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Pocket formula for alpha decay energies and half-lives of actinide nuclei

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Abstract: *It is important to construct the* correct formula for alpha decay half-lives. The present work verifies the validity of Geiger–Nuttall law (GNL) for actinides and hence presenting the simple empirical formula for alpha decay energies and halflives for actinides based on the available experimental results. We have studied the variation of logarithmic halflives with $Z_d^nQ^{-1/2}$ for different values of n (0.2–1). The variation of logarithmic halflives with $Z_d^nQ^{-1/2}$ is found to be linear for which $n = 0.6$. The values produced by the present formula compared with that of experiments. The present formulae successfully produces the alpha decay half-lives and Q-value in the actinide region. This formula is specially for actinide parent nuclei only.

Keywords: actinides; alpha decay; Geiger–Nuttall law.

1 Introduction

Geiger–Nuttall law (GNL) is extremely successful and is considered to be generally valid. The linear relationship between the logarithm of α-decay half-lives ($T_{1/2}$) and the reciprocal of the square root of decay energies ($Q_\alpha^{-1/2}$) was reported in GNL [1]

$$\log T_{1/2} = A(Z)Q_\alpha^{-1/2} + B(Z) \quad (1)$$

$A(Z)$ and $B(Z)$ are the coefficients which are determined by fitting experimental data for each isotopic chain. Ren et al. [2] modified GNL by including the effects of quantum numbers of α-core relative motion and parity between parent nuclei and daughter nuclei.

$$\log T_{1/2} = a\sqrt{\mu}Z_cZ_dQ_\alpha^{-1/2} + b\sqrt{\mu}\sqrt{Z_cZ_d} + c + S + Pl(l+1) \quad (2)$$

The S, $l(l+1)$ and P are spin, centrifugal and parity terms defined clearly in Ref. [2]. Qi et al. [3, 4] also verified GNL for medium and heavy nuclei. For calculation of alpha decay half-lives, many empirical and semi-empirical formulas, with adjustable parameters, have been developed to deduce their experimental values. All of these formulas are based on different variations of Gamow theory [5] and other theories. Almost all formulae depends on the atomic number Z, mass number A and energy released Q.

Subsequently, there were a number of theoretical models to predict the alpha decay half-lives, the nuclear structure information, and also the microscopic understanding of α-decay phenomenon. Among the alpha decay models, the generalized liquid drop model is the successful model among the different macroscopic models which in turn describes the process of fusion, fission, alpha particle emission, light particle emission [6–12] and nuclear structure parameters such as nuclear radius and mass, investigation of charge asymmetry, deformation and proximity effects. The empirical coefficients appeared in GNL have a physical meaning and this law is valid for general α-decay Process [13]. From the literature [14] it is also observed that there is deviation of GNL in neutron deficient heavy nuclei. Delion and Ghinescu [15] verified GNL for nuclei in strong electromagnetic fields. Sharma et al. [16] studied the different decay process in actinides such as Plutonium. Fields et al. [17] experimentally studied the Alpha decay of ^{247}Cm. Hoff et al. [18] studied α-decay in the actinides such as ^{255}Md, ^{256}Md and ^{257}Md isotopes. Previous workers also studied the alpha decay properties in some actinide nuclei [19, 20]. Aim of the present work is verify the validity of GNL for actinides and hence presenting the simple empirical formula for alpha decay energies and half-lives of actinide nuclei.

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2 Theory

The decay half-life of parent nuclei with the emission of alpha particle is studied by

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{\nu P} \tag{3}$$

where λ is the decay constant and ν is the assault frequency and is expressed as

$$\nu = \frac{\omega}{2\pi} = \frac{2E_\nu}{h} \tag{4}$$

where Ev is the empirical vibrational energy. The penetration probability P through the potential barrier studied by the following equation

$$P = \exp\left\{-\frac{2}{\eta}\int_{R_a}^{R_b}\sqrt{2\mu\left(V_T(r) - Q\right)}dr\right\} \tag{5}$$

where μ is the reduced mass alpha decay system, R_a and R_b are the inner and outer turning points and these turning points are calculated by

$$\mathrm{V_T(R_a) = Q = V_T(R_b)} \tag{6}$$

To identify the stable superheavy nuclei, we have investigated the alpha decay process using the following theoretical framework. The concept of the coulomb and proximity model was proposed by Shi and Swiatecki [21, 22]. The proximity function was introduced by Blocky et al. [23, 24]. Santhosh and Joseph also used this model to study the decay properties [25, 26]. The potential $V(R)$ is considered as the sum of the Coulomb, the nuclear and the centrifugal potentials.

$$V(R) = V_C(R) + V_N(R) + V_{cf}(R) \tag{7}$$

Coulomb potential Vc(R) is taken as

$$V_C(R) = Z_1 Z_2 e^2 \begin{cases} \dfrac{1}{R} & (R > R_C) \\ \dfrac{1}{2R_c}\left[3 - \left(\dfrac{R}{R_c}\right)^2\right] & (R < R_C) \end{cases} \tag{8}$$

Where $R_C = 1.24 \times (R_1 + R_2)$, R_1 and R_2 are respectively the radii of the emitted alpha and daughter nuclei. Here Z_1 and Z_2 are the atomic numbers of the daughter and emitted cluster the nuclear potential $V_N(R)$ is calculated from the proximity potential defined by the previous work [27]. The Langer modified centrifugal barrier is adopted [28] in the present calculation.

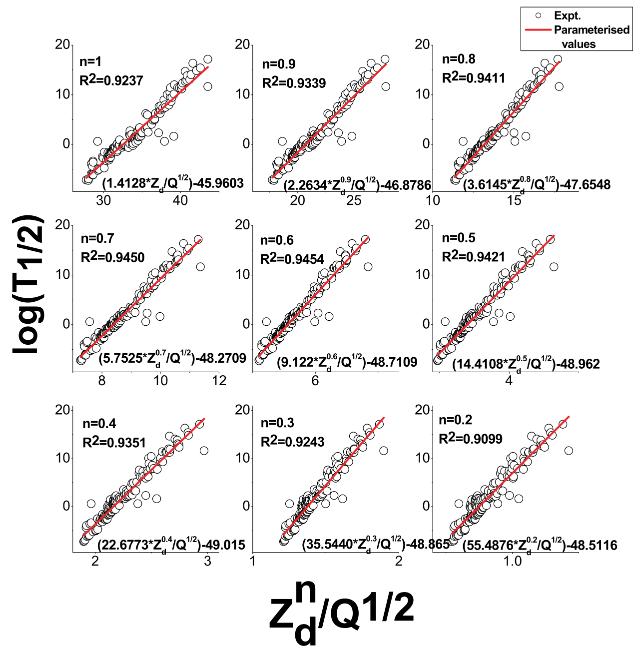

Figure 1: Variation of logarithmic halflives $\log(T_{1/2})$ as a function of $Z_d^n\,Q^{-\frac{1}{2}}$.

3 Results and discussion

The energy released during the alpha decay (Q_α) is calculated using the procedure explained in our previous work [27–29]. We have used experimental mass excess

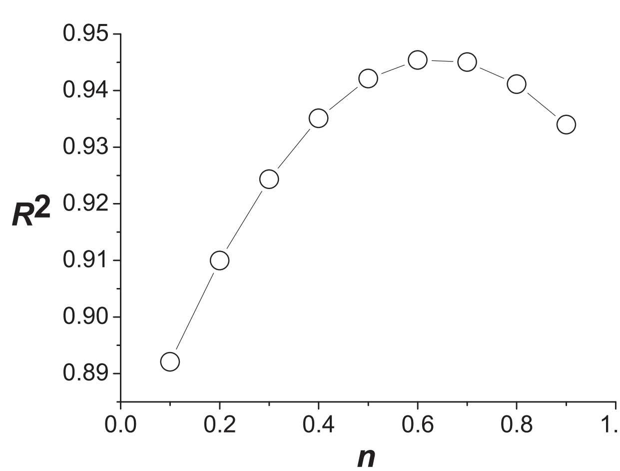

Figure 2: Comparison of coefficient of determination of experimental values for different values of n.

values [30]. For those nuclei, where experimental mass excess was unavailable, we have used recent theoretical values [30, 31]. We have calculated the alpha decay half lives of almost all actinide nuclei and these values are used for parameterization. This figure shows the variation of logarithmic half-lives as a function of $Z_d^n Q^{-1/2}$. Here we have varied the n from 1 to 0.2. Each layer of Figure 1 shows different values of n. with decrease in the value of n, effect of atomic number on the alpha decay half-life decreases. We have fitted the equation for logarithmic half-lives in terms of $Z_d^n Q^{-1/2}$. The coefficient of determination of actual values (R^2) for different **n** values are also included in the each layer of Figure 1. Figure 2 shows the variation of n with the coefficient of determination of actual values (R^2).

Table 1: Comparism of present formula with experiment.

Nuclei	Expt.	Present formula	% of deviation	Nuclei	Expt.	Present formula	% of deviation
^{205}Ac	−1.699	−1.957	15.202	^{233}U	12.701	12.546	1.219
^{206}Ac	−1.658	−1.577	4.859	^{234}U	12.889	12.867	0.17
^{208}Ac	−1.022	−0.85	16.848	^{235}U	16.346	14.038	14.123
^{209}Ac	−1.06	−0.879	17.069	^{236}U	14.869	14.755	0.768
^{210}Ac	−0.456	−0.504	10.503	^{238}U	17.149	16.968	1.054
^{213}Ac	−0.132	−0.148	12.433	^{225}Np	−2.444	−2.631	7.673
^{217}Ac	−7.161	−6.3	12.032	^{237}Np	13.831	12.65	8.537
^{218}Ac	−5.967	−5.29	11.344	^{236}Pu	7.956	8.063	1.354
^{219}Ac	−4.928	−3.959	19.663	^{238}Pu	9.442	9.437	0.062
^{221}Ac	−1.284	−1.033	19.516	^{239}Pu	11.881	11.34	4.556
^{223}Ac	2.1	2.35	11.871	^{240}Pu	11.316	11.274	0.368
^{225}Ac	5.933	5.876	0.962	^{242}Pu	13.072	12.887	1.414
^{210}Th	−1.796	−1.575	12.32	^{244}Pu	15.41	14.955	2.949
^{212}Th	−1.499	−1.247	16.824	^{241}Am	10.137	9.583	5.459
^{213}Th	−0.842	−0.882	4.841	^{243}Am	11.365	10.639	6.395
^{214}Th	−1.06	−0.851	19.731	^{240}Cm	6.367	6.364	0.058
^{216}Th	−1.585	−1.583	0.14	^{242}Cm	7.149	7.165	0.223
^{218}Th	−6.932	−6.047	12.771	^{243}Cm	8.963	7.377	17.696
^{219}Th	−5.979	−5.292	11.488	^{245}Cm	11.425	10.028	12.228
^{221}Th	−2.759	−3.121	13.111	^{246}Cm	11.173	10.824	3.129
^{225}Th	2.72	2.185	19.667	^{248}Cm	14.041	12.604	10.235
^{227}Th	6.21	5.298	14.682	^{239}Cf	1.591	1.768	11.137
228Th	7.78	8.28	6.423	^{240}Cf	1.76	2.092	18.83
229Th	11.396	10.192	10.566	^{242}Cf	2.346	2.743	16.916
230Th	12.377	12.599	1.798	^{244}Cf	3.045	3.4	11.66
^{212}Pa	−2.292	−2.253	1.706	^{246}Cf	5.111	5.146	0.694
^{213}Pa	−2.276	−2.17	4.631	^{248}Cf	7.459	7.223	3.17
^{214}Pa	−1.77	−1.834	3.637	^{250}Cf	8.616	8.276	3.943
^{215}Pa	−1.854	−1.749	5.675	^{252}Cf	7.922	7.869	0.659
^{217}Pa	−2.42	−2.442	0.909	^{240}Es	0.778	0.77	1.0162
^{219}Pa	−7.276	−6.25	14.092	^{242}Es	1.13	0.983	13.0224
^{220}Pa	−6.108	−5.315	12.985	^{243}Es	1.322	1.254	5.1908
^{221}Pa	−5.229	−4.386	16.115	^{253}Es	6.248	5.972	4.4215
^{223}Pa	−2.292	−2.003	12.624	^{254}Es	7.377	6.48	12.1554
^{226}Pa	2.033	2.29	12.614	^{247}Fm	1.041	0.993	4.6826
^{227}Pa	3.362	3.842	14.297	^{248}Fm	1.538	1.801	17.1338
^{231}Pa	12.01	10.694	10.978	^{250}Fm	3.255	3.244	0.3348
^{216}U	−2.347	−2.245	4.32	^{252}Fm	4.962	4.695	5.3803
^{217}U	−1.796	−1.969	9.646	^{254}Fm	4.068	4.127	1.4379
^{218}U	−3.292	−2.898	11.991	^{255}Fm	4.859	4.373	9.9955
^{221}U	−6.18	−5.556	10.1	^{257}Fm	6.939	5.809	16.2805
^{224}U	−3.076	−2.509	18.441	^{254}No	1.708	1.694	0.7714
^{227}U	1.82	1.745	4.116	^{259}No	3.542	2.875	18.8169
^{230}U	6.243	6.731	7.813	^{259}Lr	0.792	0.94	18.5703
^{232}U	9.336	9.619	3.029				

Table 2: Fitting parameters for Q-value formula in the atomic number range 89 < *Z* < 92.

Z	(N/Z) = 1.3–1.4				(N/Z) = 1.4–1.58			
	α_3	α_2	α_3	α_0	α_3	α_2	α_1	α_0
89	−1202.14	4907.813	−6682.86	3042.769	13,602.68	−60,537.9	89,741.19	−44,303.8
90	3603.584	−14,563.6	19,609.44	−8788.85	1073.733	−4843.22	7240.625	−3580.63
91	6459.89	−25,737.9	34,164.65	−15,100.6	680.744	−2950.41	4220.185	−1983.03
92	−19,913.2	82,054.02	−112,660	51,550.25	332.2455	−1369.67	1833.427	−783.696

Table 3: Fitting parameters for Q-value formula in the atomic number range 93 < *Z* < 101 ((N/Z) = 1.4–1.6).

Z	α_3	α_2	α_1	α_0
93	2229.152	−9745.24	14,158.25	−6828.46
94	−447.893	2094.404	−3277.47	1721.428
95	158.9766	−619.091	767.9383	−289.472
96	−301.487	1350.373	−2029.53	1029.945
97	−512.94	2250.086	−3303.85	1631.053
98	2160.656	−9742.79	14,622.77	−7298.07
99	829.7781	−3741.27	5607.114	−2785.85
100	580.1075	−2572.98	3787.838	−1842.74
101	−1521.68	6870.684	−10,323.2	5169.523

From this figure, it is surprisingly observed that coefficient of determination of actual values (R^2) increases with increase of n up to 0.6, then decreases. That is coefficient of determination is maximum for n = 0.6. The fitted equation for logarithmic alpha decay halflives for the actinide region is as follows;

$$\log\,(T) = \left[\frac{Z_d^{0.6}}{\sqrt{Q}}\right] 9.122 - 48.7109 \tag{9}$$

To validate the above empirical formula, we have compared the values produced by the present formula with that of the experiments available in the literature. The variation of logarithmic half-lives with $Z_d^n/\sqrt{Q}$ is found to be linear for which n = 0.6. The comparison of present formula with experiments is as shown in Table 1. In this table, we have also presented the percentage of deviation of present formula with experiments [32].

After the calculation of Q-value for alpha decay in actinides, we have searched for their parameterization. The expressed paratmeterized equation for Q value is as follows;

$$Q = \sum_{i=0}^{n} \alpha_i \left(\frac{N}{Z}\right)^i \tag{10}$$

In the above formula, α_i's are fitting parameters which depends on the atomic number and *n* is the degree of the polynomial. The value of n = 3 for the atomic number regions 89 < *Z* < 92 and 93 < *Z* < 101 and fitting parameters are given in Tables 2 and 3. The value of n = 2 for the atomic number regions 102 < *Z* < 103 and fitting parameters are given in Table 4. To test the validity of the present Q-value formula, we have compared the values produced by the present formula with that of the experiments [33]. This comparison is as shown in Figure 3.

Brown [34] constructed the following equation for alpha decay half-lives using GNL based on the experimental alpha decay half-lives of atomic number region 61 < *Z* < 93

$$\log_{10} T_{1/2}(s) \;=\; \left(9.54\; Z_{\mathrm{d}}^{0.6} / \sqrt{Q_\alpha}\right) - 51.37 \tag{11}$$

For the fitting this semi-empirical formula, Brown used experimental half-lives of only 8 nuclei [35]. The construction of present formula used the experimental half-lives of all available 96 nuclei [35]. To compare the present formula with that of the brown formula, we have calculated the percentage of deviation and it is shown in Figure 4. Average percentage of deviation for Brown formula is 6.98 and that of present formula is 1.73. This formula is based on the experimental half-lives of only 8

Table 4: Fitting parameters for Q-value formula in the atomic number range 102 < *Z* < 103.

Z	(N/Z) = 1.45–1.48			(N/Z) = 1.49–1.53		
	α_2	α_1	α_0	α_2	α_1	α_0
102	358.417	−1071.35	808.9402	−848.62	2563.104	−1926.78
103	−834.611	2428.495	−1757.65	−652.174	1951.201	−1450.53

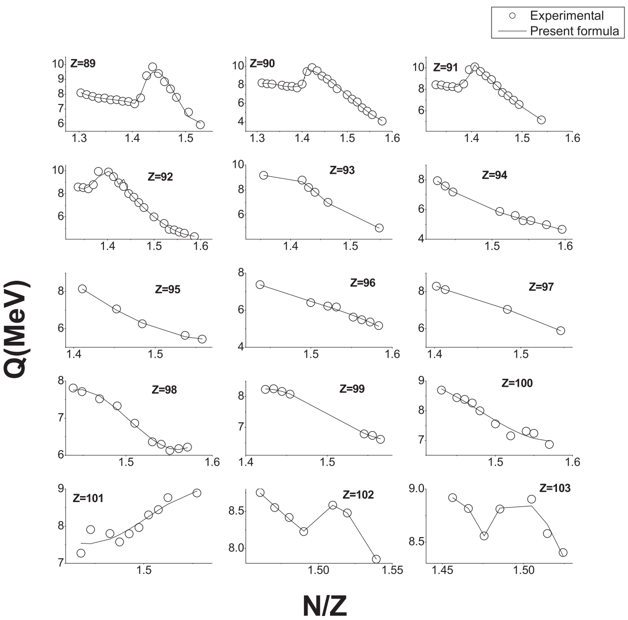

Figure 3: Comparison of values produced by the Q-value with that of the experiments.

actinide nuclei. This formula may not predict exact values of alpha decay half-lives in the actinide region. But present formula is specifically for the actinide region only and it is based on experimental alpha decay half-lives of 96 nuclei. Hence present formula predicts more accurate alpha decay half-lives than that of the Brown formula [34].

To know the deformation effects, we have evaluated the logarithmic half-lives of some deformed nuclei using

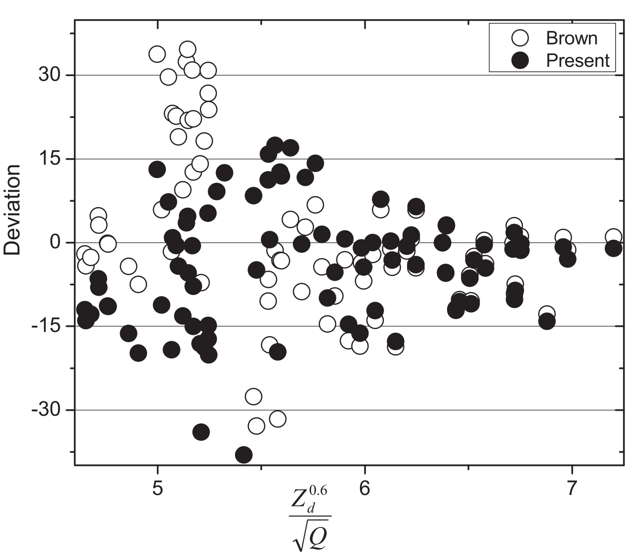

Figure 4: Percentage of deviation of Presxent formula with that of brown formula.

Table 5: Comparison of logarithmic halflives produced by the present formula with that of experiments for deformed nucleus.

Nucleus	β_2	β_4	$\log(T_{1/2})$	
			Experiment	Present formula
^{230}Th	0.192	0.900	12.38	12.60
^{234}U	0.208	0.087	12.89	12.87
^{236}U	0.224	0.078	14.87	14.76
^{238}U	0.228	0.066	17.15	16.97
^{238}Pu	0.229	0.070	9.44	9.44
^{240}Pu	0.233	0.061	11.32	11.27
^{242}Pu	0.235	0.042	13.07	12.89
^{244}Pu	0.238	0.033	15.41	14.96
^{244}Cm	0.238	0.033	8.76	8.63
^{246}Cm	0.238	0.033	11.17	10.82

present formula and compared with that of experiments and this comparison is as shown in Table 5. From this comparison, it is observed that the present formula also produces almost exact values for deformed parent nuclei. That is present formulae successfully produces the alpha decay half-lives and Q-value in the actinide region. This formula is specially for actinide parent nuclei only.

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