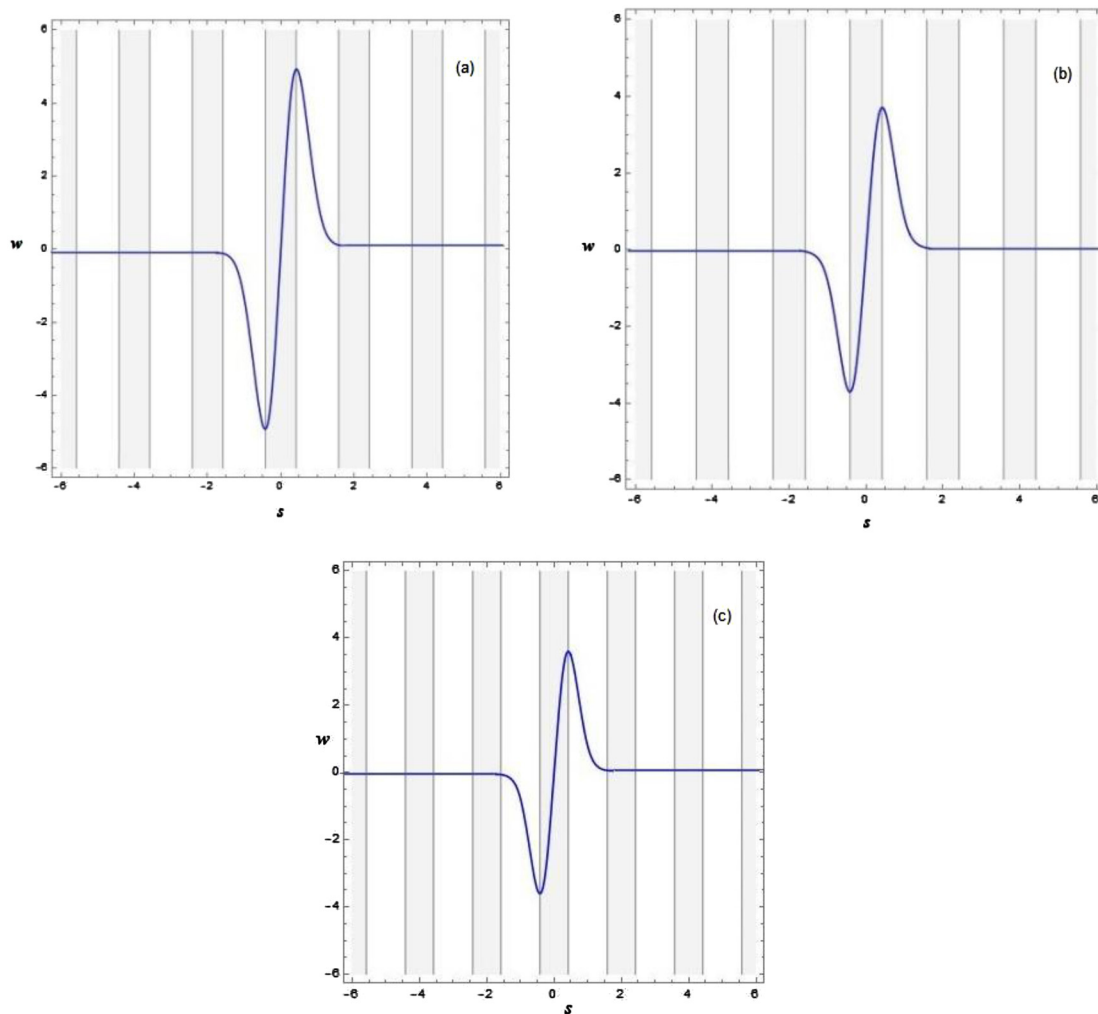


Figure 4: Spatial profile of the gap soliton in the second finite band gap for the case of (a) high power ($\mu = -4.9$), (b) moderate power ($\mu = -5.7$), (c) low power ($\mu = -6.2$).

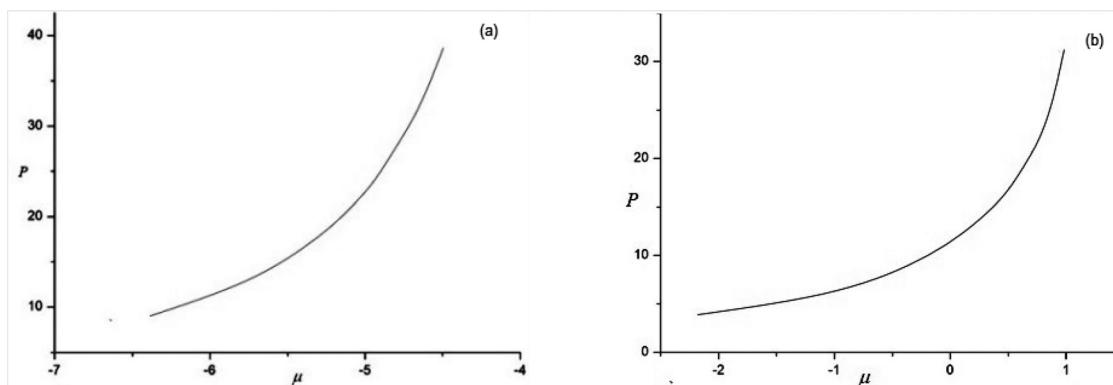


Figure 5: Power of the gap solitons as a function of the propagation constant μ for the (a) first finite band gap (b) second finite band gap.

i.e., $\delta_i \neq 0$. If only the real component exists, i.e., $\delta_i = 0$, then the gap solitons are stable. Hence, in Eq. (10), δ^2 has to be real for linear stability of the solitons.

Equation (9) signifies an eigenvalue problem. We first discretize Eq. (10) by a finite difference method, i.e., using central differences along with the boundary conditions for

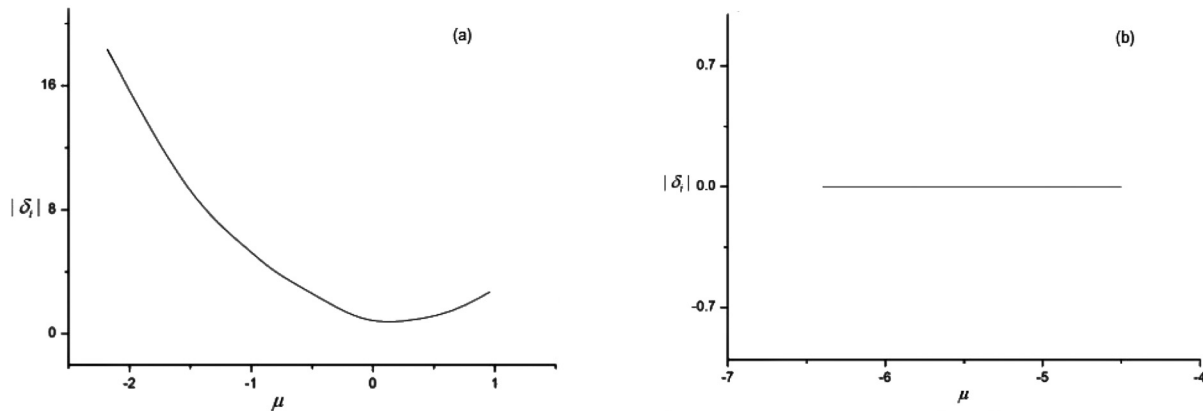


Figure 6: Imaginary part of δ for the gap solitons in the (a) first finite band gap and (b) second finite band gap.

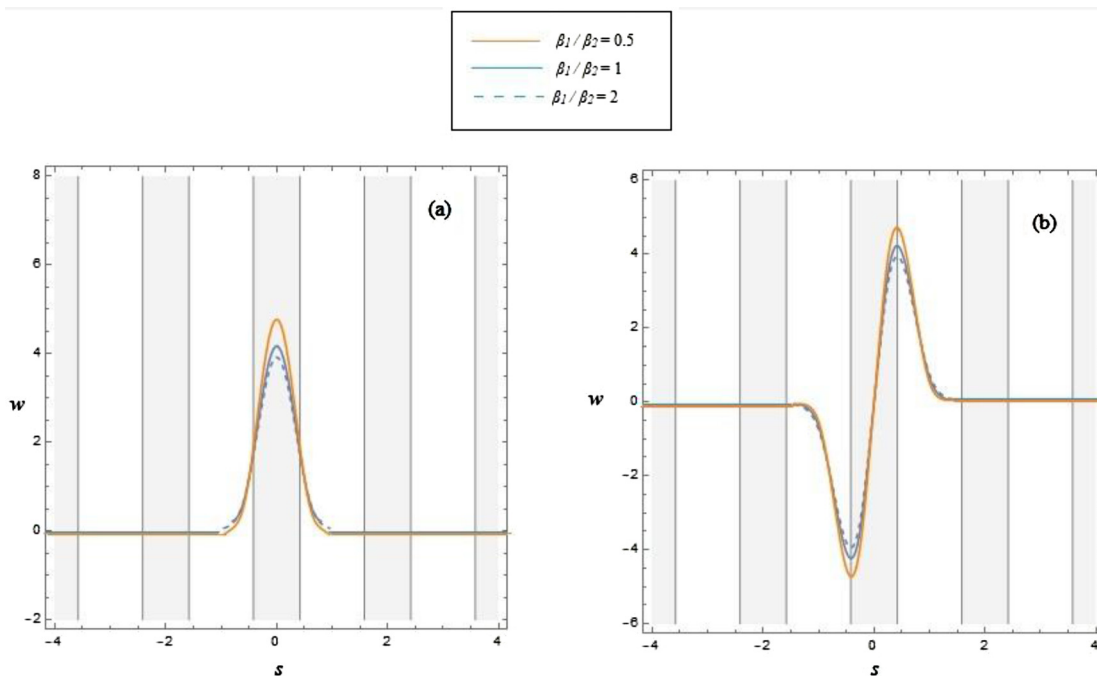


Figure 7: Gap soliton profiles in the (a) first finite band gap ($\mu = -0.4$) (b) second finite band gap ($\mu = -5.7$) for the linear to quadratic electro-optic coefficient ratio $\beta_1/\beta_2 = 0.5$, $\beta_1/\beta_2 = 1$, $\beta_1/\beta_2 = 2$.

a bright soliton solution. Since the differential equation in Eq. (10) is of fourth order, hence we obtain a pentadiagonal $N \times N$ matrix formulation, which is then solved to obtain the value of the eigenvalue δ^2 . We plot the imaginary part of δ for the gap solitons in the first and second finite band gaps in Figure 6. As discussed earlier, we need δ to be purely real for a stable gap soliton. From Figure 6a, the imaginary part of δ is non-zero throughout the first finite band gap. While the value of the imaginary part of δ decreases near $\mu=0$, it is never completely zero and hence the gap solitons are not linearly stable in this region. In Figure 6b, we can see that the imaginary part of δ is zero

throughout the second finite band gap implying stability of the gap solitons in this region. This result which we obtained for the novel PR crystals having both the linear and quadratic non-linearity is quite interesting and novel when compared to previous studies on gap solitons in various other types of nonlinear optical media [17, 21, 27, 29, 32].

2.4 Effect of electro-optic coefficients

Since we have considered novel PR materials which exhibit both the linear and quadratic electro-optic effect

simultaneously, it would be plausible to study the interplay between the two nonlinearities more deeply. We need to see how the relative magnitude of the two electro-optic coefficients changes the spatial profile and stability of the gap solitons in both the finite band gaps. Since the parameters β_1 and β_2 , which represent the strength of the nonlinearity, depend upon the linear and quadratic electro-optic coefficient respectively, we shall consider three broad cases for this study, i.e., $\beta_1/\beta_2 = 0.5$, $\beta_1/\beta_2 = 1$, $\beta_1/\beta_2 = 2$ where $\beta_1 = 15.46$. Figure 7 compares the spatial profile of the gap solitons for the three aforementioned cases. With regards to the linear stability, we again solve the eigenvalue problem Eq. (10) for the three situations. Figure 8 shows the imaginary part of the perturbation at each value of the frequency in the photonic band gap. As found before, the gap solitons are found to be unstable in the first finite band gap and stable in the second finite band gap. We can infer from Figure 8a that the degree of instability of the gap solitons in the first finite band gap increases as the ratio of the nonlinear coefficients β_1/β_2 increases. Figure 8b shows stability of the gap solitons in

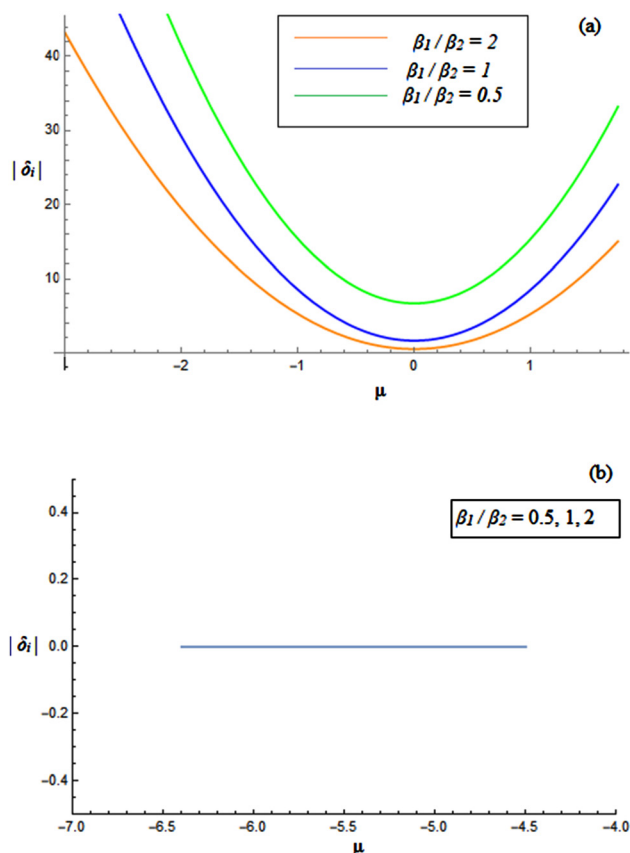


Figure 8: Imaginary part of δ for the gap solitons in the (a) first finite band gap (b) second finite band gap for the linear to quadratic electro-optic coefficient ratio $\beta_1/\beta_2 = 0.5$, $\beta_1/\beta_2 = 1$, $\beta_1/\beta_2 = 2$.

the second finite band gap for all three values of β_1/β_2 as the imaginary portion of the perturbation is zero.

3 Conclusions

We have theoretically investigated for the first time the existence of gap solitons supported by an optical lattice embedded in PR crystal exhibiting the linear and quadratic electro-optic effect simultaneously. The gap soliton profile in the first and second band gap has been obtained for the case of low, moderate and high power. The power of the gap solitons depends upon the value of the axial propagation constant. Single humped solitons are theoretically predicted in the first finite band gap while the double humped solitons are found to be supported in the second finite band gap. The power of the gap soliton depends upon the value of the axial propagation constant. We delineate three power regimes and study the gap soliton profiles in each region. The stability analysis by the VK criterion tells us that the solitons in both band gaps are stable. Since VK criterion is a necessary but not sufficient condition for stability, we have also undertaken the linear stability analysis by perturbation methods. We find that the solitons in the first finite band gap are unstable while the solitons in the second band gap are found to be stable against small perturbations. Finally, we study the effect of the interplay of the linear and quadratic electro-optic coefficients. Three distinct cases pertaining to the ratio of the linear and quadratic electro-optic coefficients are identified and the existence and stability of the gap solitons are studied in detail for each.

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