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Resonant interactions between the fundamental and higher harmonic of positron acoustic waves in quantum plasma

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Abstract: In this paper, we study the second harmonic generation and its interaction with the fundamental mode in a magnetised dense positron-ion plasma interacting with laser pulses. It has been shown that different harmonics propagate with different phase velocities. The gradual evolution of the fundamental wave into higher harmonics is studied, and the conversion efficiency is calculated. Dependence of conversion efficiency on wavenumber shifts and the applied magnetic field has also been examined.

Keywords: laser beam; positron acoustic wave; positron-ion plasma; plasma-vacuum interface; QHD; second harmonic generation.

1 Introduction

In recent years, particle acceleration by the interaction of high-intensity laser pulse with plasma has become of great interest because of its tremendous applicability in fundamental research and industrial use. Such laser plasmas have vast applications in medical and biological fields. The development for trapping and storing positrons [1, 2] allows a large number of low-temperature positrons to form a plasma. The physical properties of such plasma may vary from the conventional ones, but the collective behaviours shown by them are complimentary to electrons in plasmas.

Positrons are fundamentally important in the study of physics because of their recombination with electron to form neutral plasmas with dynamical symmetry between charged particles [3, 4]. Such positron-hydrogen interactions have been often observed in astrophysical

environments. Many wave modes and associated instabilities are found to occur in hydrogen plasmas and solar system plasmas with an excess of positrons [5–7].

Many a time high energy electron beam propagates through a dense positron plasma. A part of them recombines releasing a tremendous amount of energy that causes newer wave modes which may or may not survive nonlinear Landau damping. Under such conditions, oblique propagation of waves is conventional [7]. If there is a magnetic field present in such a homogeneous plasma environment, both electrons and positrons may often transform from a neutral plasma to two oppositely charged species. Such a flip of state can be obtained by meticulously varying the particle densities. The easy availability of low energy positron is helpful in experimental studies of magnetic confinement, positron-atom and positron-molecule interaction, as well as often fundamental particles [8]. Because of the plasma ray astronomical studies, such interactions are of immense value [9, 10]. Slow positrons can be easily obtained from ^{22}Na isotope (with half-life 2.6 years) used along with a tungsten moderator. This ^{22}Na isotope emits positron with an energy of 545 KeV which can be slowed down to 2 eV (nonrelativistic) by a tungsten crystal [11, 12]. Now, that such a positron plasma has been contained by any physical mean, there exists a plasma vacuum boundary interface. When a wave is generated in such a plasma due to nonlinear interactions, higher-order harmonics are often found to originate. The resonant interactions between the original wave and the newly generated harmonics give rise to damping-like effects, as well as energy conversion between them. In order to study plasma harmonic generation by free particles, without the competing effect of atomic harmonic generation by bound particles, a preformed plasma is required [13].

In the present paper, we consider a laser pulse with a phase velocity comparable to the velocity of light in a dense magnetised positron plasma with low energy. We further study the conversion efficiency of the harmonic generated and the energy distribution arising out of such resonant interactions.

The paper is organized in the following way: in section 2, starting from the fundamental equations, we derive the

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linear dispersion relation of second harmonics. In the next section, we study the generation and interaction of the second harmonics with the fundamental mode. Here, we study the conversion efficiency and resonant interaction with associated energy distributions. Finally, we conclude with some remark on the scope for further study and its possible applications.

2 Basic equations and model

We consider a magnetised quantum plasma consisting of positrons and negatively charged ions occupying half-space ($x > 0$) and bounded by vacuum ($x < 0$), as shown in Figure 1. Wave is travelling along the x -axis, and the plasma is considered collision less (almost) due to the Pauli blocking mechanism. Electrostatic waves are considered to be travelling in such a completely degenerate dense plasma. The following basic equations govern the dynamics of such a plasma:

$$\frac{\partial n_p}{\partial t} + \vec{\nabla} \cdot (n_p \vec{u}_p) = 0 \quad (1)$$

$$\left(\frac{\partial}{\partial t} + \vec{u}_p \cdot \vec{\nabla} \right) (\vec{u}_p) = \frac{e\vec{E}}{m} + \frac{e}{m_p} \vec{u}_p \times (\vec{B} + \vec{b}) - \frac{1}{m_p n_p} \vec{\nabla} P_p + \frac{\hbar^2}{2m_p} \vec{\nabla} \left(\frac{\nabla^2 \sqrt{n_p}}{\sqrt{n_p}} \right) \quad (2)$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi e (n_p - Z_{ie} n_i) \quad (3)$$

where n_i = density of the negative ions, Z_{ie} = charge of the negative ions. The Laplace equation in vacuum is given as follows:

$$\nabla \phi_v = 0 \quad (4)$$

Here, u_p , P_p , $\hbar$, E , B , b , n_i and ϕ_v are the fluid velocity, degeneracy pressure, the Planck constant, electric field, radiation magnetic field, applied magnetic field, ion density and electric potential in vacuum, respectively. The last term of Eq. (2) is Bohm potential term. According to Chandrasekhar [14], one can write the degeneracy pressure in the following form:

$$P_{\text{degeneracy}} = \frac{\pi m_e^4 c^5}{3\hbar^3} [R(2R^2 - 3)\sqrt{1 + R^2} + 3 \sinh^{-1} R] \quad (5)$$

where 'c' is the speed of light in vacuum and $R = P_F/mc$ ($P_F \Rightarrow$ Fermi relativistic momentum).

Now, the basic set of Eqs. (1)–(4) is normalised by employing the following scheme (for simplicity, we will confine ourselves in one dimension in the linear analysis)

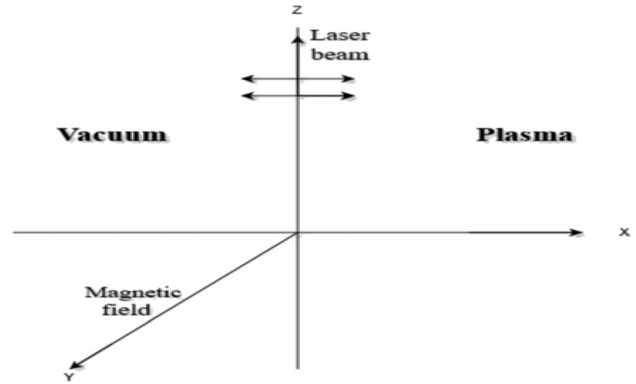


Figure 1: Schematic of polarized electromagnetic wave propagating in homogeneous plasma.

$x \rightarrow \frac{x\omega_p}{c_s}$, $t \rightarrow t\omega_p$, $\omega_j \rightarrow \frac{\omega_j}{\omega_p}$, $\phi \rightarrow \frac{e\phi}{2k_B T_F}$, $n_p \rightarrow \frac{n_p}{n_{p0}}$, $n_i \rightarrow \frac{n_i}{n_{i0}}$ and $u_e \rightarrow \frac{u_e}{c_s}$. Here, $\omega_p = \sqrt{\frac{4\pi n_{e0} e^2}{m_e}}$ is the plasma oscillation frequency and $c_s = \sqrt{\frac{2k_B T_F}{m_e}}$ is the positron acoustic speed. We obtained the following equations:

$$\frac{\partial n_p}{\partial t} + \frac{\partial n_p u_p}{\partial x} = 0 \quad (6)$$

$$\left[\frac{\partial}{\partial t} + u_p \frac{\partial}{\partial x} \right] u_p = \frac{\partial \phi}{\partial x} - \omega_c u_p - G_p \frac{\partial n_p}{\partial x} + \frac{H^2}{2} \frac{\partial}{\partial x} \left[\frac{1}{\sqrt{n_p}} \frac{\partial^2 \sqrt{n_p}}{\partial x^2} \right] \quad (7)$$

$$\frac{\partial^2 \phi}{\partial x^2} = (n_p - Z_{ie} n_i) \quad (8)$$

$$\frac{\partial^2 \phi_v}{\partial x^2} = 0 \quad (9)$$

where ω_c is the cyclotron frequency normalised with respect to plasma oscillation frequency, H is the quantum diffraction parameter defined as $H = \hbar\omega_p/2k_B T_{Fe}$, ($T_{Fe} \rightarrow$ Fermi temperature) and $G_p = \frac{m_p c^2}{6k_B T_{Fe}} (R_0^2/\sqrt{1+R_0^2} + R_0^2) [R_0 \rightarrow (n/n_0)^{(1/3)}]$

3 Linear analysis

We assume that every field quantity (f) has the following form:

$$f = f_0 + f(x) \exp[i(kz - \omega t)] + c.c \quad (10)$$

Using Eq. (10) in Eqs. (6)–(9) and solving the resulting equation under the boundary condition that electric potential is continuous across the interface, we obtain the following linear dispersion relation in one dimension:

$$\omega = \frac{1}{\sqrt{2}} \left(1 + \frac{k}{\sqrt{2}} \sqrt{\frac{\Lambda R_0}{3} + \frac{H^2 k^2}{4}} \right) \quad (11)$$

where $\Lambda = \frac{m_e c^2}{2k_B T_{Fe} Z_{ie}}$. The normalised frequency (ω) depends on the wavenumber (k), electron Fermi temperature (T_{Fe}), Quantum diffraction parameter (H) and relativistic degeneracy factor (R_0).

4 Generation of second harmonic

Now, consider a linearly polarized (along the x -axis) laser pulse propagating along the z -direction in a uniform quantum plasma containing positive and negative ions. The plasma is embedded in a transverse magnetic field in y -direction. The electric field component of the laser beam is given as follows:

$$\vec{E}_l = \frac{1}{2} \hat{e}_x E_0 e^{i(k_0 z - \omega_0 t)} + c.c \quad (12)$$

where $\omega_0 = \frac{\omega_p}{\sqrt{1-\mu^2}}$, μ is the refractive index of plasma, ω_p is plasma frequency and $\hat{e}_x$ is a unit vector along x -direction. As the wave propagates in the plasma, transverse current densities at double the fundamental frequency are generated due to nonlinear laser-plasma interactions. Now, we can write the corresponding electric fields for the fundamental and double fundamental frequencies (ω_0 and $2\omega_0$) as follows:

$$\vec{E}_1 = \frac{1}{2} \hat{e}_x E_1 e^{i(k_1 z - \omega_0 t)} + c.c \quad (13)$$

$$\vec{E}_2 = \frac{1}{2} \hat{e}_x E_2 e^{i(k_2 z - 2\omega_0 t)} + c.c \quad (14)$$

Here, E_1 and E_2 are the amplitudes; k_1 and k_2 are the propagation vectors which are given as follows:

$$k_1 = \frac{\omega_0}{c} \mu_1 \quad (15)$$

$$k_2 = 2 \frac{\omega_0}{c} \mu_2 \quad (16)$$

where μ_1 and μ_2 are the corresponding wave refractive indices for the plasma medium.

The wave equation governing the propagation of laser beam in the plasma is given by as follows:

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \vec{E} = \frac{4\pi}{c^2} \frac{\partial \vec{J}}{\partial t} \quad (17)$$

The total electric field vector due to interaction is given as follows:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 \quad (18)$$

The plasma current density due to positrons is given as follows:

$$\vec{J} = en \vec{u}_p \quad (19)$$

Using reductive perturbation expansion and following the standard procedure, we obtain the first-order transverse and longitudinal velocities as follows:

$$u_{p_x}^{(1)} = -\frac{i}{2} \left[\frac{c\lambda_1 \omega_0^2}{\omega_0^2 - \omega_c^2} e^{i(k_1 z - \omega_0 t)} + 2 \frac{c\lambda_2 \omega_0^4}{4\omega_0^2 - \omega_c^2} e^{i(k_2 z - 2\omega_0 t)} - \frac{\lambda_2 (2\omega_0 + i\omega_c) n^{(1,2)}}{(4\omega_0^2 - \omega_c^2)} - \frac{\lambda_1 (\omega_0 + i\omega_c) \Theta_{q1} n^{(1,1)}}{(\omega_0^2 - \omega_c^2)} \right] + \text{constant} \quad (20)$$

$$u_{p_z}^{(1)} = -\frac{1}{2} \left[\frac{c\lambda_1 \omega_0^2}{\omega_0^2 - \omega_c^2} e^{i(k_1 z - \omega_0 t)} + 4 \frac{c\lambda_2 \omega_0^4}{4\omega_0^2 - \omega_c^2} e^{i(k_2 z - 2\omega_0 t)} + \frac{\lambda_1 (\omega_0 - i\omega_c) \Theta_{q2} n^{(1,1)}}{(\omega_0^2 - \omega_c^2)} + \frac{\lambda_2 (2\omega_0 - i\omega_c) n^{(1,2)}}{(4\omega_0^2 - \omega_c^2)} \right] + \text{constant} \quad (21)$$

$$\left. \begin{aligned} \lambda_1 &= -\frac{eE_1}{mc\omega_0} \\ \lambda_2 &= -\frac{eE_2}{mc\omega_0} \\ \omega_c &= -\frac{eb}{mc} \\ \Theta_{q1} &= \left\{ \frac{V_F^2}{n_0} + \frac{\hbar^2}{4m^2} \frac{1}{n_0} k_1^2 \right\} k_1 \\ \Theta_{q2} &= \left\{ \frac{V_F^2}{n_0} + \frac{\hbar^2}{4m^2} \frac{1}{n_0} k_2^2 \right\} k_2 \end{aligned} \right\} \quad (22)$$

While obtaining above velocity components (Eqs. 20 and 21), we assumed the first-order perturbed plasma density $n^{(1)}$ containing only the contribution from the fundamentals and first harmonic terms $n^{(1)} = n^{(1,1)} + n^{(1,2)}$, where $n^{(1,1)}$ and $n^{(1,2)}$ vary as $n_1 e^{i(k_1 z - \omega_0 t)}$ and $n_2 e^{i(k_2 z - 2\omega_0 t)}$. Now, substituting perturbed positron density into continuity equation, we derive the first-order plasma positron density as follows:

$$n^{(1)} = \frac{1}{2} \left[\frac{(c\omega_c \omega_0 - 2\omega_0 \Theta_{q1} n_1 + 2i\omega_c \Theta_{q1} n_1) k_1 n_0 \lambda_1}{(\omega_0^3 - \omega_c^2 \omega_0)} e^{i(k_1 z - \omega_0 t)} + \frac{(c\omega_c \omega_0 - 4\omega_0 \Theta_{q2} n_1 + 2i\omega_c \Theta_{q2} n_2) k_2 n_0 \lambda_2}{4(\omega_0^3 - \omega_c^2 \omega_0)} e^{i(k_2 z - 2\omega_0 t)} \right] + \text{constant} \quad (23)$$

in which

$$n_1 = \frac{-ck_1 n_0 \omega_c \omega_0^2}{2[\omega_0^2(\omega_0^2 - \omega_c^2) + n_0 \omega_0 k_1(\omega_0 - i\omega_c)\Theta_{q_1}]}$$

$$n_2 = \frac{-2ck_2 n_0 \omega_c \omega_0^2}{2[4\omega_0^2(4\omega_0^2 - \omega_c^2) + n_0 \omega_0 k_2(\omega_0 - i\omega_c)\Theta_{q_2}]}$$

Similarly, we can calculate the second ordered transverse and longitudinal velocities as follows:

$$u_{p_x}^{(2)} = \left[\frac{i}{4} \frac{c^2 k_1 \omega_0^2 \omega_c (\omega_0^2 - 4\omega_c^2) \lambda_1^2}{(\omega_0^2 - \omega_c^2)(4\omega_0^2 - \omega_c^2)} + \frac{\lambda_1^2 k_1 \{p_2 \Theta_{q_2} n_1^2 - p_1 \Theta_{q_1} n_1\}}{(\omega_0^2 - \omega_c^2)(4\omega_0^2 - \omega_c^2)} \right] e^{2i(k_1 z - \omega_0 t)} \quad (24)$$

$$u_{p_z}^{(2)} = \frac{\lambda_1^2 k_1 c^2 \{3\omega_0^3 \omega_c^2 + (\omega_0^3 - \omega_c^2 \omega_0)(4\omega_0^2 - 2\omega_c^2)\}}{4(\omega_0^2 - \omega_c^2)(4\omega_0^2 - \omega_c^2)} e^{2i(k_1 z - \omega_0 t)} - \left[\frac{2\lambda_1^2 k_1 (\omega_0 \omega_c - i\omega_c^2)(c\omega_0^2 + 2\omega_0^2 - 2\omega_c^2)\Theta_{q_1} n_1}{(\omega_0^2 - \omega_c^2)(4\omega_0^2 - \omega_c^2)} + \frac{2(\omega_0^2 - i\omega_c \omega_0)(c\omega_c^2 - \omega_c^2 + 2\omega_0^2)\lambda_1^2 k_1 \Theta_{q_1} n_1^2}{(\omega_0^2 - \omega_c^2)(4\omega_0^2 - \omega_c^2)} \right] e^{2i(k_1 z - \omega_0 t)} \quad (25)$$

The second-order x -component of velocity is generated due to uniform magnetic field and reduces to zero in its absence. However, the z -component of velocity is due to the magnetic vector of the radiation field.

Substituting first-order densities and second-order velocities in the continuity equation, we get the following equation:

$$n^{(2)} = \left[\frac{3\lambda_1^2 k_1^2 n_0 c^2 \omega_0^2 \omega_c^2}{4(4\omega_0^2 - \omega_c^2)(\omega_0^2 - \omega_c^2)} + \frac{\lambda_1^2 k_1^2 n_0 c^2 (\omega_0^2 - \omega_c^2)(4\omega_0^2 - 2\omega_c^2)}{4(4\omega_0^2 - \omega_c^2)(\omega_0^2 - \omega_c^2)} \frac{n_0 k_1^2}{\omega_0(4\omega_0^2 - \omega_c^2)(\omega_0^2 - \omega_c^2)} \right. \\ \left. \left\{ \omega_c(\omega_0 - i\omega_c) \left(\omega_0^2 - \omega_c^2 + \frac{c\omega_0^2}{2} \right) \Theta_{q_1} n_1 + i\omega_0(\omega_0 - i\omega_c) \left(\omega_0^2 - \omega_c^2 + \frac{c\omega_c^2}{2} \right) \Theta_{q_1} n_1^2 \right\} \right] e^{2i(k_1 z - \omega_0 t)} \quad (26)$$

The perturbed velocities and densities are used to obtain the transverse current density from Eq. (19)

$$J_x = \left\{ \frac{ien_0 c \lambda_1 \omega_0^2}{2(\omega_0^2 - \omega_c^2)} - \frac{en_0 \lambda_1 (\omega_0 + i\omega_c) n_1 \Theta_{q_1}}{\omega_0^2 - \omega_c^2} \right\} e^{i(k_1 z - \omega_0 t)} - \left\{ \frac{iecn_0 \omega_0^2 \lambda_2}{(4\omega_0^2 - \omega_c^2)} - \frac{en_0 \lambda_2 (2\omega_0 + i\omega_c) n_2 \Theta_{q_2}}{4\omega_0^2 - \omega_c^2} \right\} e^{i(k_1 z - 2\omega_0 t)} + \beta e^{2i(k_1 z - \omega_0 t)} + \text{constant} \quad (27)$$

where

$$\beta = \frac{ek_1 n_0}{(4\omega_0^2 - \omega_c^2)(\omega_0^2 - \omega_c^2)} \left[-\frac{i}{4} \lambda_1^2 c^2 \omega_0^2 \omega_c (\omega_0^2 + \omega_c^2) - \lambda_1^2 n_2 \Theta_{q_1} \left\{ p_1 + \left(\frac{c(4\omega_0^2 - \omega_c^2)}{2} \right) \times (i(\omega_0 - i\omega_c)\omega_0 - (\omega_0 + i\omega_c)\omega_c) \right\} + \{(\omega_0^2 + \omega_c^2)(4\omega_0^2 - \omega_c^2) - \omega_0 p_2\} \right] \times \left(\frac{\Theta_{q_1}^2 n_1^2}{\omega_0} \right)$$

$$p_1 = \omega_0(\omega_0 - i\omega_c) \{2i(\omega_0^2 - \omega_c^2) + ic(\omega_0^2 + \omega_c^2) + (\omega_0 + i\omega_c)(\omega_0^2 - \omega_c^2 + c\omega_0 \omega_c)\}$$

$$p_2 = 2\omega_0(\omega_0^2 + \omega_c^2) + i\omega_c(\omega_0 - i\omega_c)^2$$

Using J_x , we obtain the first and second harmonic dispersion relations as follows:

$$c^2 k_1^2 = \omega_0^2 - \frac{\omega_0^2 \omega_p^2}{(\omega_0^2 - \omega_c^2)} \quad (28)$$

$$c^2 k_2^2 = 4\omega_0^2 - \frac{4\omega_0^2 \omega_p^2}{(4\omega_0^2 - \omega_c^2)} \quad (29)$$

To obtain the amplitude of the second harmonic term, we substitute the current density (Eq. 19) in the wave Eq. (17), equate the second harmonic terms and then after proper manipulation, we obtain the following equation:

$$\frac{\partial \lambda_2(z)}{\partial z} = \frac{3i\lambda_1^2 k_1}{2c} \frac{\omega_p^2 \omega_0^2 \omega_c (\omega_0^2 + \omega_c^2)}{k_2(\omega_0^2 - \omega_c^2)(4\omega_0^2 - \omega_c^2)} e^{i\Delta k z} \quad (30)$$

$$\lambda_2 = \left[\frac{3\lambda_1^2 \omega_c \omega_p^2}{8c\omega_0^2} \left\{ \frac{\left\{ 1 - \frac{\omega_p^2}{\omega_0^2} \left(1 - \frac{\omega_c^2}{\omega_0^2} \right) \right\}^{1/2} \left(1 + \frac{\omega_c^2}{\omega_0^2} \right)}{\left\{ 1 - \frac{\omega_p^2}{4\omega_0^2} \left(1 - \frac{\omega_c^2}{4\omega_0^2} \right) \right\}^{-1/2} \left(1 - \frac{\omega_c^2}{\omega_0^2} \right)^2 \left(1 - \frac{\omega_c^2}{4\omega_0^2} \right)} \right\} - \frac{4\omega_p^2 \lambda_1 k_1 n_1 \Theta_{q_1}}{c^3 k^2 (4\omega_0^2 - \omega_c^2)(\omega_0^2 - \omega_c^2)^2} \Gamma \right] e^{i\Delta k z/2} \frac{\sin(\Delta k z/2)}{\Delta k} \quad (31)$$

where, wavenumber shift Δk is given as follows:

$$\Delta k = k_2 - 2k_1 \quad (32)$$

The second harmonic conversion efficiency is given as follows:

$$\eta = \frac{\mu_1 |\lambda_2|^2}{\mu_2 |\lambda_1|^2} \quad (33)$$

$$\eta = \frac{9\lambda_1^2 \omega_c^2 \omega_p^4}{16c^2 \omega_0^4} \left[\frac{\left\{ 1 - \frac{\omega_p^2}{\omega_0^2} \left(1 - \frac{\omega_c^2}{\omega_0^2} \right) \right\}^{1/2} \left(1 + \frac{\omega_c^2}{\omega_0^2} \right)}{\left\{ 1 - \frac{\omega_p^2}{4\omega_0^2} \left(1 - \frac{\omega_c^2}{4\omega_0^2} \right) \right\}^{1/2} \left(1 - \frac{\omega_c^2}{\omega_0^2} \right)^2 \left(1 - \frac{\omega_c^2}{4\omega_0^2} \right)} + \frac{4\omega_p^4 k_1 n_1 \Theta_{q_1} \Gamma}{2c^6 k_2 (\omega_0^2 - \omega_c^2)^4 (4\omega_0^2 - \omega_c^2)^2} \right] \frac{\sin^2(\Delta k z / 2)}{\Delta k} \quad (34)$$

$$\eta_{\max} = \frac{9\lambda_1^2 \omega_c^2 \omega_p^4}{16c^2 \omega_0^4} \left[\frac{\left\{ 1 - \frac{\omega_p^2}{\omega_0^2} \left(1 - \frac{\omega_c^2}{\omega_0^2} \right) \right\}^{1/2} \left(1 + \frac{\omega_c^2}{\omega_0^2} \right)}{\left\{ 1 - \frac{\omega_p^2}{4\omega_0^2} \left(1 - \frac{\omega_c^2}{4\omega_0^2} \right) \right\}^{1/2} \left(1 - \frac{\omega_c^2}{\omega_0^2} \right)^2 \left(1 - \frac{\omega_c^2}{4\omega_0^2} \right)} + \frac{4\omega_p^4 k_1 n_1 \Theta_{q_1} \Gamma}{2c^6 k_2 (\omega_0^2 - \omega_c^2)^4 (4\omega_0^2 - \omega_c^2)^2} \right] \quad (35)$$

in which

$$\Gamma = \left[n_1 \Theta_{q_1} (\omega_0^2 + \omega_c^2) (4\omega_0^2 - \omega_c^2) - \omega_0 p_2 + p_1 + \frac{c}{2} (4\omega_0^2 - \omega_c^2) \times \{ i(\omega_0 - i\omega_c)\omega_0 - (\omega_0 + i\omega_c)\omega_c \} \right]$$

5 Results

We have calculated the conversion efficiency of the fundamental into the second harmonic. We have studied the relationship between the frequency and wavenumber shift (Δk) and its dependence on the applied magnetic field. The corresponding results are shown graphically in Figures 2 and 3. Figure 4 shows the dependence of conversion efficiency on

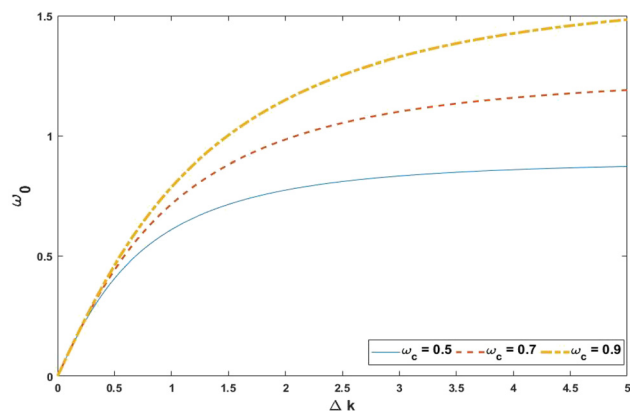


Figure 2: η_0 versus Δk with a varying magnetic field.

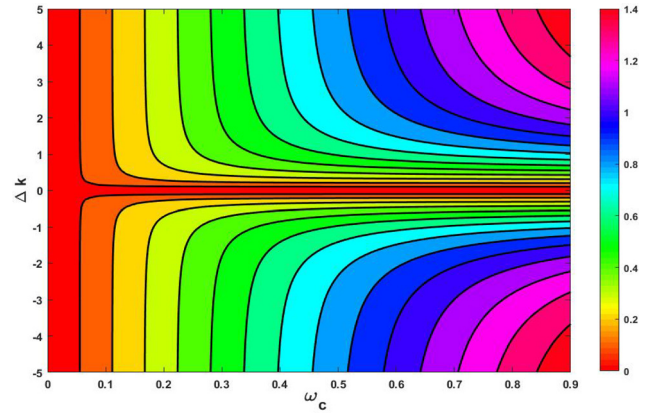


Figure 3: A contour plot of ω_0 in $(\Delta k - \omega_c)$ plane.

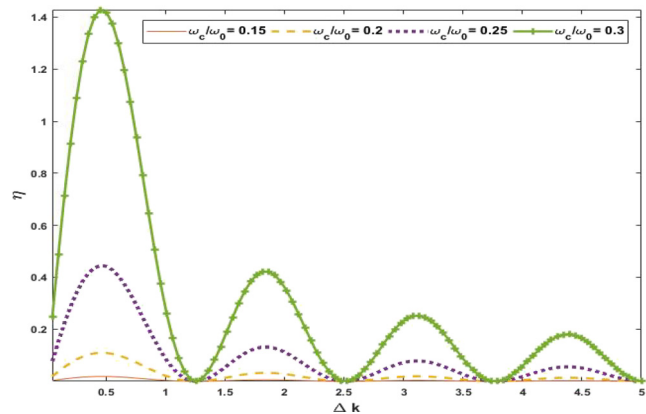


Figure 4: η versus Δk with varying $\frac{\omega_c}{\omega_0}$.

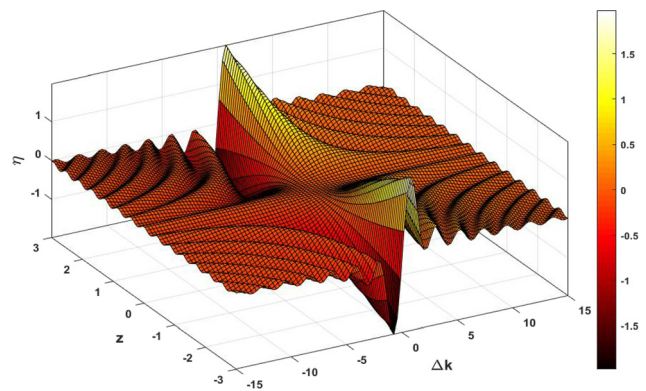


Figure 5: Variation of conversion efficiency (η) with $(z - \Delta k)$.

the wavenumber shift for various values of the applied magnetic field. It shows that conversion efficiency is large for small wavenumber shift. Figure 5 shows the conversion efficiency length of wavenumber shift ($\eta - z - \Delta k$) plots. The figure is symmetric and there is gradual decay along the length and flip in directionality due to the phase factor of the

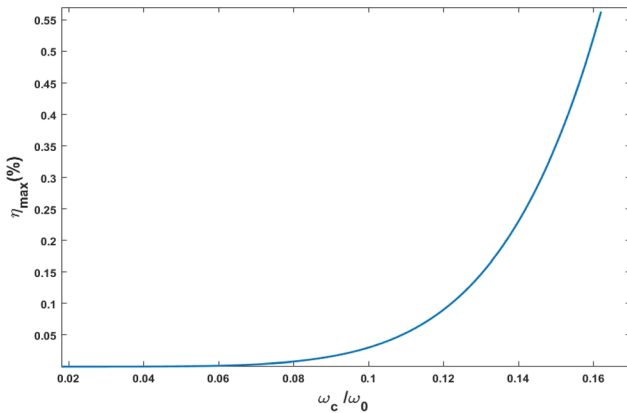


Figure 6: Variation of maximum conversion efficiency η_{max} with $\frac{\omega_c}{\omega_0}$.

propagating wave. As we know harmonics carry energy with it, the energy exchange during such a harmonic conversion becomes important to study losses in the dispersive media. Figure 6 shows the variation of maximum conversion efficiency (η_{max}) with the magnetic field. It is evident that for low magnetic field strength, η_{max} increases very slowly with magnetic field, but for strong field, it increases rapidly.

6 Conclusions

In this work, analytical study of second harmonic generation by a polarized laser pulse in homogeneous dense positron-ion plasma has been presented. The Lorentz force acting on plasma positrons introduces changes in relativistic mass and causes positron density perturbations, leading to change in the propagation characteristics of the laser beam. The wave equation governing the evolution of second harmonic is set up by using nonlinear current density arising due to the fundamental radiation. The slowly varying second harmonic amplitude and the conversion efficiency are obtained by solving the wave equation. This present study reveals that the maximum conversion efficiency increases with magnetic field strength and decreases with wave-number shift. Our finding will help in the study of laser-plasma interaction, as well as laser beam interacting, with any nonlinear optical medium.

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