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Dynamical properties of nonlinear ion-acoustic waves based on the nonlinear Schrödinger equation in a multi-pair nonextensive plasma

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Abstract: Dynamical properties of nonlinear ion-acoustic waves (IAWs) in multi-pair plasmas (MPPs) constituting adiabatic ion fluids of positive and negative charges, and q-nonextensive electrons and positrons are examined. The nonlinear Schrödinger equation (NLSE) is considered to study the dynamics of IAWs in a nonextensive MPP system. Bifurcation of the dynamical system obtained from the NLSE shows that the system supports various wave forms such as, nonlinear periodic wave, kink and anti-kink waves in different ranges of q. The analytical solutions for ion-acoustic nonlinear periodic wave, kink and anti-kink waves are obtained. The impacts of system parameters such as, nonextensive parameter (q), mass ratio of negative and positive ions (μ_1), number density ratio of positive and negative ions (μ_2), number density ratio of positrons and negative ions (μ_p), temperature ratio of positive ions and electrons (σ_2) and temperature ratio of electrons and positrons (δ) on IAW solutions are bestowed. The results of this study are applicable to understand different dynamical behaviors of nonlinear IAWs found in the Earth's ionosphere, such as, D-region [H^+, O_2^-] and F-region [H^+, H^-] and multipair plasma system laboratory [C^+, C^-].

Keywords: bifurcation; dynamical system; kink and anti-kink waves; nonlinear periodic wave.

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1 Introduction

Over the past few years, studies on presence of pair-ion (PI) of positive and negative charged ions in plasmas have contributed greatly in research works. The existence of such pair of ions was experimentally observed in absence of electrons [1]. The difference in masses between negative and positive charged ion particles significantly produces temporal and spatial variations in plasma systems. The three electrostatic modes propagating in PI plasma systems are IAWs, ion plasma waves and intermediate-frequency waves [2–5]. Rasheed et al. [6] studied degenerated PI plasma systems applicable to space research such as, white dwarf and dense neutron stars. The study of PI plasmas with other pair of particles such as, electron and positron (e-p) pair are called multi-pair plasmas (MPPs) [7]. Some researchers studied electron and positron pair plasmas both in stationary and rotating cases [8–14]. Plasma research involves study of IAWs in PI plasma system for which PI mass density provides inertial moment while, the thermal pressure of positrons and electrons generates restoring force. In MPPs, the presence of positive ions immensely effects charge neutrality condition [15], dispersion relation, nonlinearity of IAWs, etc. The adiabatic property of fluid ions enriches features of nonlinear forms such as, shock [16], solitary [17] etc. The MPPs mark their existence through laboratory observations [18, 19] and space environments such as higher region of Titan's atmosphere [20], etc.

The non-Maxwellian distributions (kappa, nonextensive and nonthermal distributions) are followed by particles of plasma systems due to their high-range of collisions or interactions. These particles diverge from Maxwellian distribution and thereby, follow non-Maxwellian distributions. Tsallis [21] introduced the nonextensive distribution by proposing the generalized Boltzmann-Gibbs-Shannon (BGS) entropy. Tsallis extended the theory for systems that support long-range interactions and collisions [22, 23]. The nonextensive parameter q represents the strength of nonextensivity and ranges from $-1 < q < 1$. It results into Maxwellian limit for $q \to 1$, and is not normalizable for $q < -1$. The implementations of nonextensive plasmas are extensively found in astrophysical and cosmological scenarios [24], and plasma dynamics [25, 26], Hamiltonian systems [27] with long-range interaction, and nonlinear gravitational model [28]. Later, Lima et al. [25] and Liu et al. [29] reported that the nonextensive distribution supported characteristics of non-Maxwellian particles. Jannat et al [30] studied ion-acoustic solitons under the KdV, mKdV and Gardner equations in nonextensive multipair plasmas. In plasma systems, the impacts of nonextensive electrons, ions and positrons on different traveling waves were reported [31–34].

In dynamical systems, the qualitative change in flow of structure that changes with parameters is termed as bifurcation [35]. Bifurcation analyses through phase plane analysis, show different transitions occurring in dynamical systems. Samanta et al. [36] were the first to report study of plasma waves employing bifurcation analysis. Recently, Khondaker et al. [37] studied rogue waves in nonextensive MPPs but not nonlinear periodic, kink and anti-kink waves. This is the motivation to examine dynamical properties of nonlinear (periodic, kink and anti-kink) IAWs under the NLSE in an MPP.

The article is structured as: In section 2, model equations are discussed. In section 3, the NLSE is considered. In section 4, dynamical properties of IAWs are examined for distinct ranges of q. Analytical solutions for IAWs and effects of system parameters on traveling wave solutions of IAWs are depicted for ranges $-1 < q < 0$ and $0 < q < 1$. Lastly, in section 5, conclusions of the study are drawn.

2 Model equations

An MPP system comprising of pair-ions (inertial adiabatic negative and positive ions) and inertia-less q-nonextensive distributed positrons and electrons is considered. Here, the charge of negative ions is $q_- = -eZ_-$ while, the charge is $q_+ = +eZ_+$ for positive ions, where $Z_{+,-}$ denotes state of charge for positive and negative ions. At equilibrium condition, charge neutrality holds $n_{+0}Z_+ + n_{p0} = n_{-0}Z_+ + n_{e0}$, with "0" denoting unperturbed quantity. The normalized model equations are given by [37]

$$\frac{\partial n_-}{\partial t} + \frac{\partial}{\partial x}(n_- u_-) = 0, \tag{1}$$

$$\frac{\partial u_-}{\partial t} + u_- \frac{\partial u_-}{\partial x} + 3\sigma_1 n_- \frac{\partial n_-}{\partial x} = \frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial n_+}{\partial t} + \frac{\partial}{\partial x}(n_+ u_+) = 0, \tag{3}$$

$$\frac{\partial u_+}{\partial t} + u_+ \frac{\partial u_+}{\partial x} + 3\sigma_2 n_+ \frac{\partial n_+}{\partial x} = -\mu_1 \frac{\partial \phi}{\partial x}, \tag{4}$$

$$\frac{\partial^2 \phi}{\partial x^2} = (\mu_2 + \mu_p - 1)n_e - \mu_p n_p + n_- - \mu_2 n_+, \tag{5}$$

where n_-, n_+, n_p and n_e stand for number densities of negative ions, positive ions, positrons and electrons, respectively. Here, u_- (u_+) is velocity of negative (positive) fluid ions and ϕ is electrostatic potential. Here, pressures $P_+ = \left(\frac{N_+}{n_{+0}}\right)^\gamma P_{+0}, P_- = \left(\frac{N_-}{n_{-0}}\right)^\gamma P_{-0}, P_{+0} = n_{+0}k_BT_+, P_{-0} = n_{-0}k_BT_-$, where P_{-0} (P_{+0}) is the adiabatic pressure of negative (positive) ions at equilibrium and $\gamma = N + 2/N$ with N being the degree of freedom taken as 1 for 1-dimensional adiabatic case, hence, $\gamma = 3$. Here, N_+ and N_- are unnormalized number densities of positive ions and negative ions. T_- (T_+) is temperature of negative (positive) ions, T_p (T_e) denotes temperature of positrons (electrons). Here, t and x denote time and space variable, respectively. Here, k_B is Boltzmann constant. Also, $\mu_1 = m_-Z_+/m_+Z_-$, $\mu_2 = n_{+0}Z_+/n_{-0}Z_-$, $\mu_p = n_{p0}/Z_-n_{-0}$, $\sigma_1 = T_-/Z_-T_e$ and $\sigma_2 = T_+m_-/Z_-T_em_+$. Here, number density and mass of positive ions are higher than number density and mass of negative ions ($n_{+0} > n_{-0}$ and $m_+ > m_-$), and also T_e and T_p are higher than T_- and T_+, respectively. The plasma

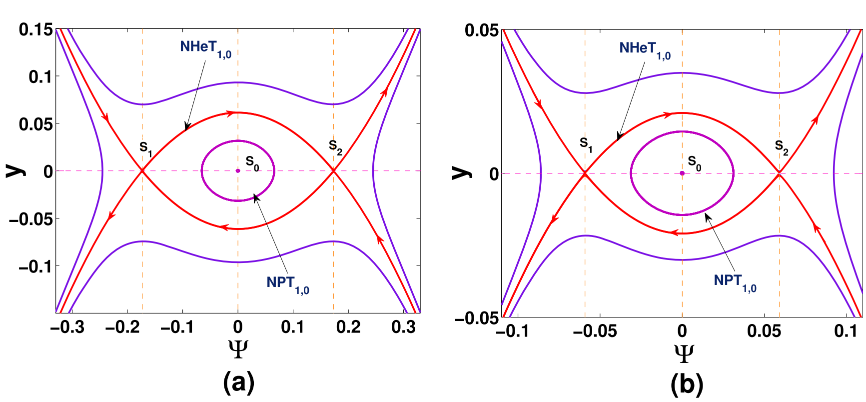

Figure 1: Phase plots of Eq. (12) in (a) $q = -0.4$, $\mu_p = 0.35$ and (b) $q = 0.07$, $\mu_p = 0.2$ with $\mu_1 = 0.45$, $\mu_2 = 1.2$, $\sigma_1 = 0.004$, $\sigma_2 = 0.05$, $\delta = 1.2$, $\beta = 0.5$, $k = 0.35$ and $l = 0.3$.

system is normalized as: n_+, n_-, n_p and n_e are normalized to n_{+0}, n_{-0}, n_{p0} and n_{e0}, respectively. Here, u_- and u_+ are normalized by $C_- = (Z_- k_B T_e/m_-)^{\frac{1}{2}}$ and $C_+ = (Z_+ k_B T_e/m_+)^{\frac{1}{2}}$, respectively. Here, ϕ, t and x are normalized by $k_B T_e/e$, $\omega_{p-} = (4\pi e^2 Z_-^2 n_{-0}/m_-)^{\frac{1}{2}}$ and $\lambda_{D-} = (k_B T_e/4\pi e^2 Z_- n_{-0})^{\frac{1}{2}}$, respectively. The normalized nonextensive electron and positron number densities are [37]:

$$\begin{cases} n_e = \{1 + (q-1)\phi\}^{\frac{1}{q-1}+\frac{1}{2}} = 1 + n_1\phi + n_2\phi^2 + n_3\phi^3, \\ n_p = \{1 - (q-1)\delta\phi\}^{\frac{1}{q-1}+\frac{1}{2}} = 1 - n_1\delta\phi + n_2\delta^2\phi^2 - n_3\delta^3\phi^3, \end{cases} \tag{6}$$

where $n_1 = \frac{1+q}{2}$, $n_2 = \frac{(1+q)(3-q)}{8}$, $n_3 = \frac{(1+q)(3-q)(5-3q)}{48}$ and $\delta = \frac{T_e}{T_p}$. Using Eq. (6) into Eq. (5), one can get

$$\frac{\partial^2\phi}{\partial x^2} = (\mu_2 - 1) + n_- - \mu_2 n_+ + \gamma_1\phi + \gamma_2\phi^2 + \gamma_3\phi^3 \cdots, \tag{7}$$

where $\gamma_1 = n_1(\mu_2 + \mu_p - 1 + \mu_p\delta)$, $\gamma_2 = n_2(\mu_2 + \mu_p - 1 - \mu_p\delta^2)$ and $\gamma_3 = n_3(\mu_2 + \mu_p - 1 + \mu_p\delta^3)$.

3 The nonlinear schrödinger equation (NLSE)

We examine IAWs in MPPs under the following NLSE [37]

$$i\frac{\partial\Phi}{\partial\tau} + P\frac{\partial^2\Phi}{\partial\xi^2} + Q|\Phi^2|\Phi = 0, \tag{8}$$

where $\Phi = \phi_1$. The coefficients of dispersion P and nonlinear Q are expressed as

$$P = \frac{F_2 - A^3S^3}{2AS\omega k^2(A^2 + \mu_1\mu_2S^2)}$$

and

$$Q = \frac{A^2S^2[2\gamma_2(C_5 + C_{10}) + 3\gamma_3 - F_3]}{2\omega k^2(A^2 + \mu_1\mu_2S^2)},$$

where

$$\omega^2 = \frac{M + k^2\sqrt{M^2 - 4GN}}{2G}, \; v_g = \frac{F_1 - 2A^2S^2 - AS(A - \mu_1\mu_2S)}{2\omega k(A^2 + \mu_1\mu_2S^2)},$$

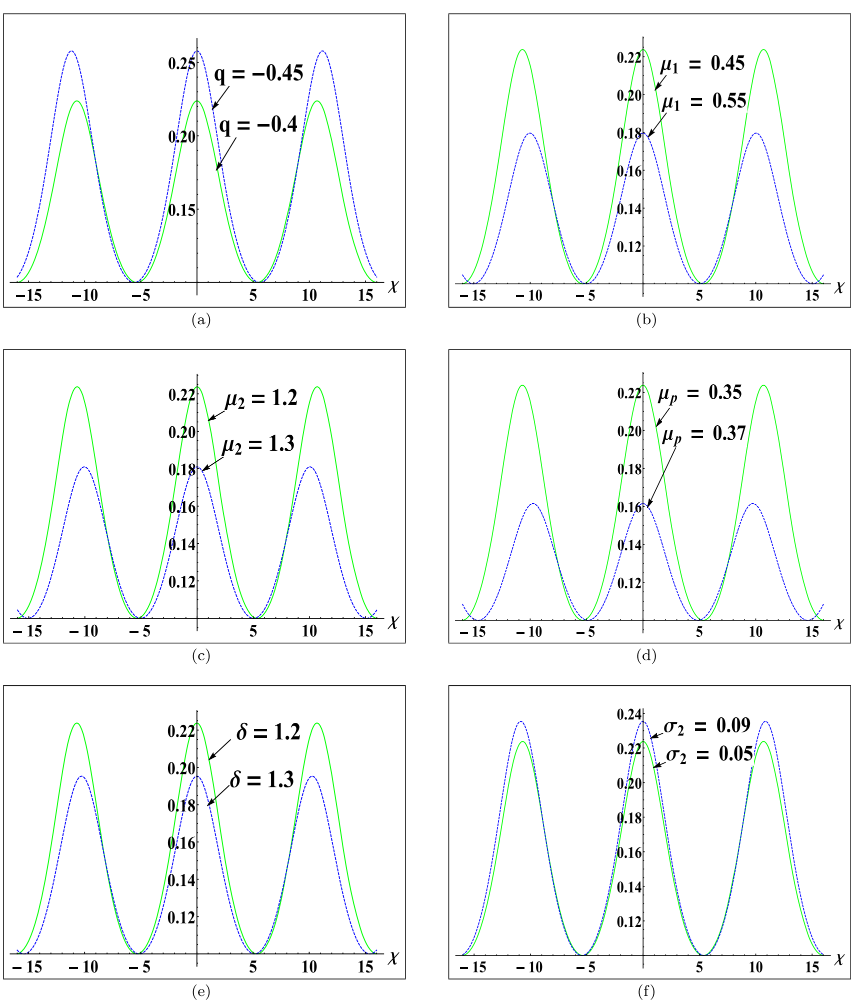

Figure 2: Effects of parameters (a) q, (b) μ_1, (c) μ_2, (d) μ_p, (e) δ and (f) σ_2 on NPIAW with other parameters same as in Figure 1a.

$$G = (\gamma_1 + k^2),\ M = (1 + \mu_1\mu_2 + \gamma_1\lambda_1 + \gamma_1\lambda_2 + \lambda_1 k^2 + \lambda_2 k^2),$$
$$N = (\lambda_2 + \mu_1\mu_2\lambda_1 + \gamma_1\lambda_1\lambda_2 + \lambda_1\lambda_2 k^2),$$

$$\lambda_1 = 3\sigma_1,\ \lambda_2 = 3\sigma_2,\ A = \omega^2 - \lambda_2 k^2,$$

$$S = \lambda_1 k^2 - \omega^2,$$

$$F_1 = \omega^2 (A^2 + \mu_1\mu_2 S^2) + k^2 (\lambda_1 A^2 + \mu_1\mu_2\lambda_2 S^2),$$

$$\begin{aligned} F_2 = A^3 &[(\omega v_g - \lambda_1 k)(\lambda_1 k^3 + k\omega^2 - 2\omega v_g k^2 - kS) \\ &+ (v_g k - \omega)(\omega^3 + \lambda_1 \omega k^2 - 2\omega^2 v_g k - v_g kS)] \\ &- \mu_1\mu_2 S^3 [(\omega v_g - \lambda_2 k)(\lambda_2 k^3 + k\omega^2 - 2\omega v_g k^2 + kA) \\ &+ (v_g k - \omega)(\omega^3 + \lambda_2 \omega k^2 - 2\omega^2 v_g k + v_g kA)], \end{aligned}$$

$$\begin{aligned} F_3 = &\frac{2\omega k^3 (C_2 + C_7)}{S^2} + \frac{(\omega^2 k^2 + \lambda_1 k^4)(C_1 + C_6)}{S^2} \\ &+ \frac{\mu_1\mu_2 (\omega^2 k^2 + \lambda_2 k^4)(C_3 + C_8)}{A^2} + \frac{2\mu_1\mu_2 \omega k^3 (C_4 + C_9)}{A^2}, \end{aligned}$$

$$C_1 = \frac{2C_5 k^2 S^2 - 3\omega^2 k^4 - \lambda_1 k^6}{2S^3},$$

$$C_2 = \frac{\omega (C_1 S^2 - k^4)}{kS^2},$$

$$C_3 = \frac{2C_5 \mu_1 k^2 A^2 + \mu_1^2 (\lambda_2 k^6 + 3\omega^2 k^4)}{2A^3},$$

$$C_4 = \frac{\omega (C_3 A^2 - \mu_1^2 k^4)}{kA^2},$$

$$C_5 = \frac{A^3 (3\omega^2 k^4 + \lambda_1 k^6 - 2\gamma_2 S^3) + \mu_2 S^3 (3\omega^2 \mu_1^2 k^4 + \lambda_2 \mu_1^2 k^6)}{2A^3 S^3 (4k^2 + \gamma_1) + 2k^2 S^2 A^3 - \mu_1\mu_2 k^2 A^2 S^3},$$

$$C_6 = \frac{k^2\omega^2 + 2v_g \omega k^3 + \lambda_1 k^4 - C_{10} S^2}{S^2 \left(v_g^2 - \lambda_1\right)},$$

$$C_7 = \frac{v_g C_6 S^2 - 2\omega k^3}{S^2},$$

$$C_8 = \frac{\mu_1^2 (2\omega v_g k^3 + k^2\omega^2 + \lambda_2 k^4) - \mu_1 C_{10} A^2}{A^2 \left(v_g^2 - \lambda_2\right)},$$

$$C_9 = \frac{v_g C_8 A^2 - 2\omega \mu_1^2 k^3}{A^2},$$

$$C_{10} = \frac{F_4 + 2\gamma_2 A^2 S^2 \left(v_g^2 - \lambda_1\right)\left(v_g^2 - \lambda_2\right)}{A^2 S^2 \left(v_g^2 - \lambda_2\right) + \mu_1\mu_2 A^2 S^2 \left(v_g^2 - \lambda_1\right) - \gamma_1 A^2 S^2 \left(v_g^2 - \lambda_1\right)\left(v_g^2 - \lambda_2\right)},$$

$$\begin{aligned} F_4 = &A^2 (k^2\omega^2 + 2v_g \omega k^3 + \lambda_1 k^4)\left(v_g^2 - \lambda_2\right) - \mu_2 S^2 (\mu_1^2 k^2 \omega^2 \\ &+ 2v_g \omega \mu_1^2 k^3 + \lambda_2 \mu_1^2 k^4)\left(v_g^2 - \lambda_1\right). \end{aligned}$$

For the derivation of the above NLSE, one can refer the work [37]. In the work [37], IAW solutions such as, periodic, kink and anti-kink wave solutions were not reported.

4 Dynamical properties

To investigate the dynamical properties of IAWs, we consider transformations $\chi = l\xi - V\tau$ and $\Phi(\chi) = \Psi(\chi)\exp(i\beta\chi)$ in the NLSE (8). Then, we obtain

$$Pl^2 \frac{d^2\Psi}{d\chi^2} + \left(\beta V\Psi - \beta^2 Pl^2 \Psi + Q|\Psi^2|\Psi\right) + i\left(-V + 2Pl^2\beta\right)\frac{d\Psi}{d\chi} = 0, \tag{9}$$

where $0 < l < 1$ and V is speed of traveling wave. Equating the imaginary part of the system, we get

$$V = 2Pl^2\beta. \tag{10}$$

Equating the real part of Eq. (9), we obtain the following equation

$$\frac{d^2\Psi}{d\chi^2} = \left(\beta^2 - \frac{1}{Pl^2}\beta V\right)\Psi - \frac{Q}{Pl^2}\Psi^3. \tag{11}$$

Then, we represent Eq. (11) as the following dynamical system [38]

$$\begin{cases} \dfrac{d\Psi}{d\chi} = y, \\ \dfrac{dy}{d\chi} = L_1\Psi - L_2\Psi^3, \end{cases} \tag{12}$$

where $L_1 = \left(\beta^2 - \frac{1}{Pl^2}\beta V\right)$ and $L_2 = \frac{Q}{Pl^2}$. Implementing the concept of dynamical systems [38], qualitative phase portraits for the system (12) are presented with parameters q, μ_1, μ_2, μ_p, $\sigma_1\sigma_2$, δ, β, l and k. Here, singular points of the system (12) are obtained as $S_0(\Phi_0, 0)$, $S_1(\Phi_1, 0)$ and $S_2(\Phi_2, 0)$, where $\Phi_0 = 0$ and $\Phi_1 = -\sqrt{\frac{L_1}{L_2}}$, and $\Phi_2 = \sqrt{\frac{L_1}{L_2}}$. Let J be the Jacobian matrix of the system (12). Then, $D = \det J(\phi_i, 0) = -L_1 + 3L_2\Phi_i^2$, where $i = 0,1,2$. The singular point $S_i(\Phi_i, 0)$ is a saddle point for $D < 0$, and a center for $D > 0$ [35]. Bifurcation analysis is applicable to examine dynamical motions of nonlinear waves. Any significant trajectories in phase plot correspond to wave solutions [39, 40]. Specifically, in a phase plot, nonlinear heteroclinic trajectory is associated with nonlinear kink and anti-kink wave solutions, nonlinear periodic trajectories are associated with nonlinear periodic wave solutions. Through phase plane plots, we show bifurcation analyses for IAWs in Figure 1a,b for the system (12) by varying q in ranges $-1 < q < 0$ and $0 < q < 1$, respectively along with other

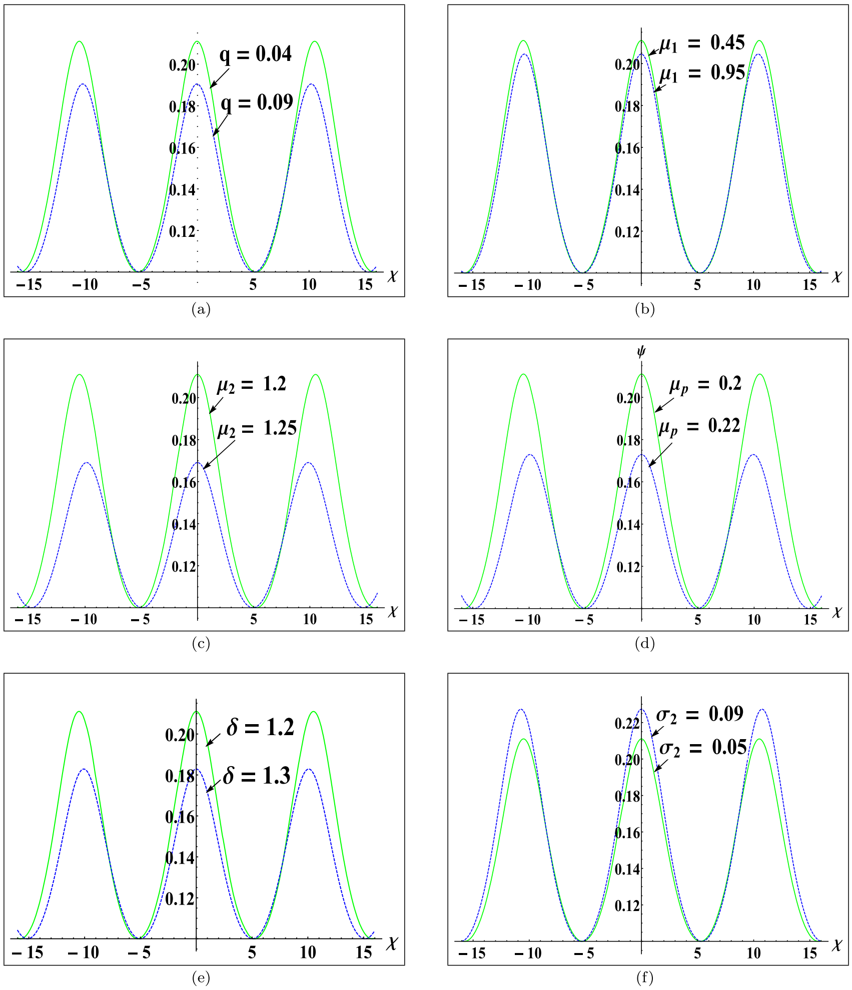

Figure 3: Effects of parameters (a) q, (b) μ_1, (c) μ_2, (d) μ_p, (e) δ and (f) σ_2 on NPIAW with other parameters same as in Figure 1b.

parameters μ_1, μ_2, μ_p, σ_1, σ_2, δ, β, l and k as fixed. Here, it is important to note that the phase plot of equation (12) in the range $q > 1$ shows similar feature as in the ranges $-1 < q < 0$ and $0 < q < 1$. Therefore, considering only the cases with ranges of q in $-1 < q < 0$ and $0 < q < 1$, we investigate the following dynamical properties of IAWs in the MPP system.

Figure 1 shows bifurcation of IAWs for the system (12) in (a) $q = -0.4$, $\mu_p = 0.35$ and (b) $q = 0.4$, $\mu_p = 0.2$ with $\mu_1 = 0.45$, $\mu_2 = 1.2$, $\sigma_1 = 0.004$, $\sigma_2 = 0.05$, $\delta = 1.2$, $\beta = 0.5$, $k = 0.35$ and $l = 0.3$. Here, Figure 1a,b show that the system (12) have center at singular point S_0 and a pair of saddle points at singular points S_1 and S_2 for both the range $-1 < q < 0$ and $0 < q < 1$. The nonlinear periodic trajectory ($NPT_{1,0}$) enclosing S_0 corresponds to a nonlinear periodic wave solution. Trajectories about a pair of saddle points connect two singular points S_1 and S_2, and form nonlinear heteroclinic trajectory ($NHeT_{1,0}$). The $NHeT_{1,0}$ corresponds to kink and anti-kink IAW solutions of the system (12) obtained based on the NLSE (8). Therefore, from Figure 1a, it is perceived that the system (12) based on the NLSE (8) supports nonlinear ion-acoustic periodic wave solution (NIAPWS), ion-acoustic kink wave solution (IAKWS) and ion-acoustic anti-kink wave solution (IAAKWS). Furthermore, the effects of q, μ_1, μ_2, μ_p, δ and σ_2 on different IAW solutions are presented graphically for $-1 < q < 0$ and $0 < q < 1$.

4.1 Analytic solutions

4.1.1 Nonlinear ion-acoustic periodic wave solution

To obtain analytical NIAPWS, we consider the Hamiltonian function $H(\psi, y)$ of the dynamical system (12) as

$$H(\psi, y) = \frac{y^2}{2} - \frac{L_1}{2}\Psi^2 + \frac{L_2}{4}\Psi^4 = h, \tag{13}$$

which simplifies to give

$$\frac{dy}{d\chi} = \sqrt{\frac{-L_2}{2}}\sqrt{(a-\Psi)(\Psi-b)(\Psi-c)(\Psi-d)}, \tag{14}$$

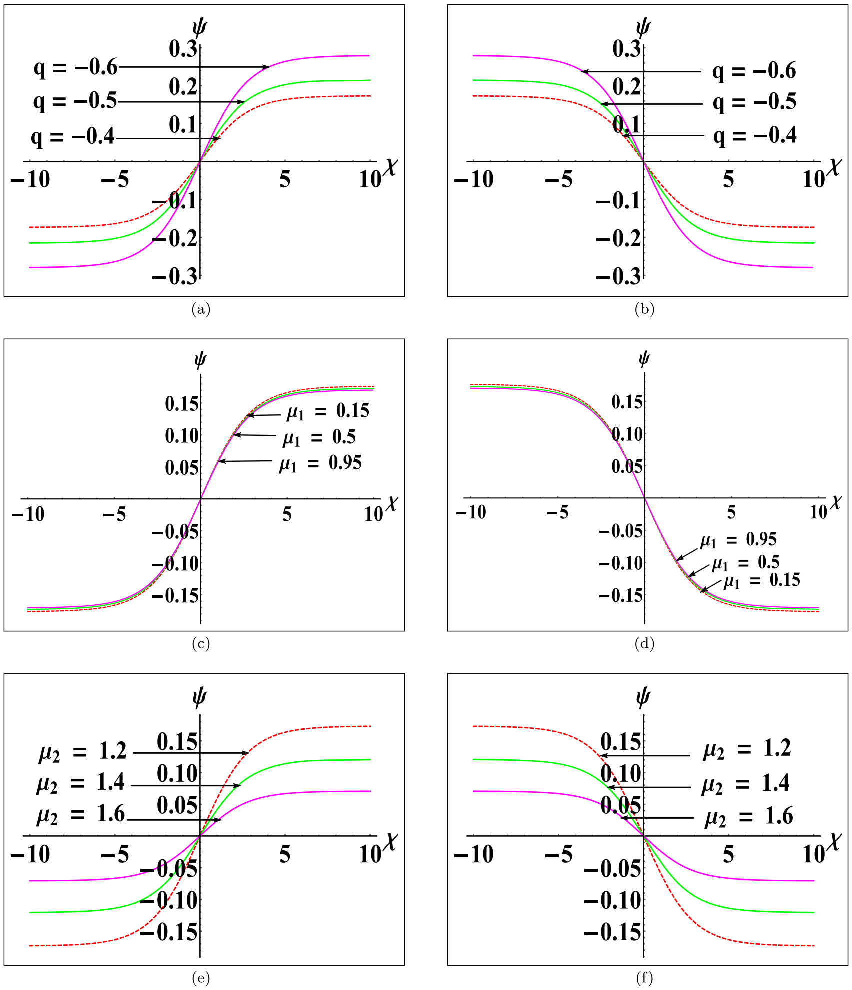

Figure 4: Effects of different parameters q, μ_1 and μ_2 on IAKWS and IAAKWS for the range $-1 < q < 0$ with other parameters same as in Figure 1a.

where a, b, c and d are roots of $h_i + \frac{-L_2}{2}\left(\Psi^4 - \frac{2L_1}{L_2}\Psi^2\right) = 0$. Using Eq. (14) in Eq. (13), we get NIAPW solution as

$$\Psi = \frac{a + d\left\{\frac{a-b}{b-d} sn^2\left(\frac{1}{g}\sqrt{\frac{-L_2}{2}}\chi, z\right)\right\}}{1 + \frac{a-b}{b-d} sn^2\left(\frac{1}{g}\sqrt{\frac{-L_2}{2}}\chi, z\right)}, \tag{15}$$

where sn is the Jacobi elliptic function [41], $g = \frac{2}{\sqrt{(a-c)(b-d)}}$ and $z = \sqrt{\frac{(a-b)(c-d)}{(a-c)(b-d)}}$.

Figure 2 shows variation of NIAPWs by changing q, μ_1, μ_2, μ_p, δ and σ_2 in the range $-1 < q < 0$ with other parameters considered same as Figure 1a. Here, we observe that the amplitude and width of NIAPW increase gradually with increase in nonextensive parameter (q) in the range $-1 < q < 0$. As observed from Figure 2b, when mass of negative ions is higher than mass of positive ions, the mass ratio μ_1 of negative and positive ions increases and hence, NIAPWs become smooth. From Figure 2c, it is observed that when number density of positive ions is relatively higher than that of negative ions, the number density ratio μ_2 of positive and negative ions increases, resulting into decrease in amplitude and width of NIAPWs. Also, it can be seen from Figure 2d that for the higher values of positron number density compared to negative ions, the number density ratio μ_p of positrons and negative ions increases and hence, the NIAPWs become smooth. From Figure 2e, we observe that with higher temperature of electrons compared to positrons, the temperature ratio of electrons and positrons (δ) increases, and this results into decrease in amplitude and width of NIAPWs. However, as observed from Figure 2f, both amplitude and width increase with higher temperature of positive ions compared to temperature of electrons via σ_2.

Figure 3 illustrates variation of NIAPWs by changing q, μ_1, μ_2, μ_p, δ and σ_2 in the range $0 < q < 1$ with other parameters considered same as Figure 1b. It is observed that when the nonextensive parameter q tends to Maxwellian distribution ($q \to 1$) in the range $0 < q < 1$, then the amplitude and width of NIAPW decrease. When mass of negative ions is comparatively higher than mass of positive ions, the mass ratio of negative and positive ions μ_1

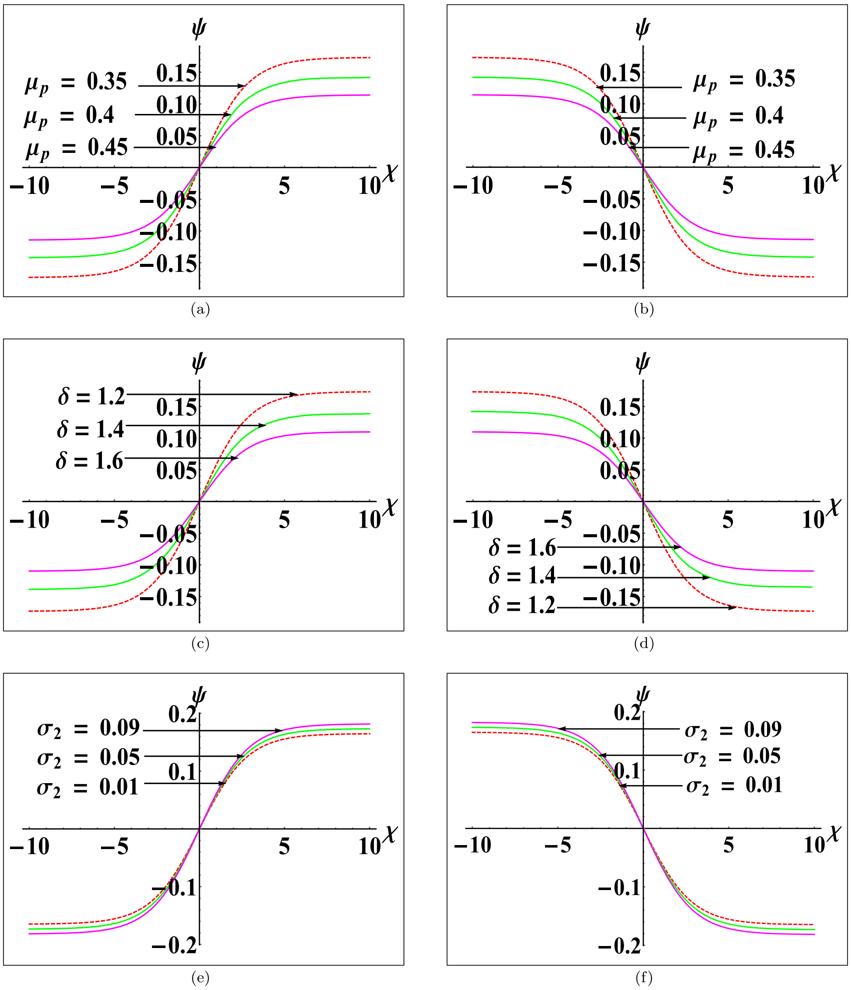

Figure 5: Effects of different parameters μ_{p}, δ *and* σ_2 on IAKWS and IAAKWS for the range $-1 < q < 0$ with other parameters same as in Figure 1a.

increases resulting into smoothness of NIAPWs. When number density of positive ions increases, the number density ratio of positive and negative ions μ_2 increases resulting into decrease in amplitude and width of NIAPWs. For higher values of positron number density than negative ion number density, the number density ratio of positrons and negative ions (μ_{p}) increases and hence, NIAPW becomes smooth. With higher temperature of electrons compared to temperature of positrons, the temperature ratio of electrons and positrons (δ), the amplitude and width of NIAPWs decrease. However, both amplitude and width increase with increase in temperature of positive ions compared to temperature of electrons in the temperature ratio of positive ions and electrons (σ_2).

4.1.2 IAKWS and IAAKWS

The analytical forms of IAKWS and IAAKWS corresponding to NHeT$_{1,0}$ (shown in Figure 1a,b) are given respectively by

$$\Psi = \pm\sqrt{\frac{L_1}{L_2}}tanh\left(\sqrt{\frac{-L_1}{2}}\chi\right). \tag{16}$$

We demonstrate the effects of parameters q, μ_1 and μ_2 on IAKWS in Figure 4a,c,e, and IAAKWS in Figure 4b,d,f for the range $-1 < q < 0$. Here, the values of other parameters are same as Figure 1a. It is observed from Figure 4a,b that when nonextensive parameter q, in the range $-1 < q < 0$, moves away from the Maxwellian limit ($q \to 1$), then smoothness of both IAKWS and IAAKWS decreases while the amplitude increases. When the mass of negative ions is comparatively higher than mass of positive ions then the mass ratio of negative and positive ions μ_1 increases. This results into decrease in amplitudes of both IAKWS and IAAKWS, and increase in smoothness as can be observed from Figure 4c,d. Also, when number density of negative ions is lesser than positive ions, then the number density ratio of positive and negative ions μ_2 grows, then smoothness of both IAKWS and IAAKWS increases as shown in Figure 4e,f.

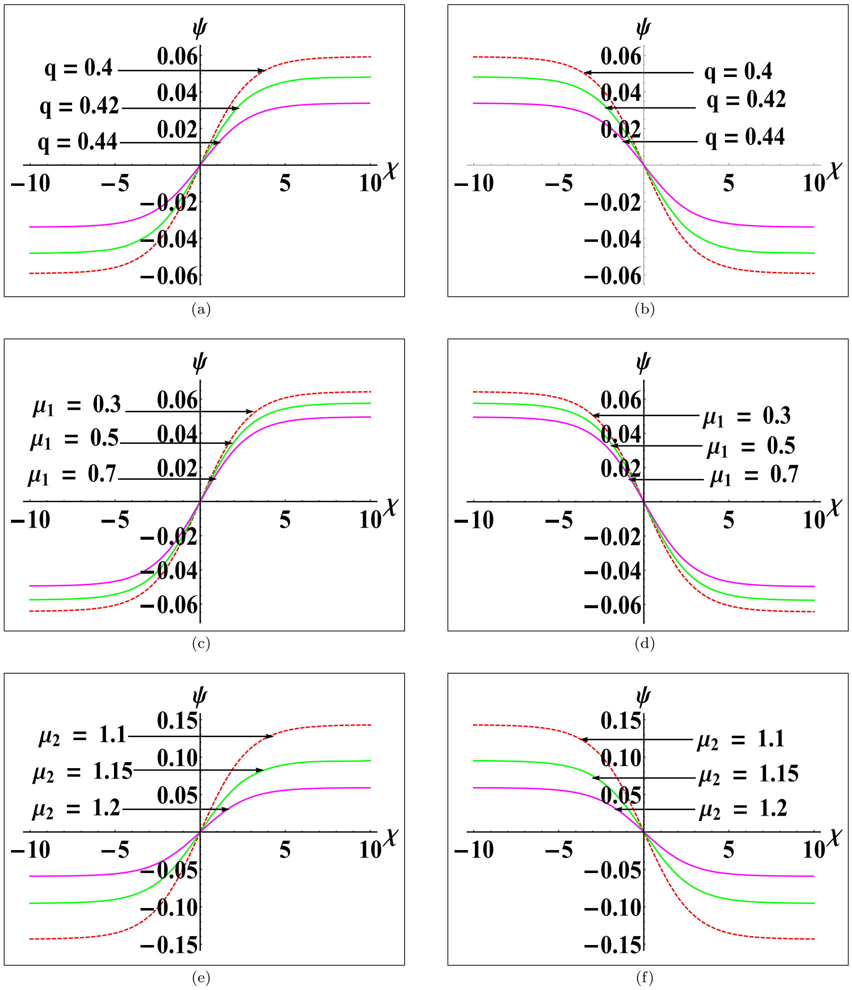

Figure 6: Effects of different parameters q, μ_1 and μ_2 on IAKWS and IAAKWS for the range $0 < q < 1$ with other parameters same as in Figure 1a.

We study the effects of parameters μ_p, δ *and* σ_2 on IAKWS in Figure 5a,c,e, and IAAKWS in Figure 5b,d,f for the range $-1 < q < 0$. Here, the values of other parameters in the range $-1 < q < 0$ are same as Figure 1a. From Figure 5a,b, we observe that amplitudes of both IAKWS and IAAKWS decrease while the smoothness increases, when number density of positrons ions is greater than number density of negative ions. When the temperature of electrons is higher than temperature of positrons, the parameter δ increases and hence, amplitudes of both IAKWS and IAAKWS decrease and the smoothness increases as can be seen Figure 5c,d. However, from Figure 5e,f, it is observed that with temperature of positive ions higher than temperature of electrons, the ratio σ_2 increases and this results into increment in amplitudes of both IAKWS and IAAKWS and decrease in smoothness.

We examine the effects of parameters q, μ_1 and μ_2 on IAKWS in Figure 6a,c,e, and IAAKWS in Figure 6b,d,f for the range $0 < q < 1$. Here, the values of other parameters are same as Figure 1b. In Figure 6a,b, we observe that when nonextensive parameter q tends to the Maxwellian limit ($q \to 1$), then amplitudes of both IAKWS and IAAKWS decrease while smoothness increases. From Figure 6c,d, it is observed that when mass of negative ions is higher than mass of positive ions, then μ_1 increases resulting into decrease in amplitudes of both IAKWS and IAAKWS. Furthermore, when number density of positive ions is greater than that of negative ions, then the number density ratio parameter of positive and negative ions μ_2 increases, and hence, smoothness of both IAKWS and IAAKWS increases as shown in Figure 6e,f.

We investigate the effects of parameters μ_p, δ *and* σ_2 on IAKWS in Figure 7a,c,e, and IAAKWS in Figure 7b,d,f for the range $0 < q < 1$. Here, the values of other parameters are same as Figure 1b. From Figure 7a,b, we observe that amplitudes of both IAKWS and IAAKWS decrease while the smoothness increases when number density of positrons ions is greater than number density of negative ions. Also, when the temperature of electrons is higher than temperature of positrons, the parameter δ increases

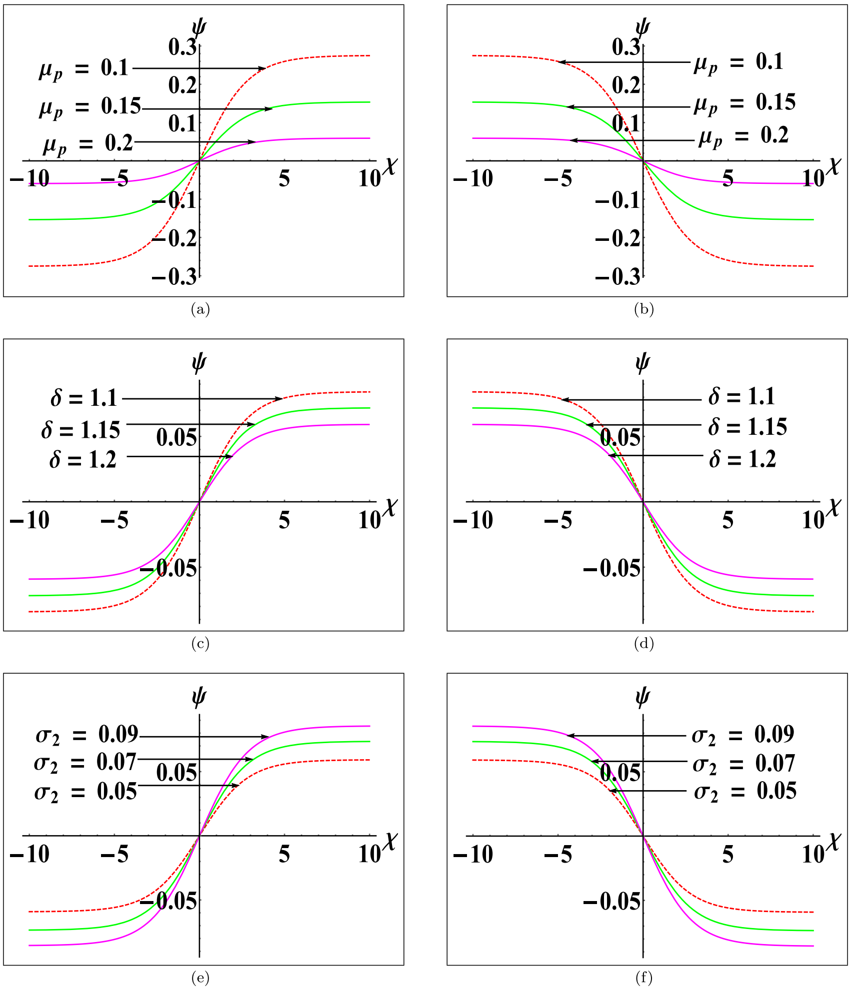

Figure 7: Effects of different parameters μ_p, δ *and* σ_2 on IAKWS and IAAKWS for the range $0 < q < 1$ with other parameters same as in Figure 1b.

and hence, amplitudes of both IAKWS and IAAKWS decrease and the smoothness increases as observed from Figure 7c,d. However, from Figure 7e,f, it is observed that with temperature of positive ions higher than temperature of electrons, the ratio (σ_2) increases and this result into increment in amplitudes of both IAKWS and IAAKWS and decrease in their smoothness.

5 Conclusions

Propagation of small-amplitude IAWs in an MPP which consists of inertial adiabatic positive and negative ions, q-nonextensive positrons and electrons, has been examined under the NLSE. By using wave transformation, the NLSE has been reduced to dynamical system. Dynamical properties of IAWs has been examined through bifurcation analyses. The phase plots have been presented for various ranges of nonextensivity q. It has been observed that the NLSE supported NIAPW, IAKW and IAAKW solutions in the range $-1 < q < 0$ and $0 < q < 1$. The effects of parameters q, μ_1, μ_2, μ_p, δ and σ_2 on different traveling IAW solutions are discussed below:

(1) NPIAWS in the ranges $-1 < q < 0$ and $0 < q < 1$: The NPIAW solution has been presented for the ranges $-1 < q < 0$ and $0 < q < 1$. It has been interesting to observed that the nonextensive parameter q shows distinctive effects on NPIAW on both the ranges $0 < q < 1$ and $-1 < q < 0$. The amplitude and width of NPIAWs have been increased when q moves away from the Maxwellian limit (i.e., from the range $-1 < q < 0$) and decreased when q tends to towards the Maxwellian limit ($0 < q < 1$). It has been observed that with increase in number densities of positive charges such as, positive ions and positrons, amplitudes and widths of NPIAW have been decreased via μ_2 and μ_p. However, with higher values of negative ion mass and electron temperature, amplitude and width of NPIAW have been increased via μ_1 and δ in both the ranges of q. However, it has been observed that the amplitude and

width of NPIAW have been increased with increase in temperature of positive ions via σ_2 for both the ranges of q.

(2) IAKWS and IAAKWS in the ranges $-1 < q < 0$ and $0 < q < 1$: The IAKWS and IAAKWS have been presented for the ranges $-1 < q < 0$ and $0 < q < 1$. The nonextensive parameter q depicts different effects on IAKWS and IAAKWS on both the ranges $0 < q < 1$ and $-1 < q < 0$. The amplitudes of IAKWS and IAAKWS have been increased when q diverge from the Maxwellian limit (i.e., from the range $-1 < q < 0$) and decreased when q converge to the Maxwellian limit (i.e., from the range $0 < q < 1$). Also, IAKWS and IAAKWS in the ranges $0 < q < 1$ and $-1 < q < 0$ have been smoothed with increase in negative ion mass (via. μ_1), positive number density (via. μ_2), positron number density (via. μ_p) and electron temperature (via. δ). However, the amplitudes of IAKWS and IAAKWS have been increased with increase in temperature of positive ions (via. σ_2) in both the ranges $-1 < q < 0$ and $0 < q < 1$.

The outcomes of this study are applicable to understand different dynamical behaviors of nonlinear IAWs in multi-pair nonextensive plasma systems. The NPIAWS, IAKWS and IAAKWS in our multipair plasma system are observed to be applicable in Earth's ionosphere, such as, D-region [H^+, O_2^-] and F-region [H^+,H^-] and multipair plasma system laboratory [C^+,C^-] [37].

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