Pinki Shome, Biswajit Sahu* and Swarup Poria

Nonlinear dynamics of ion-acoustic waves in quantum plasmas with exchange-correlation effects

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Abstract: Nonlinear properties of ion-acoustic waves (IAWs) are studied in electron-ion (EI) degenerate plasma with the electron exchange-correlation effects by using the quantum hydrodynamic (QHD) model. To investigate arbitrary amplitude IAWs, we have reduced the model equations into a system of ordinary differential equations using a traveling wave transformation. Computational investigations have been performed to examine the combined effect of Bohm potential and exchange-correlation potential significantly modifies the dynamics of IAWs by employing the concept of dynamical systems. The equilibrium points of the model are determined and its stability natures are analyzed. The phase portrait and Poincaré return map of the dynamical system are displayed numerically. Quasiperiodic as well as chaotic dynamics of the system are confirmed through the Poincaré return map diagrams.

Keywords: degenerate plasma; exchange-correlation effects; ion-acoustic waves; nonlinear structures.

1 Introduction

Studies in quantum plasmas have become important due to their potential applications in the context of quantum nano-diodes, nanophotonics and nanowires, nanoplasmonics [1], spintronics [2], microplasma systems, and small semiconductor devices, such as quantum wells [3], piezomagnetic quantum dots [4] and microelectronics [5], nonlinear optics [6], astrophysics [7], and solid density target experiments [8]. The quantum plasmas were first studied by Pines [9] in regimes where we have a high density and a low temperature as compared to classical plasmas. In quantum plasmas, since the de Broglie wavelength of the charge carriers is larger than the Debye wavelength and is near to the Fermi wavelength, quantum effects associated with the strong density correlation play a crucial role in plasma dynamics. The quantum hydrodynamic (QHD) model is a popular approach to describe the charged particle systems [10, 11] in quantum regime. The QHD model consists of a set of equations that include quantum effects through the so-called Bohm potential [12] and thus fully deserve the qualification of “QHDs” [3]. Several theoretical attempts have been made to investigate collective processes in the field of various quantum plasma systems [13–18]. Ali et al. [19] investigated the linear and nonlinear properties of the ion-acoustic waves (IAWs) using the QHD equations together with the Poisson equation in a three-component quantum electron-positron-ion plasma. Misra et al. [20] studied the nonlinear propagation of two-dimensional quantum IAWs (QIAWs) in electron-ion (EI) quantum plasma.

The quantum mechanics play a crucial role in the nonlinear dynamics of plasmas due to the overlapping of electron wave functions owing to the Heisenberg uncertainty principle, leading to electron tunneling though the quantum Bohm potential [12], and electron exchange and electron correlations [21] because of the electron-one-half spin effect [22, 23]. Inclusion of these forces in the collective behavior of the dense quantum plasma plays an important role, since the physical phenomena appear on the atomic and nanoscales. Quantum dispersive effects can also be important for diagnostics of inertial fusion plasmas [24]. The interactions between the electrons can be separated into a Hartree term due to the electrostatic potential of the total electron density and an electron exchange-correlation term [25]. The electron exchange-correlation term for the first time has been considered in QHD by Crouseilles et al. [26]. Recently, Shukla and Eliasson [27] discussed that the influence of the electron-exchange due to the electron −1/2 spin plays an important role in the electric potential and plasma dielectric function in degenerate quantum plasmas. Also, it has been shown that the velocity associated with the electron exchange effect alters the quantum

***Corresponding author: Biswajit Sahu**, Department of Mathematics, West Bengal State University, Barasat, Kolkata, 700126, India, E-mail: biswajit_sahu@yahoo.co.in. https://orcid.org/0000-0002-3394-0723
Pinki Shome and Swarup Poria: Department of Applied Mathematics, University of Calcutta, 92 APC Road, Kolkata, 700009, India

recoil effect in degenerate quantum plasmas. Plasmas where exchange effects can be important occur in e.g., laser-plasma interaction experiments on solid targets, such as in inertial confinement fusion schemes.

Thus, it would be expected that the nonlinear dynamics of the plasma waves in dense quantum plasmas including the electron exchange-correlation effects are quite different from those in classical plasmas since the influence of electron exchange-correlation and the Bohm potential alters the plasma dielectric function in quantum plasmas. Many authors have studied the influence of electron exchange-correlation term and Bohm force on the propagation characteristics of several waves [28–35] in plasmas and showed the relevance of exchange-correlation potential in context with solid-state plasmas, inertial confinement fusion plasmas and in an astrophysical region such as white dwarf stars.

The influence of electron-exchange and quantum screening on the collisional entanglement fidelity for the elastic EI collision is investigated by Hong and Jung [36]. Khan et al. [37] studied a theory for the long range oscillatory wake potential with exchange-correlation due to the motion of a test charge in strongly magnetized quantum plasmas. However, since both laboratory and space plasmas have finite ion temperatures, it is necessary to investigate the existence of nonlinear plasma waves for finite ion and electron temperatures. Chatterjee et al. [38] studied the effect of ion temperature on the arbitrary amplitude IA solitary waves in quantum EI plasma. Recently, a number of works on nonlinear waves in plasmas have been reported to study the quasiperiodic and chaotic behavior of the plasma system [39–42]. Purpose of this paper is to present an investigation of the nonlinear dynamics of QIAWs in quantum plasmas including quantum recoil effects, e.g., tunneling of degenerate plasma species through the Bohm potential, as well as interaction of exchange and correlation effects via number density. Dynamical system governing the plasma system has been formulated. Equilibrium points and their local stability natures are determined. Numerical simulation results are presented to show wide variety of qualitatively different dynamics in the model.

2 Theoretical model

We consider homogeneous, unmagnetized, quantum plasmas consisting of degenerate electrons, and nondegenerate ions with the effects of exchange-correlation potential to study the nonlinear propagation of ion acoustic waves. The basic equations describing the nonlinear dynamics of the low phase speed ($kV_{Fi} \ll \omega \ll kV_{Fe}$) QIAWs in one-dimensional quantum plasma are governed by [28, 43]

$$\frac{\partial n_e}{\partial t} + \frac{\partial (n_e u_e)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = 0 \tag{2}$$

$$\frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} = \frac{e}{m_e}\frac{\partial \phi}{\partial x} - \frac{1}{m_e n_e}\frac{\partial p_e}{\partial x} + \frac{\hbar^2}{2m_e^2}\frac{\partial}{\partial x}\left(\frac{\partial^2 \sqrt{n_e}/\partial x^2}{\sqrt{n_e}}\right) - \frac{1}{m_e}\frac{\partial V_{xc}}{\partial x} \tag{3}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{e}{m_i}\frac{\partial \phi}{\partial x} \tag{4}$$

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e (n_e - n_i) \tag{5}$$

Here, $n_e(n_i)$ is the electrons (ions) number density, $u_e(u_i)$ the fluid velocity of the electrons (ions), $m_e(m_i)$ the mass of electrons (ions), p_e the pressure of electrons, e the magnitude of the electron charge, ϕ the electrostatic potential, $\hbar$ the Planck constant divided by 2π. The second term in the right-hand side of Eq. (3) is known as the quantum statistical effect due to the Fermionic behavior of the plasma particles and the third term represents the Bohm potential effect due to the influence of the quantum-diffraction [10]. This potential arises directly from the Schr ö dinger equation, and it is responsible for quantum-like behavior involving tunneling and wave-packet spreading and comes from the nonlinear coupling between the scalar potential associated with the space charge electric field and the electron/ion wave function [10, 11]. The last term in the right-hand side of Equation (3) represents the additional potential due to the influence of the electron-exchange caused by the electron-spin. Such an exchange-correlation potential for the electrons is given by [21] $V_{xc} = -0.985\frac{e^2}{\epsilon}n_e^{1/3}\left[1 + \frac{0.034}{a_B^* n_e^{1/3}} ln(1 + 18.37 a_B^* n_e^{1/3})\right]$, where $a_B^* = \epsilon\hbar^2/m_e e^2$, ϵ is the effective dielectric permeability of the material. Because $18.37 a_B^* n_e^{1/3} \ll 1$, for the sake of simplicity we can take $V_{xc} = -1.6\frac{e^2}{\epsilon}n_e^{1/3} n_e^{1/3} + 5.65\frac{\hbar^2}{m_e}n_e^{2/3}$.

The pressure for electrons are given by

$$p_e = \frac{m_e V_{Fe}^2}{3n_0^2} n_e^3 \tag{6}$$

where n_0 is the equilibrium density for both electrons and ions, and v_{Fe} is the electron Fermi velocity. It is connected to the Fermi temperature by $m_e v_{Fe}^2/2 = k_B T_{Fe}$, k_B is the Boltzmann's constant.

Now, to avoid inaccuracy in defining the Mach number, before the normalization procedure, let us follow the investigation by Dubinov [44]. The linear dispersion relation can be obtained from linearized form of Eqs. (1)–(5) by considering the perturbations of all dynamical variables of the form $e^{i(kx-\omega t)}$, where ω and k are the wave frequency and wave number of perturbations. Thus, we obtain the dispersion relation as

$$1-\frac{\omega_{pe}^2}{\omega^2-v_{Fe}^2k^2-\frac{\hbar^2k^4}{4m_e^2}+\Gamma_e k^2}-\frac{\omega_{pi}^2}{\omega^2}=0, \tag{7}$$

where $\omega_{pj}=\left(\frac{4\pi n_0 e^2}{m_j}\right)^{1/2}$ is the plasma frequency for the jth particle and $\Gamma_e=\frac{0.53e^2n_0^{1/3}}{m_e\epsilon}-\frac{3.77\hbar^2n_0^{2/3}}{m_e^2}$ is the term corresponding to electron exchange-correlation. Carrying out some simple algebra, one can obtain the dispersion relation which contains two branches:

$$\omega^2=\frac{1}{2}\left[\left(\omega_{pe}^2+\omega_{pi}^2+C_1\right)\pm\sqrt{\left(\omega_{pe}^2+\omega_{pi}^2+C_1\right)^2-4C_1\omega_{pi}^2}\,\right], \tag{8}$$

where $C_1=v_{Fe}^2k^2+\frac{\hbar^2k^4}{4m_e^2}+\Gamma_e k^2$. The plus and minus sign before the square root in Equation (8), respectively, correspond to the plasmon branch, for which the electron and ion oscillations in wave are anti-phase and the acoustic branch, for which the electron and ion oscillations are in phase. Thus, the IAW can have two-tones, which propagate with identical phase velocities [45, 46]. Therefore, it may be concluded that periodic IAWs in the quantum plasma system show two-tones, i.e. they are synchronous superpositions of ion-plasma oscillations and free quantum ion oscillations. Equation (8) shows that the dispersion curves get significantly modified by the effects of the exchange-correlation coefficient. Now, the expression for velocity of sound for the acoustic branch can be determined by:

$$C_s=\lim_{k\to 0}\frac{\omega}{k}=\omega_{pi}\sqrt{\lambda_{Fe}^2+\frac{\Gamma_e}{\omega_{pe}^2}}, \tag{9}$$

where $\lambda_{Fe}=v_{Fe}/\omega_{pe}$.

Now we introduce the following normalization. $\bar{x}=\omega_{pi}x/c_i$, $\bar{t}=\omega_{pi}t$, $\bar{u}_{e,i}=u_{e,i}/(2k_BT_{Fe}/m_i)^{1/2}$, $\bar{\phi}=e\phi/2k_BT_{Fe}$, $\bar{n}_e=n_e/n_0$, $\bar{n}_i=n_i/n_0$, where $\omega_{pe}=\left(\frac{4\pi e^2 n_0}{m_e}\right)^{1/2}$ and $\omega_{pi}=\left(\frac{4\pi e^2 n_0}{m_i}\right)^{1/2}$. Using these new variables and dropping the bars, we have from equation (3) and (4) respectively,

$$\frac{m_e}{m_i}\left(\frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x}\right)=\frac{\partial\phi}{\partial x}-n_e\frac{\partial n_e}{\partial x}+\frac{H^2}{2}\frac{\partial}{\partial x}\left(\frac{\partial^2\sqrt{n_e}/\partial x^2}{\sqrt{n_e}}\right)-\lambda\frac{\partial n_e^{2/3}}{\partial x}+\gamma\frac{\partial n_e^{1/3}}{\partial x} \tag{10}$$

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}=-\frac{\partial\phi}{\partial x} \tag{11}$$

where

$$H=\frac{\hbar\omega_{pe}}{2k_BT_{Fe}},\quad \lambda=\frac{5.65\hbar^2n_0^{2/3}}{2k_BT_{Fe}m_e},\quad \gamma=\frac{1.6e^2n_0^{1/3}}{2\epsilon k_BT_{Fe}}. \tag{12}$$

The parameter H measures the effects of quantum diffraction. Physically, H is essentially the ratio between the electron plasmon energy and the electron Fermi energy. The parameters λ and γ are two parameters, due to the exchange-correlation potential.

Neglecting $m_e/m_i(\ll 1)$ and using the boundary condition $n_e=1$ and $\phi=0$ at infinity, we have the normalized equations

$$\frac{\partial n_i}{\partial t}+\frac{\partial(n_iu_i)}{\partial x}=0 \tag{13}$$

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}=-\frac{\partial\phi}{\partial x} \tag{14}$$

$$\frac{\partial^2\phi}{\partial x^2}=n_e-n_i \tag{15}$$

$$\phi=-\frac{1}{2}+\frac{n_e^2}{2}-\frac{H^2}{2}\frac{1}{\sqrt{n_e}}\frac{\partial^2\sqrt{n_e}}{\partial x^2}+\lambda n_e^{\frac{2}{3}}-\gamma n_e^{\frac{1}{3}}-(\lambda-\gamma) \tag{16}$$

To study the arbitrary amplitude IAWs, we suppose that all the quantities depend on $\xi=x-Mt$, where M is the normalized nonlinear wave velocity. It should be mentioned that M is not the Mach number [44], since we have normalized it with a speed different from (9).

Now, under the appropriate boundary conditions, viz. $u_i\to 0$, $n_i\to 1$, $\phi\to 0$ as $\xi\to\pm\infty$, integration of equations (13) and (14) yield the following

$$n_i=\frac{M}{\sqrt{M^2-2\phi}} \tag{17}$$

Defining $n_e=A^2$ and using (15), (16) and (17), we obtain the system of second order differential equations:

$$\frac{d^2A}{d\xi^2}=\frac{A}{H^2}\left[A^4+2\lambda A^{\frac{4}{3}}-2\gamma A^{\frac{2}{3}}-2\phi-1-2(\lambda-\gamma)\right], \tag{18}$$

$$\frac{d^2\phi}{d\xi^2}=A^2-\frac{M}{\sqrt{M^2-2\phi}}.$$

3 Nonlinear analysis

The system (18) cannot be solved analytically to get solutions in closed form. If we write $A = X_1$, $dA/d\xi = X_2$, $\phi = X_3$, $d\phi/d\xi = X_4$, then system (18) takes the following form,

$$\frac{dX}{d\xi} = F(X) \tag{19}$$

where

$$X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} \quad \text{and} F = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix}$$

with

$$f_1 = X_2,$$

$$f_2 = \frac{X_1}{H^2}\left[X_1^4 + 2\lambda X_1^{\frac{4}{3}} - 2\gamma X_1^{\frac{2}{3}} - 2X_3 - 1 - 2(\lambda - \gamma)\right],$$

$$f_3 = X_4,$$

$$f_4 = X_1^2 - \left[\frac{M}{\sqrt{M^2 - 2X_3}}\right].$$

The equilibrium points of system (19) are given by

$$X_2^0 = 0, X_4^0 = 0 \tag{20}$$

and the real solutions of the system

$$X_1^4 = 2\lambda X_1^{\frac{4}{3}} - 2\gamma X_1^{\frac{2}{3}} - 2X_3 - 1 - 2(\lambda - \gamma) = 0, \tag{21}$$

$$X_1^2 = \left[\frac{M}{\sqrt{M^2 - 2X_3}}\right].$$

The variational matrix at the equilibrium point $(A_0, 0, \phi_0, 0)$ is

$$J_{var} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \Delta_1 & 0 & \frac{-2A_0}{H^2} & 0 \\ 0 & 0 & 0 & 1 \\ 2A_0 & 0 & \Delta_2 & 0 \end{pmatrix}$$

where

$$\Delta_1 = \frac{1}{H^2}\left[4A_0^4 + \frac{8}{3}\lambda A_0^{\frac{4}{3}} - \frac{4}{3}\gamma A_0^{\frac{2}{3}}\right]$$

$$\Delta_2 = \left[-M(M^2 - 2\phi_0)^{\frac{-3}{2}}\right].$$

The corresponding characteristic equation (assuming linearized solution of the form $e^{i\Lambda\xi}$) is given by

$$(\Lambda^2 - \Delta_1)(\Lambda^2 - \Delta_2) + \frac{4A_0^2}{H^2} = 0. \tag{22}$$

Now $A_0 = 1$ and $\phi_0 = 0$ implies

$$\Lambda^2 = \frac{1}{2}\left[\frac{4(3 + 2\lambda - \gamma)}{3H^2} - \frac{1}{M^2} \pm \sqrt{\left(\frac{4(3 + 2\lambda - \gamma)}{3H^2} + \frac{1}{M^2}\right)^2 - \frac{16}{H^2}}\right]. \tag{23}$$

The equilibrium point will be of non-hyperbolic if

$$\frac{4(3 + 2\lambda - \gamma)}{3H^2} - \frac{1}{M^2} = 0. \tag{24}$$

Then eigen values will be two pairs of imaginary number and therefore possibility of bifurcation is observed when

$$\left(\frac{4(3 + 2\lambda - \gamma)}{3H^2} + \frac{1}{M^2}\right)^2 < \frac{16}{H^2}. \tag{25}$$

The condition for purely oscillatory solutions near the equilibrium point is given by both equation (24) and (25) , not only one. From equation (24) we get

$$\frac{1}{M^2} = \frac{4(3 + 2\lambda - \gamma)}{3H^2}. \tag{26}$$

Therefore, a necessary condition for bifurcation in model (19) is given by condition (26), but it is not sufficient condition for bifurcation. This condition gives critical values of Mach number M depending on the value of H.

4 Numerical simulation results

In this section, we have investigated numerically the qualitative dynamics of the system (18) to get insight of the complexity of the wave profile with the help of numerical simulation results in MATLAB R2018a, in the traveling wave frame ξ. For illustration, we consider the values of electron number-densities relevant to dense astrophysical plasmas and metallic nanostructures [32], [47, 49] $n_0 \sim 10^{29} m^{-3}$. In Figure 1(a), we have plotted ϕ against ξ of the system (18) for $n_0 = 2.35 \times 10^{29} m^{-3}$ $(H = 0.65, \gamma = 6.43)$, and $M = 3.25$. Figure 1(b), (c) show phase diagrams $dA/d\xi$ versus ϕ and ϕ versus $d\phi/d\xi$, respectively, for the same data set. Figure 1(d) shows Poincaré return map of the system (18). The chaotic nature of the nonlinear system (18) is observed from the Figure 1(d). The irregular set of points in the whole Poincaré plane guarantees the existence of chaos. Chaos is

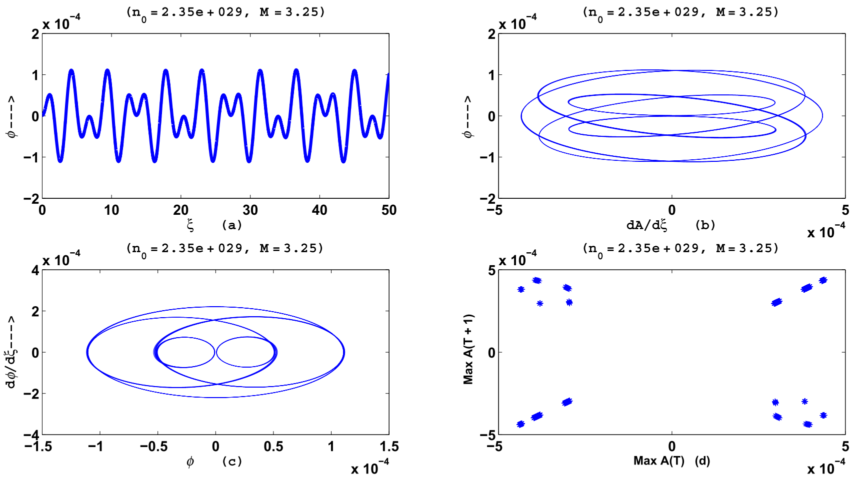

Figure 1: For $n_0 = 2.35{\times}10^{29} m^{-3}$ ($H = 0.65$, $\gamma = 6.43$), $M = 3.25$, (a) plot ϕ versus ξ of the nonlinear dynamical system (18) (b) phase diagram $dA/d\xi$ versus ϕ (c) phase diagram ϕ versus $d\phi/d\xi$ (d) Poincare return map of the system (18).

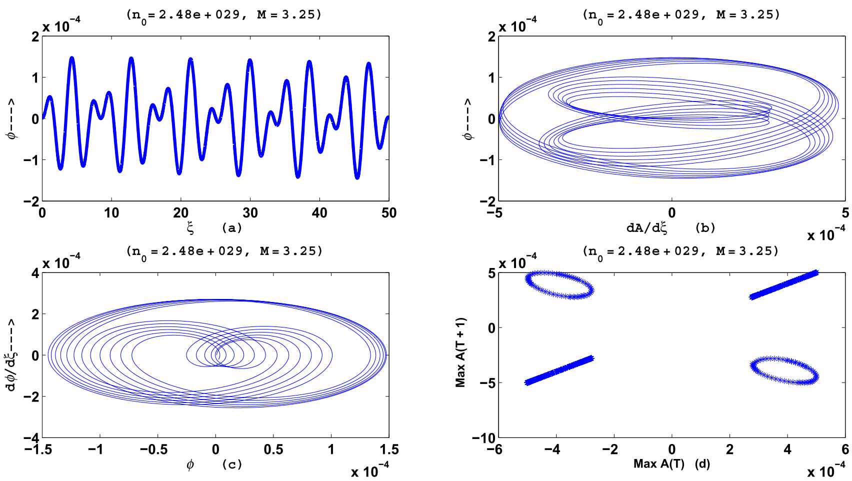

Figure 2: For $n_0 = 2.48 \times 10^{29} m^{-3}$ ($H = 0.64$, $\gamma = 6.32$), $M = 3.25$, (a) plot ϕ versus ξ of the nonlinear dynamical system (18) (b) phase diagram $dA/d\xi$ versus ϕ (c) phase diagram ϕ versus $d\phi/d\xi$ (d) Poincare return map of the system (18).

aperiodic long-term behavior of a system which has sensitive dependence on initial conditions. In presence of chaos, a small change in initial conditions causes unpredictable variation of oscillation pattern of the state variables. Therefore state variables have unpredictable bounded oscillation in the phase space for a chaotic system.

Now, taking $n_0 = 2.48{\times}10^{29} m^{-3}$ ($H = 0.64$, $\gamma = 6.32$) and keeping M fixed as $M = 3.25$, we have plotted the solution of the system (18) in Figure 2. The phase diagrams of the system are displayed in Figure 2(b), (c). The quasiperiodic nature of the dynamics is observed from the Figure 2(d) as the points in the return map lie on smooth closed curves. Quasiperiodicity is the simplest form of dynamics exhibiting nontrivial recurrence with low complexity. It is often termed as a precursor to turbulence. Quasiperiodic behavior is a pattern of recurrence with a component of unpredictability that does not lend itself to the precise measurement. The technique of Poincaré section diagrams has been very helpful in studying qualitative properties of a dynamical system, as it allows one to trace the properties of quasiperiodic orbits of the original higher-dimensional system, as projected on a lower-dimensional space (Poincaré surface). The Poincaré return map plots (Figure 2(d)) confirm the existence of quasiperiodic behavior in the model for the above choice of physical parametric values. When the number density n_0 is allowed to further increase (i.e., quantum diffraction H decrease), the system shows the destabilizing natures and the pseudo recurrence (quasiperiodic) behavior disappears and the chaotic state is observed as seen from Figure 3. The route from quasiperiodic to chaos is studied for the system by varying the number density n_0. Thus we observe that transition from chaotic $\longrightarrow$ quasiperiodic $\longrightarrow$ chaotic behavior occur as the value of number density n_0 increases. The quasiperiodic route to chaos is very different from that studied in most low-dimensional systems such as the Lorenz system and logistic map, where the period doubling route to chaos is common. Therefore, it is very difficult to show the Lyapunov exponent diagram clearly. We only show the Poincaré

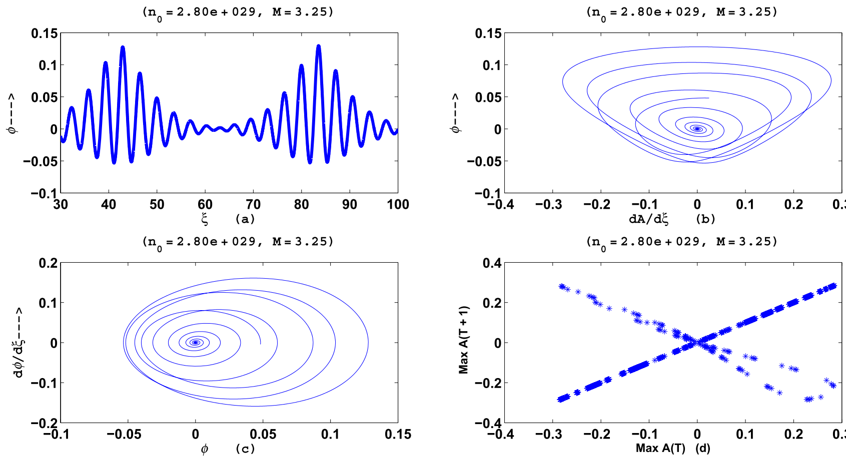

Figure 3: For n_0=2.80 × $10^{29}m^{-3}$(H = 0.63, γ = 6.06), M = 3.25, (a) plot ϕ versus ξ of the nonlinear dynamical system (18) (b) phase portrait $dA/d\xi$ versus ϕ (c) phase portrait ϕ versus $d\phi/d\xi$ (d) Poincare return map of the system (18).

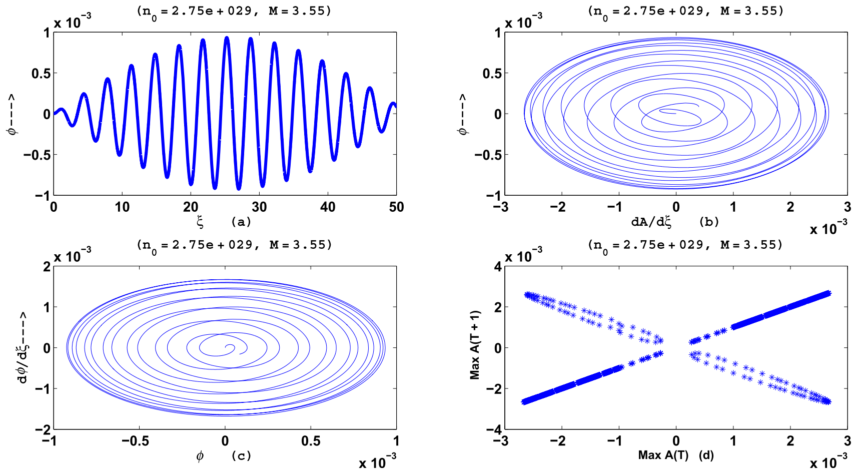

Figure 4: For n_0 = 2.75 × $10^{29}m^{-3}$(H = 0.63, γ = 6.10), M = 3.55, (a) plot ϕ versus ξ of the nonlinear dynamical system (18) (b) phase portrait $dA/d\xi$ versus ϕ (c) phase portrait ϕ versus $d\phi/d\xi$ (d) Poincare return map of the system (18).

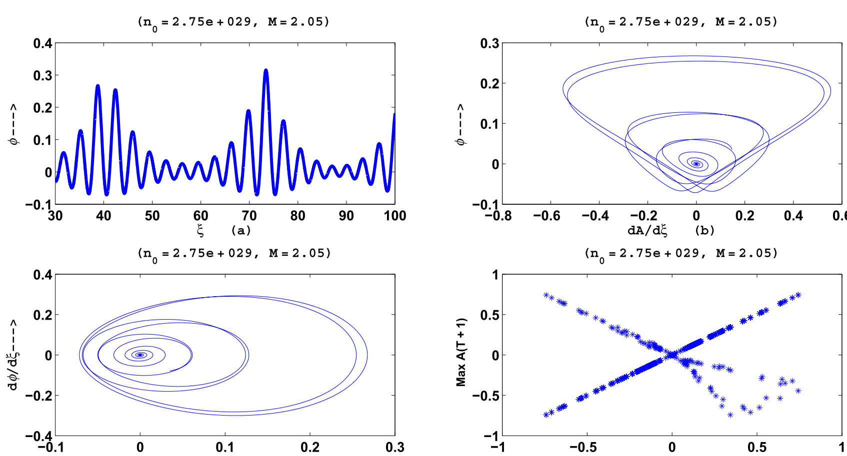

Figure 5: For n_0 = 2.75 × $10^{29}m^{-3}$(H = 0.63, γ = 6.10), M = 2.05, (a) plot ϕ versus ξ of the nonlinear dynamical system (18) (b) phase diagram $dA/d\xi$ versus ϕ (c) phase diagram ϕ versus $d\phi/d\xi$ (d) Poincare return map of the system (18).

return map diagram. It should be mentioned that Poincaré return map plot is sufficient to show quasiperiodic behavior and chaos. It is a well-established method to show quasiperiodicity and chaos [50, 51]. To see the effect of M on the dynamical properties of wave profile, phase portraits and Poincare return map are explored in Figures 4–6 for different values of M. From Figure 4, the quasiperiodic nature of the dynamics has been observed. The phase diagrams of the system are depicted in Figure 4(b), (c). Existence of quasiperiodic behavior is obvious from the Poincaré return map diagram presented in Figure 4(d). Figure 5 represents the qualitative behaviors

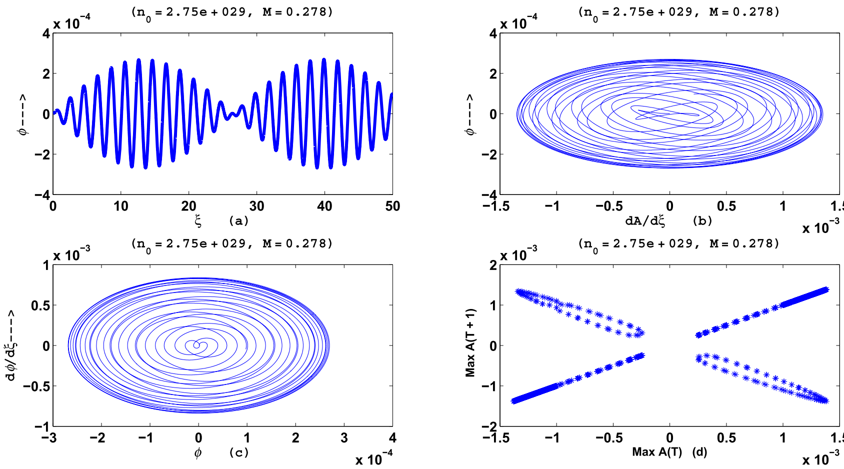

Figure 6: For $n_0 = 2.75 \times 10^{29} m^{-3}$($H = 0.63$, $\gamma = 6.10$), $M = 0.278$, (a) plot ϕ versus ξ of the nonlinear dynamical system (18) (b) phase diagram $dA/d\xi$ versus ϕ (c) phase diagram ϕ versus $d\phi/d\xi$ (d) Poincare return map of the system (18).

of the system (18) for smaller value of M than the value considered in Figure 4. The chaotic nature of the solution is observed from the Figure 5(d). If we further decrease the value of M, we again observe the quasiperiodic nature of the dynamics as the points lie on smooth closed curves (Figure 6(d)). Thus, for different values of M transition from quasiperiodic oscillations $\rightarrow$ chaotic nature $\rightarrow$ quasiperiodic oscillations is observed. Therefore, depending on the values of Mach number M, quantum diffraction H or exchange-correlation parameter γ bifurcations are observed numerically through quasiperiodic to chaotic solution and chaotic to quasiperiodic solution. In this study, the theoretical analyses in support of the numerically observed bifurcations are not provided. Hence, we can summarize that the plasma system exhibits either quasiperiodic or chaotic oscillations depending on the values of the physical parameters and the transition from quasiperiodic to chaotic oscillation or chaotic to quasiperiodic oscillations is possible in the present plasma model.

5 Conclusions

We have addressed the dynamical behavior of IAWs in quantum plasmas with exchange-correlation effects by using the QHD model. After deriving a set of coupled equations for arbitrary amplitude IAWs, we have analyzed the stability of equilibrium points employing the theory of dynamical systems. We have also obtained the bifurcation condition for the transition from quasiperiodic to chaotic oscillation or chaotic to quasiperiodic oscillations. Numerically it is shown that quasiperiodic as well as chaotic behavior exist in the system depending on the values of Mach number M and quantum diffraction H or exchange-correlation parameter γ. Therefore, quantum diffraction or electron exchange-correlation along with the Mach number plays an significant role in the dynamics of the model. Existence of these types of nonlinear structures is confirmed by the plots of Poincare map. Transition from quasiperiodic oscillation to chaotic motion or chaotic to quasiperiodic nature is possible in such model with the variation of values of the relevant parameters. The results of the present investigation may have relevance to understand the salient features of dense astrophysical as well as laboratory plasmas.

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